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Lecture 6: The Intertemporal CAPM (ICAPM): A Multifactor Model and Empirical Evidence

I. Reading.
   A. BKM, Chapter 11, Sections 11.1, 11.6 then 11.5.
   B. BKM, Chapter 13, Sections 13.2 and 13.3.

II. ICAPM Assumptions.
   1. Same as CAPM except cannot represent individual tastes and preferences in \(\{E[R], \sigma[R]\}\) space.

III. When do individuals care about more than expected return and standard deviation?
   A. single period setting:
      1. Returns are not normally distributed and individual utility depends on more than expected portfolio return and standard deviation.

   B. multiperiod setting:
      1. Returns are not normally distributed and individual utility depends on more than expected portfolio return and standard deviation.
      2. Expected return and covariances of returns in future periods depend on the state of the world at the end of this period; e.g., predictable returns.
      3. Individual preferences in the future depend on the state of the world at the end of this period.
      4. Individual receives labor income.
IV. Examples

A. Predictable Returns.
1. It has been empirically documented that expected stock returns over a period depend on variables known at the start of the period: e.g. dividend yield on the S&P 500 at the start of period t, DP(start t): see Lecture 3.
2. A high S&P500 dividend yield at the start of this month implies high expected returns on stocks this month.
3. So a high S&P500 dividend yield at the end of this month implies high expected returns on stocks next month.
4. Thus, S&P500 dividend yield at the end of this month is a state variable that individuals care about when making portfolio decisions today.

B. Human Capital Value.
1. An unexpectedly poorer economy at the end of the month implies a negative shock to human capital value over the month
   a. the negative shock to human wealth is due to an increased probability of a low bonus or, worse, job loss.
2. Thus, the state of the economy at the end of period t is positively related to the shock to human capital value over period t.
3. Suppose a macroeconomic indicator MI(end t) summarizes the state of the economy at the end of period t:
   a. the economy at the end of period t is better for higher MI(end t).
   b. examples of such indicators include # of help wanted positions and # of building permits issued.
4. A sufficiently risk averse individual likes a portfolio whose return over period t, Rp(t), has a low or negative covariance with
   a. the shock to the individual’s human capital over period t;
   b. the state of the economy at the end of period t;
   c. MI(end t).
5. The macroeconomic indicator, MI(end t), is a state variable the individual cares about when making portfolio decisions at the start of period t.

V. Tastes and Preferences with a Long-term Investment Horizon.

A. In general, if an individual cares about a macroeconomic indicator MI(end t) then can only fully represent an individual’s tastes and preferences for her period t portfolio return using \{E[R(t)], \sigma[R(t)], \text{cov}[R(t), MI(end t)]\}.

B. Even more generally, if individuals care about a set of K state variables s1(end t), ..., sK(end t), then can only fully represent an individual’s tastes and preferences for her period t portfolio return using \{E[R(t)], \sigma[R(t)], \text{cov}[R(t), s1(end t)], ..., \text{cov}[R(t), sK(end t)]\}.
VI. Portfolio Choice.
   A. Since individual’s care about more than expected return and standard deviation of return, individuals no longer hold combinations of the riskfree asset and the tangency portfolio:
      1. i.e., individuals no longer hold portfolios on efficient part of the MVF for the N risky assets and the riskless.
      2. i.e., individuals no longer hold portfolios on the Capital Allocation Line for the tangency portfolio.
   B. Thus, in the ICAPM, since individuals no longer necessarily hold combinations of the riskfree asset and the tangency portfolio, the market portfolio is no longer necessarily the tangency portfolio.
   C. Example: Human Capital Value.
      1. The tangency portfolio on the MVF for the N risky assets may have return over period t whose covariance with MI at the end of period t is high.
      2. Thus, an individual may prefer to hold a portfolio in period t below the capital allocation line for the tangency portfolio but which has a very low covariance with MI over period t.
      3. It is possible to show all individuals hold combinations of
         a. the riskfree asset.
         b. the market portfolio.
         c. a portfolio whose return $R_{M(t)}$ hedges shocks to human capital value over period t.
   D. More generally, it is possible to show that in equilibrium all individuals irrespective of tastes and preferences hold a combination of:
      1. the riskfree asset.
      2. the market portfolio.
      3. K hedging portfolios, $R_n$, $R_{n2}$, ..., $R_{nK}$, one for each state variable.
   E. Thus, the ICAPM is a generalization of the CAPM.
VII. Individual Assets.

A. Recall that the market portfolio is no longer necessarily the tangency portfolio: so the market need not lie on the positive sloped part of the MVF for the N risky assets.

B. Minimum variance mathematics then tells us that there need not be a linear relation between expected return and Beta with respect to the market portfolio; i.e., assets need not all lie on the SML:

\[ E[R_i] - R_f = \beta_{i,M} \{E[R_M] - R_f \}. \]

C. Example (cont): Human Capital Value.

1. If individuals care about covariance of portfolio return over t with MI(end t) and asset returns over t and MI(end t) are multivariate normally distributed, the following holds for all assets:

\[ E[R_i(t)] = R_f + \beta^*_{i,M} \lambda^*_{M} + \beta^*_{i,MI} \lambda^*_{MI} \]

where:

\[ \lambda^*_{M} = E[R_M] - R_f = E[r_M] \]

\[ \lambda^*_{MI} = E[R_{MI}] - R_f = E[r_{MI}] \]

are constants that are the same for all assets and portfolios; and

\[ \beta^*_{i,MI} \] and \[ \beta^*_{i,M} \] are regression coefficients from a multivariate regression of \[ r_i(t) \] on \[ r_M(t) \] and MI(end t):

\[ r_i(t) = a_{i,0} + \beta^*_{i,M} r_M(t) + \beta^*_{i,MI} MI(t) + e_i(t). \]

2. The hedging portfolio for MI(end t) can be used instead of MI(end t):

\[ a. \] In the multiple regression to determine risk loadings, replace MI(end t) with \[ r_{MI}(t) = [R_{MI}(t) - R_f] \], the excess return on the portfolio that hedges shocks to human capital value over t:

\[ r_i(t) = a_{i,0} + \beta^*_{i,M} r_M(t) + \beta^*_{i,MI} r_{MI}(t) + e_i(t). \]

\[ b. \] Then the following expression holds for all assets and portfolios of assets:

\[ E[R_i] = R_f + \beta^*_{i,M} \lambda^*_{M} + \beta^*_{i,MI} \lambda^*_{hMI} \]

where:

\[ \lambda^*_{M} = E[R_M] - R_f = E[r_M] \]

\[ \lambda^*_{hMI} = E[R_{hMI}] - R_f = E[r_{hMI}] \]

are constants that are the same for all assets and portfolios.
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D. Generally, if individuals care about the covariance of portfolio return with a set of state variables $s_1, s_2, ..., s_K$, returns and the state variables are multivariate normally distributed then can show that the following holds for all assets and portfolios of assets:

$$E[R_i] = R_f + \beta_{i,M}^* \lambda_{M}^* + \beta_{i,s1}^* \lambda_{s1}^* + \beta_{i,s2}^* \lambda_{s2}^* + ... + \beta_{i,sK}^* \lambda_{sK}^*$$

where:

- $\lambda_{M}^*, \lambda_{s1}^*, \lambda_{s2}^*, ..., \lambda_{sK}^*$ are constants that are the same for all assets and portfolios; and
- $\beta_{i,sk}^*$ for $k=1,2,...,K$, and $\beta_{i,M}^*$ are regression coefficients from a multivariate regression of $r_i$ on $r_M$, $s_1$, $s_2$, ... and $s_K$:

$$r_i = a_{i,0} + \beta_{i,M}^* r_M + \beta_{i,s1}^* s_1 + \beta_{i,s2}^* s_2 + ... + \beta_{i,sK}^* s_K + e_i$$

E. Note:

1. $r_i = R_i - R_f$ and $r_M = R_M - R_f$.
2. $\beta_{i,sk}^*$ for $k=1,2,...,K$, and $\beta_{i,M}^*$ are referred to as risk loadings and vary across assets; they measure the sensitivity of asset $i$ to each of the risks that individuals care about.
3. $\lambda_{M}^*, \lambda_{s1}^*, \lambda_{s2}^*, ..., \lambda_{sK}^*$ are referred to as risk premia and measure the expected return compensation an individual must receive to bear one unit of the relevant risk.
4. $\lambda_{M}^* = E[R_M]-R_f = E[r_M]$ since when $r_M$ is regressed on $r_M$, $s_1$, $s_2$, ... and $s_K$ we get $\beta_{M,M}^* = 1$ and $\beta_{M,s1}^* = \beta_{M,s2}^* = ... = \beta_{M,sK}^* = 0$.

F. The $K$ hedging portfolios can be used instead of the $K$ state variables:

1. Replace $s_1$, $s_2$, ..., $s_K$ with $[R_{h1}-R_f]$, $[R_{h2}-R_f]$, ..., $[R_{hK}-R_f]$ in the multiple regression.
2. With this substitution, we get risk premia that satisfy:

$$\lambda_{h1}^* = E[R_{h1}]-R_f, \lambda_{h2}^* = E[R_{h2}]-R_f, ..., \lambda_{hK}^* = E[R_{hK}]-R_f.$$ 

VIII. CAPM vs ICAPM

A. It can easily be seen that the CAPM is a special case of this ICAPM model.

B. In particular, the expression for expected return on any asset in VII. D. above reduces to the CAPM when $K=0$; i.e., when individuals only care about $E[R]$ and $\sigma[R]$. 
IX. Numerical Example. Let GIP(Jan) be the January growth rate of industrial production. Suppose each individual cares about \{E[R_p(Jan)], \sigma[R_p(Jan)], \sigma[R_p(Jan), GIP(Jan)]\} when forming his/her portfolio \( p \) for January. The following additional information is available:

<table>
<thead>
<tr>
<th>i</th>
<th>( E[R_i(Jan)] )</th>
<th>( \beta^*_{i,M} )</th>
<th>( \beta^*_{i,GIP} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pink</td>
<td>1.73%</td>
<td>1.3</td>
<td>0.25</td>
</tr>
<tr>
<td>Grey</td>
<td>1.34%</td>
<td>0.9</td>
<td>0.10</td>
</tr>
<tr>
<td>Black</td>
<td>?</td>
<td>0.9</td>
<td>0.05</td>
</tr>
</tbody>
</table>

where \( \beta^*_{i,M} \) and \( \beta^*_{i,GIP} \) are regression coefficients from a multiple regression (time-series) of \( R_i(t) \) on \( R_M(t) \) and GIP(t):

\[
R_i(t) = \phi_{i,0} + \beta^*_{i,M} R_M(t) + \beta^*_{i,GIP} GIP(t) + \epsilon_i(t).
\]

Also know that riskless rate for January, \( R_f(Jan) \), 0.7%.

1. What is the risk premium for bearing \( \beta^*_{i,M} \) risk?

Know ICAPM holds. So all assets lie on

\[
E[R_i(Jan)] = R_f(Jan) + \beta^*_{i,M} \lambda^*_M + \beta^*_{i,GIP} \lambda^*_GIP
\]

where \( \lambda^*_M = E[R_M(Jan)] - R_f(Jan) \).

Using this formula for Pink and Grey:

Pink: \( 1.73 = 0.7 + 1.3 \lambda^*_M + 0.25 \lambda^*_GIP \)

Grey: \( 1.34 = 0.7 + 0.9 \lambda^*_M + 0.10 \lambda^*_GIP \)

Now Pink \( \Rightarrow \lambda^*_GIP = 4 \times (1.03\% - 1.3 \lambda^*_M) \)

which can be substituted into Grey to obtain

\( 1.34 = 0.7 + 0.9 \lambda^*_M + 0.10 \times 4 \times (1.03\% - 1.3 \lambda^*_M) \).

It follows that \( \lambda^*_M = 0.6\% \) and \( \lambda^*_GIP = 1\% \).

So the risk premium for bearing \( \beta^*_{i,M} \) risk \( \lambda^*_M \) is 0.6%.

2. What is the expected January return on the market portfolio \( E[R_M(Jan)] \)?

\[
E[R_M(Jan)] = \lambda^*_M + R_f(Jan) = 0.6\% + 0.7\% = 1.3\%.
\]
3. What is the risk premium for bearing $\beta^*_{GIP}$ risk?

From above, the risk premium for bearing $\beta^*_{GIP}$ risk $\lambda^*_{GIP}$ is 1%.

4. Is the market portfolio on the minimum variance frontier of the risky assets in the economy? Why or why not?

Not necessarily. The reason is that individuals care about more than just $E[R]$ and $\sigma[R]$.

5. What is the expected return on Black?

Know that Black satisfies:

$$E[R_{Black(Jan)}] = R_f(Jan) + \beta^*_{Black,M} \lambda^*_{M} + \beta^*_{Black,GIP} \lambda^*_{GIP}$$
$$= 0.7 + \beta^*_{Black,M} 0.6 + \beta^*_{Black,GIP} 1$$
$$= 0.7 + 0.9 \times 0.6 + 0.05 \times 1$$
$$= 1.29\%$$
X. ICAPM Empirically: The Fama and French [1993] 3-Factor Model:

A. ICAPM interpretation of CAPM’s empirical failure:
1. Interpret size and book-to-market, for asset $i$ as proxying for risk loadings $(\beta_{i,sk}^*)$ on state variables that individuals care about.

B. Two hedging portfolios in Fama-French model: excess returns
1. SMB zero-investment portfolio: long small and short big stocks, while being book-to-market neutral.
2. HML zero-investment portfolio: long high and short low book-to-market stocks, while being size neutral.

C. Implications of Fama-French model for expected returns:
1. All assets satisfy:

$$E[R_i] = R_f + \beta_{i,M}^* E[r_M] + \beta_{i,SMB}^* E[r_{SMB}] + \beta_{i,HML}^* E[r_{HML}]$$

where:
- $r_i(t)$ is the excess return on portfolio $i$ in month $t$.
- $r_M(t)$ is the excess return on market portfolio in month $t$.
- $r_{SMB}(t)$ is the return on the SML portfolio in month $t$.
- $r_{HML}(t)$ is the return on the HML portfolio in month $t$.
- $\beta_{i,M}^*$, $\beta_{i,SMB}^*$, and $\beta_{i,HML}^*$ are the regression coefficients from the following regression:

$$r_i(t) = \alpha_{i,3} + \beta_{i,M}^* r_M(t) + \beta_{i,SMB}^* r_{SMB}(t) + \beta_{i,HML}^* r_{HML}(t) + u_i(t)$$

2. Intuition:
   a. Value stocks are poor hedges against risks that typical investors care about ($\beta_{i,HML}^*$ for value stocks is high): so value stocks require a higher expected return than growth stocks (all else equal) to induce investors to hold them.
   b. Small stocks are poor hedges against risks that typical investors care about ($\beta_{i,SMB}^*$ for small stocks is high): so small stocks require a higher expected return than large stocks (all else equal) to induce investors to hold them.
D. 25 portfolios:
1. quintile break-points calculated on the basis of size and book-to-market.
2. form 25 value-weighted portfolios based on these breakpoints.

E. Results
1. Deviations from the Fama-French ICAPM expected return equation much smaller than deviations from the SML
   a. Value portfolios: largest deviation only 0.13% per annum
      (compared to a largest deviation of 0.57% per annum from SML)
Lecture 6: Valuation Models (with an Introduction to Capital Budgeting).

I. Reading.
   A. BKM, Chapter 18, except Section 18.6.
   B. RWJ, Chapter 8, Section 8.1 and skim Sections 8.2-8.6.

II. Introduction.
   A. Definition of Valuation.
      1. Valuation is the art/science of determining what a security or asset is worth.
         a. sometimes we can observe a market value for a security and we are interested in assessing whether it is over or under valued (e.g., stock analysts).
         b. sometimes there is no market value and we are trying to construct one for bargaining or transaction purposes (e.g., a corporation is interested in selling a division).
         c. sometimes we have a project that we are deciding whether to accept or reject.
      2. The value of a security or asset is going to depend crucially on the asset pricing model we choose. (The effect is through the appropriate discount rate.)
      3. The most common kinds of valuation problem are
         a. equity valuation.
            (1) seasoned equity.
            (2) IPOs.
         b. firm valuation.
         c. capital budgeting: project valuation.
   B. Three Valuation Approaches.
      1. Discounted Cash Flow (DCF) Models: values an asset by calculating the present value of all future cash flows
      2. Relative Valuation: values an asset by looking at the prices of comparable assets and using multiples such as price/earnings (P/E).
      3. Contingent Claim Valuation: uses option pricing tools to value assets with option features.
III. Discounted Cash Flow Models.
   A. General Approach.
      1. The intrinsic value of an asset $P_t^i$ is the present value of expected cash
         flows $E[D_t^i]$ on the asset discounted by the required rate of return on the
         asset $E[R_t^i]$:

         $$P_0^i = \sum_{\tau=1}^{\infty} \frac{E[CF_\tau^i]}{(1 + E[R_\tau^i])^\tau}.$$  

   B. Two items affect the intrinsic value of an asset.
      1. Expected Return on the asset.

   C. Discussion.
      1. This formula highlights the relation between expected return and price and
         why we call a model that tells us something about expected return an asset
         pricing model.
      2. We can see that holding expected cash flows fixed, asset price today is
         decreasing in expected asset return; the higher the expected return needed
         to compensate for the asset’s risk the lower the asset’s price.
D. Equity Valuation: Dividend Discount Model (DDM)

1. DDM is an example of a discounted cash flow model.
2. DDM assumes that the stock is bought, held for some time (dividends are collected), and then sold.
3. The share is valued as the present value of the expected dividends and the expected proceeds from the sale.
4. Assume that dividends are paid annually and that the time 0 dividend has just been paid.
5. If the stock is held one year, the return on the stock is

\[ R^i = \frac{D^i_1 + P^i_1}{P^i_0} - 1 \]

where \( D^i_t \) is firm i’s dividend per share at time t and \( P^i_t \) is the stock price of the firm at t. Taking expectations and rearranging gives

\[ P^i_0 = \frac{E[D^i_1 + P^i_1]}{1 + E[R^i]} \]

6. Notice that \( E[R^i] \) here applies to the firm’s equity not the firm’s assets.
7. If the stock is held for two years, the present value is given by

\[ P^i_0 = \frac{E[D^i_1]}{1 + E[R^i]} + \frac{E[D^i_2 + P^i_2]}{(1 + E[R^i])^2} \]

8. If the stock is held until the company is liquidated, the present value is given by

\[ P^i_0 = \frac{E[D^i_1]}{1 + E[R^i]} + \frac{E[D^i_2]}{(1 + E[R^i])^2} + \cdots + \frac{E[D^i_\tau]}{(1 + E[R^i])^\tau} + \cdots \]

\[ = \sum_{\tau=-1}^{\infty} \frac{E[D^i_\tau]}{(1 + E[R^i])^\tau} \]

which is known as a dividend discount model (DDM).
E. Capital Budgeting Decisions.

1. A firm is constantly deciding whether to undertake various projects available to it: these are capital budgeting decisions.

2. The firm’s claimholders want the firm to undertake a project if its value to the firm is positive.

3. The discounted cash flow approach says to discount the expected cash flows from the project to the present using the appropriate expected return for the cash flows.

4. The sum of present values (including any initial outlay) is known as the net present value NPV of the project:

\[
NPV_i^0 = \sum_{t=0}^{\infty} \frac{E[CF_t^i]}{(1 + E[R^i])^t}
\]

5. The firm should only undertake projects with positive NPVs.

6. The project cash flows include all incremental cash flows as a result of undertaking the project.

7. The appropriate expected return for discounting back the expected project cash flows depends on their riskiness.

8. Example. ZDF Co. is considering an expansion into a new line of business producing widgets. The new line will require an outlay of $20 million today and will generate an expected net cash flow of $5 million per year for the next 10 years. Each year’s cash flow will be received at the end of the year. The required return on the firm’s equity is currently 20% p.a. while the required return on the firm’s assets is 18% p.a.. The required return on the assets for firms currently producing widgets is 15% p.a. Headquarters expects to use all its current idle administrative capacity to oversee the new business line. Headquarter overhead is $6 million per year and idle capacity is currently 25% of total capacity. Should ZDF Co. expand into the new line of business?

a. Use the required return on assets in the new business line of 15% p.a..

b. Ignore headquarter overhead since it is a sunk cost.

c. Calculate the NPV of the expansion:

\[
NPV_0 = -20 + 5 \times PVAF_{15\%,10} = -20 + 5 \times 5.0187 = 5.09.
\]

d. Since the NPV \( > 0 \), ZDF Co. should undertake the expansion.
Lecture 6 Foundations of Finance

IV. Expected Return Determination.
A. Approaches:
1. In a CAPM framework, use the SML; this approach allows you to explicitly make adjustments to your Beta estimate to reflect your assessment of the future Beta of the stock.
2. If valuing existing equity, can also use a historical average return as an estimate of expected return.
3. Can also adjust the estimate to take into account the predictability of returns and to allow for the sensitivity of the stock to other sources of risk (in an I-CAPM setting); we will not focus on these adjustments here.
4. For simplicity, we will ignore tax considerations.

B. Equity Beta versus Firm Beta.
1. Can think of the firm as a portfolio of assets/projects or a portfolio of claims on those assets:

\[ V = A_1 + A_2 + \ldots + A_J \]
and
\[ V = S + B \]

where
\( V \) is the value of the firm;
\( A_j \) is the value of the jth asset of the firm;
\( S \) is the market value of the firm’s equity;
\( B \) is the market value of the firm’s debt.

2. Recall that Beta with respect to the market for a portfolio is a weighted average of the Betas of the assets that comprise the portfolio where the weights are the portfolio weights. In an I-CAPM context, the same is true for Beta with respect to other variables individuals care about.

3. It follows that for Beta with respect to any variable (which of course includes Beta with respect to the market):

\[ \beta_V = \frac{A_1}{V} \beta_{A_1} + \frac{A_2}{V} \beta_{A_2} + \ldots + \frac{A_J}{V} \beta_{A_J} \]

and
\[ \beta_V = \frac{S}{V} \beta_S + \frac{B}{V} \beta_B \]

where
\( \beta_V \) is the Beta of the firm;
\( \beta_{A_j} \) is the Beta of the jth asset of the firm;
\( \beta_S \) is the Beta of the firm’s equity;
\( \beta_B \) is the Beta of the firm’s debt.
4. Note
   a. If the firm’s assets are unchanged, then firm Beta with respect to any variable is unchanged.
   b. Each asset or project of the firm can have a different Beta from the firm Beta.
   c. Equity Beta can be calculated by rewriting the above formula:

   \[
   \beta_s = \frac{V}{S} \beta_V - \frac{B}{S} \beta_B = \beta_V + \frac{B}{S} [\beta_V - \beta_B]
   \]

   d. Can see that equity Beta depends on:
      (1) the Betas of the firm’s assets;
      (2) the level of debt of the firm; and
      (3) the Beta of the firm’s debt.
   e. If the firm’s debt is riskless, debt Beta with respect to any variable is 0 and so equity Beta can be calculated:

   \[
   \beta_s = \frac{V}{S} \beta_V
   \]

5. Deciding which Beta to use for a particular valuation problem.
   a. When using DCF methods to value the firm directly, use firm Beta to calculate the expected return on the firm’s assets.
   b. When using DCF methods to value the firm’s equity directly, use equity Beta to calculate the expected return on the firm’s equity.
   c. When valuing or evaluating a specific project, always use the Beta of the project (which could be different from the firm’s Beta).

C. Examples.
1. Suppose ZX company has a two assets. The first has a Beta with respect to the market of 1.5 while the second has a Beta with respect to the market of 0.9. The first asset is worth $12M and the second is worth $8M. The firm has $4M of riskless debt. The CAPM holds for the economy, the riskless rate is 5% p.a. and the expected return on the market portfolio is 13% p.a. What is the expected return on ZX’s equity?

   a. First, get the Beta of the firm:

   \[
   \beta_{V,M} = \frac{A_1}{V} \beta_{A1,M} + \frac{A_2}{V} \beta_{A2,M} = \frac{12}{12+8} 1.5 + \frac{8}{12+8} 0.9 = 1.26.
   \]

   b. Second, get the Beta of the equity:

   \[
   \beta_{S,M} = \frac{V}{S} \beta_{V,M} = \frac{20}{20-4} 1.26 = 1.575.
   \]
c. Third, use the SML to get the expected return on the equity:
\[ E[R_{s}] = R_f + \{E[R_M] - R_f\} \beta_{S,M} = 5\% + \{13\%-5\%\} 1.575 = 17.6\%. \]

2. Project Beta is likely to differ from firm Beta when:
   a. The firm is a conglomerate.
   b. The project represents an entry into a new industry by the firm.

3. IBM Example: Value IBM equity as of the end of 12/04.
   a. To use DCF techniques, need an estimate of the expected return on IBM stock: \( E[R^{S-IBM}] \)
   b. CAPM
      (1) Inputs:
         (a) market expected return (\( E[R^M] \))
            i) average monthly return on the S&P 500 for
               the period 1/65 to 12/04 is 0.94%
            ii) annualized gives 11.88\% = (1+0.0094)^{12}-1.
         (b) \( R^f \): based on yield curve at end of 12/04: use 2.74\%.
         (c) \( \beta_{S-IBM,M} \) can be obtained from the Bloomberg screen:
             1.01 is the Beta obtained using weekly data from
             1/03 to 12/04.
      (2) Using the SML:
         \[ E[R^{S-IBM}] = 2.74\% + 1.01 \times (11.88 - 2.74)\% = 11.88\% \]
   c. Could also use an estimate of expected equity return based on historical average return.
V. Constant Growth DDM.
A. Assumption:
   1. \( g_i \), the growth rate of the expected dividend, is assumed constant.
   2. So \( E[D_{1,i}] = D_{0,i} (1 + g_i) \), \( E[D_{2,i}] = E[D_{1,i}] (1 + g_i) \), ..., 
      \( E[D_{\tau+1,i}] = E[D_{\tau,i}] (1 + g_i) \).

B. DDM can be written:

\[
P_0^i = \frac{D_0^i (1 + g_i)}{E[R_i] - g_i} = \frac{E[D_i^i]}{E[R_i] - g_i}
\]
which is valid so long as \( E[R_i] > g_i \).

C. IBM Example:
1. Aim is to value IBM stock as at 12/31/04.
2. Note:
   a. The total dividends paid in 2004 were $0.70 per share.
   b. Using the CAPM gives a discount rate of 11.88% p.a.
   c. The Earnings Estimates table indicates a growth rate in annual earnings over the next 5 years of 10.68%; this will be our estimate of \( g_{IBM} \).
3. Inputs:
   a. \( D_{0,IBM} = 0.70 \)
   b. \( E[R_{IBM}] = 11.88\% \)
   c. \( g_{IBM} = 10.68\% \)
4. Using the constant growth DDM:

\[
P_{0,IBM} = D_{0,IBM} (1 + g_{IBM}) / (E[R_{IBM}] - g_{IBM}) = 0.70 \times (1 + 0.1068) / (0.1188 - 0.1068) = 64.56.
\]

which can be compared to the price of IBM at the end of December 2004 of $98.58. If we had full faith in our valuation we would consider IBM to be overvalued and issue a sell recommendation.
D. Other Implications of the Constant Growth DDM.

1. Can rewrite the basic model:

\[
E[R_i] = \frac{E[D_{i+1}]}{P_0} + g_i
\]

which breaks required return into the expected dividend yield plus expected capital gain.

2. The expected capital gain on the stock is \( g_i \) (assuming no stock splits or stock dividends):

\[
E[P_{i+1}] = P_0(1 + g_i).
\]

3. If we assume the stock is correctly valued, we can use the stock’s dividend yield and earnings growth rate to calculate an estimate of expected return.
VI. Investment Opportunities.

A. Introduction.
1. Let $K_{\tau}^i$ be the book value of a share of equity at time $\tau$.
2. The book value per share evolves through time in the following way:
   $$K_{\tau}^i = K_{\tau-1}^i + (E_\tau^i - D_\tau^i)$$
   where $E_\tau^i$ are firm i’s earnings (after interest) per share in period $\tau$. Any earnings not paid out as a dividend get added to the book value.

B. Assumptions.
1. The constant growth DDM correctly values stock: so $g^i$, the growth rate of the expected dividend, is assumed constant.
2. Each year firm i’s assets generate an expected after interest cash flow which is a constant fraction $ROE^i$ of the book value of the equity at the start of the year:
   $$E[E_{\tau+1}^i] = K_{\tau}^i \cdot ROE^i$$
   where this $ROE^i$ is known as the expected return on book equity.
3. Firm i pays a constant fraction $(1-b^i)$ of its earnings as a dividend. So
   $$D_{\tau+1}^i = (1-b^i) E_{\tau+1}^i$$
   for any $\tau$.
   a. $(1-b^i)$ is called the payout ratio.
b. $b^i$ is called the plowback or retention ratio.

C. Implications.
1. Constant payout ratio means expected earnings per share also grow at $g^i$:
   $$\frac{E[E_{\tau}^i]}{E_0^i} = 1 + g^i$$
2. Fixed ROE means book value per share is also expected to grow at $g^i$:
   $$\frac{E[K_{\tau}^i]}{K_0^i} = 1 + g^i$$
3. Earnings growth depends on retention rate and ROE:
   $$b^i \cdot ROE^i = g^i.$$
D. IBM Example.
   1. Note:
      a. Taking the earnings growth estimate as our estimate of $g_{IBM}$ is consistent with this constant payout model.
      b. Earnings per share for IBM for 2004 was $4.94.
   2. Inputs:
      a. $E_0^{IBM} = $4.94
      b. $D_0^{IBM} = $0.70.
      c. $g_{IBM} = 10.68\%$.
   3. Calculations:
      \[ b_{IBM} = 1 - b_{IBM} = \frac{D_0^{IBM}}{E_0^{IBM}} = \frac{0.70}{4.94} = 14.17\% \text{ and } b_{IBM} = 85.83\%. \]
      \[ E[E_1^{IBM}] = E_0^{IBM} [1 + g_{IBM}] = 4.94 [1 + 0.1068] = 5.48. \]
      \[ ROE_{IBM} = \frac{g_{IBM}}{b_{IBM}} = \frac{0.1068}{0.8583} = 12.45\%. \]
   4. $K_0^{IBM}$ implied by the model:
      \[ K_0^{IBM} = \frac{E[E_1^{IBM}]}{ROE_{IBM}} = \frac{5.48}{0.1245} = $44.02 \]
      which can be compared with the actual book value at the end of 2004 of $18.08$. The difference between the two is a measure of the extent to which the assumptions about the evolution of book value hold for IBM.
E. Uses of the Model.

1. Valuation.
   a. Can easily show that the following formula must hold for the stock price of firm i:

   \[ P_i^0 = E_0(1 + b^i ROE^i)(1 - b^i) = \frac{E[E_i^i](1 - b^i)}{E[R_i^i] - b^i ROE^i}. \]

2. Optimal Plowback Ratio.
   a. If firm i paid out all its earnings as a dividend \((b^i = 0)\), its stock price at time zero would be \(E[E_1^i]/E[R_i]\). The difference between this value and the constant growth DDM value is due to growth. Thus,

   \[ P_i^0 = \frac{E[E_1^i]}{E[R_i]} + PVGO_0^i = \frac{E_0(1 + g^i)}{E[R_i]} + PVGO_0^i. \]
   
   where \(PVGO_0^i\) is the value at time 0 of firm i’s growth opportunities.

   b. Note that:
      (1) If \(E[R_i] > ROE^i\):
         a) \(PVGO_0^i \geq 0\); and,  
         b) \(b^i = 0\) maximizes \(P_i^0\).
      (2) If \(E[R_i] < ROE^i\):
         a) \(PVGO_0^i \leq 0\); and,  
         b) \(P_i^0\) is increasing in \(b^i\).
      (3) If \(E[R_i] = ROE^i\):
         a) \(PVGO_0^i = 0\); and,  
         b) \(P_i^0\) is unaffected by choice of \(b^i\).

F. IBM Example.

1. Inputs.
   a. \(P_{0, IBM} = $64.56\).
   b. \(E[E_1, IBM] = $5.48\).
   c. \(E[R_{IBM}] = 11.88\%\).
   d. \(ROE_{IBM} = 12.45\%\).

2. Can calculate \(PVGO_0^{IBM}\):

   \[ PVGO_0^{IBM} = P_{0, IBM} - \frac{E[E_1, IBM]}{E[R_{IBM}]} = $64.56 - $5.48/0.1188 = $64.56 - $46.13 = $18.43. \]

   Note that \(PVGO_0^{IBM} \geq 0\) as would be expected since \(11.88\% = E[R_{IBM}] < ROE_{IBM} = 12.45\%\).
G. Proofs of implications and valuation formula:

1. Since dividend per share is a fixed fraction of earnings per share, it follows that expected earnings per share also grow at $g_i$:

$$\frac{E[E_{1,i}]}{E_{0,i}} = \frac{E[D_{1,i}]}{(1 - b_i)D_{0,i}} = \frac{E[D_{1,i}]}{D_{0,i}} = (1 + g_i)$$

2. Can show that the book value per share is also expected to grow at $g_i$:

$$\frac{E[K_{1,i}]}{K_{0,i}} = \frac{E[E_{1,i}]}{ROE_i E[E_{1,i}]} = \frac{E[E_{2,i}]}{E[E_{1,i}]} = 1 + g_i.$$

3. What is the expected book value per share at time 1?

$$E[K_{1,i}] = E[K_{0,i} + (E_{1,i} - D_{1,i})] = K_{0,i} + E_{1,i} \cdot ROE_i - K_{0,i} \cdot ROE_i \cdot (1-b) = K_{0,i} (1 + ROE_i \cdot b_i).$$

Thus, have shown that $b_i \cdot ROE_i = g_i$.

4. Can easily show that the following formula must hold for the stock price of firm $i$:

$$P_{0,i} = \frac{D_{0,i} \{1 + g_i \}}{E[R_{1,i}] - g_i} = \frac{E_{0,i} \{1 + g_i \} \{1 - b_i \}}{E[R_{1,i}] - b_i \cdot ROE_i} = \frac{E[E_{1,i}] \{1 - b_i \}}{E[R_{1,i}] - b_i \cdot ROE_i}.$$
VII. Relative Valuation Approaches.

A. Definition of P/E ratio.
   1. The Price/Earnings or P/E ratio is defined as the price per share divided by the earnings per share (after interest).
   2. IBM Example: As of the end of December 2004, the P/E ratio for IBM is given as 19.44 by Bloomberg.
   3. The P/E ratio is sometimes used to describe the price as “IBM is selling at 19.44 times earnings”.

B. Logic:
   1. Investment opportunities model above implies:
      \[
      \frac{P_0^i}{E_0^i} = \frac{(1 + g^i)(1 - b^i)}{E[R^i] - g^i}.
      \]

      2. Consider stock i and let \((P/E)_C^i\) be the P/E ratio of a comparable firm where comparable means:
         a. Similar required return.
         b. Similar payout rate.
         c. Similar growth rate for expected dividends.

      3. Then:
      \[
      P_0^i = E_0^i \cdot (P/E)_C^i.
      \]

C. Use of P/E ratio.
   1. The P/E ratio is sometimes used to get a rough measure of the intrinsic value of a company that is not publicly traded.
   2. When valuing the equity of a firm, the approach requires a set of comparable firms to be identified.
   3. An average P/E ratio is calculated for the set of comparable firms.
   4. The current earnings of the firm are multiplied by this average P/E to obtain an estimate of the firm’s intrinsic value.

D. Advantages of relative valuation.
   1. Simple and quick.

E. Disadvantages of relative valuation.
   1. Definition of a comparable firm is subjective.
   2. Accounting earnings are subject to distortions across firms due to unstable accounting practices.
### HISTORICAL BETA

**IBM US Equity**

<table>
<thead>
<tr>
<th>Relative Index</th>
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**Period**: Weekly  
**Range**: 1/3/03 To 12/31/04  
**Market**: Trade

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<td>Raw Beta</td>
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<table>
<thead>
<tr>
<th>Alpha (Intercept)</th>
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<tr>
<td>R2 (Correlation)</td>
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<td>Std Dev of Error</td>
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<tr>
<td>Std Error of Beta</td>
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**Equity BETA**

**INTL BUSINESS MACHINES CORP**

**S&P 500 INDEX**

*Identifies latest observation*

\[
Y = 1.01X - 0.08
\]

---

ADJ BETA = (0.67) * RAW BETA + (0.33) * 1.0
**HISTORICAL YIELD CURVE**

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<th>4/26/05</th>
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<td>0.5953</td>
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<td>2 YEAR</td>
<td>3.055</td>
<td>3.622</td>
<td>0.5667</td>
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<tr>
<td>3 YEAR</td>
<td>3.218</td>
<td>3.723</td>
<td>0.5051</td>
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<tr>
<td>5 YEAR</td>
<td>3.607</td>
<td>3.930</td>
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<td>10 YEAR</td>
<td>4.218</td>
<td>4.241</td>
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<td>30 YEAR</td>
<td>4.826</td>
<td>4.548</td>
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P/E
INTL BUSINESS MACHINES C (IBM US)  PRICE 74.61 N $ DELAYED

Range 6/30/04 to 3/31/05 Period Monthly
Market T Trade

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<td>12/04</td>
<td><strong>19.4438</strong></td>
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<td>11/04</td>
<td><strong>19.5519</strong></td>
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<td>10/04</td>
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<td>6/04</td>
<td>18.8758</td>
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N151 Equity HE
**IBM US**

**PRICE TABLE**

**THIS PAGE: 3/31/05 TO: 6/30/04**

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<td>11/04</td>
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<td>10/04</td>
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<td>8/04</td>
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<tr>
<td>7/04</td>
<td>87.07</td>
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<tr>
<td>6/04</td>
<td>88.15</td>
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EARNINGS ESTIMATES

IBM  US  International Business Machines Corp

WALL STREET ESTIMATES

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<th></th>
<th>MEAN</th>
<th>HIGH</th>
<th>LOW</th>
<th>NUMBER EST</th>
<th>MEAN CHG LAST MNTH ($)</th>
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<tr>
<td>FISC YR END 0512</td>
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<td>5.35</td>
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<td>FISC YR END 0612</td>
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<tr>
<td>NEXT 5 YR GRTH (%)</td>
<td><strong>10.68</strong></td>
<td>15.00</td>
<td>8.00</td>
<td>11</td>
<td>0.14</td>
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ALL ESTIMATES ARE FOR DILUTED EPS FROM CONTINUING OPERATIONS
<table>
<thead>
<tr>
<th>Selected Financial Items</th>
<th>2000</th>
<th>1Q 2001</th>
<th>2Q 2002</th>
<th>3Q 2003</th>
<th>4Q 2004</th>
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<tbody>
<tr>
<td>Diluted EPS Cont Ops</td>
<td>4.44</td>
<td>4.59</td>
<td>3.95</td>
<td>4.34</td>
<td>5.05</td>
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<tr>
<td>Diluted EPS bef XD</td>
<td>4.44</td>
<td>4.59</td>
<td>3.07</td>
<td>4.34</td>
<td>4.94</td>
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<td>Diluted EPS</td>
<td>4.44</td>
<td>4.35</td>
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<td># Shrs Diluted EPS</td>
<td>1812.12</td>
<td>1771.23</td>
<td>1730.94</td>
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<td>Book value/share</td>
<td>11.56</td>
<td>13.70</td>
<td>13.23</td>
<td>16.44</td>
<td>16.08</td>
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<tr>
<td>Dividends/share</td>
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<td>.550</td>
<td>.590</td>
<td>.630</td>
<td>.700</td>
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<td>Cash Flow/Basic shr.</td>
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<td>Net fixed assets</td>
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<td>16504.00</td>
<td>14440.00</td>
<td>14689.00</td>
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<tr>
<td>Total assets</td>
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<td>88313.00</td>
<td>96484.00</td>
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<tr>
<td>Cur liabilities</td>
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<td>35119.00</td>
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<tr>
<td>Total liabilities</td>
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<td>64699.00</td>
<td>73702.00</td>
<td>76593.00</td>
<td>79436.00</td>
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<td>Shareholder equity</td>
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<td>Net sales</td>
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<td>11175.00</td>
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<td>15890.00</td>
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<tr>
<td>Net income (loss)</td>
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<td>7723.00</td>
<td>3579.00</td>
<td>7583.00</td>
<td>9430.00</td>
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</tbody>
</table>

Note: Per share values in USD, all other amounts in millions of USD.