Lecture 9: Theories of the Yield Curve.

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Lecture 9

Lecture 9: Theories of the Yield Curve.

I. Reading.
   A. BKM, Chapter 15.3-15.4.
II. Expectations Hypothesis

A. Setting:
1. Individuals do not care about interest rate risk.
2. So expected return over any future period is the same for all discount bonds and forward contracts.

B. Statements of the expectations hypothesis: 3 approximately equivalent (if using continuously compounded rates then the 3 are exactly equivalent)
1. The yield on an n-year discount bond is equal to the geometric average of the expected \( \frac{1}{2} \)-year spot rates for the next n years:

\[
1 + \frac{y_n(0)}{2} = \left( 1 + \frac{y_{\frac{1}{2}}(0)}{2} \right) \left( 1 + \frac{E_{\text{at time } 0}[y_{\frac{1}{2}}(\frac{1}{2})]}{2} \right) \cdots \left( 1 + \frac{E_{\text{at time } 0}[y_{\frac{1}{2}}(n-\frac{1}{2})]}{2} \right)^{\frac{1}{2n}}
\]

2. Expected return over the next \( \frac{1}{2} \)-year on a discount bond of any maturity is equal to the \( \frac{1}{2} \)-year spot rate:

\[y_{\frac{1}{2}}(0) = E_{\text{at time } 0}[h_n(\frac{1}{2})]\]

where \( h_n(\frac{1}{2}) \) is the return on a discount bond with a maturity n years in the future for the \( \frac{1}{2} \)-year ending at time \( \frac{1}{2} \).

3. Forward rate available today for any future \( \frac{1}{2} \)-year period is equal to the expected future spot rate for that period:

\[f_{n,n+\frac{1}{2}}(0) = E_{\text{at time } 0}[y_{\frac{1}{2}}(n)]\]

for any n.

C. Implications:
1. Yield curve slope and expectations about future spot rates:
   a. Upward sloping yield curve implies an expectation of higher spot rates in the future.
   b. Downward sloping yield curve implies an expectation of lower spot rates in the future.
2. Yield curve slope and expectations about future economic activity:
   a. A very positive slope is expected to predicts strong future GDP growth: Low short-term interest rates generate much investment today which in turn creates an expectation of strong future GDP growth.
   b. Empirically:
      (1) the slope of the yield curve is one of the best available predictors for the growth rate of the economy.
      (2) a very positive slope predicts strong future GDP growth.
D. Examples:
   a. In April 2004 the US economy was struggling to get out the recession (employment was slow to pick up, “jobless recovery”) but investors were confident that things would improve (GDP grew at a fast pace in 2003). Short term interest rates were low and the expectation was that the easy monetary policy of Fed would be successful in stimulating growth. Investors were expecting the Fed to raise interest rates, hence long rates were higher than short rates.

   a. Economy was doing very well (stock market “bubble”). Fed was worried about inflation. To combat inflation it had raised short term rates to a very high level (6.36%). Investors expected the economy to cool down, especially since the “bubble” was bursting, and investors expected the Fed to lower rates in the future. That gave a downward sloping yield curve.

   a. The central bank was worried about deflation (= decline in the price level) and a prolonged recession. To stimulate the economy it drove down the short term interest rate all the way to zero. People expected that this would be successful and that the Japanese economy would recover. Expected future short rates were higher than current rates.
a. Market expected future spot rates to be higher: a year later, the yield curve has shifted up, suggesting that the market’s expectation of higher future spot rates was realized.
b. Upward sloping yield curve suggests strong future GDP growth can be expected: GDP growth in the first quarter of 2006 was strongest since the third quarter of 2003.

5. U.S. 06/2006: yield curve flat at around 5%
   a. Suggests that market is expecting future spot rates to remain at 5% into the future.
   b. Suggests that future GDP growth not expected to be high.
III. Liquidity Preference Theory.
   A. Setting:
      1. Individuals require a liquidity premium to hold less liquid, longer maturity bonds: there is an associated price discount.
      2. The percentage change in the associated price discount going to the next longest maturity is always positive.
   B. Statement:
      1. The yield on an n-year discount bond is always higher than the geometric average of the expected ½-year spot rates for the next n years:

\[
\left(1 + \frac{y_n(0)}{2}\right) = \left(b_{\frac{1}{2}}(0) b_{1}(0) ... b_{n}(0)\right)^\frac{1}{2n}
\]

\[
\left(1 + \frac{y_{\frac{1}{2}}(0)}{2}\right) \left(1 + \frac{E_{at\ time\ 0}[y_{\frac{1}{2}}(\frac{1}{2})]}{2}\right) ... \left(1 + \frac{E_{at\ time\ 0}[y_{\frac{1}{2}}(n-\frac{1}{2})]}{2}\right)\right)^\frac{1}{2n}
\]

where \( b_{\frac{1}{2}} = 1 \) and the theory says \( b_t > 1 \) for all \( t > \frac{1}{2} \).
   C. Implications:
      1. More likely to see upward-sloping yield curves.
      2. Forward rates are upward biased predictors of future ½-year spot rates:

\[
f_{n,n+\frac{1}{2}}(0) > E_{at\ time\ 0}[y_{\frac{1}{2}}(n)] \quad \text{for all } n > \frac{1}{2}.
\]
      3. Yield curve slope and expectations about future spot rates:
         a. Upward sloping yield curve is consistent with the market expecting higher or lower spot rates in the future.
         b. Downward sloping yield curve implies that the market is expecting lower spot rates in the future.

IV. Preferred Habitat Theory.
   A. Setting:
      1. Yields on different securities are determined by the supply and demand for that security.
      2. Securities with similar maturities may not be close substitutes.
   B. Statement:
      1. The yield on an n-year discount bond can be higher or lower than the geometric average of the expected ½-year spot rates for the next n years: \( b_t > 1 \) or \( < 1 \) for all \( t > \frac{1}{2} \).
   C. Implications: Slope of the yield curve tells you little about whether the market is expecting higher or lower spot rates in the future.
Lecture 9: Bond Portfolio Management.

I. Reading.
A. BKM, Chapter 16, Sections 16.1 and 16.3.

II. Risks associated with Fixed Income Investments.
A. Reinvestment Risk.
1. If an individual has a particular time horizon $T$ and holds an instrument with a fixed cash flow received prior to $T$, then the investor faces uncertainty about what yields will prevail at the time of the cash flow. This uncertainty is known as reinvestment risk.
2. Example: Suppose an investor has to meet an obligation of $5M in two years time. If she buys a two year coupon bond to meet this obligation, there is uncertainty about the rate at which the coupons on the bond can be invested. This uncertainty is an example of reinvestment risk.

B. Liquidation Risk.
1. If an individual has a particular time horizon $T$ and holds an instrument which generates cash flows that are received after $T$, then the investor faces uncertainty about the price of the instrument at time $T$. This uncertainty is known as liquidation risk.
2. Example: Suppose an investor has to meet an obligation of $5M in two years time. If she buys a five year discount bond to meet this obligation, there is uncertainty about the price at which this bond will sell in two years time. This uncertainty is an example of liquidation risk.
III. Duration.  
A. Definition.  
   1. Macaulay Duration.  
      a. Let 1 period=1 year.  
      b. Consider a fixed income instrument \( i \) which generates the following stream of certain cash flows:  
         
         \[
         \begin{array}{cccccccc}
         0 & \frac{1}{2} & 1 & 1\frac{1}{2} & 2 & 2\frac{1}{2} & \ldots & N-\frac{1}{2} & N \\
         C_i(\frac{1}{2}) & C_i(1) & C_i(1\frac{1}{2}) & C_i(2) & C_i(2\frac{1}{2}) & \ldots & C_i(N-\frac{1}{2}) & C_i(N) \\
         \end{array}
         \]

         Let \( y_i(0) \) be the YTM at time 0 on the instrument expressed as an APR with semi-annual compounding.  
         c. Macaulay duration is defined as  
            \[
            D_i(0) = \frac{1}{2} k_i(\frac{1}{2}) + 1 k_i(1) + 1\frac{1}{2} k_i(1\frac{1}{2}) + \ldots + N k_i(N) 
            \]
            where  
            \[
            k_i(t) = \frac{1}{P_i(0)} \frac{C_i(t)}{[1+y_i(0)/2]^t}. 
            \]
            d. Note that the \( k_i(t) \)'s sum to 1:  
            \[
            1 = k_i(\frac{1}{2}) + k_i(1) + k_i(1\frac{1}{2}) + \ldots + k_i(N). 
            \]
   2. “Modified” Duration.  
      a. “Modified” duration \( D_i^*(0) \) is defined to be Macaulay duration divided by \( [1+y_i(0)/2] \):  
         \[
         D_i^*(0) = D_i(0) /[1+y_i(0)/2]. 
         \]
      b. “Modified” duration is often used in the industry.
3. Example. 2/15/06. The 3 Feb 07 note has a price of 97.1298 and a YTM expressed as an APR with semi-annual compounding of 6%.

a. Its k’s can be calculated:

\[
k_{3 \text{ Feb 07}(\frac{1}{2})} = \frac{1.5}{1.03}/97.1298 = 1.4563/97.1298 = 0.0150.
\]

\[
k_{3 \text{ Feb 07}(1)} = \frac{101.5}{(1.03)^2}/97.1298 = 95.6735/97.1298 = 0.9850.
\]

b. Its duration can be calculated:

\[
D_{3 \text{ Feb 07}(0)} = \frac{1}{2} \times k_{3 \text{ Feb 07}(\frac{1}{2})} + 1 \times k_{3 \text{ Feb 07}(1)}
\]

\[
= \frac{1}{2} \times 0.0150 + 1 \times 0.9850 = 0.9925 \text{ years}.
\]

c. Its modified duration can be calculated:

\[
D^*_{3 \text{ Feb 07}(0)} = \frac{0.9925}{(1+0.03)} = 0.9636.
\]
B. Duration can be interpreted as the effective maturity of the instrument.
1. If the instrument is a zero coupon bond, then its duration is equal to the
time until maturity.
2. For an instrument with cash flows prior to maturity, its duration is less
than the time until maturity.
3. Holding maturity and YTM constant, the larger the earlier payments
relative to the later payments then the shorter the duration.
4. Example: 2/15/06. Suppose the 10 Feb 07 coupon bond has a price of
103.8270 and a YTM expressed as an APR with semi-annual
compounding of 6%.
   a. Its k’s can be calculated:
      \[ k_{10 \text{ Feb } 07(\frac{1}{2})} = \frac{5}{1.03}/103.8270 = 4.8544/103.8270 = 0.0468. \]
      \[ k_{10 \text{ Feb } 07(1)} = \frac{105/1.03^2}{103.8270} = 98.9726/103.8270 = 0.9532. \]
   b. Its duration can be calculated:
      \[ D_{10 \text{ Feb } 07(0)} = \frac{1}{2} \times k_{10 \text{ Feb } 07(\frac{1}{2})} + 1 \times k_{10 \text{ Feb } 07(1)} = \frac{1}{2} \times 0.0468 + 1 \times 0.9532 = 0.9766 \text{ years}. \]
   c. So the 10 Feb 07 bond with larger earlier cash flows relative to
later cash flows has a shorter duration than the 3 Feb 07 bond.

C. Duration of a Portfolio.
1. If a portfolio C has \( \omega_{A,C} \) invested in asset A at time 0 and \( \omega_{B,C} \) invested in
asset B at time 0 and the YTMs of A and B are the same, the duration of C
is given by:
   \[ D_C(0) = \omega_{A,C} \times D_A(0) + \omega_{B,C} \times D_B(0). \]
2. Example (cont): 2/15/06. Suppose Tom forms a portfolio on 2/15/06 with
40% in 3 Feb 07 coupon bonds and the rest in 10 Feb 07 coupon bonds.
   a. The portfolio’s duration can be calculated:
      \[ D_P(0) = \omega_{3 \text{ Feb } 07,P} \times D_{3 \text{ Feb } 07(0)} + \omega_{10 \text{ Feb } 07,P} \times D_{10 \text{ Feb } 07(0)} \]
      \[ = 0.4 \times 0.9925 + (1-0.4) \times 0.9766 = 0.98296. \]

<table>
<thead>
<tr>
<th>Instrument</th>
<th>YTM(2/15/06)</th>
<th>Duration(2/15/06)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feb 07 Strip</td>
<td>6%</td>
<td>1.0000</td>
</tr>
<tr>
<td>3 Feb 07 Bond</td>
<td>6%</td>
<td>0.9925</td>
</tr>
<tr>
<td>10 Feb 07 Bond</td>
<td>6%</td>
<td>0.9766</td>
</tr>
<tr>
<td>40% of 3 Feb 07 and 60% of 10 Feb 07</td>
<td>6%</td>
<td>0.9925 x 0.4 + 0.9766 x 0.6 = 0.98296</td>
</tr>
</tbody>
</table>
D. Duration as a measure of yield sensitivity.

1. The price of bond i described above is given by:

\[
P^i(0) = \frac{C^i(\frac{1}{2})}{[1 + y^i(0)/2]^1} + \frac{C^i(1)}{[1 + y^i(0)/2]^2} + \frac{C^i(1\frac{1}{2})}{[1 + y^i(0)/2]^3} + \ldots + \frac{C^i(N)}{[1 + y^i(0)/2]^{2N}}.
\]

2. To assess the impact of the price of bond i to a shift in \(y^i(0)\), differentiate the above expression with respect to \(y^i(0)\):

\[
\frac{dP^i(0)}{dy^i(0)} = -\frac{1}{2} \frac{C^i(\frac{1}{2})}{[1 + y^i(0)/2]^2} - 1 \frac{C^i(1)}{[1 + y^i(0)/2]^3} - \frac{1}{2} \frac{C^i(1\frac{1}{2})}{[1 + y^i(0)/2]^4} - \ldots - N \frac{C^i(N)}{[1 + y^i(0)/2]^{2N+1}}.
\]

3. Thus, bond i’s modified Macaulay duration is related to the sensitivity of its price to shifts in \(y^i(0)\) as follows:

\[
\frac{dP^i(0)}{dy^i(0)} = -P^i(0) \cdot D^i(0).
\]

4. Thus, bond i’s modified Macaulay duration measures its price sensitivity to a change in its yield (where price sensitivity is measured by the percentage change in the price).

5. In particular, a small change in \(y^i(0)\), \(\Delta y^i(0)\), causes a change in \(P^i(0)\), \(\Delta P^i(0)\), which satisfies:

\[
\Delta P^i(0) = -P^i(0) \cdot D^i(0) \cdot \Delta y^i(0).
\]

6. For coupon bonds and strips, price is a convex function of yield: so the duration-based price-change approximation overstates the price decline when the YTM increases and understates the price increase when the YTM decreases.
7. Example (cont): 2/15/06. The 3 Feb 07 note has a price of 97.1298 and a YTM expressed as an APR with semi-annual compounding of 6%.
   a. Using the formula for approximate price change involving modified duration above:

   YTM increase: \( \Delta P^{3 \ Feb \ 07(0)} = -97.1298 \times 0.9636 \times 0.01 = -0.9359 \).
   YTM decrease: \( \Delta P^{3 \ Feb \ 07(0)} = -97.1298 \times 0.9636 \times -0.01 = 0.9359 \).

   b. Can compare the approximate price changes to the actual price changes:

<table>
<thead>
<tr>
<th>YTM</th>
<th>P(0)</th>
<th>Actual ΔP(0)</th>
<th>YTM = 6%</th>
<th>ΔYTM</th>
<th>Approx ΔP(0) = -P(0)D*(0)ΔYTM</th>
</tr>
</thead>
<tbody>
<tr>
<td>7%</td>
<td>96.2006</td>
<td>-0.9292</td>
<td>97.1298</td>
<td>0.01</td>
<td>-0.9359</td>
</tr>
<tr>
<td>6%</td>
<td>97.1298</td>
<td>0</td>
<td>97.1298</td>
<td>0.9636</td>
<td>0</td>
</tr>
<tr>
<td>5%</td>
<td>98.0725</td>
<td>+0.9427</td>
<td>0</td>
<td>0.01</td>
<td>+0.9359</td>
</tr>
</tbody>
</table>

![Duration and Price Sensitivity to Yield Shifts](image-url)
Duration and Price Sensitivity to Yield Shifts:
30 Yr Strip Price at y=6% is normalized to 100

Duration and Price Sensitivity to Yield Shifts:
10 Yr Strip Price at y=6% is normalized to 100
IV. Immunization.

A. Dedication Strategies.
1. A dedication strategy ensures that a stream of liabilities can be met from available assets by holding a portfolio of fixed income assets whose cash flows exactly match the stream of fixed outflows.
2. Note that both the asset portfolio and the liability stream have the same current value and duration.
3. Example: It is 2/15/06 and QX must pay $5M on 8/15/06 and $10M on 2/15/07. A dedication strategy would involve buying Aug 06 U.S. strips with a face value of $5M and Feb 07 U.S. strips with a face value of $10M.

B. Target Date Immunization.
1. Assumptions.
   a. the yield curve is flat at y (APR with semiannual compounding).
   b. any shift in the curve keeps it flat.
2. Target date immunization ensures that a stream of fixed outflows can be met from available assets by holding a portfolio of fixed income assets:
   a. with the same current value as the liability stream; and,
   b. with the same modified duration.
3. Note:
   a. given the flat yield curve, all bonds have the same YTM, y.
   b. for this reason, a small shift in the yield curve will have the same effect on the value of the immunizing assets and on the value of the liabilities (using the definition of duration).
   c. and so, the assets will still be sufficient to meet the stream of fixed outflows.
   d. further, it is easy to work out the current value of the liability stream using y.
   e. equating the modified durations of the assets and the liabilities is the same as equating the durations.
Example: Firm GF is required to make a $5M payment in 1 year and a $4M payment in 3 years. The yield curve is flat at 10% APR with semi-annual compounding. Firm GF wants to form a portfolio using 1-year and 4-year U.S. strips to fund the payments. How much of each strip must the portfolio contain for it to still be able to fund the payments after a shift in the yield curve?

a. The value of the liabilities is given by:
\[ L(0) = \frac{5M}{(1+0.1/2)^2} + \frac{4M}{(1+0.1/2)^6} = 4.5351M + 2.9849M = 7.5200M. \]

b. The duration of the liabilities is given by:
\[ D_L(0) = 1 \times \frac{4.5351}{7.5200} + 3 \times \frac{2.9849}{7.5200} = 1.7938 \text{ years}. \]

c. Let \( A_1(0) \) be the portfolio’s dollar investment in the 1-year strips and \( A_4(0) \) be the portfolio’s dollar investment in the 4-year strips.
d. The dollar value of the portfolio must equal the value of the liabilities. So \( A_1(0) + A_4(0) = 7.5200M. \)
e. The duration of the portfolio equals
\[ D_P(0) = \omega_1^{P} D_1(0) + (1-\omega_1^{P}) D_4(0) \]
where \( \omega_1^{P} = A_1(0)/7.5200M. \) The duration of the 1-year strip is 1 and the duration of the 4-year strip is 4.
f. Setting the duration of the portfolio equal to the duration of the liabilities gives:
\[ 1.7938 = \omega_1^{P} D_1(0) + (1-\omega_1^{P}) D_4(0) = \omega_1^{P} 1 + (1-\omega_1^{P}) 4 \Rightarrow \omega_1^{P} = 0.7354. \]
g. Thus,
\[ A_1(0) = 0.7354 \times 7.5200M = 5.5302M; \text{ and, } A_4(0) = 7.5200M - 5.5302M = 1.9898M. \]
5. Generalizations.
a. The assumption of a flat yield curve has undesirable properties.
b. When the yield curve is allowed to take more general shapes, target-date immunization is still possible, but modified Macaulay duration is not appropriate for measuring the impact of a yield curve shift on price. Need to use a more general duration measure.

C. Comparison.
1. Dedication strategies are a particular type of target date immunization.
2. The one advantage of a dedication strategy is that there is no need to rebalance through time. Almost all other immunization strategies involve reimmunizing over time.
Lecture 9-10: Derivatives: Definitions and Payoffs.

I. Readings.
   A. Options:
      1. BKM, Chapter 20, Sections 20.1 - 20.4.
   B. Forward and Futures Contracts.
      1. BKM, Chapter 22, Sections 22.1 - 22.3.

II. Preliminaries.
   A. Payoff Diagrams.
      1. Let $S(t)$ be the value of one unit of an asset at time $t$.
      2. Payoff diagram graphs the value of a portfolio at time $T$ as a function of $S(T)$.
      3. Long 1 unit of the asset is represented by a $45^\circ$ line through the origin.
      4. Short 1 unit of the asset is represented by a $-45^\circ$ line through the origin.
      5. Long a discount bond with a face value of $50$ maturing at $T$ is represented by a horizontal line at $50$: its value at $T$ is $50$ irrespective of the value of the asset.
      6. Short a discount bond with a face value of $50$ maturing at $T$ is represented by a horizontal line at $-50$. 
Long 1 Stock:
Payoff at T

S(T)

0 10 20 30 40 50 60 70 80 90 100

-100 0 20 40 60 80 100
Short 1 Stock:
Payoff at T
Long 1 Discount Bond maturing at $T$ with $\$50$ face value:
Payoff at $T$
Short 1 Discount Bond maturing at $T$ with $\$50$ face value:
Payoff at $T$

$-B_1(T) = -50$

$S(T)$
III. Options.
   A. Call Option.
      1. Definition.
         a. A call option gives its holder the right (but not the obligation) to buy the option’s underlying asset at a specified price.
         b. The specified price is known as the exercise or strike price.
         c. A European option can only be exercised at the expiration date of the option. An American option can be exercised any time prior to the expiration of the option.

<table>
<thead>
<tr>
<th>Position opened at time 0</th>
<th>Time 0</th>
<th>Time T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long 1 European Call Option on the Stock expiring at T with an exercise price of $50</td>
<td>$-C_{50,T}(0)$</td>
<td>0</td>
</tr>
</tbody>
</table>
2. Payoff at the Expiration Date to the Holder of a European Option.
   a. Let $T$ be the expiration date, $X=50$ be the strike price and $S(t)$ be the value of the option’s underlying asset at time $t$.
   b. Since the holder of the option is not obligated to buy the asset for $50$, she will only do so when the payoff from doing so is greater than zero.

   (1) The price paid by the holder prior to $T$ for the option is irrelevant to the decision to exercise since it is a sunk cost.
   (2) If $[S(T)-50] > 0$, the holder wants to exercise the option as it allows her to buy for $50$ an asset worth more than $50$.
   (3) If $[S(T)-50] < 0$, the holder does not want to exercise the option since she would be paying $50$ for an asset she could buy in the market for less than $50$. So her payoff is zero.
   (4) Thus, the holder’s payoff from the call option is $\max\{S(T)-50, 0\}$.
   (5) Since the payoff from the option is non-negative, no arbitrage tells you that the call option has a non-negative value at any time $t$, $C_{50}(t)$, prior to $T$.
   (6) At the expiration date, $C_{50}(T) = \max\{S(T)-50, 0\}$. 
3. Writer of the Call Option.
   a. The payoff to the option writer is just the negative of the payoff to the option holder.
   b. In return, the option writer receives the option’s value when she first writes the call option and sells it.
   c. Suppose the call is written at time 0 and both the writer and the option buyer keep their positions until expiration. Can see that the sum of the writer’s and the holder’s payoff is zero.

<table>
<thead>
<tr>
<th>Position opened at time 0</th>
<th>Time 0</th>
<th>Time T</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>S(T)&lt;50</td>
</tr>
<tr>
<td>Long 1 European Call Option on the Stock expiring at T with an exercise price of $50</td>
<td>-C_{50,T}(0)</td>
<td>0</td>
</tr>
<tr>
<td>Short 1 European Call Option on the Stock expiring at T with an exercise price of $50</td>
<td>C_{50,T}(0)</td>
<td>0</td>
</tr>
<tr>
<td>Sum</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
d. If the option holder exercises the option, the option writer must sell the asset to the holder for $50. Her payoff is $-\[S(T) - 50\].

e. If the option holder does not exercise the option, the writer is not obliged to do anything. Her payoff is 0.
Lecture 9-10

B. Put Option.

1. Definition.
   a. A put option gives its holder the right (but not the obligation) to sell the option’s underlying asset at a specified price (also called the exercise or strike price).
   b. European and American have the same connotations for puts as for calls.

<table>
<thead>
<tr>
<th>Position opened at time 0</th>
<th>Time 0</th>
<th>Time T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long 1 European Put Option on the Stock expiring at T with an exercise price of $50</td>
<td>(-P_{50,T}(0))</td>
<td>(50-S(T))</td>
</tr>
</tbody>
</table>

2. Payoff at the Expiration Date to the Holder of the Option.
a. Again, let $T$ be the expiration date, $X=50$ be the strike price and $S(t)$ be the value of the option’s underlying asset at time $t$.

b. Since the holder of the option is not obligated to sell the asset for $50$, she will only do so when the payoff from doing so is greater than zero.

1. The price paid by the holder prior to $T$ for the option is irrelevant to the decision to exercise since it is a sunk cost.

2. If $[50-S(T)] > 0$, the holder wants to exercise the option since it allows her to sell for $50$ an asset worth less than $50$.

3. If $[50-S(T)] < 0$, the holder does not want to exercise the option since she would be receiving $50$ for an asset she could sell in the market for more than $50$. So her payoff is zero.

4. Thus, the holder’s payoff from the put option is $\max \{50-S(T), 0\}$.

5. Since the payoff from the option is non-negative, no arbitrage tells you that the put option has a non-negative value at any time $t$, $P_{50}(t)$, prior to $T$.

6. At the expiration date, $P_{50}(T) = \max \{50-S(T), 0\}$.

7. Note that holding a put is not the same as writing a call.
3. **Writer of the Option.**
   a. The payoff to the option writer is just the negative of the payoff to the option holder.
   b. In return, the option writer receives the option’s value when she first writes the put option and sells it.
   c. Suppose the call is written at time 0 and both the writer and the option buyer keep their positions until expiration. Can see that the sum of the writer’s and the holder’s payoff is zero.

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C. Markets for Options.
   1. Chicago Board Options Exchange (CBOE).
      a. Most options on individual stocks are traded on the CBOE which was started in 1973.
      b. The CBOE is both a primary and secondary market for options.
   2. Which options have volume?
      a. Presently, there is very little trading or interest in options on individual stocks.
      b. Interest in options on indices, both exchange traded and over the counter is booming. Part of the reason is the growth of index funds.

D. Standardization: Publicly traded options are standardized in several respects.
   1. Size of Contract.
      a. Traded options on stocks are usually for 100 shares of the stock, although prices are quoted on a per share basis.
   2. Maturities.
      a. Options expire on a three month cycle.
      b. The convention is that the option expires on the Saturday following the third Friday of the month.
   3. Exercise Prices.
      a. These are in multiples of $2\frac{1}{2}, $5 or $10 depending on the prevailing stock price.

E. Examples.
   1. European call and put options on the S&P 500 index from the Bloomberg screen on 4/6/05.
   2. American call and put options on Dell stock from the Bloomberg screen on 4/6/05.
### CALL SUMMARY PAGE

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### Lecture 9-10 Foundations of Finance

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