

# Labor Income Dynamics at Business-cycle Frequencies: Implications for Portfolio Choice \*

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## Abstract

A large recent literature has focused on multiperiod portfolio choice with labor income, and while the models are elaborate along several dimensions, they almost all assume that the joint distribution of shocks to labor income and asset returns is i.i.d.. Calibrating this joint distribution to U.S. data, these papers obtain three results not found empirically for U.S. households: young agents choose a higher stock allocation than old agents; young agents choose a higher stock allocation when poor than when rich; and, young agents always hold some stock. This paper asks whether allowing the conditional joint distribution to depend on the business cycle can allow the model to generate equity holdings that better match those of U.S. households, while keeping the unconditional distribution the same as in the data. Calibrating the first two conditional moments of labor income growth to match the countercyclical volatility and procyclical mean found in U.S. data leads to large reductions in stock holdings by young agents with low wealth-income ratios. The countercyclical volatility is the more important of the two, inducing reductions which are so large that young, poor agents now hold less stock than both young, rich agents and old agents, and also hold no stock a large fraction of the time. Our results suggest that the predictability of labor-income growth at a business-cycle frequency, particularly the countercyclical variation in volatility, plays an important role in a young agent's decision-making about her portfolio's stock holding.

# 1 Introduction

A large recent literature has focused on multiperiod portfolio choice with labor income, and while the models are elaborate along several dimensions, they almost all assume that the joint distribution of shocks to labor income and asset returns is i.i.d..<sup>1</sup> Calibrating this joint distribution to U.S. data, these papers obtain three results not found empirically for U.S. households: young agents choose a higher stock allocation than old agents; young agents choose a higher stock allocation when poor than when rich; and, young agents always hold some stock.<sup>2,3</sup> This paper asks whether allowing the conditional joint distribution to depend on the business cycle can allow the model to generate equity holdings that better match those of U.S. households, while keeping the unconditional distribution the same as in the data. Calibrating the first two moments of labor income growth to match the countercyclical volatility and procyclical mean found in U.S. data leads to large reductions in stock holdings by young agents with low wealth-income ratios. The countercyclical volatility is the more important of the two, inducing reductions which are so large that young, poor agents now hold less stock than both young, rich agents and old agents, and also hold no stock a large fraction of the time. Our results suggest that the predictability of labor-income growth at a business-cycle frequency, particularly the countercyclical variation in volatility, plays an important role in a young agent's decision-making about her portfolio's stock holding.

Enriching labor-income dynamics along this dimension can be motivated by recent evidence that the first and second moments of labor income growth are predictable at business-cycle frequencies. In a recent paper, Storesletten, Telmer and Yaron (2004), hereafter Storesletten et.al., using household-level labor-earnings data from PSID, estimate that the standard deviation of shocks to permanent log labor income increases by around 75% as the macroeconomy moves from peak to trough. Further, economic intuition strongly suggests that labor income growth is higher in good times than in bad. We estimate the magnitude of this effect by taking the changes in log aggregate labor income and covarying this series with the lagged value of the 12-month dividend yield on the

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<sup>1</sup>See Zeldes (1989), Heaton and Lucas (1997), Davis and Willen (2000a), Viceira (2001), Cocco, Gomes and Maenhout (2002), and Gomes and Michaelides (2003).

<sup>2</sup>These findings are robust to the presence of reasonable transactions costs in the stock market (see Heaton and Lucas, 1997) but possibly not habit formation preferences. Polkovnichenko (2006) finds that additive habit formation preferences can induce lower stock holdings as the agent's wealth-income ratio declines.

<sup>3</sup>The empirical papers that report contradictory stock holding patterns by U.S. households include Friend and Blume (1975), Poterba (1993), Bertaut (1994), Blume and Zeldes (1994), Heaton and Lucas (2000a) and (2000b), Vissing-Jorgensen (2002) and Curcuru, Heaton, Lucas and Moore (2004). Calvet, Campbell and Sodini (2006), who look at asset holdings of Swedish individuals, report substantial non-participation and find that stock holdings and participation both increase in wealth after controlling for income

value weighted NYSE index. When the aggregate labor income measure is either monthly earnings for the retail sales industry or the total private sector (both from the Bureau of Labor Statistics tables), the point estimate of this covariance is negative and strongly significant. Since dividend yield is counter-cyclical, this point estimate implies that the change in log aggregate labor income is pro-cyclical, which is consistent with intuition.

Both the pro-cyclical behavior of mean labor income growth (state-dependent mean channel) and the counter-cyclical behavior of labor-income volatility (state-dependent volatility channel) can reduce stock holding by young low wealth-income ratio agents. This result can be explained by a static diversification story which says that positive (negative) covariance between stock returns and human capital causes the agent to hold less (more) stock. When risk aversion is greater than 1, both channels produce human capital that covaries more positively with stock return than when both channels are switched off, which means that the static diversification intuition implies lower stock holdings. However, the deeper question is why, when risk aversion is greater than 1, the two channels produce human capital that is more positively correlated with stock return than when both channels are switched off. The hedging-demand intuition of Merton (1973) can be used to answer this question. Merton (1973) shows that for CRRA investors with risk aversion greater than 1, positive correlation between return and future investment opportunities leads to reductions in stock holdings by young investors. Empirically, realized stock return is low when the probability of entering or remaining in a recession increases. But in recessions expected income growth is low and the volatility of income growth is high. So a low stock return this period means low expected income growth and high volatility of income growth in the next period and future periods. Thus, stock returns and future “labor income” opportunities are positively correlated. Therefore business-cycle variation in the first two moments of income growth causes reductions in stock holdings by young investors. Moreover, these reductions are more pronounced for poor young investors, for whom future labor income is more important. This mechanism is the flipside of the one by which return predictability increases the stock holdings of young agents with risk aversion greater than 1. These young agents hold more stock than myopic agents because of the negative correlation between stock return and future opportunity sets induced by the predictability. Consistent with the hedging-demand intuition, when risk aversion is less than 1, both channels are found to produce human capital that covaries negatively with stock return, and the young low wealth-income ratio agent is found to hold more stock when either or both channels are switched on than when both channels are switched off.

Our goal is to quantify the effects of these two labor-income channels on portfolio allocations by young investors. To do so, we formulate a dynamic life-cycle portfolio choice problem and calibrate the stock return and labor income processes to U.S. data. Simple VAR dynamics are used to incorporate both mechanisms, with dividend yield, a counter-cyclical business-cycle variable, being used as the predictor for both labor-income growth and stock return. Robustness checks indicate that the VAR does a good job of capturing both the high and low frequency income growth predictability in the data. The agent starts work at age 22 and retires at 65, receiving social security payments of 93.8% of her retirement permanent income until death, as reported in Cocco, Gomes and Maenhout (2002) for college graduates. Death probabilities for the agent are taken from the U.S. Life Tables, 2001, provided by the NCHS. The agent has power utility and risk aversion of 6. The state-dependent mean SDM channel is incorporated by calibrating the covariance of labor income growth and lagged dividend yield to that for aggregate monthly wages in the retail trade industry. The state-dependent volatility SDV channel is incorporated by allowing the second moment to be predictable using a bifurcation of dividend yield. We bifurcate the quadrature’s dividend yield variable using a cutoff value such that the unconditional probabilities of the resulting recession state and expansion state match the unconditional probabilities in the data for the NBER-based expansions and recessions. The ratio of the innovation volatility for permanent labor income growth in recessions relative to expansions is matched to 1.75, the value reported in Storesletten et.al. using the NBER variable to determine the timing of expansions and recessions. At the same time, the unconditional volatility of permanent labor income growth itself is always matched to the 15% per annum reported in Gakidis (1997) based on PSID data for professionals and managers not self-employed under age 45. We use a 3rd order polynomial to approximate an agent’s typically hump shaped life-cycle earnings profile, taking the parameter point estimates in Cocco, Gomes and Maenhout (2002) for college graduates.

In this base case, the simultaneous presence of the two business-cycle channels calibrated to data causes the agent’s stock allocation to drop from near the boundary of 100% to an average allocation of less than 25% for a young agent’s whose financial wealth is less than 30 times her monthly wage. The magnitude of the reduction is only increased by considering smaller wealth-income ratios. However, while both business-cycle channels are important, the volatility channel is definitely the more important. When financial wealth is 10 times the young agent’s monthly labor income, the average stock allocation decreases by an allocation of 17% when the mean channel is switched on but the volatility channel is left off and by an allocation of 68% when the volatility

channel is switched on but the mean channel is left off. It is the volatility channel's presence that causes the relation between average stock allocation and wealth-income ratio to flip from the negative relation in the theoretical literature to a positive one as in the data. Turning to stock allocations as a function of age for an agent with zero financial wealth at age 22, the average stock allocation is a negative function of age from age 22 to 57 and lower at retirement than at age 22 when both channels are switched off. Switching on the two business-cycle channels causes the function to become hump-shaped from age 22 to age 54 and the average stock allocation to be much higher at retirement than at age 22, consistent with the data. Turning to the non-participation results, both channels switched off leads to participation in the stock market virtually all the time, irrespective of age or wealth-income ratio. Switching on the two business-cycle channels results in substantial non-participation by agents in their first month and the non-participation steadily declines as the agent gets older. For example, an agent with a wealth-income ratio of 0 in the first month decides not to participate in the stock market 79% of the time in the first month; and after ten years, this probability has declined to a fraction that is still above 26%.

Importantly, the volatility channel's affect on allocations is robust to using the NBER expansion-recession variable directly to calibrate the expansion-recession state, which means that this channel is able generate large reductions in the average stock allocations of young, poor agents without relying on the large negative contemporaneous correlation between dividend yield and stock returns. Moreover, recognizing that agents may not have sufficient information to infer the NBER expansion-recession variable at the start of each month, we use the empirical relation between dividend yield and the NBER expansion-recession variable to calculate the probability of an NBER expansion conditional on the value of the dividend yield. Taking the volatility conditional on the NBER variable as given by Storesletten et.al., this probability allows us to calculate analytically the volatility conditional on the dividend yield value. As would be expected, the effect of the volatility channel on allocations is attenuated relative to when the agent uses NBER expansion-recession variable directly. But it is still the same qualitatively, since the reduction in the average stock allocations of young, poor agents is sufficiently large that in combination with the state-dependent mean channel, these agents still hold less stock on average than young wealthy agents.

Our model is able to generate realistic wealth accumulation by the agent over her life. And a number of robustness checks and extensions are also performed. The ability of the two business cycle channels to reduce the stock holdings of poor young agents is largely unaffected by whether stock returns are i.i.d. or predictable, the presence of social security, the introduction of a realistic

probability of unemployment, a flat rather than hump-shaped labor income profile or the presence of temporary shocks to labor income.

Positive conditional correlation between today's realized return and today's labor growth innovation can also reduce equity holdings (see Davis and Willen, 2000a and 2000b and Michaelides, 2003). This is a diversification-like channel and is available even when stock return and labor income growth are i.i.d. processes. Consequently, it is a channel that is quite distinct from the two we are considering. However, the contemporaneous correlation between returns and labor income growth appears to be small in the data.<sup>4</sup> This small unconditional correlation is an important stylized fact that restricts the ability of the return correlation channel to reduce equity holdings (see Viceira, 2001).

Benzoni, Collin-Dufresne and Goldstein (2006) is independent work that considers a setting in which the resulting generating process for labor income has some features that are similar to the one we use. They assume that aggregate labor income and stock dividend are cointegrated to obtain their predictive variable which is the difference between the logs of the two. This difference is a stationary variable given the assumed cointegrating relation. However, the meaningfulness of their calibration relies heavily on the cointegrating relation holding, and while there is good intuition for such a relation holding (which makes their paper interesting), the empirical evidence is rather weak. In contrast, we don't need to assume such a co-integrating relation to identify our predictive variable. All we need is pro-cyclical expected labor income growth which is consistent with intuition and strongly supported by the data. Note that the resulting reductions in stock holdings from the two calibrations, theirs and ours, are likely to be quantitatively different and in fact they are. Finally, and most importantly, their setup does not allow for income growth heteroskedasticity which we find to be a much more important channel than our state dependent mean channel anyway.

Section 2 presents our formulation of the problem and describes the two channels through which we allow labor income to affect the stock holdings of young agents, while section 3 describes how the return and labor income processes are calibrated to the data. Section 4 discusses our results and section 5 concludes.

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<sup>4</sup>Davis and Willen (2000b) use individual level Current Population Survey data and find the correlations between S&P 500 returns and labor income shocks to be very close to zero or negative for all but the most educated group. Fama and Schwert (1977) report near zero correlations between the value weighted portfolio of NYSE stocks and measures of aggregate labor income. Botazzi, Pesenti and Wincoop (1996) provide corroborating international evidence.

## 2 Formulation and Solution of the Problem

### 2.1 Processes

Extending the specification in Carroll (1996) and (1997), labor income is specified to have both permanent and temporary components as follows:

$$y_{t+1} = y_{t+1}^P + \epsilon_{t+1}, \quad (1)$$

$$g_{t+1} \equiv y_{t+1}^P - y_t^P = \bar{g} + b_g d_t + u_{t+1}, \quad (2)$$

where  $y_t \equiv \ln(Y_t)$  is log labor income received at  $t$ ,  $y_t^P \equiv \ln(Y_t^P)$  is log permanent income at  $t$ ,  $\epsilon_{t+1}$  is log temporary labor income at  $t$ ,  $d_t \equiv \ln(1 + D_t)$ , and  $\epsilon_{t+1}$  and  $u_{t+1}$  are uncorrelated i.i.d. processes.  $D$  is the mean reverting predictor to proxy for the business cycle, which we take to be the 12-month dividend yield on the value-weighted NYSE index.  $d_t$  is normalized to be zero mean and unit variance.  $\epsilon_{t+1}$  contains no information about future returns  $(R_{t+1}, R_{t+2}, \dots)$  or about future  $D$  values  $(D_{t+1}, D_{t+2}, \dots)$ . To allow the agent's income process to be age-dependent, we allow the  $\bar{g}$  to be age-dependant with  $\bar{g}_t$  defined to be the  $\bar{g}$  value at age  $t$ .

We also specify a VAR for the log market return and dividend yield for which lagged dividend yield is the only predictor:

$$r_{t+1} = a_r + b_r d_t + e_{t+1}, \quad (3)$$

$$d_{t+1} = a_d + b_d d_t + w_{t+1}, \quad (4)$$

where  $r_{t+1} \equiv \ln(R_{t+1})$  is the log market return,  $a_r$  and  $a_d$  are intercepts,  $b_r$  and  $b_d$  are coefficients and  $[u \ e \ w]$  is a vector of mean-zero, multivariate normal disturbances, with unconditional covariance matrix  $\Sigma$ , whose conditional covariance matrix might possibility depend on the state. Let  $\sigma_{kj}$  be the unconditional covariance of  $k$  with  $j$  where  $k, j$  can be  $u, e$  or  $w$ . Similarly, let  $\sigma_k$  be the unconditional standard deviation of  $k$  where  $k$  can be  $u, e$  or  $w$ .

It is instructive to compare our labor income specification (with  $\bar{g}_t$  constant and equal to  $\bar{g}$  each period for simplicity) to that used in standard lifecycle models.<sup>5</sup> In those models, the permanent component of log labor income is modelled as a random walk with a drift and so is not a stationary process. But the change in the log of the permanent component is a stationary process, i.i.d. with a non-zero mean:

$$y_{t+1}^P - y_t^P = \bar{g} + u_{t+1}, \quad (5)$$

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<sup>5</sup>See for example Carroll (1996) and (1997), Gakidis (1997) and Viceira (2001).

Adding a temporary component as in eq. (1), log labor income at time  $t + \tau$  can be written as a function of the log of the permanent component of labor income at time  $t$  and the shocks as follows:

$$y_{t+\tau} = y_t^P + \tau\bar{g} + \sum_{j=1}^{\tau} u_{t+j} + \epsilon_{t+\tau}, \quad (6)$$

Thus, each shock to the change in the permanent component  $u_{t+j}$  has a permanent impact on log labor income at  $t + \tau$  for all  $\tau \geq j$ .

The same is true for our specification which nests this standard lifecycle one as a special case ( $b_g = 0$ ). The change in the permanent component of log labor income remains a stationary process in our specification and log labor income remains non-stationary. Analogously to eq. (6), log labor income at time  $t + \tau$  for our specification can be written as a function of log labor income at time  $t$ , the dividend yield at  $t$  and the shocks between  $t$  and  $t + \tau$  as follows:

$$y_{t+\tau} = y_t^P + \tau\bar{g} + b_g \frac{1 - b_d^\tau}{1 - b_d} d_t + b_g \sum_{j=1}^{\tau-1} \frac{1 - b_d^{\tau-j}}{1 - b_d} w_{t+j} + \sum_{j=1}^{\tau} u_{t+j} + \epsilon_{t+\tau}, \quad (7)$$

This decomposition shows that, as in the standard specification eq. (5), each shock to the change in the permanent component  $u_{t+j}$  has a permanent impact on log labor income at  $t + \tau$  for all  $\tau \geq j$ . As before, the temporary component shock,  $\epsilon_{t+\tau}$  only affects log labor income in the period of the shock, namely  $t + \tau$ . This decomposition also shows that each shock to dividend yield  $w_{t+j}$  has an effect on log labor income at  $t + \tau$  for all  $\tau > j$ . This effect grows with  $\tau$ , though at a declining rate governed by both  $b_g$  and  $b_d$ .

## 2.2 Problem and Solution Technique

We consider the optimal portfolio problem of a investor with a finite life of  $T$  periods and utility over intermediate consumption. Preferences are time separable and exhibit constant relative risk aversion (CRRA):

$$E \left[ \sum_{t=1}^T p_{1,t} \delta^t \frac{c_t^{1-\gamma}}{1-\gamma} \middle| D_1, W_1, Y_1^P \right], \quad (8)$$

where  $\gamma$  is the relative-risk-aversion coefficient,  $\delta$  is the time-discount parameter,  $W_t$  is wealth at time- $t$ ,  $c_t$  is consumption at time- $t$  and  $p_{\tau,t}$  is the probability that the agent is still alive at time- $t$  given that she is alive at time- $\tau$ . Note that the expected lifetime utility depends on the state of the economy at time 1.

In the base case, the agent retires at some time  $S$  and thereafter receives social security income each period equal to a fraction  $x$  of her permanent income at time  $S$  through until the terminal

date. So  $Y_{S+1}^P = xY_S^P$ ,  $Y_{t+1}^P = Y_t^P$  for  $t = S + 1, \dots, T - 1$  and  $\epsilon_{t+1} = 0$  for  $t = S, \dots, T - 1$ . With income, the law of motion for the investor's wealth,  $W$ , is given by

$$W_{t+1} = (W_t + Y_t - c_t) \left[ \alpha_t (R_{t+1} - R_t^f) + R_t^f \right] \quad \text{for } t = 1, \dots, T - 1, \quad (9)$$

where  $Y_t$  is labor or social security income received at time- $t$ . The law of motion for the investor's wealth,  $W$ , can be rewritten as

$$\Gamma_{t+1} = (\Gamma_t - \hat{\kappa}_t + \exp\{\epsilon_t\}) \exp\{-g_{t+1}\} \left[ \alpha_t (R_{t+1} - R_t^f) + R_t^f \right] \quad \text{for } t = 1, \dots, T - 1. \quad (10)$$

where  $\hat{\kappa}_t \equiv \frac{c_t}{Y_t^P}$  and  $\Gamma_t$  is the ratio of financial wealth at  $t$  to lagged permanent income.

Given this specification of the agent's problem with labor and social security income, the value function at  $t$  for all  $t = 1, 2, \dots, T - 1$  is homogenous in  $Y_t^P$  and has an additional state variable: the ratio of financial wealth at  $t$  to lagged permanent income  $\Gamma_t$ . Given our parametric assumptions, the Bellman equation faced by the investor is given by

$$\begin{aligned} \frac{a(\Gamma_t, D_t, t)(Y_t^P)^{1-\gamma}}{1-\gamma} &= E \left[ \max_{\hat{\kappa}(\Gamma_t, D_t, \epsilon_t, t), \alpha(\Gamma_t, D_t, \epsilon_t, t)} \left\{ \frac{\hat{\kappa}_t^{1-\gamma} (Y_t^P)^{1-\gamma}}{1-\gamma} \right. \right. \\ &+ p_{t,t+1} \delta \frac{(Y_t^P)^{1-\gamma}}{1-\gamma} E \left[ a(\Gamma_{t+1}, D_{t+1}, t+1) (\exp\{g_{t+1}\})^{1-\gamma} | \Gamma_t, D_t, g_t, \epsilon_t \right] \left. \right\} | \Gamma_t, D_t, \epsilon_t \right], \\ &\quad \text{for } t = 1, \dots, T - 1, \end{aligned} \quad (11)$$

where  $\alpha_t \equiv \alpha(\Gamma_t, D_t, \epsilon_t, t)$  and  $\hat{\kappa}_t \equiv \hat{\kappa}(\Gamma_t, D_t, \epsilon_t, t)$ . This recursion is solved by backward iteration starting with  $t = T - 1$  and  $a(\Gamma, D, T) = \Gamma^{1-\gamma}$ .

The holdings of both the risky and the riskless assets are always constrained to be non-negative. Compared to the standard portfolio choice problem, the presence of an additional state variable, the wealth to lagged permanent income ratio, considerably complicates the methodology needed to obtain accurate solutions in a manageable time-frame. Building on the numerical approach in Gourinchas and Parker (2002), we develop a new numerical methodology that allows the number of grid points to vary across ranges of the wealth-income ratio and chooses the number for each range to ensure that the resulting numerical errors in the policy functions are within prespecified bounds. An appendix contains a detailed description of the methodology employed.

### 2.3 The Two Channels and Their Marginal Impact on Allocations

The paper focuses on two channels through which labor income can affect the stock holdings of young agents and it is easy to describe them in the context of the VAR framework presented above in equations (2) to (4). The state-dependent mean channel (SDM) requires expected  $g$  in this and future periods to depend on current  $d$  in such a way that it is higher in expansions than in recessions. Since  $d$  is countercyclical, this is equivalent to  $b_g < 0$ . This channel is switched off by setting  $b_g = 0$ . The state-dependent volatility channel (SDV) requires the conditional volatility of  $g$  this period to depend on current  $d$  so that it's higher in recessions than expansions. Following Storesletten et.al., we parameterize this by allowing the conditional volatility of  $g$  to take two values, depending on the current  $d$  value. In particular, since  $d$  is countercyclical, we allow  $\sigma[u_{t+1}|d_t \leq d^*] < \sigma[u_{t+1}|d_t > d^*]$  for  $d^*$  the highest  $d$  value for which the economy is still in an expansion. This channel is switched off by setting  $\sigma[u_{t+1}|d_t] = \sigma_u$  for all  $d_t$ .

It is easy to see that all pairwise combinations of these two channels switched on and off are implementable, resulting in 4 specifications: 1) Both Effects: SDM, SDV; 2) SDM, no SDV; 3) no SDM, SDV; 4) Neither: no SDM, no SDV. Either channel can be switched on when the other channel is present or not. Thus, for each channel, there are 2 comparisons that generate 2 measures of that channel's incremental effect on stock holdings. The 2 comparisons hold all else constant, including return predictability when present, so that any change in stock holdings can only be due to the effect of the channel in question.

## 3 Calibration

We use the one-month Treasury-bill rate to obtain a proxy for the risk-free rate, the value-weighted return of all stocks on the NYSE, AMEX and NASDAQ as the market return, and the 12-month dividend yield on the value-weighted NYSE index as a proxy for the predictive variable  $D$ . Aggregate labor income data is used to obtain point estimates of some moments of interest. Wage earnings data is from the Bureau of Labor Statistics website. We use either Retail Trade which is series CEU4200000004 or Total Private which is series CEU0500000004. We don't use Total Public since the income received by government employees is likely to be much less sensitive to the business cycle than the income received by private sector employees. All data is per capita and measured at a monthly frequency. The Retail Trade income data starts from January 1972 and the Total Private income data starts from January 1964, while the return on the market and dividend yield start from January 1927. All data series end in January 2004. Income and return data are disinflated using

a CPI measure, series CPIAUCNS, available from U.S. Department of Labor: Bureau of Labor Statistics.

We estimate the VAR for the market return and dividend yield in (3) and (4): so  $d$  is always normalized to be zero mean and unit variance. The data VAR for return and dividend yield is estimated using ordinary least squares (OLS) and discretized using a variation of Tauchen and Hussey's (1991) Gaussian quadrature method; the variation is designed to ensure that  $d$  is the only state variable (see Balduzzi and Lynch (1999) for details). However, following and extending Lynch (2000), this study implements the discretization in a manner that produces exact matches for important moments for portfolio choice; in particular, we match the correlation between the innovations to return and dividend yield and the volatility of the innovation to return in each state to the unconditional volatility of the innovation and correlation between the innovations in the data. We choose 19 quadrature points for the dividend yield and 3 points for the stock-return innovations since Balduzzi and Lynch (1999) find that the resulting approximation is able to capture important dimensions of the return predictability in the data. Data point estimates and quadrature parameters are reported in Panel A of Table 1. The only parameter that the quadrature cannot match is the persistence parameter for dividend yield: the quadrature value is a little lower than the point estimate in the data. In the base case, stock return and dividend yield dynamics are kept as in data and the same regardless of the subset of the two channels switched on. We also consider two cases with i.i.d. stock returns.

Turning to the labor income process, the volatilities for log labor income are set to the baseline values in Viceira (1997, 2001) who describes these values as consistent with those obtained by Gakidis (1997) based on PSID data for professionals and managers not self-employed under age 45.<sup>6</sup> Viceira's baseline value for the standard deviation of the change in log permanent labor income is 15% per year. To get the monthly value, we utilize a loglinear approximation to relate these monthly parameters to their annual counterparts while explicitly recognizing the predictive dynamics at a monthly frequency. There is no temporary shock in the base case.

For our base case, the agent starts work at age 22 and retires at 65, and receives 93.8% of her retirement permanent income each month until death, which is the amount that Cocco, Gomes and Maenhout (2002) report as the average social security payout for college graduates. The agent dies with probability 1 at age 100, and death probabilities are taken from the U.S. Life Tables,

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<sup>6</sup>A number of papers (see, for example, Chamberlain and Hirano, 1997 and Carroll and Samwick, 1997) have estimated labor income parameters and a range of values are reported across these studies. However, the Gakidis values seem to lie within this range, which makes them reasonable to use.

2001, provided by the NCHS. There is no unemployment. We use a 3rd order polynomial for  $\bar{g}$  to approximate the hump shaped life-cycle earnings profile as in Campbell and Cocco (2003). We use as parameters for the polynomial the point estimates in Cocco, Gomes and Maenhaut (2002) for college graduates. Figure 1 plots  $\exp\{\sum_{\tau=1}^t \bar{g}_\tau\}$  as a function of age,  $t$ . Parameter values are reported in Panel B of Table 1 for the 4 quadrature approximations for labor income, each specification a possible combination of the two channels switched on and off. Panel B shows that the mean and volatility of log monthly permanent labor income growth is kept constant across the 4 specifications.

Monthly aggregate labor income data is used to compute covariances between permanent labor income growth and lagged dividend yield, contemporaneous dividend yield and contemporaneous market return. It is reasonable to use aggregate data to estimate covariance if the idiosyncratic component of individual labor income growth is uncorrelated with these two series. In the base case using Retail Trade data, we calibrate the SDM channel by matching  $b_g = \sigma_{gd-1}$  to the point estimate for log Retail Trade income growth (expressed in percent) covaried with lagged dividend yield reported in Panel A of Table 1. The point estimate for  $b_g$  is -0.164 and is highly significant using Newey West standard errors with 3 or 12 months of lags.<sup>7</sup> Given a per annum volatility for  $g$  of 15%, the  $b_g$  value of -0.164 implies  $\rho_{gd-1} = -3.13\%$  in the model, and a monthly volatility for  $g$  of 5.259%. When the SDM channel is switched off,  $b_g$  is set equal to zero and  $\sigma_u$  is adjusted to keep  $\sigma_g$  equal to 5.259%.

Panel A of Table 1 reports the point estimate for  $\sigma_{gr}$ , the covariance of aggregate Retail Trade wage income growth with stock return, both expressed in percent, to be 0.089 which implies, using the point estimates for  $b_g$  and  $b_r$  and the fact that the dividend yield is normalized to have a variance of 1, a  $\sigma_{ue}$  value of 0.139 as reported in Panel A of Table 1. Taking the standard deviation of the labor income growth residual to be 5.259% as in the calibration and the standard deviation of the return residual to be the data value of 5.507%, this  $\sigma_{ue}$  value implies a  $\rho_{ue}$  value of 0.48% as reported in Panel B of Table 1. In the calibration,  $\rho_{ue}$  conditional on the dividend yield state, is matched to this value, state by state. Panel A of Table 1 reports the point estimate for  $\sigma_{gd}$ , the covariance of aggregate Retail Trade wage income growth (expressed in percent) with normalized dividend yield, to be -0.171 which implies, using the point estimates for  $b_g$  and  $b_d$  and the fact that the dividend yield is normalized to have a variance of 1, a  $\sigma_{uw}$  value of -0.010. Taking the

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<sup>7</sup>When log Total Private income growth (expressed in percent) is covaried with lagged dividend yield, the (unreported) point estimate for  $b_g$  is -0.105 and is highly significant using Newey West standard errors with 3 or 12 months of lags.

standard deviation of the labor income growth residual to be 5.259% as in the calibration and the standard deviation of the dividend yield residual to be the data value of 0.206, the  $\sigma_{uw}$  value implies a  $\rho_{uw}$  value of -0.91% as reported in Panel B of Table 1. In the calibration,  $\rho_{uw}$  conditional on the dividend yield state, is matched to this value, state by state.

Turning to the state-dependent volatility channel, Storesletten et.al. find that the conditional volatility of permanent labor income growth is 1.75 higher in recessions than expansions, using NBER business-cycle cutoffs to define the two. To incorporate this heteroscedasticity without increasing the state space, we need a way to define the two periods as function of the dividend yield state. Storesletten et.al.'s business cycle specification implies a 68% probability of expansion and 32% probability of recession. We bifurcate the quadrature's dividend yield variable to obtain recession and expansions states with the cutoff value chosen to match these unconditional probabilities. Interestingly, we obtain similar transition matrices to Storesletten et.al. for the two state transition probability matrix at a yearly frequency. In particular, the probability of remaining in the expansion state is found to be 76% in the data and 82% in our calibration, values which are quite close to each other. There is more of a disparity for the probability of remaining in a recession but 50% in the data and 63% for our calibration are still quite close. Further, we find that the Spearman correlation between our recession variable and NBER recessions is 64.24%. In summary, our procedure for creating recession and expansion states has produced a two-state Markov chain which replicates key features of the expansion and recession states that Storesletten et.al. found in the data. When the SDV channel is switched on, the conditional volatility of  $g$  in recession states is allowed to be 1.75 times its value in expansion states; otherwise the ratio is 1. We also examine what happens when the SDV channel is gradually switched off by allowing this volatility ratio to decline gradually from 1.75 down to 1.

The VAR specification for labor income growth and dividend yield is very parsimonious especially since the only predictive variable for income growth is lagged dividend yield. However, it also implies a particular pattern of predictability for  $y_{t+T} - y_t$  using  $d_t$  as the predictive variable. A natural concern associated with using the VAR is the possibility of misspecification which, if present, would be expected to drive a wedge between the VAR-implied and the actual predictability of  $y_{t+T} - y_t$  that increases with  $T$ . To assess whether this is an issue, we derive the moments associated with such a predictive regression for an arbitrary horizon  $T$ . Starting with the moments for  $T = 1$  which were used to obtain the parameter estimates in Table 1, we add moments for one or more other  $T$ s all greater than 1. We add  $T = 12$  to obtain one GMM system,  $T = 36$

to obtain another, and both  $T = 12$  and  $T = 36$  to obtain a third. We do not report the results but they are available from the authors upon request. In short, the  $b_g$  point estimate is always negative, similar in magnitude to the value reported in Table 1 and highly significant. The resulting GMM systems are overidentified but the GMM J statistic is always insignificant. The results are similar for Newey-West standard errors obtained using 3 or 12 lags. Thus, it appears that the VAR specification is doing a good job of capturing income growth predictability at both low and high frequencies.

Our income specification does not allow the temporary component of log income to be predictable using dividend yield. One concern is that our estimate of  $|b_g|$  might be overstated if in fact this temporary component is predictable. To check this possibility, we allow the temporary component to be predictable:

$$\epsilon_{t+1} = b_\epsilon d_t + \nu_{t+1}, \tag{12}$$

where  $\nu_{t+1}$  is i.i.d. and orthogonal to all other shocks. We then derive expressions for the moments associated with the predictive regression of  $y_{t+T} - y_t$  on  $d_t$  in terms of the underlying parameters including  $b_d$  and  $b_\epsilon$ . Again, we start with the moments for  $T = 1$  and then add those for  $T = 12$  to obtain one GMM system, those for  $T = 36$  to obtain another, and those for both  $T = 12$  and  $T = 36$  to obtain a third. In unreported results, the  $b_\epsilon$  estimate is always small in magnitude and insignificant, and the  $b_g$  estimate is similar to that obtained for the same system with  $b_\epsilon$  set to zero. The third GMM system is overidentified but the GMM J statistic is insignificant. Again, the results are similar for Newey-West standard errors obtained using 3 or 12 lags. These results suggest it is unlikely that predictability of the temporary component is contaminating our estimates of  $b_g$ .

## 4 Results

This section reports policy functions for the various problems described above. Simulation results are also reported.

### 4.1 Base Case: Age-dependent Profile, Retirement and Death Probabilities

Table 2 reports asset allocation and incremental effect results for the base case described in Section 3. The agent has CRRA preferences with a coefficient of risk aversion of 6 and results are reported for a range of wealth to permanent income ratios from 0 to  $\infty$ . The agent has access to the market portfolio and to a riskless bond. Panel A reports average stock holdings when both channels are

present or neither is present. Panel B reports the incremental effects on stock holdings of switching on one of the two channels SDM or SDV. Each of these channels can be switched on when the other channel is present or not and the 2 rows of each channel’s subpanel report the incremental stock-holding reductions for these 2 cases. The calibration of the 4 problems needed to do the comparisons is detailed in section 3 above. Subfigure 2.a plots stock holdings for first-month agents as a function of wealth to permanent income ratios from 0 to 500 with both channels off, both channels on, only SDM switched on and only SDV switched on.<sup>8</sup>

Subfigure 2.a shows that in the absence of the two channels, there is a negative relation between average stock allocation and wealth-income ratio, as has been documented in prior studies. Panel A of Table 2 shows that the simultaneous presence of both channels leads to large reductions in average holdings for young agents with low wealth-income ratios. At zero wealth, the average holding drops from 97.5% to 20.3% but even at a wealth-income ratio of 30, the drop is still substantial, from 92.9% to 24.6%. Figure 2.a shows that when both channels are switched on, the relation between average stock allocation and wealth-income ratio goes from negative to positive which is consistent with the empirical evidence.

It is important to understand the intuition for why the SDV and SDM channels cause reductions in stock holdings. This result can be explained by a static diversification story which says that positive (negative) covariance between stock returns and human capital causes the agent to hold less (more) stock. When risk aversion is greater than 1, both the SDV and SDM channels produce human capital that covaries more positively with stock return than when both channels are switched off, which means that the static diversification intuition implies lower stock holdings.<sup>9</sup>

However, the deeper question is why, when risk aversion is greater than 1, the SDV and SDM channels produce human capital that covaries more positively with stock return than when both channels are switched off. The hedging-demand intuition of Merton (1973) can be used to answer this question. With risk aversion greater than 1, positive correlation between stock return and future opportunity sets reduces the stock holding of a young agent relative to that of a myopic agent. Lower mean labor income growth and higher volatility both mean poorer future opportunity sets. Stock returns are low when the probability the economy enters or remains in a recession increases (i.e a positive shock to dividend yield), and so lower mean labor income growth and higher volatility

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<sup>8</sup>A range of 0 to 500 was chosen for the wealth-income ratio in Figure 2.a and all other figures plotting average first-month stock holdings as a function of wealth-income ratio since this range brackets the empirically relevant range for agents aged 22: at that age, only extremely wealthy agents, constituting an extremely small fraction of the population, have wealth to monthly permanent income ratios higher than 500.

<sup>9</sup>The magnitudes of the covariances with the channels on and off are discussed later in the section.

in bad states both reduce the stock holding of a young agent. Note that these reductions in stock holdings can be regarded as the flipside of the effect of mean stock return predictability on portfolio allocation. Since expected stock returns are positively related to dividend yield (see, for example, Fama and French, 1988 and 1989), the negative correlation between today's dividend yield innovation and today's return shock also means that today's stock returns are high when expected future stock returns are low, which induces a positive hedging demand for stock. This is one of the key results from the recent literature exploring portfolio choices by a multiperiod agent in the presence of return predictability: see, for example, Campbell and Viceira (1999), Barberis (2000) and Balduzzi and Lynch (1999).

Consistent with the hedging-demand intuition, we find that when risk aversion is less than 1, both the SDV and SDM channels produce human capital that covaries negatively with stock return, and the young low wealth-income ratio agent holds more stock with either or both channels switched on than when both channels are switched off. In unreported results, when we consider the base case with an agent whose risk aversion is 0.8 and who faces less stringent borrowing restrictions, switching on the two business cycle channels increases the young, poor agent's stock allocations, consistent with the hedging demand intuition, and causes the covariance of stock return with human capital to become negative, which is consistent with the static diversification intuition. So the hedging-demand intuition is useful because it allows us to understand how the two channels affect the correlation between human capital and stock return and thus how the two channels affect the stock holdings of young low wealth-income ratio agents.

An important question is the contribution of each channel to the overall effect documented in Panel A. Panel B of Table 2 shows that the SDV channel is much more important than the SDM channel at low wealth income ratios, irrespective of whether the other channel is present or not. At zero wealth, switching on SDV with no SDM causes the agent's average first-month stock holding to drop by 71.2% while switching on SDM with no SDV causes a much more modest drop of 17.8%. Even at a wealth-income ratio as high as 30, the drop is 41.0% for adding SDV but only 17.2% for adding SDM. Figure 2.a also shows that for a wealth income ratio as little as 175, the drop for adding SDV is virtually the same as for adding SDM, and Panel B of Table 2 confirms this also holds for a wealth-income ratio of 1000. Thus, Figure 2.a shows that switching on the SDM channel alone does not change the direction of the relation between average stock allocation and wealth-income ratio. However, when the SDV channel alone is switched on, the relation becomes positive, as Figure 2.a shows. So while both the SDM and SDV channels have a considerable

effect on the young agent’s stock holdings, it is the SDV channel that changes the direction of the relation between stock holding and wealth-income ratio. Interestingly, at low wealth-income ratios, the effect of the SDV channel on average first-month stock holdings is very sensitive to the size of the wealth-income ratio, while the effect of the SDM channel is largely invariant to value of this ratio. As the wealth-income increases from 0 to 100, the drop for adding SDV declines from 71.2% down to only 21.3%, while the drop for adding SDM hovers around 16% to 18% over this entire range of wealth-income values.

One way to assess the magnitude of the reductions in stock holdings induced by the two channels is to examine the average conditional covariance between stock return and human capital value.<sup>10</sup> The intuition for why the two channels decrease the stock allocation of young agents with low wealth income ratios makes it clear that, in the absence of short-selling constraints, the magnitude of the hedging demand induced by these channels is driven by how large and positive this covariance is. Human capital value at time 1 can be calculated as follows:

$$H_1(D_1, \Gamma_1, Y_1^P) = E \left[ \sum_{t=1}^T p_{1,t} \delta^t \left( \frac{C_t}{C_1} \right)^\gamma Y_t \mid D_1, W_1, Y_1^P \right], \quad (13)$$

Consider an agent with a wealth-income ratio of 0 and hold fixed the current level of permanent labor income: switching the two channels on increases the average conditional covariance between stock return and human capital value by a factor of 81.<sup>11</sup> Switching on only the SDV channel increases the average conditional covariance by a factor of 55 while switching on only the SDM channel increases it by a factor of only 29. These results explain why the SDV channel causes a much larger decline in the average stock allocation of a young agent with a zero wealth-income ratio than the SDM channel. Further, at a wealth income ratio of 500, the average covariance between stock return and human capital value is about the same with the SDV or the SDM channel switched on, which is consistent with the associated drops in average stock allocation being about the same for each too.

Interestingly, the drop in average stock holding from adding either channel is less when the other is present rather than not at low wealth-income ratios, while the converse is true at higher wealth income ratios. For zero wealth, the reduction in average holding for either channel is around 12% of portfolio value less when the other channel is present rather than not while for a wealth

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<sup>10</sup>The other determinants of the magnitudes of the reductions in stock holdings induced by the two channels are the two no short-selling constraints. But understanding their impact does not help in understanding why the SDV channel has a greater effect than the SDM channel so a discussion of their impact is postponed till later.

<sup>11</sup>Covariances between stock return and human capital value are only reported in the text.

income ratio of 100, the reduction in average holding for either channel is around 10% of portfolio value more when the other channel is present rather than not. This result from Panel B of Table 2 suggests that the two channels do not have independent impacts on holdings.

The presence of the two short-selling constraints plays a major role in determining how the presence or not of the other channel affects how each channel reduces the age-22 agent's stock holdings. Recall that in the absence of any short-selling constraints, both channels induce reductions in stock holdings. The no bond-short-selling constraint is more likely to bind before a channel is switched on when the other channel is switched off rather than on, irrespective of the size of each channel's effect. Thus, the no bond-short-selling constraint causes the drop in average stock holding from adding either channel to be more when the other channel is switched on rather than off. The no stock-short-selling constraint is more likely to bind after a channel is switched on when the other channel is switched on rather than off and the effects of both channels together are large. Thus, the no stock-short-selling constraint causes the drop in average stock holding from adding either channel to be less when the other channel is switched on rather than off. Moreover, since the effects of both channels together are likely to be larger for lower wealth income ratios, this impact of the no stock-short-selling constraint increases as the wealth-income ratio declines. Thus, at higher wealth-income ratios, the drop in average stock holding from adding either channel is more when the other is switched on rather than off because of the impact of the no bond-short-selling constraint. At low wealth-income ratios, the converse is true because the impact of the no stock-short-selling constraint is able to outweigh that of the no bond-short-selling constraint.

It is also interesting to look at the average conditional covariance between stock return and human capital since this covariance largely determines the magnitude of the hedging demand induced by whichever business-cycle channels are on, in the absence of any short-sale constraints. For an agent with zero financial wealth, switching on the SDV channel alone increases this covariance by a factor of 55 while switching it on in addition to the SDM channel only increases it by a factor of 52 (relative to its value with both channels switched off). Similarly, switching on the SDM channel alone increases this covariance by a factor of 29 while switching it on in addition to the SDV channel only increases it by a factor of 26 (again relative to its value with both channels switched off). So for an agent with zero financial wealth, switching on either channel increases this covariance by less with the other channel already switched on than with the other channel not switched on. Thus, even in the absence of short-sale constraints, the drop in average stock holding from adding either channel would be expected to be less when the other is switched on rather than off for an agent

with zero financial wealth. For an agent with a wealth-income ratio of 100, switching on the SDV channel alone increases this covariance by a factor of 58 while switching it on in addition to the SDM channel increases it by a factor of 61 (relative to its value with both channels switched off). Similarly, switching on the SDM channel alone increases this covariance by a factor of 25 while switching it on in addition to the SDV channel increases it by a factor of 28 (again relative to its value with both channels switched off). So for an agent with a wealth-income ratio of 100, switching on either channel increases this covariance by more with the other channel already switched on than with the other channel not switched on. Thus, in the absence of short-sale constraints, the drop in average stock holding from adding either channel would be expected to be larger when the other is switched on rather than off for an agent with a wealth-income ratio of 100.

When the agent is about to turn 100 years of age, switching on the two channels has virtually no effect on her allocation to stock, which is decreasing in the wealth-income ratio. This result (unreported) indicates that the large effects on stock holdings reported in Table 2 caused by the state-dependent mean and volatility of permanent labor income growth are coming from the long horizon of the young agent. This finding is consistent with the intuition for the reduced holdings we just presented because the old agent has very little human capital left and so the covariance of human capital with stock return is very small.

Another question of interest is how stock allocations vary over an agent's life. To address this question, Figure 2.b plots average stock allocation as a function of age for an agent with an initial wealth to permanent income ratio of 0. Results are obtained via simulation and 500000 paths are simulated for each case of the 4 cases under consideration, and average allocations at each age are recorded. Initial dividend yield states are drawn from their unconditional distribution. This figure shows that for an initial wealth-income ratio of 0 and with both effects switched off, stock holding is counterfactually declining in age from age 22 to age 57 and is much higher at age 22 than at retirement. However, once the two business-cycle channels are switched on, the relation becomes hump-shaped from age 22 to 54, with a much lower average stock allocation at age 22 than at age 65, which is consistent with the data. Moreover, the figure shows that when the wealth-income ratio is 0, the SDV channel alone is enough to obtain a hump-shaped relation from age 22 to age 54 with a lower average stock allocation at age 22 than at age 65, but SDM alone is not.

An important question is the affect of these two channels on the stock market participation rates of poor young agents. Figure 3.a plots the probability of non-participation in the first month as a function of the agent's first-month wealth-income ratio. With both channels switched off,

non-participation is a zero- or near zero-probability event in the first month, irrespective of the agent's wealth-income ratio. With both business-cycle channels switched on, the probability of non-participation becomes as high as 79% for low wealth-income ratios but is less than 10% for all wealth-income ratios greater than 200. With only SDV channel switched on, the probability of non-participation is still over 70% when the wealth income ratio is zero, while the SDM channel alone has very little effect on the level of non-participation relative to the case with both channels switched off, with a probability of non-participation that is never above 21%. We are also interested in non-participation as a function of age. Figure 3.b provides results on this point plotting the probability of non-participation as a function of age for an agent whose initial wealth income ratio is 0. The figure indicates that both channels switched off leads to participation in the stock market virtually all the time, irrespective of age or initial wealth-income ratio. Switching on the two business-cycle channels results in substantial non-participation by agents in their first month and that non-participation steadily declines as the agent gets older. According to the figure, an agent with an initial wealth-income ratio of 0 decides not to participate in the stock market 79% of the time in the first month; and after ten years, this probability has declined to a fraction that is still about 26%. The implication is that the business-cycle variation in the first 2 moments of permanent labor income growth, particularly the countercyclical variation in the second moment, can cause young, poor agents not to participate in the stock market a large fraction of the time. This result can be contrasted with the virtual 100% participation rate obtained for agent's irrespective of age or wealth-income ratio when both channels are switched off.

Figure 4.a plots an agent's average financial wealth to permanent income ratio as a function of age, and Figure 4.b plots an agent's average consumption to permanent income ratio as a function of age, both for an agent with an initial wealth to permanent income ratio of 0. Figure 4.a shows that the average financial wealth to permanent monthly income ratio increases monotonically from 0 at age 22 to somewhere between 112 and 124 at age 65, depending on which channels are switched on, and then declines monotonically back to zero from age 65 to age 100. Switching on either or both of the channels causes almost no variation in the average wealth to permanent income profile. The shape of the relation and the magnitude of the wealth accumulation is consistent with that documented by Gomes and Michaelides (2005) using the 2001 Survey of Consumer Finance (SCF) data. They report a median wealth to annual income ratio of 0.287 for households with an age between 20 and 35, 2.170 for households aged between 36 and 65 and 7.931 for households aged 65 or older, and a 90th-percentile wealth to annual income ratio of 2.702 for households with an

age between 20 and 35, 10.648 for households aged between 36 and 65 and 33.363 for households aged 65 or older. Our model produces average wealth income ratios that lie roughly between the median and 90th-percentile accumulation values that they report. For example, at the midpoint of the 36-65 age range, the agent has an average financial wealth to permanent monthly income ratio of around 80 which lies between the median value of 26 and the 90th-percentile value of 128 reported by Gomes and Michaelides (2005) for this age range. Since the income profile for our agent is calibrated to college graduates, we would expect her wealth accumulation to lie above the median wealth accumulation reported by Gomes and Michaelides. As a second example, the agent's average financial wealth to permanent monthly income ratio from age 65 to 72 lies between the median value of 95 and the 90th percentile value of 400 reported by Gomes and Michaelides for the 65 and over age range. Moreover, at all ages within each age range, the level of wealth accumulation generated by our model is lower than the 90th percentile of wealth accumulation that they report.

Figure 4.b shows that the average consumption to permanent monthly income ratio starts at around 0.9 at age 22 and remains at that level until age 32 and then increases monotonically from around 0.9 at age 32 to around 1.6 at age 65 before declining over the remainder of the agent's life; the relation is a convex one between age 22 and age 60 and after age 75. Switching on either or both of the channels causes almost no variation in the average consumption to permanent income profile, though a higher wealth to permanent income ratio at a certain age is typically associated with a higher consumption to permanent income ratio at the same age. Both Gourinchas and Parker (2002) and Fernandez-Villaverde and Krueger (2004) using the Consumer Expenditure Survey document a hump-shaped total consumption profile, with the hump occurring around 45-50 years of age. Average dollar consumption as a function of age for our agent with a zero wealth-income ratio is also hump-shaped (not reported), though the hump occurs a little later than age 50.

## **4.2 State Dependent Volatility: Recognizing that Dividend Yield Business Cycles do not Exactly Coincide with NBER Business Cycles**

Throughout, we have used a volatility ratio between recessions and expansions of 1.75, the figure obtained by Storesletten et.al. using the NBER definition of these events. However, our recession-expansion variable is obtained by bifurcating aggregate dividend yield to match the unconditional probabilities of recessions and expansions as per the NBER variable. In the data, this dividend yield classification of expansions and recessions does not exactly coincide with that obtained from the NBER variable, which opens the possibility that the applicable volatility ratio could differ

across for the two classifications. In particular, if the true ratio for the dividend yield classification is lower than 1.75, then the reductions in stock holdings reported above for the SDV channel are likely overstated.<sup>12</sup> We address this concern in three ways. In the first subsection, we calibrate the expansion-recession state variable directly to the NBER business cycle variable. This specification has the advantage of using the same expansion-recession variable that Storesletten et.al. used to obtain a volatility ratio between recessions and expansions of 1.75, though it does require an assumption that investors have access to information that allows them to determine the NBER state at the beginning of each month. We find that the volatility channel's affect on allocations is robust to using the NBER expansion-recession variable directly to calibrate the expansion-recession state, which means that this channel is able to generate large reductions in the average stock allocations of young, poor agents without relying on the large negative contemporaneous correlation between dividend yield and stock returns. In the second subsection, we recognize that agents may not have sufficient information to infer the NBER expansion-recession variable at the start of each month. Instead, we use the empirical relation between dividend yield and the NBER expansion-recession variable to calculate the probability of an NBER expansion conditional on the value of the dividend yield. Taking the volatility conditional on the NBER variable as given by Storesletten et.al., this probability allows us to calculate analytically the volatility of the change in log permanent labor income conditional on the dividend yield value. As would be expected, the effect of the volatility channel on allocations is attenuated relative to when the agent uses NBER expansion-recession variable directly. But it is still the same qualitatively, since the reduction in the average stock allocations of young, poor agents is sufficiently large that in combination with the state-dependent mean channel, these agents hold still less stock on average than young wealthy agents. In the third subsection, we return to the expansion-recession variable obtained by bifurcating dividend yield, but we reduce the volatility ratio between recessions and expansions to account for the less than perfect correlation between this expansion-recession variable and the NBER variable in the data.

#### **4.2.1 Using the NBER Business Cycle Variable Rather Than Bifurcated Dividend Yield as the Expansion-Recession State Variable**

In this subsection, we use the NBER expansion-recession variable to calibrate our business cycle variable, which we allow to have three states: recession; early expansion; and, late expansion. The NBER variable is available for January 1929 onwards so the data for this calibration starts in

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<sup>12</sup>It is worth mentioning that the true ratio could just as easily be higher than 1.75 since the dividend yield could generate expansion and recession states which identify the underlying volatility states more accurately than the NBER variable.

January 1929. An expansion on average lasts for 52 months in the data so we define the early expansion state in the data to be the first 26 months of any expansion. If an expansion lasts for no more than 26 months, there is no late expansion associated with that expansion. In the quadrature, stock returns and the change in log permanent income are calibrated to the value-weighted return of all stocks on the NYSE, AMEX and NASDAQ and the Retail Trade aggregate labor income series respectively, as before. The mean and volatility of log permanent labor income growth are allowed to depend on the state, while its unconditional mean and volatility are set equal to the values used in the initial calibration reported in Table 1. For simplicity, returns are calibrated to be i.i.d. and conditionally uncorrelated with contemporaneous labor income, with mean and volatility as in the data.<sup>13</sup>

There are three grid points per state for log permanent labor income growth and three grid points for the stock return. For each state, return bins are constructed in the data to coincide with the probability of each of the three quadrature return nodes. We now describe the algorithm for calculating the quadrature probability of going from the recession state and a given return node to the early expansion state to illustrate how the quadrature transition probability matrix going from the state and return this month to the state next month is constructed. The probability of being in the early expansion state next month conditional on being in the recession state this month and having a return realization this month that lies in the  $k$ th bin is calculated from the data. This value becomes, in the quadrature, the probability of being in the early expansion state next month conditional on being in the recession state this month and having the  $k$ th grid point as the return realization this month. The other elements of this return-and-state transition matrix are constructed in the same way. This calibration approach has the appealing property that the resulting quadrature approximation matches the 3x3 transition probability matrix for the 3 expansion-recession states to that in the data.

Table 3 reports data values matched by the quadrature and the resulting quadrature values. Panel A reports the return-and-state transition matrix, the 3x3 transition matrix for the state, and the unconditional probabilities of the 3 states, all for the data and for the quadrature. Panel B reports the unconditional mean and the volatility of the stock return for the data and for the quadrature. Panel C reports parameters of the change in log permanent labor income growth both in the data and for the quadrature. Notice in Panel A that the probability of a recession next month conditional on a recession this month and the return bin this month decreases monotonically with

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<sup>13</sup>The results are qualitatively similar if the mean and volatility of the stock return are allowed to vary across the states based on the point estimates in the data.

the return bin, and the same is true for the probability of a recession next month conditional on late expansion this month and the return bin this month. At the same time, the probability of an early expansion next month conditional on a recession this month and the return bin this month increases monotonically with the return bin. These results suggest that conditioning on the state this month, a higher stock return this month is positively related to the state of the economy next month, with a high return increasing the likelihood of an expansion next month and reducing the likelihood of a recession next month. Given that the the volatility of log permanent labor income growth is higher in recessions and its mean is lower, this is precisely the relation needed for the two channels to reduce the stock allocations of young, poor agents when risk aversion is greater than 1.

Table 4 reports the analogous first month allocation results to Table 2 for the cases that use the NBER variable to calibrate the expansion-recession state variable. Subfigure 5.a plots average stock holdings for first-month agents as a function of wealth to permanent income ratios from 0 to 500 with both channels off, both channels on, only SDM switched on and only SDV switched on. Subfigure 5.a and Panel A of Table 4 show that, with both channels switched on, the average stock holdings of agents are increasing in wealth-income ratio as the wealth-income ratio goes from 10 to 500 and are much lower for wealth-income ratios below 10 than for those above 10. Thus, poor young agents continue to hold less stock than wealthy young agents even when the NBER variable is used to calibrate the expansion-recession state variable rather than dividend yield. The presence of both channels causes the average stock allocation of a young agent with a wealth income ratio of 0 to drop from 100.0% to 34.2%, which is only a little smaller than the reduction from 97.5% down to 20.3% when dividend yield is the state variable. Not surprisingly, both the SDM and the SDV channels have a smaller effect on stock allocations than in the base case, but the effect of SDV is still large. The presence of the SDV channel alone causes the average stock allocation of a young agent with a wealth income ratio of 0 to drop by an amount of 61.6%, which is smaller than the reduction when dividend yield is the state variable, but still very large in absolute terms. Subfigure 5.b plots average stock allocation as a function of age for an agent with an initial wealth to permanent income ratio of 0. As in the case with dividend yield as the state variable, the absence of both the SDM and SDV channels causes the average allocation to decrease monotonically from age 22 to 57 and to be higher at age 22 than retirement age while the simultaneous presence of both instead causes the average allocation to be hump-shaped from age 22 to age 54 and to be much lower at age 22 than retirement age.<sup>14</sup>

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<sup>14</sup>It is worth noting that the allocation results just described are robust to calibrating a two-state expansion-recession variable to the NBER variable rather than a three-state variable. However, the third state is needed to get

Turning to non-participation, Subfigure 6.a plots the probability of non-participation in the first month as a function of the agent’s first-month wealth-income ratio, while Subfigure 6.b plots the probability of non-participation as a function of age for an agent whose initial wealth income ratio is 0. These subfigures show that the presence of both the SDM and SDV channels still leads to non-participation, and it is in excess of 35% for young agents when their wealth-income ratios are below 75. The SDV channel alone can also generate non-participation in excess of 35% for young agents when their wealth-income ratios are below 50. For a 22 year-old agent with a zero wealth-income ratio, the two channels generate non-participation above 30% until at least age 29. Thus, considerable non-participation in stocks by the young can be obtained using the NBER variable to calibrate the expansion-recession state variable. To sum up, the overall message of this subsection is that our main results, especially those pertaining to the effect of the SDV channel on average stock allocations, are robust to allowing the expansion-recession variable to be calibrated to the NBER business cycle variable rather a bifurcation of dividend yield.

#### 4.2.2 Calibrating the Volatility of Log Permanent Labor Income Growth Conditional on the Dividend Yield Value

One concern with the analysis in the previous section is the assumption that the agent has access to sufficient information to be able to infer the value of the NBER expansion/recession variable at the start of each month. This assumption is a concern since the NBER variable is not published in real-time. This section addresses this concern by calibrating the volatility of the change in log permanent labor income conditional on the dividend yield value using an analytical expression for this conditional volatility that depends on: 1)the probability distribution of the NBER expansion-recession variable conditional on the value of the dividend yield, which we calibrate to data; and, 2)the volatility of the change in log permanent labor income conditional on the NBER variable, for which we use the ratio of 1.75 reported in Storesletten et.al..

We first obtain analytically an expression for the volatility of the change in log permanent labor income conditional on the dividend yield. The analytical result follows once we make the following innocuous assumptions:

$$\begin{aligned} E[u_{t+1}|d_t, D_t^B] &= 0 \\ E[u_{t+1}^2|d_t, NBER_t] &= E[u_{t+1}^2|D_t^B] \end{aligned} \tag{14}$$

where  $D_t^B$  is the value of the NBER expansion/recession variable for the month that ends at time-  


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any non-participation.

$(t + 1)$  and recall that  $u_{t+1}$  is the shock to  $g_{t+1}$ . Letting  $d_t$  be discrete with  $N_d$  possible nodes  $d^1, d^2, \dots, d^{N_d}$ , the following is then true analytically:

$$\begin{aligned} \sigma^2[u_{t+1}|d_t = d^i] &= E[u_{t+1}^2|d_t = d^i] \\ &= p[D_t^B = 1|d_t = d^i]E[u_{t+1}^2|D_t^B = 1] + p[D_t^B = 0|d_t = d^i]E[u_{t+1}^2|D_t^B = 0] \end{aligned} \quad (15)$$

where  $D_t^B$  takes a value of 1 if the month ending at time- $(t + 1)$  is an expansion and 0 otherwise. We can use this expression and the data to calculate the volatility of log permanent labor income growth conditional on each dividend yield node. The first assumption just says that the shock to  $g_{t+1}$  cannot be forecast using  $D_t^B$  or  $d_t$ . The second assumption says that  $d_t$  provides no additional information about the volatility of  $g_{t+1}$  over and above that provided by  $D_t^B$ .

Equation (15), together with the empirical relation between dividend yield and the NBER expansion-recession variable and the volatility conditional on the NBER variable as given by Storesletten et.al., is then used to calibrate the volatility of the change in log permanent labor income conditional on the dividend yield value to data. Specifically, the probability distribution of the NBER expansion-recession variable conditional on each of the 19 values of the dividend yield is obtained from the data as follows. The unconditional probabilities of the 19 quadrature nodes are used to calculate 18 percentile cutoffs for dividend yield in the data that are then used to identify 19 dividend yield bins in the data, each one corresponding to a quadrature dividend-yield node. The distribution of the NBER variable conditional on a given dividend-yield node in the quadrature is taken to be the distribution of the NBER variable for the corresponding dividend-yield bin in the data. The return data for this calibration starts in January 1929. Figure 7 plots the volatilities of  $g$  conditional on the dividend-yield nodes that are obtained using this procedure, scaling each by the volatility conditional on an NBER expansion reported by Storesletten et.al.. The figure shows considerable variation in the conditional volatility across dividend nodes, which suggests that the induced reductions in stock holdings could be sizable.

Table 5 reports the analogous first month allocation results to Table 2 for the cases that condition the volatility of  $g$  on the dividend yield node and so allow the volatility to be state-dependent node-by-node.<sup>15</sup> Subfigure 8.a plots average stock holdings for first-month agents as a function of wealth to permanent income ratios from 0 to 500 with both channels off, both channels on, only SDM switched on and only SDV switched on. Subfigure 8.a and Panel A of Table 5 show that, with

<sup>15</sup>Notice that in Panel A of Table 5 the average stock allocations with both channels switched off are virtually identical to those reported in Table 2, which shows that the use of a slightly shorter data period for returns because of the availability of the NBER expansion/recession variable is having virtually no effect on the results.

both channels switched on, the average stock holdings of agents with wealth income ratios between 10 and 150 are increasing monotonically in wealth-income ratio, and the average stock holdings of agents with wealth income ratios below 10 (which is a reasonable definition of poor for young agents) are lower than those for agents with wealth income ratios between 50 and 500 (which is a reasonable definition of wealthy for young agents). Thus, poor young agents continue to hold less stock than wealthy young agents even when the volatility of  $g$  is conditioned on the dividend yield value rather than on a bifurcation of dividend yield that is assumed to capture the same business-cycle variation in the volatility of  $g$  that is captured by the NBER expansion/recession variable. The presence of both channels causes the average stock allocation of a young agent with a wealth income ratio of 0 to drop from 97.5% to 42.4%, which is a smaller drop than when the bifurcation of the dividend yield is used to identify expansions and recessions, but still a substantial drop none the less. Not surprisingly, the SDM channel's effect on allocations is similar to that in the base case, while the SDV channel has an effect that is smaller than that in the base case, though still large. The presence of the SDV channel alone causes the average stock allocation of a young agent with a wealth income ratio of 0 to drop by an amount of 51.9%, which is smaller than the reduction when either a bifurcation of dividend yield or the NBER variable is used to identify expansions and recessions, but still very large in absolute terms. Subfigure 8.b plots average stock allocation as a function of age for an agent with an initial wealth to permanent income ratio of 0. As in the cases with the dividend-yield bifurcation or the NBER variable as the expansion/recession indicator, the simultaneous presence of both the SDM and SDV channels causes the average stock allocation to be hump-shaped from age 22 to age 54 and much lower at age 22 than at retirement age.

Figure 7 shows that the relation between the conditional volatility of log permanent labor income growth and dividend yield is u-shaped. So the relation between volatility and dividend yield is negative for low dividend yield states, which makes stock more attractive in those states because volatility goes down when when return is low. Average stock holdings still drop substantially when both channels are switched on because in many of the high dividend-yield states for which the relation between volatility and dividend yield is positive, the agent holds zero or less than 100% stock. Using the hedging-demand intuition, the positive relation between volatility and dividend yield makes stock less attractive in these high dividend-yield states.

Turning to non-participation, Subfigure 9.a plots the probability of non-participation in the first month as a function of the agent's first-month wealth-income ratio, while Subfigure 9.b plots the probability of non-participation as a function of age for an agent whose initial wealth income ratio

is 0. These subfigures show that the presence of both the SDM and SDV channels still leads to non-participation, and it is in excess of 30% for young agents with zero wealth-income ratios. The SDV channel alone can also generate non-participation in excess of 25% for young agents with zero wealth-income ratios. For a 22 year-old agent with a zero wealth-income ratio, the two channels generate non-participation above 25% until at least age 29. Again, nontrivial non-participation in stocks by the young can be obtained conditioning the volatility of  $g$  on the dividend yield value. To sum up, the overall message of this subsection is that our main results, especially those pertaining to the effect of the SDV channel on average stock allocations, are robust to conditioning the volatility of  $g$  on the dividend yield value rather than on a bifurcation of dividend yield that is assumed to capture the same business-cycle variation in the volatility of  $g$  as that captured by the NBER expansion/recession variable.

#### **4.2.3 A Conservative Calibration of the Volatility Ratio between Recessions and Expansions keeping Bifurcated Dividend Yield as the Expansion/Recession State Variable**

In this subsection, we use the result in equation (15) applied to the volatility of  $g$  conditional on a set of dividend yield nodes rather than on a single node and then calculate the conditional volatility ratio of  $g$  for the dividend yield bifurcation, given the probabilities of being in an NBER expansion or recession conditional on being in a dividend yield expansion or recession. These conditional probabilities are calculated directly from the data subject to an adjustment that allows for the possibility that dividend yield is measured with error. The adjustment involves replacing the recession-expansion cutoff value with a range (that includes the cutoff) within which the dividend yield state is assumed to always match the NBER state. The range occurs 10% of the time based on the quadrature probabilities. When this adjustment is implemented, a ratio of 1.33 is obtained, which can be viewed as a conservative estimate of the true ratio.

Table 6 reports the analogous first month allocation results to Table 2 except for a volatility ratio of 1.33 rather than 1.75. Panel A shows that the presence of both channels causes the average stock allocation of a young agent with a wealth income ratio of 0 to drop from 97.5% to 43.0%, which is a smaller drop than when 1.75 is used as the volatility ratio, but still a substantial drop none the less. Subfigure 10.a plots average stock holdings for first-period agents for a volatility ratio of 1.33. As reported above for the case when the volatility of  $g$  is conditioned on the dividend yield value itself, Panel A of Table 6 and Subfigure 10.a show that, with both channels switched on, the average stock holdings of agents with wealth income ratios between 10 and 150 are increasing

monotonically in wealth-income ratio, and the average stock holdings of agents with wealth income ratios below 10 (which is a reasonable definition of poor for young agents) are lower than those for agents with wealth income ratios between 50 and 500 (which is a reasonable definition of wealthy for young agents). While the effect of the SDM channel alone on allocations is similar to that in the base case, the SDV channel alone has an effect that is smaller than that in the base case and very similar to that of the SDM channel alone: the presence of either channel alone causes the average stock allocation of a young agent with a wealth income ratio of 0 to drop by an amount between 14% and 18%. Turning on either channel when the other is already switched on causes the average stock allocation of a young agent with a wealth income ratio of 0 to drop by more than 36%. Thus, poor young agents continue to hold less stock than wealthy young agents when both channels are switched on even when the volatility ratio is chosen more conservatively.

Subfigure 10.b contains time-series plots of average stock allocations for an agent with a wealth income ratio in the first month of 0. Once again, the results are similar to those for the case with a volatility ratio of 1.75, with the simultaneous presence of both channels causing the average stock allocation to be hump-shaped from age 22 to age 54 and much lower at age 22 than at retirement age. Thus, the finding that the presence of both the SDM and SDV channels causes poor young agents hold more stock when young than old continues to hold with the more conservative 1.33 volatility ratio.

Turning to non-participation, Subfigure 11.a plots the probability of non-participation in the first month as a function of the agent's first-month wealth-income ratio, while Subfigure 11.b plots the probability of non-participation as a function of age for an agent whose initial wealth income ratio is 0. These subfigures show that the presence of both the SDM and SDV channels still leads to non-participation, and it is in excess of 25% for young agents with zero wealth-income ratios. The SDV channel alone can also generate non-participation in excess of 15% for young agents with zero wealth-income ratios. For a 22 year-old agent with a zero wealth-income ratio, the two channels generate non-participation above 20% until at least age 29. So nontrivial non-participation in stocks by the young can be obtained using the more conservative volatility ratio of 1.33. To sum up, the overall message of this subsection is that our main results, especially those pertaining to the effect of the two channels together on average stock allocations, are robust to using the more conservative volatility ratio of 1.33.

### 4.3 Turning Off the SDV Channel Gradually

The results discussed above show that the SDV channel induces large reductions in the average stock allocations of young agents with low wealth-income ratios and large increases in non-participation in the stock market by these same agents. An interesting question is the sensitivity of these findings to the strength of the SDV channel. To assess this, we examine how the reduction in the average stock allocation and the increase in non-participation in the stock market is affected by varying the ratio of the volatility of the permanent income shock in high versus low dividend-yield states from 1.75, which is calibrated to data, all the way down to 1, which totally switches off the SDV channel. Figures 12.a and 12.b plot the average stock allocation of an agent in her first month as a function of this permanent income volatility ratio for wealth to monthly permanent income ratios in that month of 1, 10, 100 and 1000. Figures 13.a and 13.b plot average stock allocation as a function of age for an agent with an initial wealth to permanent income ratio of 0 when the volatility ratio is 1, 1.225, 1.375, 1.525 and 1.75. The SDM channel is switched on for Figures 12.a and 13.a but is switched off for Figure 12.b and 13.b.

Focusing on the case in which the SDM channel is switched on, Figure 12.a shows that the first-month agent's average allocation for wealth-income ratios of 1 and 10 declines monotonically in the volatility ratio, with most of the decline occurring as the ratio increases from 1 to 1.375. For a wealth to permanent income ratio of 1, the average stock allocation declines from about 80% when the volatility ratio is 1 to just over 30% when the ratio is 1.375. The average stock allocation declines less than 15% of portfolio value more when the ratio increases from 1.375 to 1.75. The pattern is very similar when the agent's wealth to permanent income ratio is 10. The implication is that the SDV channel can achieve declines in the average stock allocation of young, low wealth-income agents with a volatility ratio as low as 1.375 that are comparable to those obtained with the data volatility ratio of 1.75. This robustness of the SDV channel is an important feature of the SDV channel since it means that even if the true volatility ratio is lower than the data value of 1.75, the SDV channel can still generate large declines in the average stock holdings of young, low wealth-income agents.

The reason for this finding is likely that even a modest volatility ratio of 1.375 reduces stock holdings so much that the no short-selling constraint on stock binds in most of the dividend-yield states. When the volatility ratio is increased further, the no short-selling constraint on stock continues to bind in those dividend-yield states, which means that the average stock allocation can only decline further because of declines in the agent's stock allocation in the remaining states for

which this constraint is not yet binding. The result is very small declines in the agent's average stock allocation as the volatility ratio is increased above 1.375.

Again focusing on the case in which the SDM channel is switched on, Figure 13.a shows that with a volatility ratio as low as 1.375 the SDV channel still generates a hump-shaped average stock allocation profile from age 22 to 54 with a lower average allocation at age 22 than at age 65, for an agent with an initial wealth-income ratio of zero. These results for the average stock allocation profile as a function of age are consistent with those discussed above for the average stock allocation of the first-month agent as a function of the volatility ratio. When the SDM channel is switched on, the SDV channel can cause large reductions in the average stock holdings of young agents with low wealth-income ratios even at a volatility ratio as low as 1.375.

Turning to the case in which the SDM channel is switched off, Figure 12.b shows that increasing the volatility ratio from 1 to 1.45 causes the average stock allocation of the young agent with a wealth-income ratio of 1 or 10 to decline by less than 20%. For wealth-income ratios of 1 and 10, the first-month agent's average allocation in Figure 12.b only flattens out as a function of the volatility ratio for volatility ratios above 1.6 and 1.675 respectively. The intuition for this finding is as follows. For a given volatility ratio, the SDV channel alone generates a smaller decline in stock holdings than the SDV and SDM channels together, and so the no short-selling constraint on stock binds in fewer dividend-yield states. Consequently, the volatility ratio needs to be a lot higher with the SDM channel switched off than on for an increase in the volatility ratio not to reduce average stock allocations very much: when the agent's wealth income ratio is 1 or 10, the ratio needs to be above 1.6 or 1.675 respectively when the SDM channel is switched off but only above 1.375 in either case when the SDM channel is switched on. Figure 13.b shows that when the agent has a zero-wealth income ratio in the first month and the SDM channel is off, the agent's first month allocation is only below her age 65 allocation if the volatility ratio is at least 1.525.

Turning to stock market participation, Figures 12.c and 12.d plot the probability of non-participation by the first-month agent as a function of the permanent income volatility ratio described above for wealth to monthly permanent income ratios in that month of 1, 10, 100 and 1000. Figures 13.c and 13.d plot the probability of non-participation as a function of age for an agent with an initial wealth to permanent income ratio of 0 when the volatility ratio is 1, 1.225, 1.375, 1.525 and 1.75. The SDM channel is switched on for Figures 12.c and 13.c but is switched off for Figure 12.d and 13.d.

Focusing on the case in which the SDM channel is switched on, Figure 12.c shows that the

first-month agent's probability of non-participation for wealth-income ratios of 1 and 10 increases monotonically in the volatility ratio, though most of the increase occurs as the ratio increases from 1.375 to 1.75. For a wealth to permanent income ratio of 1, the probability of non-participation increases from about 20% when the volatility ratio is 1 to under 35% when the ratio is 1.375, to over 40% once the ratio is 1.45 or more and to almost 80% when the ratio is 1.75. The pattern is very similar when the agent's wealth to permanent income ratio is 10, though the ratio needs to be 1.525 or more before probability of non-participation goes above 40%. Again focusing on the case in which the SDM channel is switched on, Figure 13.c shows that with a volatility ratio as low as 1.375 the SDV channel still generates a probability of non-participation by an agent (with an initial wealth-income ratio of 0) of more than 30% at age 22 and over 25% at age 29. The implication is that when both channels are switched on, substantial non-participation of young, low wealth-income agents can be achieved with a volatility ratio as low as 1.375, though the probability of non-participation continues to increase as the ratio increases from 1.375 up to 1.75. Figures 12.d and 13.d show that when the SDM channel is switched off, the volatility ratio must be at least 1.525 for non-participation at age 22 to be greater than 30% for an agent with an initial wealth-income ratio of zero, which is higher than the cutoff value for 30% non-participation of 1.375 when the SDM channel is also on.

#### **4.4 i.i.d. Return Case: Age-dependent Profile, Retirement and Death Probabilities**

An interesting question is how return predictability affects the results discussed above. Table 7 reports asset allocation and incremental-effect results of first-month agents for this i.i.d. return case, which is the same as the base case except that the stock return is i.i.d.. The format of Table 7 is the same as Table 2. Figure 14.a plots stock holdings for first-month agents as a function of wealth to permanent income ratios from 0 to 500 with both channels off, both channels on, only SDM switched on and only SDV switched on. Figure 14.b plots average stock allocation as a function of age for an agent with an initial wealth to permanent income ratio of 0.

Table 7 and Figure 14.a show that the qualitative and quantitative conclusions presented above regarding the stock holdings of young poor agents are robust to switching off return predictability. Doing so eliminates the positive hedging demands induced by such predictability but this effect is always present irrespective of whether a channel is switched on or off. So this effect of switching off return predictability is not likely to impact the marginal effects of either the SDM or SDV channels on stock holdings. Switching off the stock return predictability also reduces the magnitude of

the conditional negative correlation between return and dividend yield (to keep the unconditional correlation equal to that in the data) which reduces the magnitude of the reductions in stock holdings induced by the SDM and SDV channels. This effect is found to outweigh the loss of the positive hedging demands induced by return predictability, causing the average stock allocation of the age-22 agent with both channels switched on to be higher than in the predictable return case. It is also the reason why the reductions in stock holdings induced by the SDM and SDV channels are smaller with i.i.d. returns than with predictable returns. With i.i.d. returns, the average stock holding of the agent in the first month drops from 97.5% to 29.5% at zero wealth, which is slightly smaller than the drop from 97.5% to 20.3% when returns are predictable. Panel A of Table 7 and Figure 14.a show that when both channels are switched on, the relation between average stock allocation and wealth-income ratio still goes from negative to positive at least for wealth-income ratios from 1 up to 1000. Plotting the average stock allocation as a function of age for an agent with no initial wealth, Figure 14.b shows that turning on both channels when returns are i.i.d. also causes the average stock allocation to go from being monotonically decreasing up to age 57 and much lower at retirement age than age 22 to being hump-shaped from age 22 to age 54 and much lower at age 22 than retirement age. Recall that when returns are predictable the 2 channels also cause the average allocation to exhibit a similar pattern. Panel B of Table 7 and Figures 14.a and 14.b also show that the SDV channel is still much more important than the SDM channel for reducing the stock allocations of young agents with wealth-income ratios lower than 30 for young agents when returns are i.i.d., irrespective of whether the other channel is present or not. Finally, Panel B of Table 7 shows that the drop in average stock holding from adding either channel is always larger when the other channel is switched on rather than off, so long as the wealth-income ratio is 100 or lower. The smaller reduction in stock holdings (due to the reduced magnitude of the conditional negative correlation between return and dividend yield) causes the no stock-short-selling constraint to bind less often. Using the reasoning in Subsection 4.1 for how the short-selling constraints impact the effects of the two channels on stock holdings, this observation explains why in the i.i.d. return case, the drop in average stock holding from adding either channel is always larger when the other is switched on rather than off, when the wealth-income ratio is 100 or lower.

Turning to stock market participation, Figure 15.a plots the probability of non-participation in the first month as a function of the agent's first-month wealth-income ratio when returns are i.i.d. while Figure 15.b plots the probability of non-participation as a function of age for an agent

whose initial wealth income ratio is 0, again when returns are i.i.d.. These figures show that the two business cycle channels have similar qualitative effects and slightly smaller quantitative effects on stock market participation when returns are i.i.d. as compared to when returns are predictable. With both channels switched off, non-participation is a zero- or near zero-probability event in the first month, irrespective of the agent's wealth-income ratio. With both business-cycle channels switched on in Figure 15.a, the probability of non-participation becomes as high as 65% for low wealth-income ratios, which is comparable but slightly lower than the non-participation probability of almost 80% when returns are predictable. With only the SDV channel switched on, the probability of non-participation is still over 60% when the wealth income ratio is zero, which again is comparable but slightly lower than the non-participation probability when returns are predictable. As is the case when returns are predictable, the SDM channel alone has very little effect on the level of non-participation relative to the case in which both channels are switched off with a probability of non-participation that is never above 20%. Figure 15.b indicates that both channels switched off leads to participation in the stock market virtually all the time, irrespective of age or initial wealth-income ratio, which is also the finding when returns are predictable. Switching on the two business-cycle channels results in substantial non-participation by agents in their first month and that non-participation steadily declines as the agent gets older, just like the case with predictable returns. According to the Figure 15.b, an agent with an initial wealth-income ratio of 0 decides not to participate in the stock market more than 65% of the time in the first month; and after ten years, this probability has declined to a fraction that is still above 20%.

#### 4.5 Robustness Checks

An appendix available from the authors on request contains results for a number of robustness checks and extensions to the base case that we also performed. This subsection contains a brief summary of these robustness checks and extensions.

Setting the mean log growth rate of permanent labor income to 3% per annum produces stock allocations that are virtually identical to the base case considered above, which had a hump-shaped log growth rate of permanent labor income profile. Thus, the ability of the two business cycle channels to reduce the stock allocations of young, low wealth-income agents is not being driven by the hump-shaped earnings profile implemented in the base case. Setting the correlation between dividend-yield innovation and the permanent labor income growth innovation to zero produces stock allocations that are virtually identical to the base case considered above with the correlation

set to that in the data. This result is not surprising since the data correlation is quite close to zero at -3.74%. Viceira's (1997, 2001) baseline value for the standard deviation of the log temporary shock is 10% per year. Adding a 10% temporary shock to per annum labor income growth has little effect on the agent's allocations. Thus, it appears that temporary shocks to labor income growth do not materially affect allocations, at least shocks of the magnitude documented for U.S. individuals.

The impact of an unemployment state on portfolio allocations is another question of interest. Carroll (1992) uses PSID data and finds that the probability of a near-zero income realization for a year is 0.05%. The first way we incorporate this is by allowing a 0.05% chance each month of being in an unemployment state that pays only 10% of permanent labor income. This parameter choice implies that the average fraction of a year that the agent is unemployed is 0.05%, which is also implied by the Carroll number. Introducing this i.i.d. unemployment state lowers average stock allocations a little irrespective of which subset of channels is switched on, but leaves the incremental effect of the two channels on stock allocations largely unchanged. The small effect of the i.i.d. unemployment state on the stock allocations of first-month agents probably isn't so surprising given results in Viceira (1997) for transitory unemployment. On the other hand, Cocco, Gomes and Maenhout (2005) examine the effect of an unemployment state in a model with annual decision-making and they present simulation results that are suggestive, but not conclusive, that an 0.05% probability of being unemployed in any given year has a large effect on stock allocations, particularly early in life. But their model implies some degree of persistence in the unemployment state at a monthly frequency. To incorporate persistence into the unemployment state, we also consider a Markov regime switching model, with two states, employment and unemployment. By allowing the transition matrix to be age-dependent, we are able to exactly replicate the transition dynamics used by Cocco, Gomes and Maenhout and assumed by Carroll. Introducing this persistent unemployment state typically lowers average stock allocations only slightly more than introducing the i.i.d. unemployment state, but leaves the incremental effect of the two channels on stock allocations largely unchanged.

Another interesting question is the effect of social security on stock allocation decisions. To assess this, we consider two cases. First, we do not allow the agent to receive any social security payments at or after retirement. Second, we allow the agent to receive a lump sum payout equivalent to receiving 93.8% of her retirement permanent income until death, which was the average social security payout number reported in Cocco, Gomes and Maenhout (2005) for college gradu-

ates. However, abolishing social security or modifying the form of the social security payout has almost no impact on the stock holding of the young agent, though abolishing social security materially decreases the agent's stock allocation in the years just prior to retirement. When no social security payments are received, the two business cycle channels still cause the relation between stock allocation and wealth-income ratio to become positive for young agents and an agent with zero initial wealth to have a higher average stock allocation at retirement than at age 22.

The magnitude of the covariance with lagged dividend yield is lower for growth in total private sector earnings than for growth in retail trade earnings. Consequently, it is worthwhile checking whether the state-dependent mean channel continues to have a large effect when this series is used to calibrate the covariance. We find that the combined effect of the two business-cycle channels on average stock allocations in the first month is reduced but never by an amount greater than an allocation of 2% with the largest reduction occurring when the young agent's wealth is 100 times her monthly labor income.

## 5 Conclusion

This paper asks whether allowing the conditional joint distribution of labor income to depend on the business cycle can allow the model to generate equity holdings that better match those of U.S. households, while keeping the unconditional distribution the same as in the data. Calibrating the first two moments of labor income growth to match the countercyclical volatility and procyclical mean found in U.S. data leads to large reductions in stock holdings by young agents with low wealth-income ratios. The countercyclical volatility is the more important of the two, inducing reductions which are so large that young, poor agents now hold less stock than both young, rich agents and old agents, and also hold no stock a large fraction of the time. Our results suggest that the predictability of labor-income growth at a business-cycle frequency, particularly the countercyclical variation in volatility, plays an important role in a young agent's decision-making about her portfolio's stock holding.

In future work, we plan to endogenize the agent's labor supply decision. Doing so may ameliorate the effects of this business cycle variation in labor income growth on the stock allocations of young, poor agents since agents can reduce hours worked when labor opportunities are poor. On the flip side, they can increase hours worked when labor opportunities are good, which would exacerbate the effects. We conjecture that the effects on the stock allocations of poor, young agents are still likely to be large, much in the same way that hedging demands induced by stock return predictability

are large despite the fact that the agent's can choose how much to invest in stock.

## Appendix: Numerical Methodology

This appendix sketches the numerical methodology used when solving the dynamic portfolio choice problem in the presence of labor income. There are two important tasks the methodology achieves. The first is to resort to extrapolation only when the labor income problem on hand is economically sufficiently close (in a sense to be made clear below) to the otherwise identical no labor income problem for which the functional form for the value function is known. The second is to endogenize the discrete state representation of the value function with respect to wealth to permanent income ratio to help bound error propagation at each iteration.

In addressing the first task, we note that wealth to permanent labor income (wealth-income, henceforth) ratio state is unbounded on the non-negative side of the real line. To represent the value function on this dimension, this range is partitioned into three disjoint, non-degenerate intervals on which the value function of unknown form is approximated as a three-part structure. At each iteration, the lower intermediate boundary point is chosen to be the smallest value of wealth-income ratio such that an agent without the labor income for all periods to terminal date requires no more than 10% extra wealth to be equally happy as an otherwise identical agent with the labor income at that wealth-income ratio regardless of the dividend yield state. The upper intermediate boundary point is chosen to satisfy the same definition at 1%. Over the lower-end interval, the value function is approximated piecewise linearly or using the piecewise shape-preserving monotone cubic hermite interpolant of Fritsch and Carlson (1980). Over the higher-end interval, the value function is taken to be that of the otherwise identical problem without the labor income. Over the middle interval, the value function is approximated as a function of the form,  $V(W) = a \times (W - b)^c + d$ , where  $a$ ,  $b$ ,  $c$ ,  $d$  are constants in  $\mathbb{R}$ , to match the function and the first derivative values at the upper and the lower intermediate boundary points.

In addressing the second task, we suggest a dynamic gridding procedure to bound errors on policy functions at each iteration. This procedure takes the value function representation for the previous iteration as given, computes the intermediate boundary points for the iteration at hand and continues to add points in the lower-end interval in a particular way until policies at no point on a representative grid for the next iteration differ by more than prespecified magnitudes across increasingly denser representations for the iteration on hand.

The steps of this procedure for obtaining the representation for  $V_t$  are:

- 1) take value function representation for the previous iteration,  $V_{t+1}(\cdot)$ , as given.
- 2) initialize a set of grid nodes,  $X_t = \{\Gamma_{t,1}, \Gamma_{t,2}, \Gamma_{t,3}, \dots, \Gamma_{t,n}\}$ , in increasing order, where  $\Gamma_{t,1} = 0$ ,

$\Gamma_{t,n} = LP_{t,10\%}$ ,  $n$  is a constant, and  $LP_{t,10\%}$  is the lower intermediate boundary point for iteration  $t$  with the defining parameter set to 10%.

3) define  $Y_t = \{(\Gamma_{t,1} + \Gamma_{t,2})/2, (\Gamma_{t,2} + \Gamma_{t,3})/2, \dots, (\Gamma_{t,n-1} + \Gamma_{t,n})/2\}$ , in increasing order.

4) compute  $V_t(X_t, D_t)$  using the value function representation,  $V_{t+1}(\cdot)$ .

5) compute  $V_t(Y_t, D_t)$  using the value function representation,  $V_{t+1}(\cdot)$ .

6) compute the lower intermediate point,  $LP_{t-1,10\%}$  for iteration  $t - 1$  using  $V_t$  as represented by  $X_t \cup Y_t$ .

7) define  $A_{t-1} = \{(i - 1) \times (LP_{t-1,10\%})/q\}_{i=1}^{q+1}$ , where  $q \geq 2$  is a constant.

8) compute  $V_{t-1}(A_{t-1}, D_{t-1})$  using  $V_t$  as represented by  $X_t$ .

9) compute  $V_{t-1}(A_{t-1}, D_{t-1})$  using  $V_t$  as represented by  $X_t \cup Y_t$ .

10) define  $Z_{t-1} \subseteq A_{t-1} = \{x \in A_{t-1} \mid \text{policy functions at } x \text{ using representations in steps 8) and 9) differ more than prespecified magnitudes for any } D_{t-1}\}$ .

11) if  $Z_{t-1} = \emptyset$ , accept  $X_t \cup Y_t$  as sufficient representation for  $V_t$ , exit procedure.

12) compute the range of wealth-income,  $\Gamma_t$ , range the system can possibly assume by starting at any point in  $Z_{t-1}$  under any realization of the return and the labor income shocks and under any allowed policy. This range is generically of the form  $[0 \ h_t]$ , for some  $h_t > 0$ .

13) update  $X_t$  as  $X_t \cup Y_t$  in increasing order.

14) update  $Y_t$  as  $\{(X_{t,1} + X_{t,2})/2, (X_{t,2} + X_{t,3})/2, \dots, (X_{t,k-1} + X_{t,k})/2\}$ ; where  $k$  is the smallest positive integer satisfying  $X_{t,k} \geq \min(h_t, LP_t)$ .

15) return to step 5).

In step 2), initialization takes advantage of the  $V_{t+1}$  representation. In steps 8) and 9), the objective function has to be jointly solved for consumption and portfolio policies subject to the short sales constraints on the T-bill and the stock. A recursive golden section algorithm is used to optimize the consumption policy,  $\hat{\kappa}$  accurate to the fourth decimal digit and the portfolio policy  $\alpha$  accurate to the third decimal digit. In step 10) the maximal absolute scaled deviation in consumption policy,  $|(\hat{\kappa}_{coarse} - \hat{\kappa}_{dense})/\hat{\kappa}_{dense}|$  is bounded from above by  $10^{-3}$  and the maximal absolute deviation in portfolio policy,  $|\alpha_{coarse} - \alpha_{dense}|$ , is bounded from above by  $10^{-2}$ .  $q$  and  $n$  are set to 50.

Given the form of the interpolant (linear or cubic hermite), this procedure naturally provides a set of grid nodes which can be expected to represent the value function at hand sufficiently well before one goes on to the next iteration. Procedures described above produce a joint state for the base case specification with predictable returns and when both effects are present (SDM and SDV) which has on average (over the lifecycle), approximately  $19 \times 450 = 8550$  grid nodes. To ensure

that results are robust to the parameters of procedures, we change the defining parameter for the lower boundary point from 10% to 9%, 8%, 7%, and 6% for the base case specification when both effects are present (SDM and SDV). The maximal  $|(\hat{\kappa}_{10\%} - \hat{\kappa}_{x\%})/\hat{\kappa}_{x\%}|$  and  $|\alpha_{10\%} - \alpha_{x\%}|$  for an equally spaced 100-node grid on  $[0, LP_{t,10\%}]$  are less than  $10^{-3}$  and  $10^{-2}$  respectively for all  $t = 1, \dots, 936$  and  $x = 6,7,8,9$ . In a further robustness check, we fix the defining parameter for the lower boundary point at 10%, but in steps 8) and 9) we change the precision on the consumption policy,  $\hat{\kappa}$ , from  $10^{-4}$  to  $10^{-6}$ . We again note that the maximal absolute scaled consumption and portfolio policies never change more than  $10^{-3}$  and  $10^{-2}$  across an equally spaced 100-node grid on  $[0, LP_{t,10\%}]$  for all  $t = 1, \dots, 936$ .

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Table 1. CALIBRATION: RETURN, DIVIDEND YIELD and RETAIL TRADE INCOME. The table reports calibration and data values for the parameters of the VAR specification introduced in section 2.1 of the text using aggregate (Retail Trade) and individual Labor Income data.

$$r_{t+1} = a_r + b_r d_t + e_{t+1},$$

$$d_{t+1} = a_d + b_d d_t + w_{t+1},$$

$$g_{t+1}^a \equiv y_{t+1}^P - y_t^P = \bar{g}^a + b_{g^a} d_t + u_{t+1}^a.$$

The dependent variables are percent logarithmic return on the Market Portfolio,  $r$ , change in aggregate labor income,  $g^a$ , and 12-month dividend yield (normalized to have zero mean and unit volatility). The independent variable is the lagged dividend yield. Panel A reports the data point estimates and the t-statistics of the regression slopes as well as means, correlations and covariances. The panel also reports quadrature values for the same.  $\mu$  refers to unconditional mean.  $\sigma$  with one (two) subscript letter(s) refer to unconditional standard deviation (covariance).  $\rho$  refers to the correlation coefficient. T-statistics are reported for the slope coefficient estimates in the *t-stat* column with Newey-West correction for 3 months and 12 months respectively. The Market portfolio is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks obtained from Kenneth French's website. Dividend yield is the 12-month dividend yield on the NYSE index.  $g^a$  is the change in log Retail Trade income. Similarly  $u_a$  denotes the residual. Labor income data is from the Bureau of Labor Statistics website. Retail Trade is series CEU4200000004. Income and return data are disinflated using a CPI measure, series CPIAUCNS, available from U.S. Department of Labor: Bureau of Labor Statistics. All data is measured at a monthly frequency. We use Retail Trade income data starting from January 1972. Return on the market and dividend yield start from January 1927. All data series end in January 2004. Returns and income growth are expressed in percent. Panel B reports values used for the individual agent's optimization problem incorporating the parameter estimates for individual data. Here  $g$  denotes the logarithmic change in individual permanent income as described in section 2.1 of the text. Individual labor income means and variances are taken from Gakidis (1997) based on PSID data for professionals and managers not self-employed under age 45.  $\mu$  refers to unconditional mean.  $\sigma$  with one (two) subscript letter(s) refer to unconditional standard deviation (covariance).  $\sigma_{u|exp}$ , ( $\sigma_{u|rec}$ ) refers to the conditional volatility of the permanent income growth shock in expansion (recession) states.  $\rho$  refers to unconditional correlation.  $-1$  used as a subscript implies the lagged value for a variable. SDM and SDV stand for state dependent mean and state dependent volatility channels respectively, for permanent income growth. A  $+$  ( $-$ ) sign means that the channel at the start of the row is present (not present) for the case in consideration. Given the data values for aggregate or individual income in the last two columns, the calibration procedure generates the values in the first four columns, one for each case considered. To determine the permanent income volatility values at monthly frequencies, we utilize a loglinear approximation, relating these parameter values to their annual counterparts, while explicitly accounting for the predictable dynamics of the permanent income growth. Details of this approximation is available from the authors.

Panel A: Return, Dividend Yield and Aggregate Labor Income											
Regression		t-stats		Std Dev (diagonal), Cov(above), Cor(below)							
Coefficients		lag		Unconditional						Av Cond	
$a_i$	$b_i$	3 mth	12 mth	$g$	$r$	$d$	$u^a$	$e$	$w$	$e$	$w$
Data											
$g^a$	-0.164	-3.07	-4.43	1.284	0.089	-0.171	$u^a$ 1.273	0.139	-0.010		
$r$	0.535	0.307	0.89	1.12	1.25%	5.516	-0.702	$e$ 1.99%	5.507	-1.002	
$d$	0	0.979	45.73	63.86	-13.30%	-12.73%	1	$w$ -3.74%	-88.35%	0.206	
Quadrature											
$r$	0.535	0.307						$e$	5.505	-1.343	5.505 -1.134
$d$	0	0.962						$w$	-89.54%	0.272	-89.91% 0.271

Table 1 cont.

Panel B: Labor Income					
SDM	+	+	-	-	
SDV	+	-	+	-	
Parameters	Quadrature				Data
	Calibrated				Ag . Indiv.
$\mu_g$	fig.1	fig.1	fig.1	fig.1	fig.1
Annual $\sigma_g$	15	15	15	15	15
$\sigma_g$	5.259	5.259	5.259	5.259	
$b_g$	-0.164	-0.164	0	0	-0.164
$\sigma_{gr}$	0.089	0.089	0.139	0.139	0.089
Other					
$\rho_{ga_{-1}}$	-3.13%	-3.13%	0	0	
$\rho_{gr}$	0.31%	0.31%	0.48%	0.48%	
$\rho_{eu}$	0.48%	0.48%	0.48%	0.48%	
$\sigma_{eu} = \text{Av. Cond } \sigma_{eu}$	0.139	0.139	0.139	0.139	
Av. Cond. $\rho_{eu}$	0.50%	0.48%	0.50%	0.48%	
$\sigma_{u exp}$	4.062	5.257	4.064	5.259	
$\sigma_{u rec}$	7.108	5.257	7.111	5.259	
$\rho_{wu}$	-0.91%	-0.91%	-0.91%	-0.91%	

Table 2. FIRST MONTH ALLOCATION RESULTS. LIFE CYCLE SPECIFICATION. AGE-DEPENDENT INCOME GROWTH PROFILE. ANNUITY RETIREMENT MONEY. SPECIFICATION-BASED PERMANENT INCOME VOLATILITY. NO TRANSITORY SHOCKS. RETAIL TRADE DATA. The table reports asset allocation results for a CRRA agent with a coefficient of risk aversion of 6 for the *first* month of her 78 year horizon, for a range of wealth to permanent income ratios from 0 to  $\infty$ . The agent starts work at age 22 and retires at 65, receiving 93.8% of his retirement permanent income until death as reported in Cocco, Gomes and Maenhout (2002). The agent dies with probability 1 at age 100, with death probabilities taken from the U.S. Life Tables, 2001, provided by the NCHS. The mean logarithmic monthly growth rate of permanent income is allowed to be age-dependent and is calibrated to the polynomial-smoothed income profile for college graduates in Cocco, Gomes and Maenhout (2002). The volatility of the logarithmic monthly growth rate of permanent income  $\sigma_g$ , is obtained from its annual counterpart (fixed at 15% from Gakidis (1997)), accounting for the assumed dynamics of the permanent income growth. There are no transitory shocks to income. Retail Trade Income (series CEU4200000004 by Bureau of Labor Statistics) is also used to calibrate labor income as described in Table 1. Agent has access to the market portfolio (value-weighted return on all NYSE, AMEX, and NASDAQ stocks) and to a riskless bond. Calibrations of these assets are as described in section 3 of the text. SDM and SDV denote, respectively, the state-dependent mean and state-dependent volatility channels through which permanent labor income growth can affect stock holdings (see section 2.3 for descriptions). Panel A reports average stock holdings when both and none of the channels are present. Panel B reports the incremental effects on stock holdings of switching on one of the channels by averaging across all states. Each of these channels can be switched on when the other channel is present or not and the two rows of each channel's subpanel report the incremental stock-holding reductions for these 2 cases. The calibration of the 4 problems needed to do the comparisons is detailed in section 3 of the text.

Panel A: Allocations Averaged Across All States								
	Wealth/Income Ratio							
	0	1	10	30	70	100	1000	$\infty$
All	0.203	0.203	0.220	0.246	0.333	0.386	0.538	0.476
None	0.975	0.975	0.960	0.929	0.882	0.862	0.660	0.476
Panel B: Incremental Effects Averaged Across All States								
	Wealth/Income Ratio							
	0	1	10	30	70	100	1000	$\infty$
State Dependent Mean								
SDV	-0.060	-0.060	-0.061	-0.274	-0.293	-0.262	-0.064	0.000
No SDV	-0.178	-0.176	-0.168	-0.172	-0.161	-0.164	-0.063	0.000
State Dependent Volatility								
SDM	-0.593	-0.596	-0.573	-0.511	-0.388	-0.312	-0.059	0.000
No SDM	-0.712	-0.712	-0.680	-0.410	-0.256	-0.213	-0.058	0.000

Table 3. CALIBRATION OF NBER BASED BUSINESS CYCLE STATES, RETURN AND INCOME. The table reports calibration and data values for the conditional and unconditional probabilities of the NBER business cycle variable, return and the income growth. The table also provides the transition probability matrix for the business cycle state. The variables are percent logarithmic return on the Market Portfolio,  $\tau$ , percent logarithmic growth rate of permanent income,  $g$  and the NBER business cycle,  $s$ . Conditional on the business cycle state, a three point gaussian quadrature is used for return and we respectively label the largest, middle and lowest quadrature returns as  $r_{big}, r_{middle}$  and  $r_{low}$ . We use the business cycle dates available from the NBER website to label each month in the data period as an expansion and a recession. Then for each expansion we label the first  $n$  months as the early expansion and any remaining months of the expansion as late expansion, so every month in the period is either a recession (Rc), and early expansion (EE) or late expansion (LE). We set  $n$  to half the average duration of expansions. Income and return data are disinflated using a CPI measure, series CPIAUCNS, available from U.S. Department of Labor: Bureau of Labor Statistics.  $\mu$  refers to unconditional mean.  $\sigma$  refers to unconditional standard deviation.  $E[\cdot|s]$  and  $\sigma[\cdot|s]$  refer to the conditional expectation and standard deviation conditional on the business cycle.  $P[\cdot|s]$  refers to conditional probability based on  $s$ .  $\rho$  refers to unconditional correlation. SDM and SDV stand for state dependent mean, covariance with return and state dependent volatility channels respectively, for permanent income growth as described in section 2.1 of the text.  $A + (-)$  sign means that the channel at the start of the row is present (not present) for the case in consideration. The Market portfolio is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks obtained from Kenneth French's website. Aggregate labor income data is from the Bureau of Labor Statistics website. Retail Trade is series CEU420000004. Individual labor income moments are taken from Gakidis (1997) based on PSID data for professionals and managers not self-employed under age 45 and for Non Farm Proprietor's income from Heaton and Lucas (2000). All data is measured at a monthly frequency. We use Retail Trade income data starting from January 1972. Return and NBER data start at Jan 1929. All data series end in January 2004. Income and return data are disinflated using a CPI measure, series CPIAUCNS, available from U.S. Department of Labor: Bureau of Labor Statistics. Per capita income values are generated by dividing all income series with a population measure, series POP, available from U.S. Department of Commerce: Census Bureau. Panel A reports the transition probabilities conditional on return and business cycle state and conditions on only the business cycle state. Panel B reports the unconditional means and standard deviations for returns. Panel C reports conditional and unconditional means and standard deviations for income growth. Given the data values for aggregate or individual income in the last two columns, the calibration procedure generates the values in the first four columns, one for each case considered.

Panel A: Return and the NBER Variable										
$s$	$p[Rc r_{low},s]$	$p[Rc r_{mid},s]$	$p[Rc r_{big},s]$	$p[EE r_{low},s]$	$p[EE r_{mid},s]$	$p[EE r_{big},s]$	$p[Rc s]$	$p[EE s]$	$p[LE s]$	$P[s]$
Data										
Rc	96.43	92.86	85.71	3.57	7.14	14.29	92.26	7.74	0.00	18.69
EE	0.00	1.38	00.00	98.15	94.95	98.18	0.92	96.02	3.06	36.48
LE	4.48	2.23	1.49	0.00	0.00	0.00	2.48	0.00	97.52	44.83
Quadrature										
Rc	96.43	92.86	85.71	3.57	7.14	14.29	92.26	7.74	0.00	18.69
EE	0.00	1.38	00.00	98.15	94.95	98.18	0.92	96.02	3.06	36.48
LE	4.48	2.23	1.49	0.00	0.00	0.00	2.48	0.00	97.52	44.83
Panel B: Stock Return										
Data										
$E[r]$	Quadrature									
$\sigma[r]$	0.478	0.478								
	5.477	5.477								
Panel C: Labor Income										
SDM	+	+	-	-	-	-	-	-	-	-
SDV	+	-	+	-	-	-	-	-	-	-
Calibrated (same as Quadrature)										
$\mu_g$	fig.1	fig.1	fig.1	fig.1	Ag	Indiv	fig.1			
$E[g Rc]-\mu_g$	-0.2634	-0.2634	0	0	-0.2634	fig.1				
$E[g EE]-\mu_g$	0.0674	0.0674	0	0	0.0674					
$E[g LE]-\mu_g$	0.0550	0.0550	0	0	0.0550					
$\sigma_g$	5.260	5.260	5.260	5.260	5.260	5.260			5.260	
$\sigma[g Rc]$	8.618	5.259	8.618	5.260	8.618	5.260				
$\sigma[g EE]$	4.925	5.259	4.926	5.260	4.926	5.260				
$\sigma[g LE]$	4.925	5.259	4.926	5.260	4.926	5.260				
$\rho[r,g s]$	0	0	0	0	0	0				

Table 4. FIRST MONTH ALLOCATION RESULTS. LIFE CYCLE SPECIFICATION. AGE-DEPENDENT INCOME GROWTH PROFILE. ANNUITY RETIREMENT MONEY. SPECIFICATION-BASED PERMANENT INCOME VOLATILITY. NO TRANSITORY SHOCKS. RETAIL TRADE DATA. DIRECTLY CALIBRATED NBER BASED BUSINESS CYCLES. IID RETURNS. The table reports asset allocation results for a CRRA agent with a coefficient of risk aversion of 6 for the *first* month of her 78 year horizon, for a range of wealth to permanent income ratios from 0 to  $\infty$ . The business cycle states are directly calibrated to NBER expansion and recession dates, as described in section 3. The agent starts work at age 22 and retires at 65, receiving 93.8% of his retirement permanent income until death as reported in Cocco, Gomes and Maenhout (2002). The agent dies with probability 1 at age 100, with death probabilities taken from the U.S. Life Tables, 2001, provided by the NCHS. Returns are iid. The mean logarithmic monthly growth rate of permanent income is allowed to be age-dependent and is calibrated to the polynomial-smoothed income profile for college graduates in Cocco, Gomes and Maenhout (2002). The volatility of the logarithmic monthly growth rate of permanent income  $\sigma_g$ , is obtained from its annual counterpart (fixed at 15% from Gakidis (1997)), accounting for the assumed dynamics of the permanent income growth. There are no transitory shocks to income. Retail Trade Income (series CEU4200000004 by Bureau of Labor Statistics) is also used to calibrate labor income as described in Table 1. Agent has access to the market portfolio (value-weighted return on all NYSE, AMEX, and NASDAQ stocks) and to a riskless bond. Calibrations of these assets are as described in section 3 of the text. SDM and SDV denote, respectively, the state-dependent mean and state-dependent volatility channels through which permanent labor income growth can affect stock holdings (see section 2.3 for descriptions). Panel A reports average stock holdings when both and none of the channels are present. Panel B reports the incremental effects on stock holdings of switching on one of the channels by averaging across all states. Each of these channels can be switched on when the other channel is present or not and the two rows of each channel's subpanel report the incremental stock-holding reductions for these 2 cases. The calibration of the 4 problems needed to do the comparisons is detailed in section 3 of the text.

Panel A: Allocations Averaged Across All States								
	Wealth/Income Ratio							
	0	1	10	30	70	100	1000	$\infty$
All	0.342	0.337	0.331	0.338	0.382	0.410	0.469	0.320
None	1.000	1.000	0.998	0.984	0.930	0.834	0.641	0.320
Panel B: Incremental Effects Averaged Across All States								
	Wealth/Income Ratio							
	0	1	10	30	70	100	1000	$\infty$
State Dependent Mean								
SDV	-0.053	-0.057	-0.120	-0.274	-0.286	-0.235	-0.140	0.000
No SDV	-0.042	-0.040	-0.062	-0.114	-0.120	-0.121	-0.046	0.000
State Dependent Volatility								
SDM	-0.616	-0.623	-0.605	-0.532	-0.428	-0.303	-0.126	0.000
No SDM	-0.605	-0.606	-0.547	-0.372	-0.262	-0.189	-0.032	0.000

Table 5. FIRST MONTH ALLOCATION RESULTS. LIFE CYCLE SPECIFICATION. AGE-DEPENDENT INCOME GROWTH PROFILE. ANNUITY RETIREMENT MONEY. SPECIFICATION-BASED PERMANENT INCOME VOLATILITY. NO TRANSITORY SHOCKS. RETAIL TRADE DATA. NODE BY NODE STATE DEPENDENT VOLATILITY SPECIFICATION. The table reports asset allocation results for a CRRA agent with a coefficient of risk aversion of 6 for the *first* month of her 78 year horizon, for a range of wealth to permanent income ratios from 0 to  $\infty$ . The agent starts work at age 22 and retires at 65, receiving 93.8% of his retirement permanent income until death as reported in Cocco, Gomes and Maenhout (2002). The agent dies with probability 1 at age 100, with death probabilities taken from the U.S. Life Tables, 2001, provided by the NCHS. The mean logarithmic monthly growth rate of permanent income is allowed to be age-dependent and is calibrated to the polynomial-smoothed income profile for college graduates in Cocco, Gomes and Maenhout (2002). The volatility of the logarithmic monthly growth rate of permanent income  $\sigma_g$ , is obtained from its annual counterpart (fixed at 15% from Gakidis (1997)), accounting for the assumed dynamics of the permanent income growth. There are no transitory shocks to income. Retail Trade Income (series CEU4200000004 by Bureau of Labor Statistics) is also used to calibrate labor income as described in Table 1. Agent has access to the market portfolio (value-weighted return on all NYSE, AMEX, and NASDAQ stocks) and to a riskless bond. Calibrations of these assets are as described in section 3 of the text. SDM and SDV denote, respectively, the state-dependent mean and state-dependent volatility channels through which permanent labor income growth can affect stock holdings (see section 2.3 for descriptions). State dependent volatility channel is implemented allowing each dividend yield state to have a different permanent income growth volatility, as described in section 3. Panel A reports average stock holdings when both and none of the channels are present. Panel B reports the incremental effects on stock holdings of switching on one of the channels by averaging across all states. Each of these channels can be switched on when the other channel is present or not and the two rows of each channel's subpanel report the incremental stock-holding reductions for these 2 cases. The calibration of the 4 problems needed to do the comparisons is detailed in section 3 of the text.

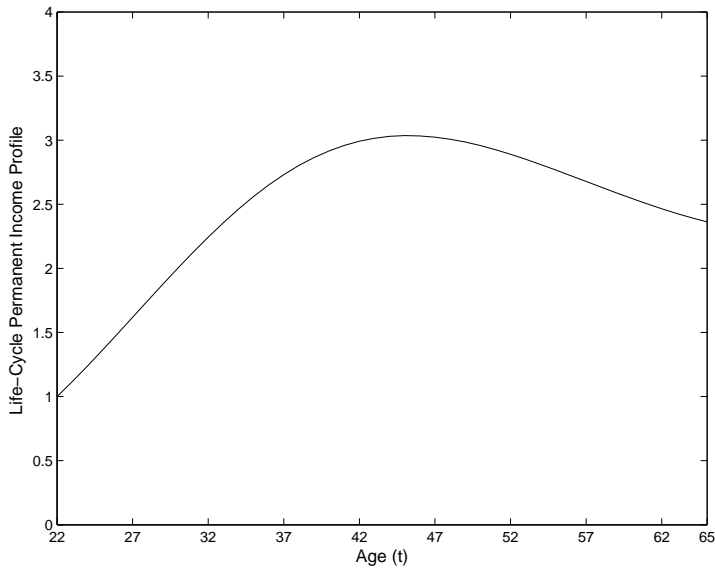
Panel A: Allocations Averaged Across All States								
	Wealth/Income Ratio							
	0	1	10	30	70	100	1000	$\infty$
All	0.424	0.421	0.422	0.435	0.504	0.537	0.582	0.483
None	0.975	0.974	0.959	0.930	0.883	0.860	0.674	0.483
Panel B: Incremental Effects Averaged Across All States								
	Wealth/Income Ratio							
	0	1	10	30	70	100	1000	$\infty$
State Dependent Mean								
SDV	-0.032	-0.037	-0.047	-0.257	-0.257	-0.244	-0.080	0.000
No SDV	-0.178	-0.175	-0.167	-0.173	-0.165	-0.163	-0.070	0.000
State Dependent Volatility								
SDM	-0.373	-0.379	-0.370	-0.323	-0.214	-0.160	-0.023	0.000
No SDM	-0.519	-0.517	-0.490	-0.239	-0.122	-0.079	-0.012	0.000

Table 6. FIRST MONTH ALLOCATION RESULTS. LIFE CYCLE SPECIFICATION. AGE-DEPENDENT INCOME GROWTH PROFILE. ANNUITY RETIREMENT MONEY. SPECIFICATION-BASED PERMANENT INCOME VOLATILITY. NO TRANSITORY SHOCKS. RETAIL TRADE DATA. CONSERVATIVE STATE DEPENDENT VOLATILITY SPECIFICATION. The table reports asset allocation results for a CRRA agent with a coefficient of risk aversion of 6 for the *first* month of her 78 year horizon, for a range of wealth to permanent income ratios from 0 to  $\infty$ . The agent starts work at age 22 and retires at 65, receiving 93.8% of his retirement permanent income until death as reported in Cocco, Gomes and Maenhout (2002). The agent dies with probability 1 at age 100, with death probabilities taken from the U.S. Life Tables, 2001, provided by the NCHS. The mean logarithmic monthly growth rate of permanent income is allowed to be age-dependent and is calibrated to the polynomial-smoothed income profile for college graduates in Cocco, Gomes and Maenhout (2002). The volatility of the logarithmic monthly growth rate of permanent income  $\sigma_g$ , is obtained from its annual counterpart (fixed at 15% from Gakidis (1997)), accounting for the assumed dynamics of the permanent income growth. There are no transitory shocks to income. Retail Trade Income (series CEU4200000004 by Bureau of Labor Statistics) is also used to calibrate labor income as described in Table 1. Agent has access to the market portfolio (value-weighted return on all NYSE, AMEX, and NASDAQ stocks) and to a riskless bond. Calibrations of these assets are as described in section 3 of the text. SDM and SDV denote, respectively, the state-dependent mean and state-dependent volatility channels through which permanent labor income growth can affect stock holdings (see section 2.3 for descriptions). State dependent volatility channel is implemented using the conservative volatility ratio. Panel A reports average stock holdings when both and none of the channels are present. Panel B reports the incremental effects on stock holdings of switching on one of the channels by averaging across all states. Each of these channels can be switched on when the other channel is present or not and the two rows of each channel's subpanel report the incremental stock-holding reductions for these 2 cases. The calibration of the 4 problems needed to do the comparisons is detailed in section 3 of the text.

Panel A: Allocations Averaged Across All States								
	Wealth/Income Ratio							
	0	1	10	30	70	100	1000	$\infty$
All	0.430	0.424	0.427	0.441	0.510	0.542	0.582	0.476
None	0.975	0.975	0.960	0.929	0.882	0.862	0.660	0.476
Panel B: Incremental Effects Averaged Across All States								
	Wealth/Income Ratio							
	0	1	10	30	70	100	1000	$\infty$
State Dependent Mean								
SDV	-0.398	-0.403	-0.399	-0.370	-0.271	-0.232	-0.048	0.000
No SDV	-0.178	-0.176	-0.168	-0.172	-0.161	-0.164	-0.063	0.000
State Dependent Volatility								
SDM	-0.367	-0.375	-0.365	-0.316	-0.211	-0.156	-0.015	0.000
No SDM	-0.147	-0.148	-0.134	-0.118	-0.101	-0.088	-0.030	0.000

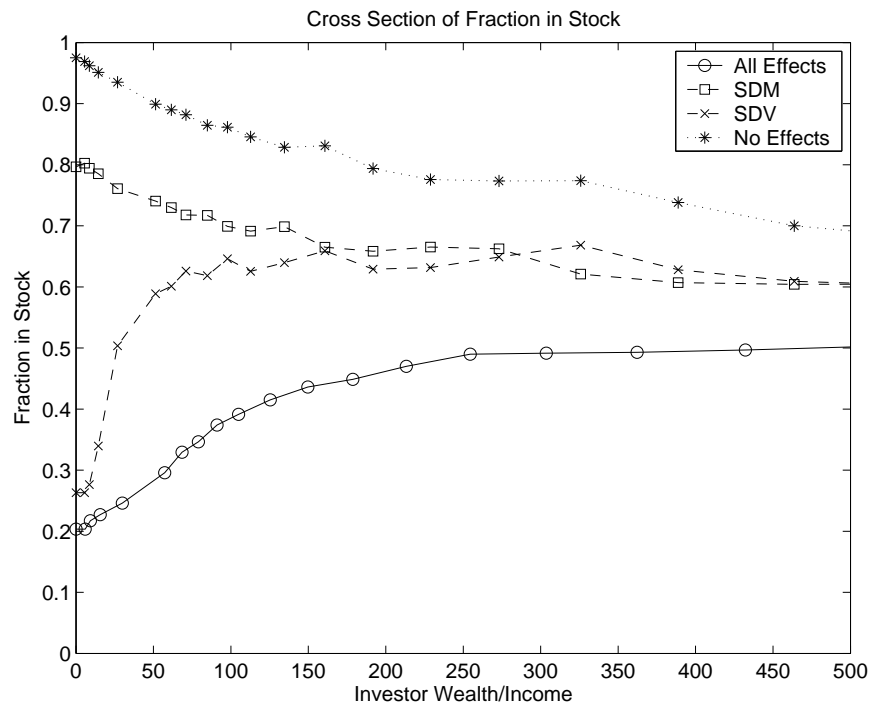
Table 7. FIRST MONTH ALLOCATION RESULTS. LIFE CYCLE SPECIFICATION. AGE-DEPENDENT INCOME GROWTH PROFILE. ANNUITY RETIREMENT MONEY. SPECIFICATION-BASED PERMANENT INCOME VOLATILITY. NO TRANSITORY SHOCKS. RETAIL TRADE DATA. IID RETURNS. The table reports asset allocation results for a CRRA agent with a coefficient of risk aversion of 6 for the *first* month of her 78 year horizon, for a range of wealth to permanent income ratios from 0 to  $\infty$ . The agent starts work at age 22 and retires at 65, receiving 93.8% of his retirement permanent income until death as reported in Cocco, Gomes and Maenhout (2002). The agent dies with probability 1 at age 100, with death probabilities taken from the U.S. Life Tables, 2001, provided by the NCHS. Returns are iid. The mean logarithmic monthly growth rate of permanent income is allowed to be age-dependent and is calibrated to the polynomial-smoothed income profile for college graduates in Cocco, Gomes and Maenhout (2002). The volatility of the logarithmic monthly growth rate of permanent income  $\sigma_g$ , is obtained from its annual counterpart (fixed at 15% from Gakidis (1997)), accounting for the assumed dynamics of the permanent income growth. There are no transitory shocks to income. Retail Trade Income (series CEU4200000004 by Bureau of Labor Statistics) is also used to calibrate labor income as described in Table 1. Agent has access to the market portfolio (value-weighted return on all NYSE, AMEX, and NASDAQ stocks) and to a riskless bond. Calibrations of these assets are as described in section 3 of the text. SDM and SDV denote, respectively, the state-dependent mean and state-dependent volatility channels through which permanent labor income growth can affect stock holdings (see section 2.3 for descriptions). Panel A reports average stock holdings when both and none of the channels are present. Panel B reports the incremental effects on stock holdings of switching on one of the channels by averaging across all states. Each of these channels can be switched on when the other channel is present or not and the two rows of each channel's subpanel report the incremental stock-holding reductions for these 2 cases. The calibration of the 4 problems needed to do the comparisons is detailed in section 3 of the text.

Panel A: Allocations Averaged Across All States								
	Wealth/Income Ratio							
	0	1	10	30	70	100	1000	$\infty$
All	0.295	0.293	0.308	0.338	0.444	0.498	0.575	0.346
None	0.975	0.974	0.955	0.924	0.872	0.846	0.651	0.346
Panel B: Incremental Effects Averaged Across All States								
	Wealth/Income Ratio							
	0	1	10	30	70	100	1000	$\infty$
State Dependent Mean								
SDV	-0.113	-0.112	-0.107	-0.273	-0.263	-0.210	0.014	0.000
No SDV	-0.099	-0.096	-0.089	-0.098	-0.094	-0.095	-0.015	0.000
State Dependent Volatility								
SDM	-0.581	-0.585	-0.558	-0.487	-0.334	-0.253	-0.060	0.000
No SDM	-0.567	-0.569	-0.539	-0.312	-0.165	-0.138	-0.090	0.000

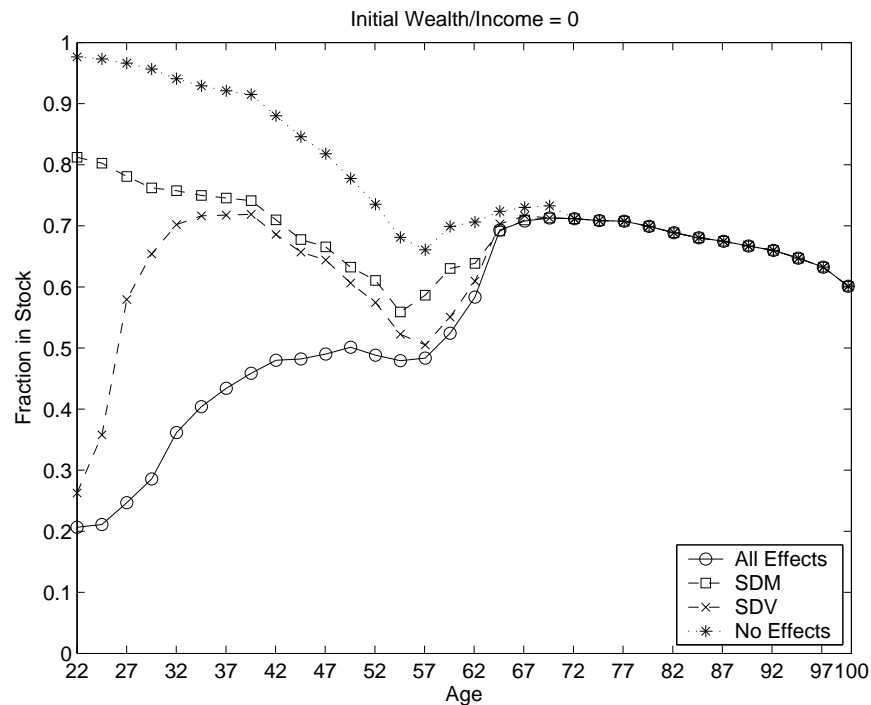


Life-Cycle Permanent Income Profile

FIGURE 1. LIFE-CYCLE PERMANENT INCOME PROFILE,  $\exp(\sum_{i=1}^t \bar{g}_i)$ . The figure provides the calibrated life-cycle permanent income profile. The profile is calibrated to the polynomial-smoothed income profile for college graduates in Cocco, Gomes and Maenhout (2002). The figure plots,  $\exp(\sum_{i=1}^t \bar{g}_i)$  where  $\bar{g}_i$  is the age dependent logarithmic growth rate of permanent income at time  $i$ . The permanent income at age 22 is set to 1. The agent is assumed to retire at 65.

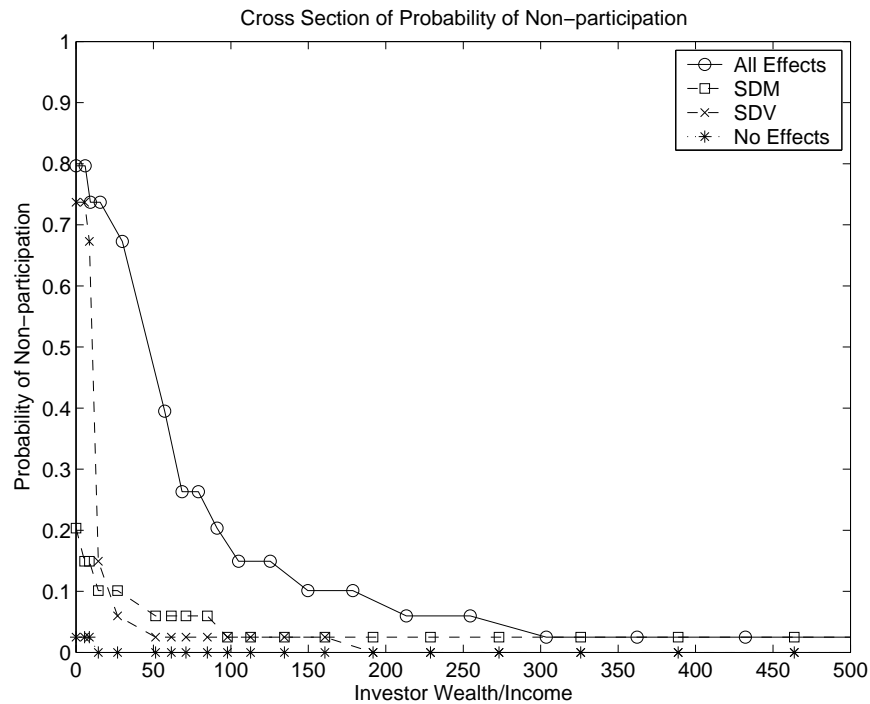


2.a Cross-sectional Fraction in Stock

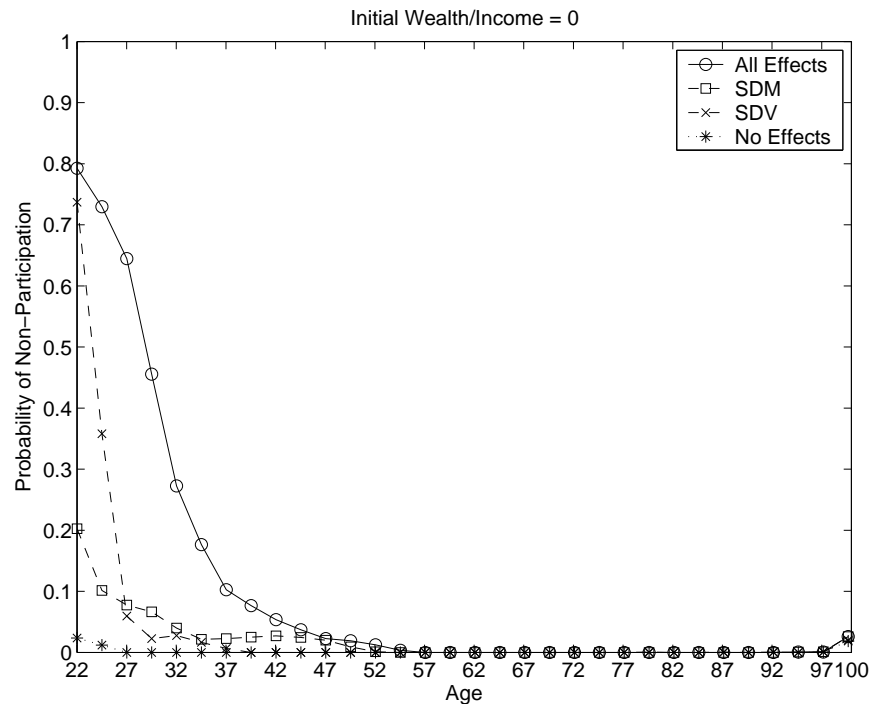


2.b Simulation Fraction in Stock

FIGURE 2. FIRST MONTH CROSS-SECTIONAL and SIMULATION ALLOCATION RESULTS. LIFE CYCLE SPECIFICATION. AGE-DEPENDENT INCOME GROWTH PROFILE. ANNUITY RETIREMENT MONEY. SPECIFICATION-BASED PERMANENT INCOME VOLATILITY. NO TRANSITORY SHOCKS. RETAIL TRADE DATA. The figure reports cross sectional allocation results for a CRRA agent with a coefficient of risk aversion of 6 for the *first* month of her 78 year horizon, for a range of wealth to permanent income ratios of 0 to 500. The figure also reports simulation allocation results through the life cycle. The agent starts work at age 22 and retires at 65, receiving 93.8% of his retirement permanent income until death as reported in Cocco, Gomes and Maenhout (2002). The agent dies with probability 1 at age 100, with death probabilities taken from the U.S. Life Tables, 2001, provided by the NCHS. The mean logarithmic monthly growth rate of permanent income is allowed to be age-dependent and is calibrated to the polynomial-smoothed income profile for college graduates in Cocco, Gomes and Maenhout (2002). The volatility of the logarithmic monthly growth rate of permanent income  $\sigma_g$ , is obtained from its annual counterpart (fixed at 15% from Gakidis (1997)), accounting for the assumed dynamics of the permanent income growth. There are no transitory shocks to income. Retail Trade Income (series CEU4200000004 by Bureau of Labor Statistics) is also used to calibrate labor income as described in Table 1. Agent has access to the market portfolio (value-weighted return on all NYSE, AMEX, and NASDAQ stocks) and to a riskless bond. Calibrations of these assets are as described in section 3 of the text. SDM and SDV denote, respectively, the state-dependent mean and state-dependent volatility channels through which permanent labor income growth can affect stock holdings (see section 2.3 for descriptions). Subfigure 2.a reports cross-sectional average stock holdings for the combinations of channels indicated in the legend. Subfigure 2.b reports average simulation stock holdings for the combinations of channels indicated in its legend, across 500000 paths when initial wealth-income ratio is zero. The calibration of the 4 problems needed to do the comparisons is detailed in section 3 of the text.



3.a Cross-sectional Probability of Non-participation



3.b Simulation Probability of Non-participation

FIGURE 3. FIRST MONTH CROSS-SECTIONAL and SIMULATION NON-PARTICIPATION RESULTS. LIFE CYCLE SPECIFICATION. AGE-DEPENDENT INCOME GROWTH PROFILE. ANNUITY RETIREMENT MONEY. SPECIFICATION-BASED PERMANENT INCOME VOLATILITY. NO TRANSITORY SHOCKS. RETAIL TRADE DATA. The figure reports cross sectional non-participation results for a CRRA agent with a coefficient of risk aversion of 6 for the *first* month of her 78 year horizon, for a range of wealth to permanent income ratios of 0 to 500. The figure also reports simulation non-participation results through the life cycle. The agent starts work at age 22 and retires at 65, receiving 93.8% of his retirement permanent income until death as reported in Cocco, Gomes and Maenhaut (2002). The agent dies with probability 1 at age 100, with death probabilities taken from the U.S. Life Tables, 2001, provided by the NCHS. The mean logarithmic monthly growth rate of permanent income is allowed to be age-dependent and is calibrated to the polynomial-smoothed income profile for college graduates in Cocco, Gomes and Maenhaut (2002). The volatility of the logarithmic monthly growth rate of permanent income  $\sigma_g$ , is obtained from its annual counterpart (fixed at 15% from Gakidis (1997)), accounting for the assumed dynamics of the permanent income growth. There are no transitory shocks to income. Retail Trade Income (series CEU4200000004 by Bureau of Labor Statistics) is also used to calibrate labor income as described in Table 1. Agent has access to the market portfolio (value-weighted return on all NYSE, AMEX, and NASDAQ stocks) and to a riskless bond. Calibrations of these assets are as described in section 3 of the text. SDM and SDV denote, respectively, the state-dependent mean and state-dependent volatility channels through which permanent labor income growth can affect stock holdings (see section 2.3 for descriptions). Subfigure 3.a reports cross-sectional probability of non-participation for the combinations of channels indicated in the legend. Subfigure 3.b reports the simulation probability of non-participation for the combinations of channels indicated in its legend, across 500000 paths when initial wealth-income ratio is zero. The calibration of the 4 problems needed to do the comparisons is detailed in section 3 of the text.

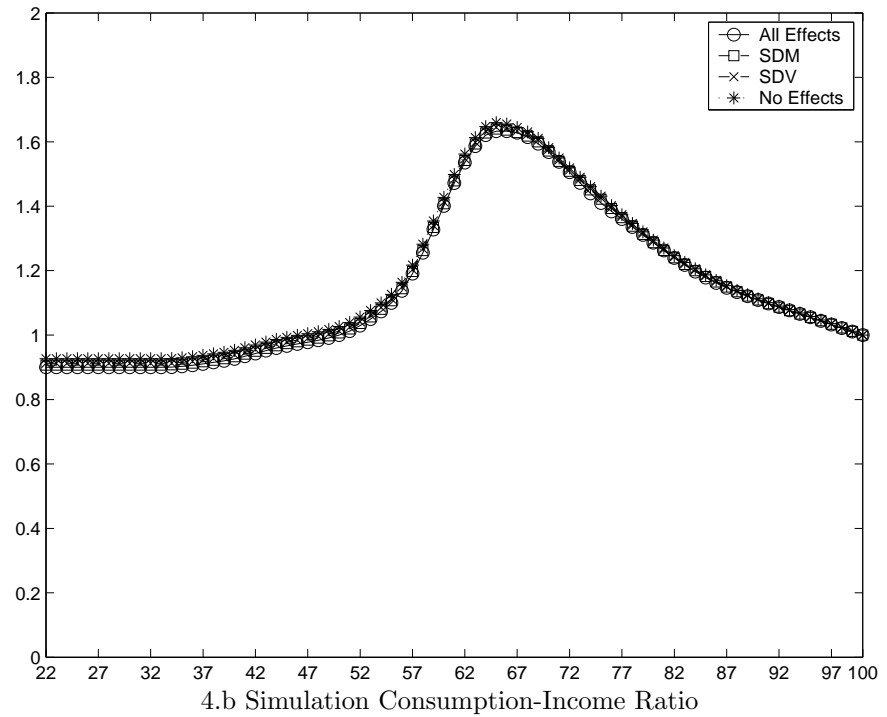
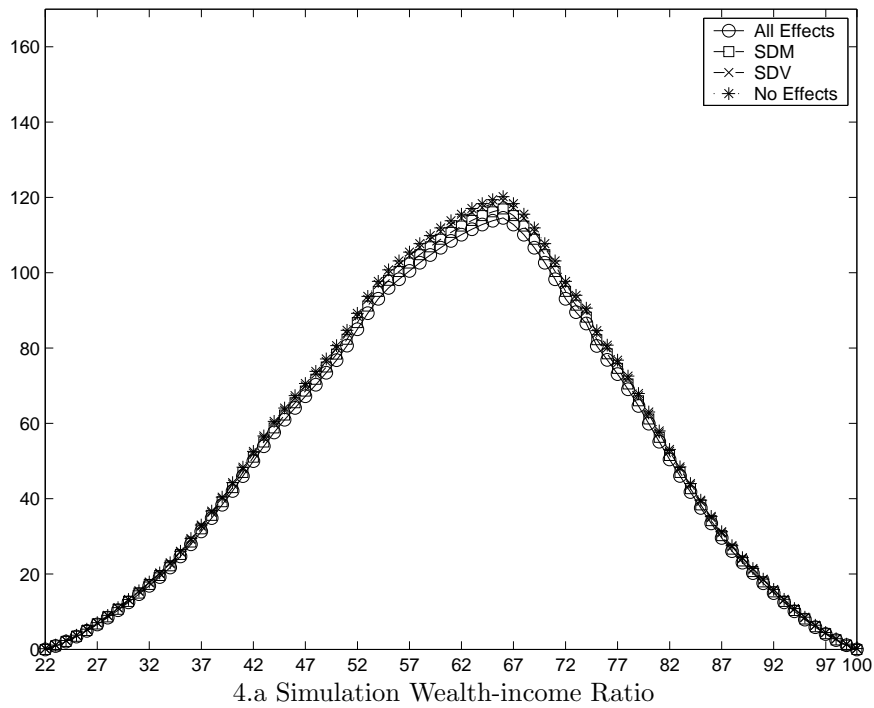
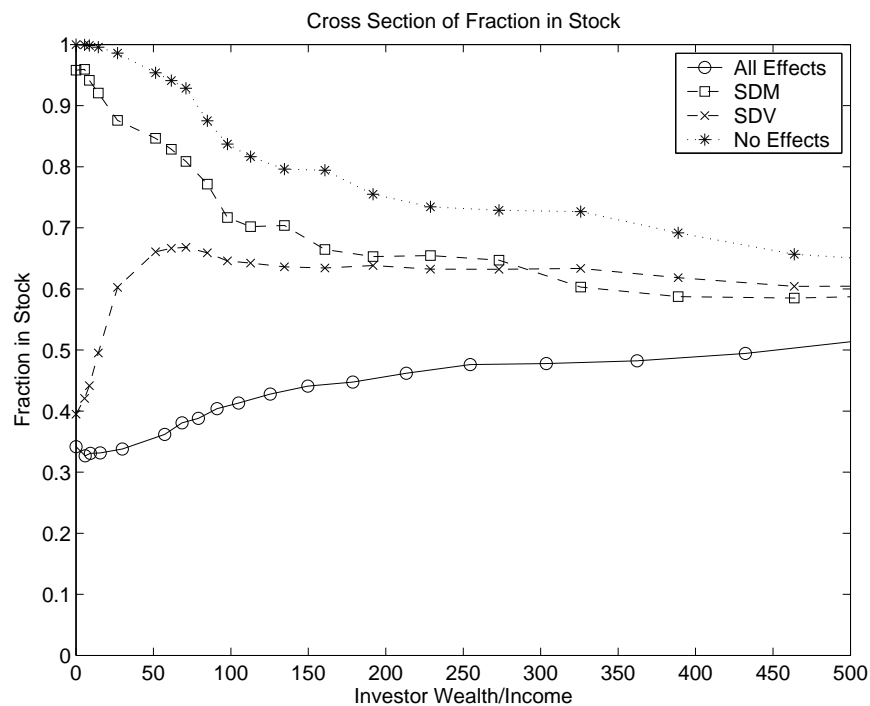
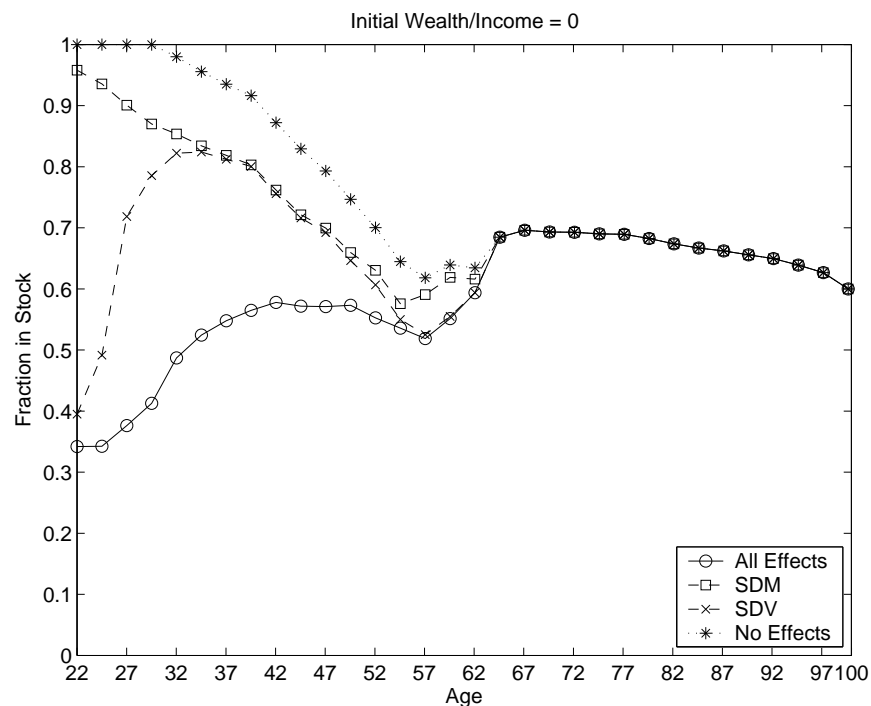


FIGURE 4. SIMULATION WEALTH-INCOME RATIO and CONSUMPTION-INCOME RATIO RESULTS. LIFE CYCLE SPECIFICATION. AGE-DEPENDENT INCOME GROWTH PROFILE. ANNUITY RETIREMENT MONEY. SPECIFICATION-BASED PERMANENT INCOME VOLATILITY. NO TRANSITORY SHOCKS. RETAIL TRADE DATA. The figure reports simulation wealth-income ratio and consumption-income ratio results for a CRRA agent with a coefficient of risk aversion of 6 through the agents working life when initial wealth-income ratio is zero. The agent starts work at age 22 and retires at 65, 5, receiving 93.8% of his retirement permanent income until death as reported in Cocco, Gomes and Maenhaut (2002). The agent dies with probability 1 at age 100, with death probabilities taken from the U.S. Life Tables, 2001, provided by the NCHS. The mean logarithmic monthly growth rate of permanent income is allowed to be age-dependent and is calibrated to the polynomial-smoothed income profile for college graduates in Cocco, Gomes and Maenhaut (2002). The volatility of the logarithmic monthly growth rate of permanent income  $\sigma_g$ , is obtained from its annual counterpart (fixed at 15% from Gakidis (1997)), accounting for the assumed dynamics of the permanent income growth. There are no transitory shocks to income. Retail Trade Income (series CEU4200000004 by Bureau of Labor Statistics) is also used to calibrate labor income as described in Table 1. Agent has access to the market portfolio (value-weighted return on all NYSE, AMEX, and NASDAQ stocks) and to a riskless bond. Calibrations of these assets are as described in section 3 of the text. SDM and SDV denote, respectively, the state-dependent mean and state-dependent volatility channels through which permanent labor income growth can affect stock holdings (see section 2.3 for descriptions). Subfigure 4.a reports simulation average wealth-income ratios for the combinations of channels indicated in the legend, across 500000 paths. Subfigure 4.b reports simulation average consumption-income ratios for the combinations of channels indicated in its legend, across 500000 paths. The calibration of the 4 problems needed to do the comparisons is detailed in section 3 of the text.

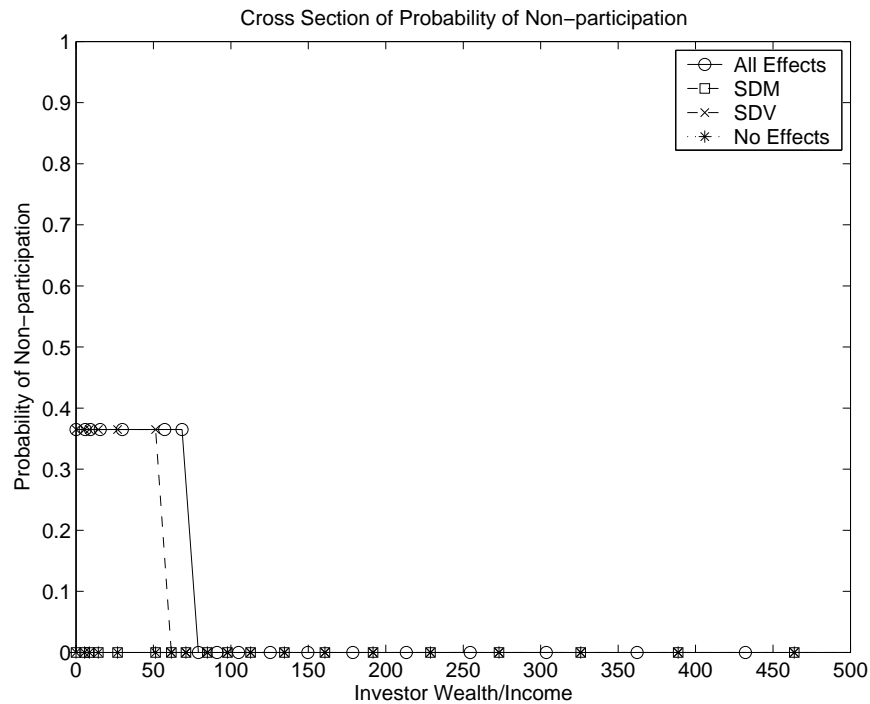


5.a Cross-sectional Fraction in Stock

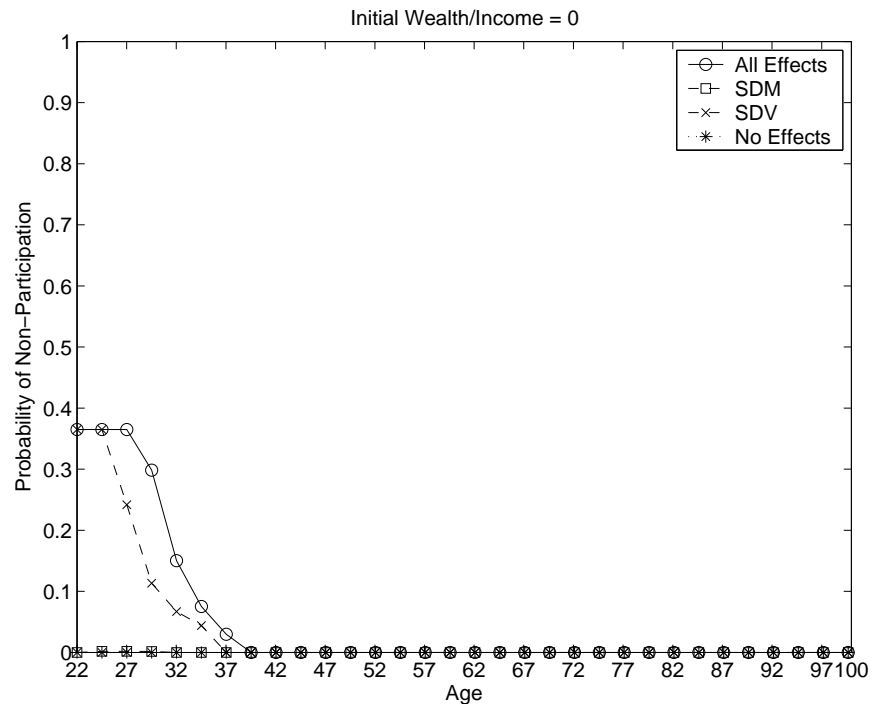


5.b Simulation Fraction in Stock

FIGURE 5. FIRST MONTH CROSS-SECTIONAL and SIMULATION ALLOCATION RESULTS. LIFE CYCLE SPECIFICATION. AGE-DEPENDENT INCOME GROWTH PROFILE. ANNUITY RETIREMENT MONEY. SPECIFICATION-BASED PERMANENT INCOME VOLATILITY. NO TRANSITORY SHOCKS. RETAIL TRADE DATA. NBER SPECIFICATION. IID RETURNS. The figure reports cross sectional allocation results for a CRRA agent with a coefficient of risk aversion of 6 for the *first* month of her 78 year horizon, for a range of wealth to permanent income ratios of 0 to 500. The figure also reports simulation allocation results through the life cycle. The agent starts work at age 22 and retires at 65, receiving 93.8% of his retirement permanent income until death as reported in Cocco, Gomes and Maenhaut (2002). The agent dies with probability 1 at age 100, with death probabilities taken from the U.S. Life Tables, 2001, provided by the NCHS. Returns are iid. The mean logarithmic monthly growth rate of permanent income is allowed to be age-dependent and is calibrated to the polynomial-smoothed income profile for college graduates in Cocco, Gomes and Maenhout (2002). The volatility of the logarithmic monthly growth rate of permanent income  $\sigma_g$ , is obtained from its annual counterpart (fixed at 15% from Gakidis (1997)), accounting for the assumed dynamics of the permanent income growth. There are no transitory shocks to income. Retail Trade Income (series CEU4200000004 by Bureau of Labor Statistics) is also used to calibrate labor income as described in Table 1. Agent has access to the market portfolio (value-weighted return on all NYSE, AMEX, and NASDAQ stocks) and to a riskless bond. Calibrations of these assets are as described in section 3 of the text. SDM and SDV denote, respectively, the state-dependent mean and state-dependent volatility channels through which permanent labor income growth can affect stock holdings (see section 2.3 for descriptions). State dependent volatility channel is implemented using the NBER specification. Subfigure 5.a reports cross-sectional average stock holdings for the combinations of channels indicated in the legend. Subfigure 5.b reports average simulation stock holdings for the combinations of channels indicated in its legend, across 500000 paths when initial wealth-income ratio is zero. The calibration of the 4 problems needed to do the comparisons is detailed in section 3 of the text.

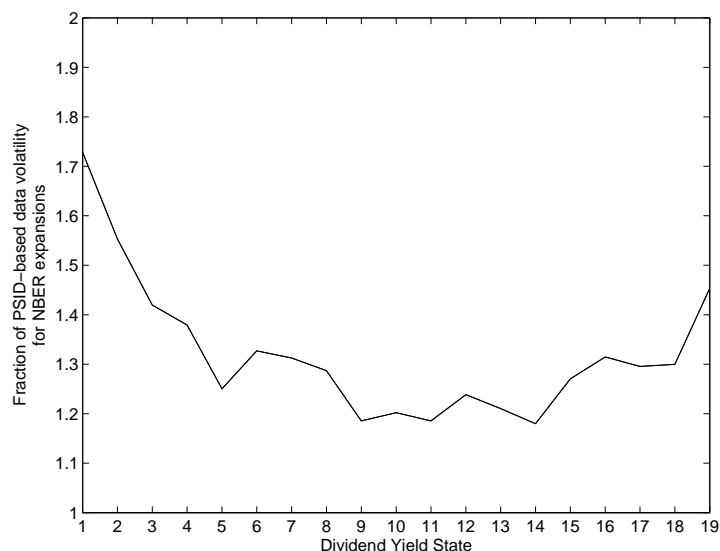


6.a Cross-sectional Probability of Non-participation



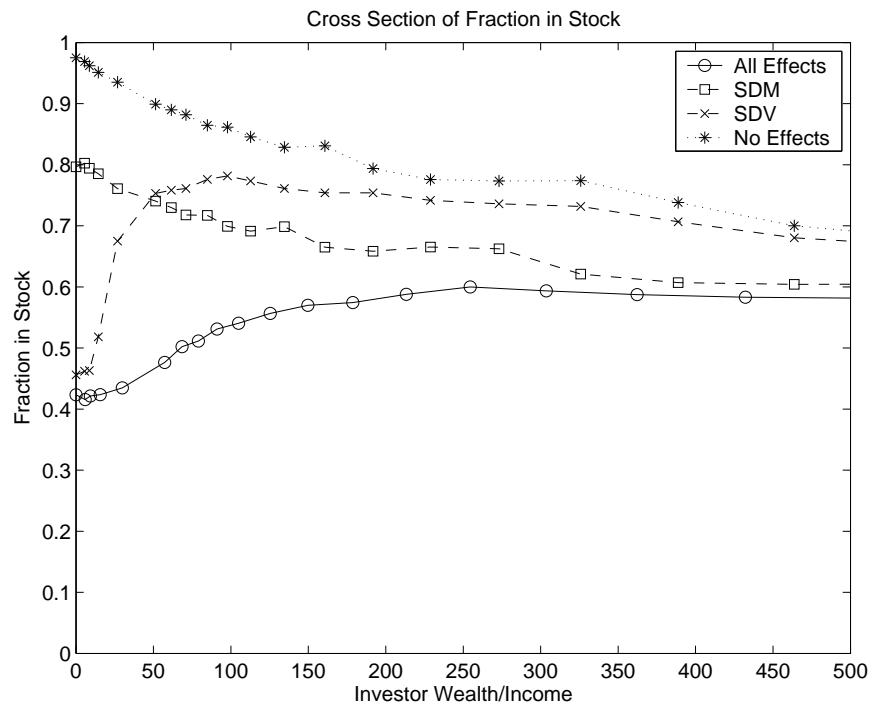
6.b Simulation Probability of Non-participation

FIGURE 6. FIRST MONTH CROSS-SECTIONAL and SIMULATION NON-PARTICIPATION RESULTS. LIFE CYCLE SPECIFICATION. AGE-DEPENDENT INCOME GROWTH PROFILE. ANNUITY RETIREMENT MONEY. SPECIFICATION-BASED PERMANENT INCOME VOLATILITY. NO TRANSITORY SHOCKS. RETAIL TRADE DATA. NBER SPECIFICATION. IID RETURNS. The figure reports cross sectional non-participation results for a CRRA agent with a coefficient of risk aversion of 6 for the *first* month of her 78 year horizon, for a range of wealth to permanent income ratios of 0 to 500. The figure also reports simulation non-participation results through the life cycle. The agent starts work at age 22 and retires at 65, receiving 93.8% of his retirement permanent income until death as reported in Cocco, Gomes and Maenhout (2002). The agent dies with probability 1 at age 100, with death probabilities taken from the U.S. Life Tables, 2001, provided by the NCHS. Returns are iid. The mean logarithmic monthly growth rate of permanent income is allowed to be age-dependent and is calibrated to the polynomial-smoothed income profile for college graduates in Cocco, Gomes and Maenhout (2002). The volatility of the logarithmic monthly growth rate of permanent income  $\sigma_g$ , is obtained from its annual counterpart (fixed at 15% from Gakidis (1997)), accounting for the assumed dynamics of the permanent income growth. There are no transitory shocks to income. Retail Trade Income (series CEU4200000004 by Bureau of Labor Statistics) is also used to calibrate labor income as described in Table 1. Agent has access to the market portfolio (value-weighted return on all NYSE, AMEX, and NASDAQ stocks) and to a riskless bond. Calibrations of these assets are as described in section 3 of the text. SDM and SDV denote, respectively, the state-dependent mean and state-dependent volatility channels through which permanent labor income growth can affect stock holdings (see section 2.3 for descriptions). State dependent volatility channel is implemented using the NBER specification. Subfigure 6.a reports cross-sectional probability of non-participation for the combinations of channels indicated in the legend. Subfigure 6.b reports the simulation probability of non-participation for the combinations of channels indicated in its legend, across 500000 paths when initial wealth-income ratio is zero. The calibration of the 4 problems needed to do the comparisons is detailed in section 3 of the text.

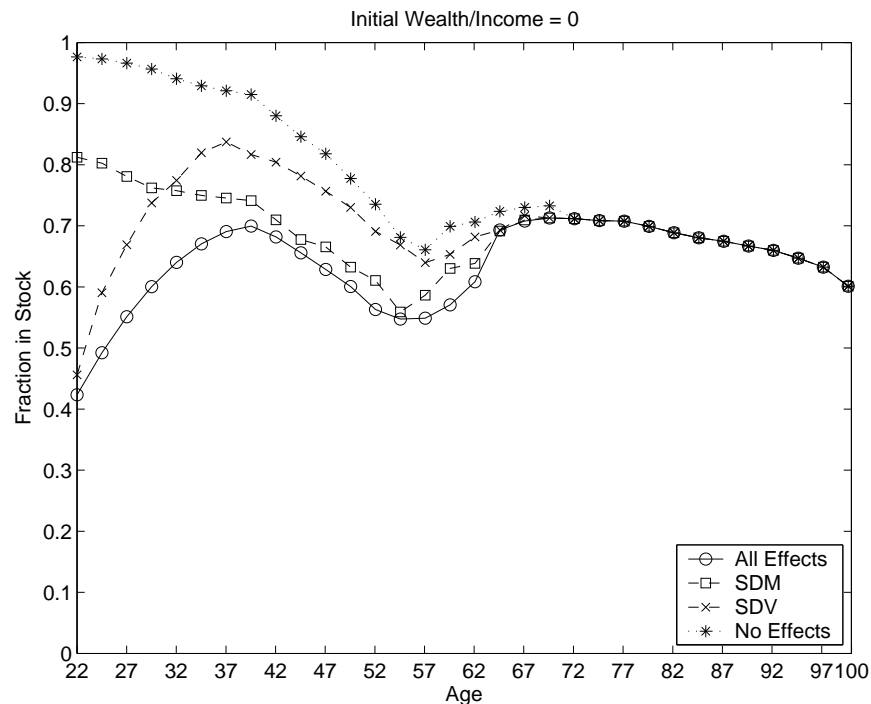


Life-Cycle Permanent Income Profile

FIGURE 7. THE PERMANENT LABOR INCOME GROWTH VOLATILITY CONDITIONAL ON THE DIVIDEND YIELD NODE AS A FRACTION OF THE PSID-BASED DATA VOLATILITY FOR NBER EXPANSIONS. The figure reports the permanent labor income growth volatility conditional on the dividend yield state as a fraction of the PSID based data volatility for NBER expansions. Storesletten, Telmer and Yaron (2004) find that the conditional volatility of permanent labor income growth is 1.75 higher in recessions than expansions, using NBER business-cycle cutoff to define the two. Storesletten, Telmer and Yaron's business cycle specification implies a 68% probability of expansion and 32% probability of recession. We bifurcate dividend yield variable to obtain recession and expansions states with the cutoff value chosen to match these unconditional probabilities. The conditional volatility for the expansion state is then calculated to match the unconditional volatility of 15% which results in 4.062%. We also implement a procedure, described in section x.xx of the paper, allowing the conditional permanent income growth volatilities for each of the 19 dividend yield states to be different. The figure reports the 19 state by state conditional permanent income growth volatilities as a fraction of the 4.062%, the conditional permanent income growth volatility based on the bifurcating of the dividend yield states.

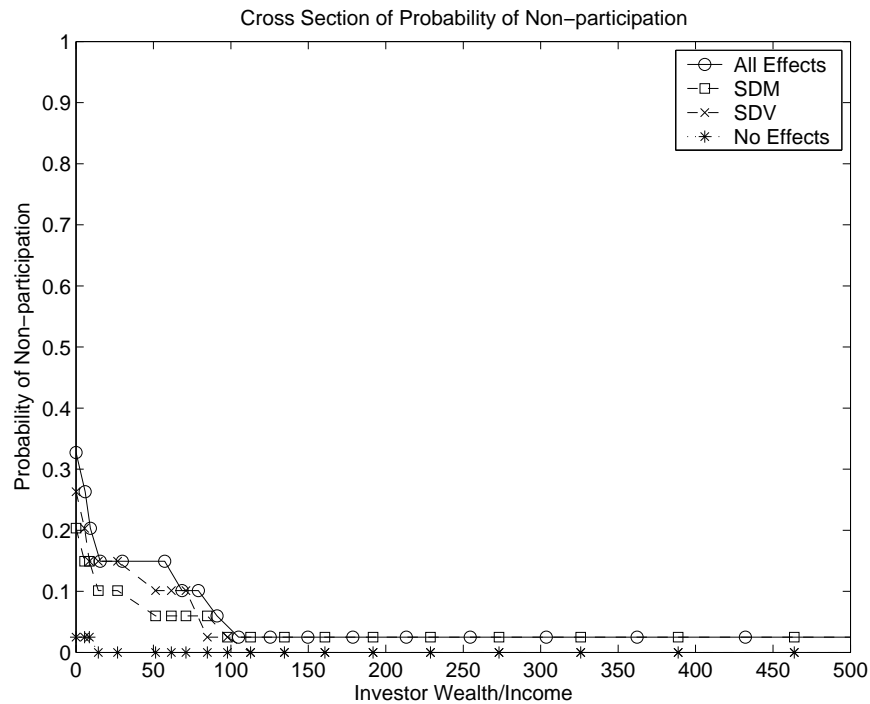


8.a Cross-sectional Fraction in Stock

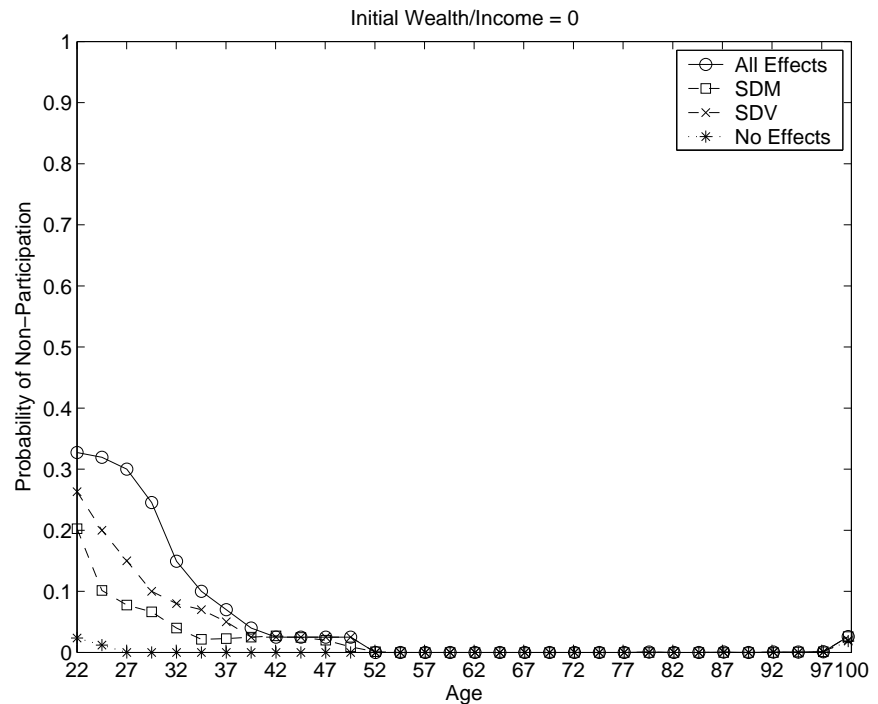


8.b Simulation Fraction in Stock

FIGURE 8. FIRST MONTH CROSS-SECTIONAL and SIMULATION ALLOCATION RESULTS. LIFE CYCLE SPECIFICATION. AGE-DEPENDENT INCOME GROWTH PROFILE. ANNUITY RETIREMENT MONEY. SPECIFICATION-BASED PERMANENT INCOME VOLATILITY. NO TRANSITORY SHOCKS. RETAIL TRADE DATA. NODE BY NODE STATE DEPENDENT VOLATILITY SPECIFICATION. The figure reports cross sectional allocation results for a CRRA agent with a coefficient of risk aversion of 6 for the *first* month of her 78 year horizon, for a range of wealth to permanent income ratios of 0 to 500. The figure also reports simulation allocation results through the life cycle. The agent starts work at age 22 and retires at 65, receiving 93.8% of his retirement permanent income until death as reported in Cocco, Gomes and Maenhout (2002). The agent dies with probability 1 at age 100, with death probabilities taken from the U.S. Life Tables, 2001, provided by the NCHS. The mean logarithmic monthly growth rate of permanent income is allowed to be age-dependent and is calibrated to the polynomial-smoothed income profile for college graduates in Cocco, Gomes and Maenhout (2002). The volatility of the logarithmic monthly growth rate of permanent income  $\sigma_g$ , is obtained from its annual counterpart (fixed at 15% from Gakidis (1997)), accounting for the assumed dynamics of the permanent income growth. There are no transitory shocks to income. Retail Trade Income (series CEU420000004 by Bureau of Labor Statistics) is also used to calibrate labor income as described in Table 1. Agent has access to the market portfolio (value-weighted return on all NYSE, AMEX, and NASDAQ stocks) and to a riskless bond. Calibrations of these assets are as described in section 3 of the text. SDM and SDV denote, respectively, the state-dependent mean and state-dependent volatility channels through which permanent labor income growth can affect stock holdings (see section 2.3 for descriptions). State dependent volatility channel is implemented allowing each dividend yield state to have a different permanent income growth volatility, as described in section 3. Subfigure 8.a reports cross-sectional average stock holdings for the combinations of channels indicated in the legend. Subfigure 8.b reports average simulation stock holdings for the combinations of channels indicated in its legend, across 500000 paths when initial wealth-income ratio is zero. The calibration of the 4 problems needed to do the comparisons is detailed in section 3 of the text.

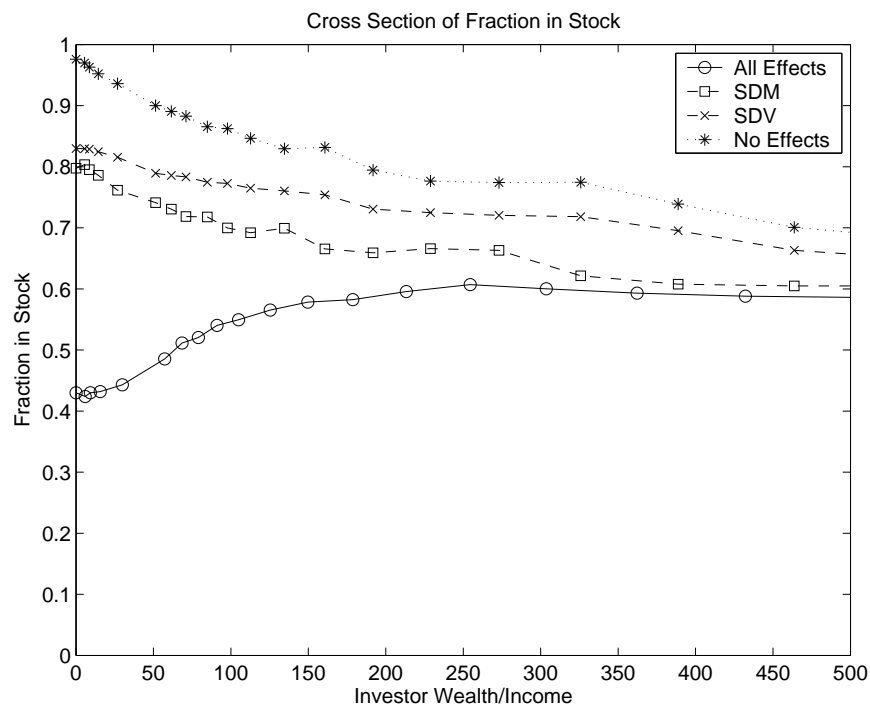


9.a Cross-sectional Probability of Non-participation

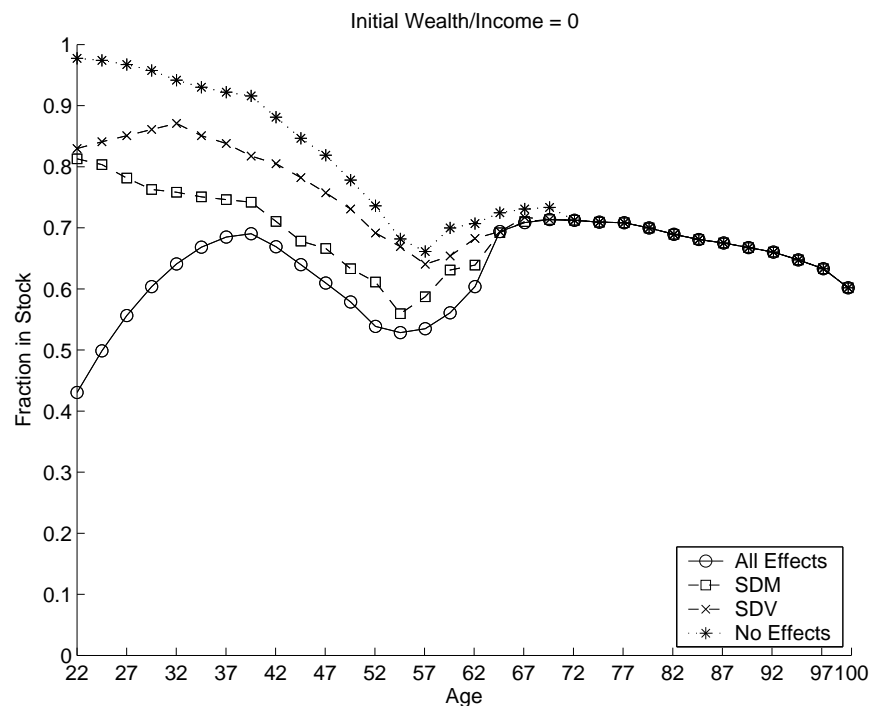


9.b Simulation Probability of Non-participation

FIGURE 9. FIRST MONTH CROSS-SECTIONAL and SIMULATION NON-PARTICIPATION RESULTS. LIFE CYCLE SPECIFICATION. AGE-DEPENDENT INCOME GROWTH PROFILE. ANNUITY RETIREMENT MONEY. SPECIFICATION-BASED PERMANENT INCOME VOLATILITY. NO TRANSITORY SHOCKS. RETAIL TRADE DATA. NODE BY NODE STATE DEPENDENT VOLATILITY SPECIFICATION. The figure reports cross sectional non-participation results for a CRRA agent with a coefficient of risk aversion of 6 for the *first* month of her 78 year horizon, for a range of wealth to permanent income ratios of 0 to 500. The figure also reports simulation non-participation results through the life cycle. The agent starts work at age 22 and retires at 65, receiving 93.8% of his retirement permanent income until death as reported in Cocco, Gomes and Maenhout (2002). The agent dies with probability 1 at age 100, with death probabilities taken from the U.S. Life Tables, 2001, provided by the NCHS. The mean logarithmic monthly growth rate of permanent income is allowed to be age-dependent and is calibrated to the polynomial-smoothed income profile for college graduates in Cocco, Gomes and Maenhout (2002). The volatility of the logarithmic monthly growth rate of permanent income  $\sigma_g$ , is obtained from its annual counterpart (fixed at 15% from Gakidis (1997)), accounting for the assumed dynamics of the permanent income growth. There are no transitory shocks to income. Retail Trade Income (series CEU4200000004 by Bureau of Labor Statistics) is also used to calibrate labor income as described in Table 1. Agent has access to the market portfolio (value-weighted return on all NYSE, AMEX, and NASDAQ stocks) and to a riskless bond. Calibrations of these assets are as described in section 3 of the text. SDM and SDV denote, respectively, the state-dependent mean and state-dependent volatility channels through which permanent labor income growth can affect stock holdings (see section 2.3 for descriptions). State dependent volatility channel is implemented allowing each dividend yield state to have a different permanent income growth volatility, as described in section 3. Subfigure 9.a reports cross-sectional probability of non-participation for the combinations of channels indicated in the legend. Subfigure 9.b reports the simulation probability of non-participation for the combinations of channels indicated in its legend, across 500000 paths when initial wealth-income ratio is zero. The calibration of the 4 problems needed to do the comparisons is detailed in section 3 of the text.

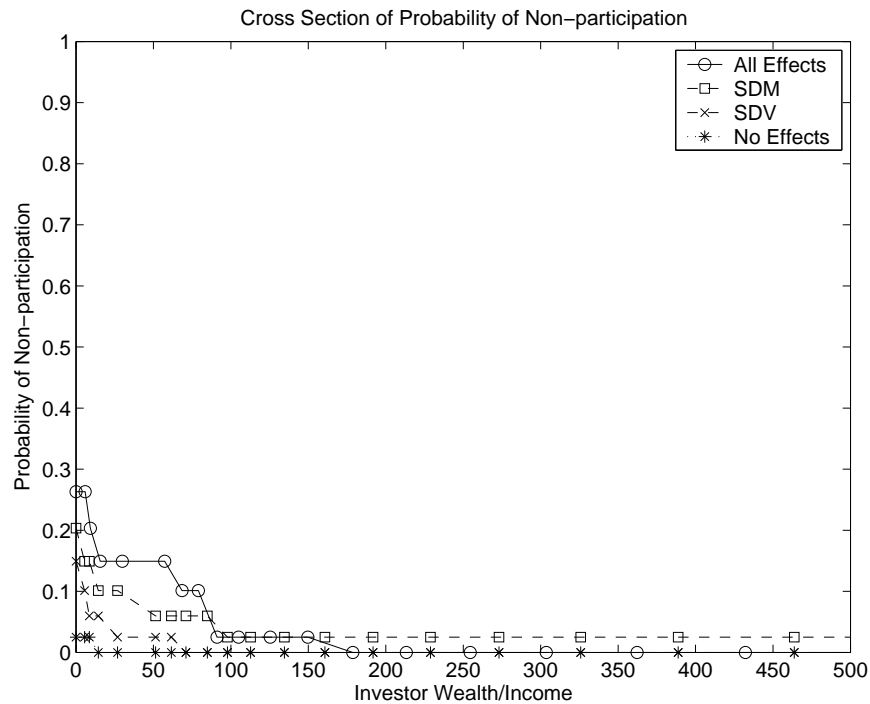


10.a Cross-sectional Fraction in Stock

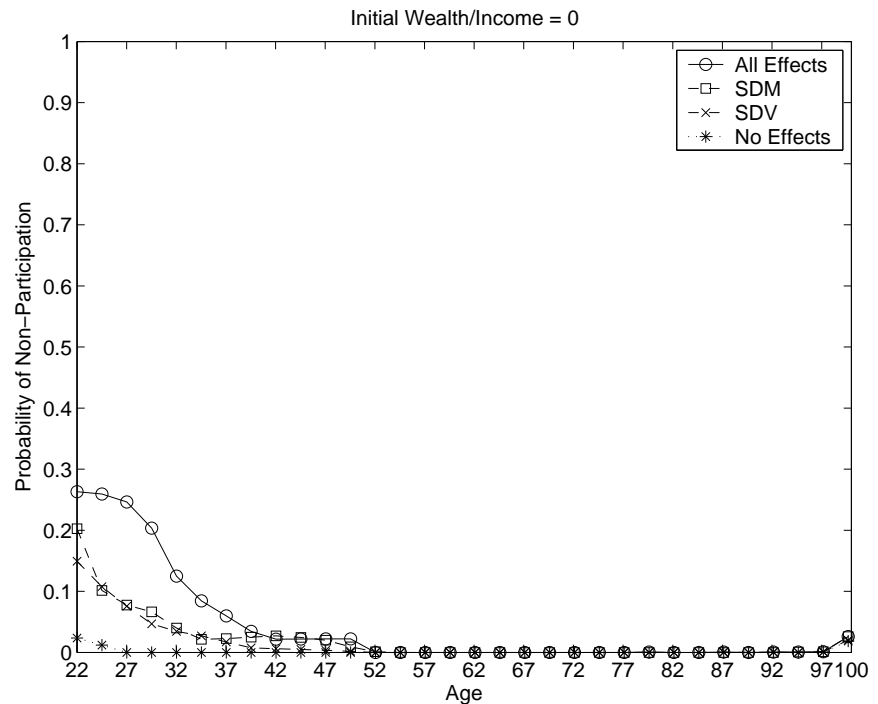


10.b Simulation Fraction in Stock

FIGURE 10. FIRST MONTH CROSS-SECTIONAL and SIMULATION ALLOCATION RESULTS. LIFE CYCLE SPECIFICATION. AGE-DEPENDENT INCOME GROWTH PROFILE. ANNUITY RETIREMENT MONEY. SPECIFICATION-BASED PERMANENT INCOME VOLATILITY. NO TRANSITORY SHOCKS. RETAIL TRADE DATA. CONSERVATIVE STATE DEPENDENT VOLATILITY SPECIFICATION. The figure reports cross sectional allocation results for a CRRA agent with a coefficient of risk aversion of 6 for the *first* month of her 78 year horizon, for a range of wealth to permanent income ratios of 0 to 500. The figure also reports simulation allocation results through the life cycle. The agent starts work at age 22 and retires at 65, receiving 93.8% of his retirement permanent income until death as reported in Cocco, Gomes and Maenhaut (2002). The agent dies with probability 1 at age 100, with death probabilities taken from the U.S. Life Tables, 2001, provided by the NCHS. The mean logarithmic monthly growth rate of permanent income is allowed to be age-dependent and is calibrated to the polynomial-smoothed income profile for college graduates in Cocco, Gomes and Maenhaut (2002). The volatility of the logarithmic monthly growth rate of permanent income  $\sigma_g$ , is obtained from its annual counterpart (fixed at 15% from Gakidis (1997)), accounting for the assumed dynamics of the permanent income growth. There are no transitory shocks to income. Retail Trade Income (series CEU4200000004 by Bureau of Labor Statistics) is also used to calibrate labor income as described in Table 1. Agent has access to the market portfolio (value-weighted return on all NYSE, AMEX, and NASDAQ stocks) and to a riskless bond. Calibrations of these assets are as described in section 3 of the text. SDM and SDV denote, respectively, the state-dependent mean and state-dependent volatility channels through which permanent labor income growth can affect stock holdings (see section 2.3 for descriptions). State dependent volatility channel is implemented using the conservative volatility ratio. Subfigure 10.a reports cross-sectional average stock holdings for the combinations of channels indicated in the legend. Subfigure 10.b reports average simulation stock holdings for the combinations of channels indicated in its legend, across 500000 paths when initial wealth-income ratio is zero. The calibration of the 4 problems needed to do the comparisons is detailed in section 3 of the text.

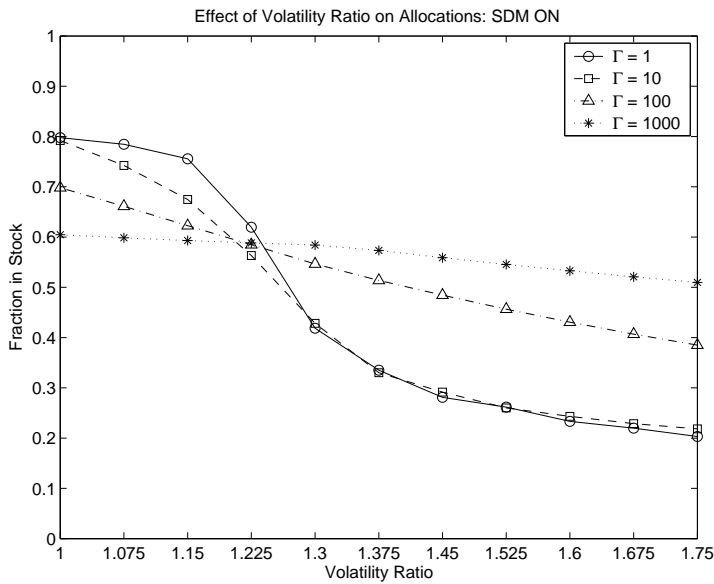


11.a Cross-sectional Probability of Non-participation

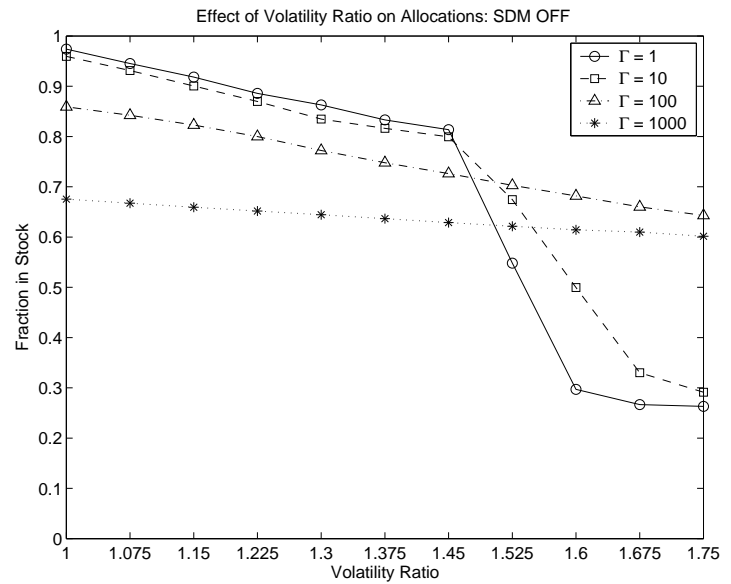


11.b Simulation Probability of Non-participation

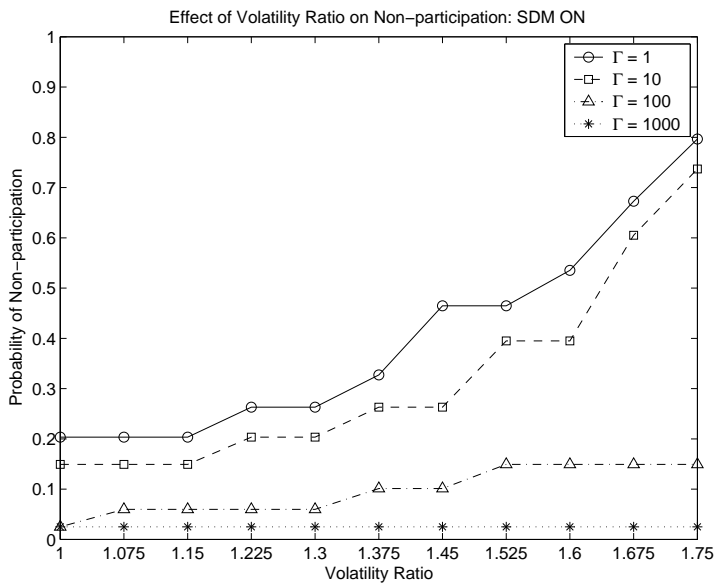
FIGURE 11. FIRST MONTH CROSS-SECTIONAL and SIMULATION NON-PARTICIPATION RESULTS. LIFE CYCLE SPECIFICATION. AGE-DEPENDENT INCOME GROWTH PROFILE. ANNUITY RETIREMENT MONEY. SPECIFICATION-BASED PERMANENT INCOME VOLATILITY. NO TRANSITORY SHOCKS. RETAIL TRADE DATA. CONSERVATIVE STATE DEPENDENT VOLATILITY SPECIFICATION. The figure reports cross sectional non-participation results for a CRRA agent with a coefficient of risk aversion of 6 for the *first* month of her 78 year horizon, for a range of wealth to permanent income ratios of 0 to 500. The figure also reports simulation non-participation results through the life cycle. The agent starts work at age 22 and retires at 65, receiving 93.8% of his retirement permanent income until death as reported in Cocco, Gomes and Maenhout (2002). The agent dies with probability 1 at age 100, with death probabilities taken from the U.S. Life Tables, 2001, provided by the NCHS. The mean logarithmic monthly growth rate of permanent income is allowed to be age-dependent and is calibrated to the polynomial-smoothed income profile for college graduates in Cocco, Gomes and Maenhout (2002). The volatility of the logarithmic monthly growth rate of permanent income  $\sigma_g$ , is obtained from its annual counterpart (fixed at 15% from Gakidis (1997)), accounting for the assumed dynamics of the permanent income growth. There are no transitory shocks to income. Retail Trade Income (series CEU4200000004 by Bureau of Labor Statistics) is also used to calibrate labor income as described in Table 1. Agent has access to the market portfolio (value-weighted return on all NYSE, AMEX, and NASDAQ stocks) and to a riskless bond. Calibrations of these assets are as described in section 3 of the text. SDM and SDV denote, respectively, the state-dependent mean and state-dependent volatility channels through which permanent labor income growth can affect stock holdings (see section 2.3 for descriptions). State dependent volatility channel is implemented using the conservative volatility ratio. Subfigure 11.a reports cross-sectional probability of non-participation for the combinations of channels indicated in the legend. Subfigure 11.b reports the simulation probability of non-participation for the combinations of channels indicated in its legend, across 500000 paths when initial wealth-income ratio is zero. The calibration of the 4 problems needed to do the comparisons is detailed in section 3 of the text.



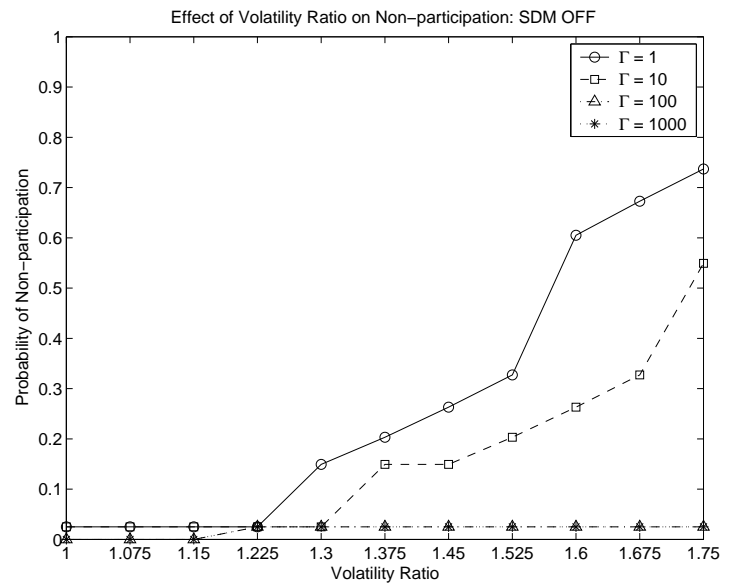
12.a Cross-sectional Fraction in Stock : SDM



12.b Cross-Sectional Fraction in Stock: No SDM

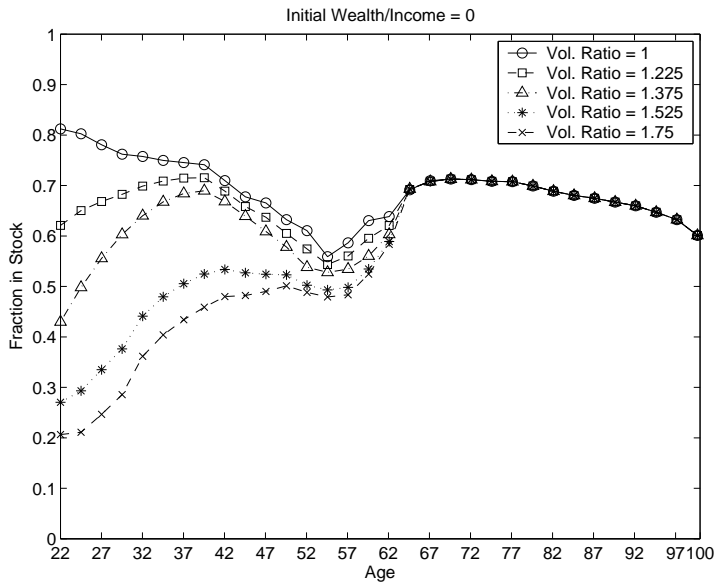


12.c Cross-sectional Probability of Non-participation : SDM

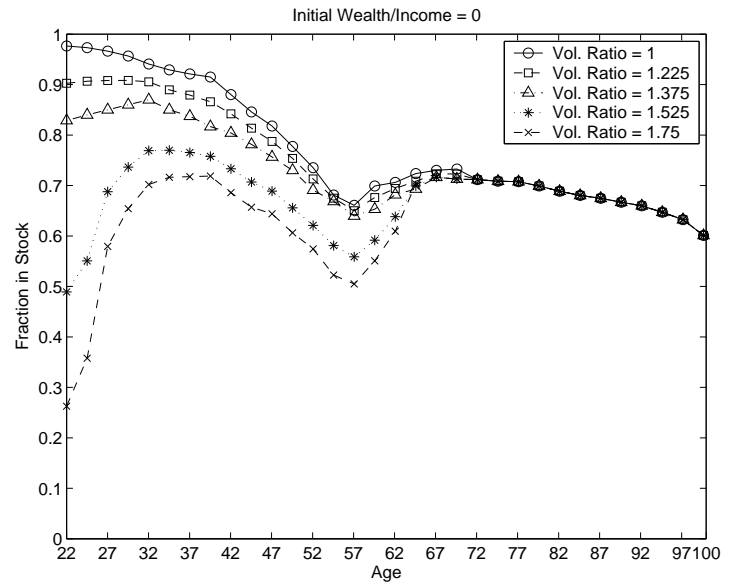


12.d Cross-sectional Probability of Non-participation : No SDM

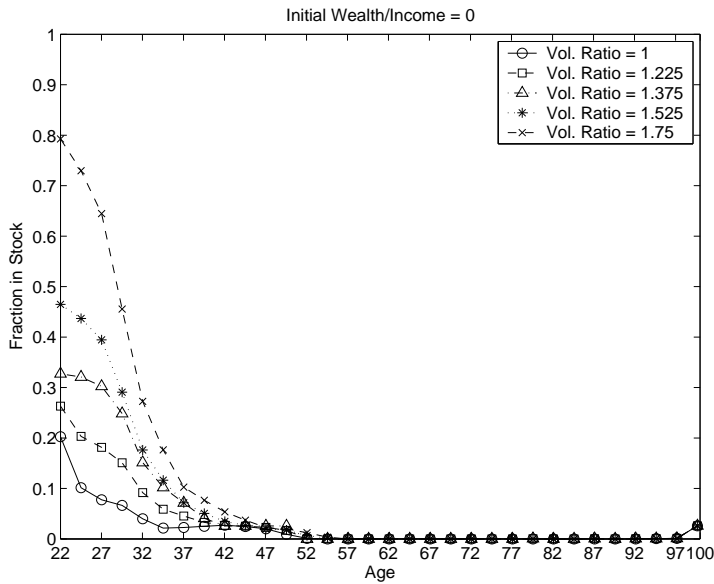
FIGURE 12. VARYING THE VOLATILITY RATIO FROM 1 to 1.75. CROSS-SECTIONAL ALLOCATION and NON-PARTICIPATION RESULTS. LIFE CYCLE SPECIFICATION. AGE-DEPENDENT INCOME GROWTH PROFILE. ANNUITY RETIREMENT MONEY. SPECIFICATION BASED PERMANENT INCOME VOLATILITY. NO TRANSITORY SHOCKS. RETAIL TRADE DATA. The figure reports the cross-sectional allocation and non-participation results for the first month of the agent's 78 year horizon for wealth income ratios of 1, 10, 100 and 1000 as the volatility ratio in the State Dependent Volatility Channel (SDV) varies from 1 to 1.75. Subfigures *a* and *c* report cross-sectional fractions in stock and probability of non-participation respectively when the State Dependent Mean Channel (SDM) is also present. Subfigures *b* and *d* report the same when SDM is not present. For descriptions of SDM and SDV see section 2.3. Results are for a CRRA agent with a coefficient of risk aversion of 6. The agent starts work at age 22 and retires at 65, receiving 93.8% of his retirement permanent until death as reported in Cocco, Gomes and Maenhaut (2002). The agent dies with probability 1 at age 100, with death probabilities taken from the U.S. Life Tables, 2001, provided by the NCHS. The mean logarithmic monthly growth rate of permanent income is allowed to be age-dependent and is calibrated to the polynomial-smoothed income profile for college graduates in Cocco, Gomes and Maenhaut (2002). The volatility of the logarithmic monthly growth rate of permanent income  $\sigma_g$ , is obtained from its annual counterpart (fixed at 15% from Gakidis (1997)), accounting for the assumed dynamics of the permanent income growth. There are no transitory shocks to income. Retail Trade Income (series CEU4200000004 by Bureau of Labor Statistics) is also used to calibrate labor income as described in Table 1. Agent has access to the market portfolio (value-weighted return on all NYSE, AMEX, and NASDAQ stocks) and to a riskless bond. Calibrations of these assets are as described in section 3 of the text. The calibration of the problems needed to do the comparisons is detailed in section 3 of the text.



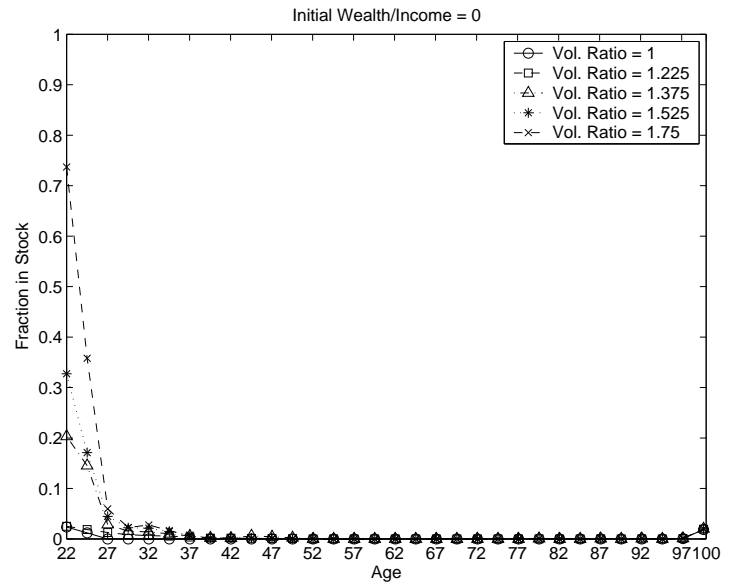
13.a Simulation Fraction in Stock : SDM



13.b Simulation Fraction in Stock: No SDM

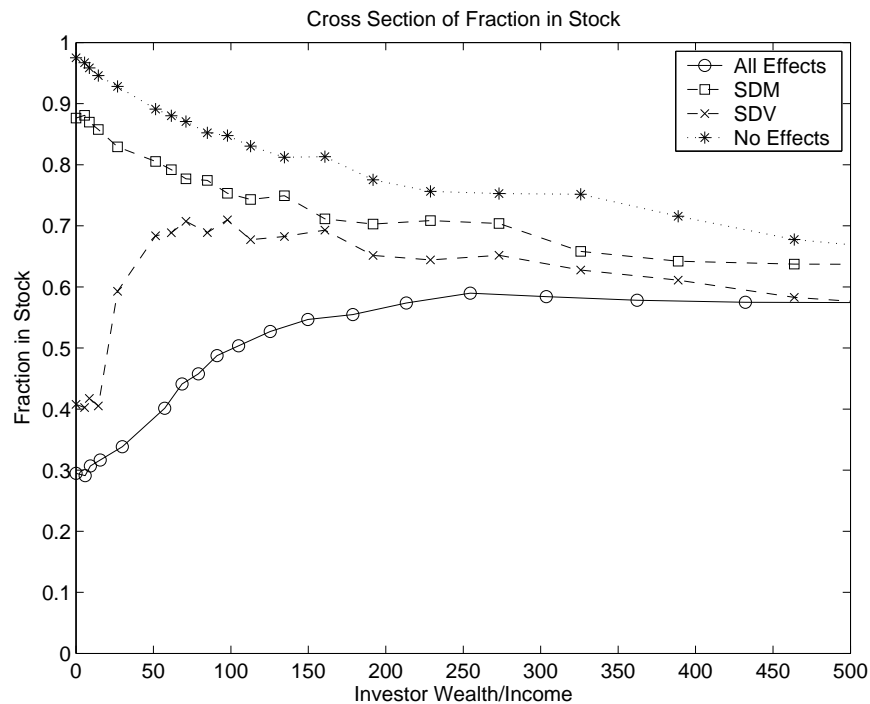


13.c Simulation Probability of Non-participation : SDM

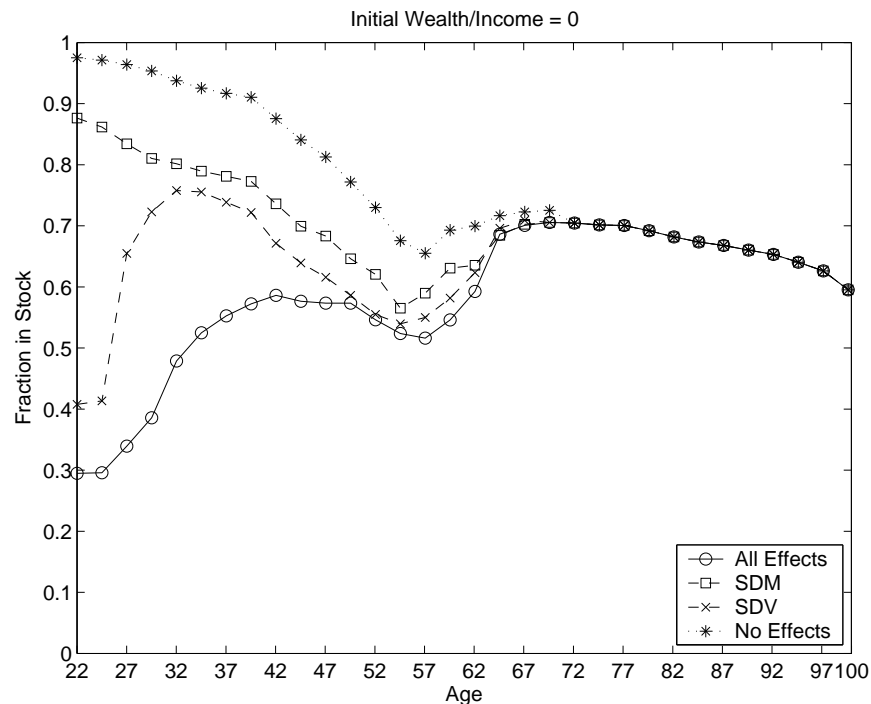


13.d Simulation Probability of Non-participation : No SDM

FIGURE 13. VARYING THE VOLATILITY RATIO FROM 1 to 1.75. SIMULATION ALLOCATION and NON-PARTICIPATION RESULTS. LIFE CYCLE SPECIFICATION. AGE-DEPENDENT INCOME GROWTH PROFILE. ANNUITY RETIREMENT MONEY. SPECIFICATION BASED PERMANENT INCOME VOLATILITY. NO TRANSITORY SHOCKS. RETAIL TRADE DATA. The figure reports the simulation allocation and non-participation results for the agent's life cycle as the volatility ratio in the State Dependent Volatility Channel (SDV) varies from 1 to 1.75. Subfigures *a* and *c* report average fractions in stock and probability of non-participation respectively when the State Dependent Mean Channel (SDM) is also present across 500000 paths. Subfigures *b* and *d* report the same when SDM is not present. For descriptions of SDM and SDV see section 2.3. Initial wealth to permanent income ratio is always set to 0. Results are for a CRRA agent with a coefficient of risk aversion of 6. The agent starts work at age 22 and retires at 65, receiving 93.8% of his retirement permanent until death as reported in Cocco, Gomes and Maenhaut (2002). The agent dies with probability 1 at age 100, with death probabilities taken from the U.S. Life Tables, 2001, provided by the NCHS. The mean logarithmic monthly growth rate of permanent income is allowed to be age-dependent and is calibrated to the polynomial-smoothed income profile for college graduates in Cocco, Gomes and Maenhaut (2002). The volatility of the logarithmic monthly growth rate of permanent income  $\sigma_g$ , is obtained from its annual counterpart (fixed at 15% from Gakidis (1997)), accounting for the assumed dynamics of the permanent income growth. There are no transitory shocks to income. Retail Trade Income (series CEU4200000004 by Bureau of Labor Statistics) is also used to calibrate labor income as described in Table 1. Agent has access to the market portfolio (value-weighted return on all NYSE, AMEX, and NASDAQ stocks) and to a riskless bond. Calibrations of these assets are as described in section 3 of the text. The calibration of the problems needed to do the comparisons is detailed in section 3 of the text.

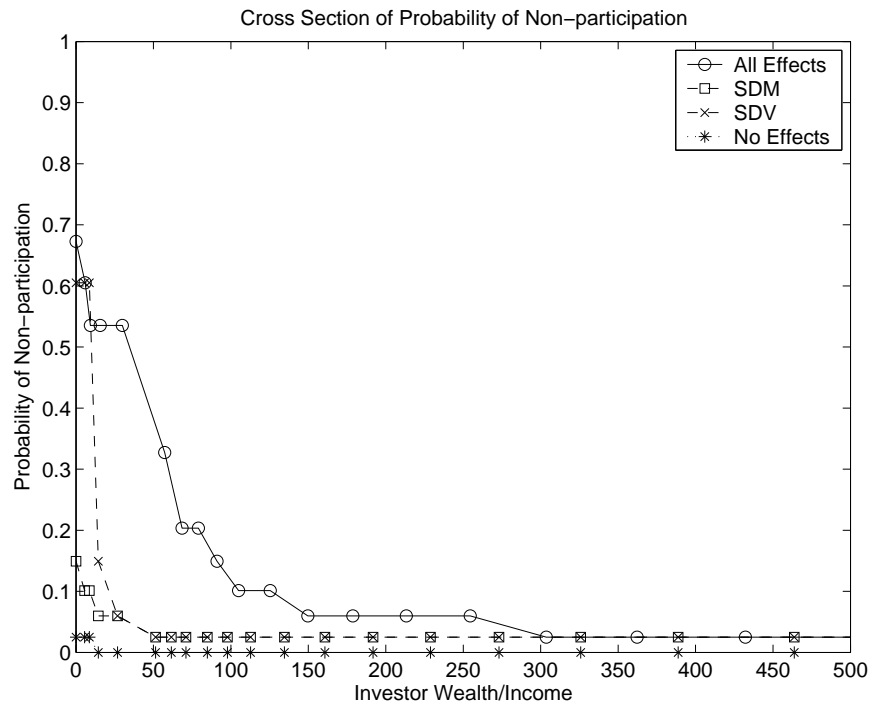


14.a Cross-sectional Fraction in Stock

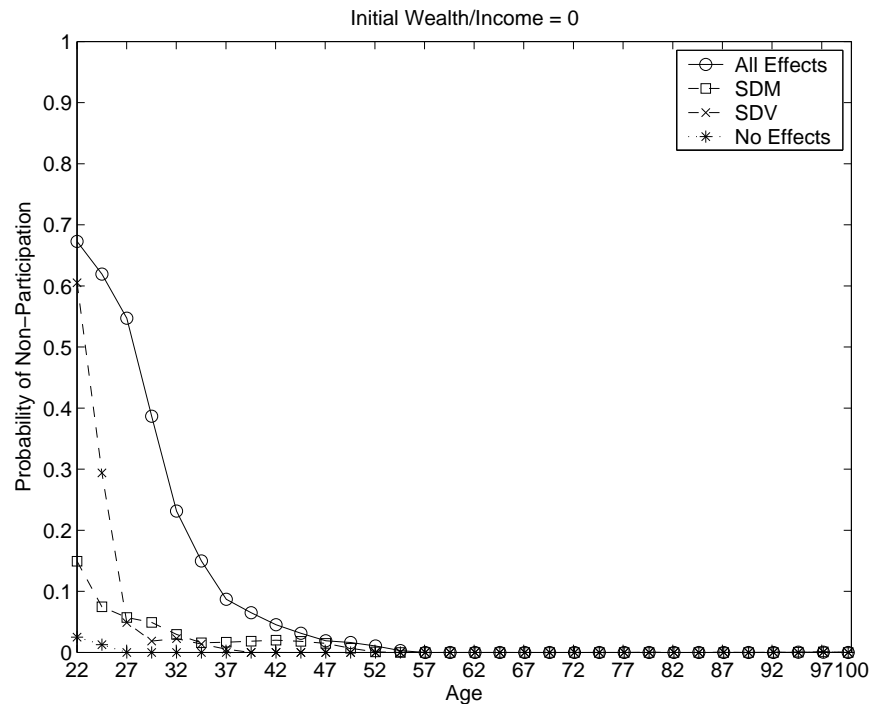


15.b Simulation Fraction in Stock

FIGURE 14. FIRST MONTH CROSS-SECTIONAL and SIMULATION ALLOCATION RESULTS. LIFE CYCLE SPECIFICATION. AGE-DEPENDENT INCOME GROWTH PROFILE. ANNUITY RETIREMENT MONEY. SPECIFICATION-BASED PERMANENT INCOME VOLATILITY. NO TRANSITORY SHOCKS. RETAIL TRADE DATA. IID RETURNS. The figure reports cross sectional allocation results for a CRRA agent with a coefficient of risk aversion of 6 for the *first* month of her 78 year horizon, for a range of wealth to permanent income ratios of 0 to 500. Returns are iid. The figure also reports simulation allocation results through the life cycle. The agent starts work at age 22 and retires at 65, receiving 93.8% of his retirement permanent income until death as reported in Cocco, Gomes and Maenhout (2002). The agent dies with probability 1 at age 100, with death probabilities taken from the U.S. Life Tables, 2001, provided by the NCHS. The mean logarithmic monthly growth rate of permanent income is allowed to be age-dependent and is calibrated to the polynomial-smoothed income profile for college graduates in Cocco, Gomes and Maenhout (2002). The volatility of the logarithmic monthly growth rate of permanent income  $\sigma_g$ , is obtained from its annual counterpart (fixed at 15% from Gakidis (1997)), accounting for the assumed dynamics of the permanent income growth. There are no transitory shocks to income. Retail Trade Income (series CEU4200000004 by Bureau of Labor Statistics) is also used to calibrate labor income as described in Table 1. Agent has access to the market portfolio (value-weighted return on all NYSE, AMEX, and NASDAQ stocks) and to a riskless bond. Calibrations of these assets are as described in section 3 of the text. SDM and SDV denote, respectively, the state-dependent mean and state-dependent volatility channels through which permanent labor income growth can affect stock holdings (see section 2.3 for descriptions). Subfigure 14.a reports cross-sectional average stock holdings for the combinations of channels indicated in the legend. Subfigure 14.b reports average simulation stock holdings for the combinations of channels indicated in its legend, across 500000 paths when initial wealth-income ratio is zero. The calibration of the 4 problems needed to do the comparisons is detailed in section 3 of the text.



15.a Cross-sectional Probability of Non-participation



15.b Simulation Probability of Non-participation

FIGURE 15. FIRST MONTH CROSS-SECTIONAL and SIMULATION NON-PARTICIPATION RESULTS. LIFE CYCLE SPECIFICATION. AGE-DEPENDENT INCOME GROWTH PROFILE. ANNUITY RETIREMENT MONEY. SPECIFICATION-BASED PERMANENT INCOME VOLATILITY. NO TRANSITORY SHOCKS. RETAIL TRADE DATA. The figure reports cross sectional non-participation results for a CRRA agent with a coefficient of risk aversion of 6 for the *first* month of her 78 year horizon, for a range of wealth to permanent income ratios of 0 to 500. Returns are iid. The figure also reports simulation non-participation results through the life cycle. The agent starts work at age 22 and retires at 65, receiving 93.8% of his retirement permanent income until death as reported in Cocco, Gomes and Maenhout (2002). The agent dies with probability 1 at age 100, with death probabilities taken from the U.S. Life Tables, 2001, provided by the NCHS. The mean logarithmic monthly growth rate of permanent income is allowed to be age-dependent and is calibrated to the polynomial-smoothed income profile for college graduates in Cocco, Gomes and Maenhout (2002). The volatility of the logarithmic monthly growth rate of permanent income  $\sigma_g$ , is obtained from its annual counterpart (fixed at 15% from Gakidis (1997)), accounting for the assumed dynamics of the permanent income growth. There are no transitory shocks to income. Retail Trade Income (series CEU4200000004 by Bureau of Labor Statistics) is also used to calibrate labor income as described in Table 1. Agent has access to the market portfolio (value-weighted return on all NYSE, AMEX, and NASDAQ stocks) and to a riskless bond. Calibrations of these assets are as described in section 3 of the text. SDM and SDV denote, respectively, the state-dependent mean and state-dependent volatility channels through which permanent labor income growth can affect stock holdings (see section 2.3 for descriptions). Subfigure 14.a reports cross-sectional probability of non-participation for the combinations of channels indicated in the legend. Subfigure 14.b reports the simulation probability of non-participation for the combinations of channels indicated in its legend, across 500000 paths when initial wealth-income ratio is zero. The calibration of the 4 problems needed to do the comparisons is detailed in section 3 of the text.