

# Explaining the Magnitude of Liquidity Premia: The Roles of Return Predictability, Wealth Shocks and State-dependent Transaction Costs\*

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This Version: 23 January 2009

Work in progress.  
Comments welcome.

\*The authors would like to thank an anonymous referee, the editor Rob Stambaugh, Viral Acharya, George Constantinides, Ned Elton, Marty Gruber, Joel Hasbrouck, John Heaton, Lasse Pedersen, Hans Stoll and the participants of the NBER Spring Asset Pricing Group Meeting, NYU Monday Finance Seminar, the NYU Macro-Finance Reading Group, the NHH Finance Seminar, the UCLA Finance Seminar, the Vanderbilt Finance Seminar and the Yale Finance Seminar for helpful comments and suggestions. All remaining errors are of course the authors' responsibility.

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# Explaining the Magnitude of Liquidity Premia: The Roles of Return Predictability, Wealth Shocks and State-dependent Transaction Costs

## Abstract

The seminal work of Constantinides (1986) documents how, when the risky return is calibrated to the U.S. market return, the impact of transaction costs on per-annum liquidity premia is an order of magnitude smaller than the cost rate itself. A number of recent papers have formed portfolios sorted on liquidity measures and found a spread in expected per-annum return that is definitely not an order of magnitude smaller than the transaction cost spread: the expected per-annum return spread is found to be around 6-7% per annum. Our paper bridges the gap between Constantinides' theoretical result and the empirical magnitude of the liquidity premium by examining dynamic portfolio choice with transaction costs in a variety of more elaborate settings that move the problem closer to the one solved by real-world investors. In particular, we allow returns to be predictable and we introduce wealth shocks, mainly labor income but also stationary multiplicative. With predictable returns, we also allow the wealth shocks and transaction costs to be state dependent. We find that adding these real world complications to the canonical problem can cause transaction costs to produce per-annum liquidity premia that are no longer an order of magnitude smaller than the rate, but are instead the same order of magnitude. For example, the presence of predictable returns and i.i.d. labor income growth uncorrelated with returns, both calibrated to data, causes the liquidity premium for an agent with a wealth to monthly labor income ratio of 0 or 10 to be 1.63% per annum and 1.08% per annum respectively; these are 20-fold and 13-fold increases, respectively, relative to that in the standard i.i.d. return case. And allowing labor income growth to exhibit the predictability observed in U.S. data causes very small reductions in these premia even for very low wealth-income ratios. We conclude that the effect of proportional transaction costs on the standard consumption and portfolio allocation problem with i.i.d. returns can be materially altered by reasonable perturbations that bring the problem closer to the one investors are actually solving.

# 1 Introduction

A number of recent papers have found a difference in expected return across portfolios sorted on liquidity measures. While the finding of a difference is not surprising, the magnitude is, with expected return differences on the order of 6-7% per annum.<sup>1</sup> The magnitude of the difference seems too large to be explained by realistic transaction costs. In particular, the seminal work of Constantinides (1986) documents how investor utility is largely insensitive to transaction costs when the investor solves the canonical problem of i.i.d returns calibrated to U.S. data, no non-financial income, and a constant proportional cost rate. For realistic proportional costs, Constantinides shows that the per-annum liquidity premium that must be offered to induce an CRRA investor to hold the transaction-cost asset instead of an otherwise identical no-transaction-cost asset is an order of magnitude smaller than the transaction cost rate itself. Constantinides also provides the intuition for this result. Investors respond to transaction costs by turning over their portfolios much less frequently than annually, because an investor's value function is insensitive to quite large deviations from the optimal no-transaction-cost portfolio allocation. Our paper bridges the gap between this theoretical result and the empirical magnitude of the liquidity premium by examining dynamic portfolio choice with transaction costs in a variety of more elaborate settings that move the problem closer to the one solved by real-world investors. In particular, we allow returns to be predictable and we introduce wealth shocks, mainly labor income but also stationary multiplicative. When returns are predictable, we also allow the wealth shocks and the transaction cost rate to be state dependent.

We find that adding these real world complications to the canonical problem can cause transaction costs to produce per-annum liquidity premia that are no longer an order of magnitude smaller than the rate, but are instead the same order of magnitude. In particular, return predictability and i.i.d. labor income growth calibrated to U.S. data are sufficient to obtain per-annum liquidity premia of the same order of magnitude as the cost rate, for realistic wealth-income ratios. For this reason, our results provide an important new insight into the effect of transaction costs on investor behavior. In particular, the effect of proportional transaction costs on the standard consumption and portfolio allocation problem with i.i.d. returns can be materially altered by reasonable perturbations that bring the problem closer to the one investors are actually solving. Moreover, since our base case agent holds a positive amount of the low liquidity assets, this agent can be regarded as a marginal or inframarginal investor in these low liquidity assets. Market clearing is an important

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<sup>1</sup>See, for example, Brennan and Subrahmanyam (1996) and Pastor and Stambaugh (2003).

consideration and we describe how heterogeneity in labor income and risk aversion and heterogeneity induced by the presence of delegated portfolio management can allow all assets to be held and net trades each month to sum to zero.

Our base case agent has power utility with a relative risk aversion coefficient of 6 and has access to a low and a high liquidity portfolio as well as a riskless asset. It is important that the agent has access to a high liquidity asset in addition to the low liquidity asset because otherwise the only asset available for trading off risk for expected return is the low liquidity asset. The low liquidity asset is calibrated to a portfolio of the 13 least liquid of 25 liquidity sorted U.S stock portfolios while the high liquidity asset is calibrated to the other 12 (see Acharya and Pedersen, 2002). Lesmond, Ogden and Trzcinka (1999) quantify the transaction costs associated with trading individual stocks and find a 3% cost for the 5 smallest size deciles and a 1% cost for the 5 largest. While investors face transaction costs on both portfolios, intuition suggests that the spread in transaction costs across the two portfolios is what is critical for the spread in expected return across the two. Consequently, we set the transaction cost rate on the high liquidity asset to zero and keep the rate on the low liquidity asset at 2%.

We find that return predictability calibrated to that in the data increases the liquidity premium on the low liquidity portfolio by a factor of 5 from 0.08% per annum to 0.43% per annum. The reason for the increase is as follows. The usual motive for trading is to rebalance the portfolio back to the optimal weights after realized risky asset returns alter the portfolio's composition from the optimal weights. Return predictability causes the optimal portfolio weights to move around through time as the agent takes advantage of the time-varying expected returns. This time variation in the optimal weights creates an additional motive for trading that causes higher liquidity premia when returns are predictable.

Labor income is an important wealth shock which we calibrate to have a permanent component, as in Carroll (1996, 1997) and Viceira (1997). We ignore the temporary component discussed in these papers because we find that this temporary component is unimportant for liquidity premia. The parameter estimates we use are those obtained from U.S. PSID data by Gakidis (1997). With i.i.d. returns and a fixed transaction cost rate, the inclusion of labor income uncorrelated with returns causes the liquidity premium on the low liquidity portfolio to become 1.42% per annum for an agent with no financial wealth, an almost 18-fold increase relative to the canonical i.i.d case. Once returns are allowed to be predictable (with the transaction cost rate remaining a constant), i.i.d. labor income growth uncorrelated with returns causes the liquidity premium to be 1.63% and

1.08% for an agent with a wealth to monthly labor income ratio of 0 and 10 respectively. These represent 20-fold and 13-fold increases, respectively, relative to the standard i.i.d case. So the introduction of i.i.d labor income growth uncorrelated with returns is enough to greatly increase the liquidity premium relative to the canonical problem, especially for agents with wealth to monthly labor income ratios lower than 10.

Labor income causes liquidity premia to increase for two reasons. First, labor income is paid in cash and earns the riskless rate unless invested in stock. Thus, labor income lowers portfolio holdings in stock, distorting them away from the optimal weights and causing the agent to trade more to rebalance back to the optimal weights. Second, shocks to labor income are permanent and so have large effects on the agent's total wealth. These total wealth shocks can lead to large shifts in the optimal weights in the agent's financial wealth portfolio particularly when the agent has a low wealth income ratio. These shifts in the optimal weights provide a strong motive for the agent to trade, leading to much higher liquidity premia than in the case with no labor income. Since adding a temporary component to the monthly shock to labor income has almost no effect on liquidity premia, the implication is that the second of these two channels is much more important than the first. It is also not surprising that return predictability still has an incremental effect, further inflating the premium, in the presence of labor income. As discussed above, return predictability causes the optimal portfolio weights to move around with the stage in the business cycle. This additional variation in the optimal weights can provide a motive for trading over and above that provided by the effects of labor income on the optimal portfolio weights.

When returns are predictable, allowing labor income growth to be procyclical, as in the data, slightly reduces the liquidity premium at very low and very high wealth income ratios, but actually increases the premium to 1.12% per annum at a wealth to monthly labor income of 10. This increase occurs because the hedging demand generated by labor income growth being procyclical reduces the optimal holdings of stock and so causes the 100% stock-holding maximum to bind less often. The agent trades more. Reductions occur because expected returns are countercyclical but labor income growth is procyclical, and so expected stock returns and the optimal portfolio weights in stock are high (low) exactly when labor income's downward pressure on the portfolio weights in stocks is small (big). The agent trades less.

Finally, liquidity premia increase even further when the transaction cost rate is allowed to be state dependent as observed in data. The reason is as follows. In the data, the transaction cost rate is countercyclical which means it's high when future expected returns are high and so the optimal

portfolio weights in stocks are high too. These are exactly the times when the agent trades the most since the effect of labor income is to lower the portfolio weights in stocks.

We find that removing access to the high liquidity asset only slightly increases the liquidity premia, and that this occurs because the base-case agent holds very little of this asset. In the U.S. economy, the high liquidity asset has the larger market capitalization so market clearing is not possible at observed prices if all agents in the economy are identical to the base case agent. However, it is likely that there is considerable heterogeneity across agents regarding their risk aversions and the income processes they receive. We examine two other agents with plausible risk aversions and plausible labor income processes, showing that for initial wealth to monthly income ratios of 10 or less, both hold more of the high than the low liquidity asset early in life. For initial wealth to monthly income ratios of 1 or lower, the second agent holds more of the high than the low liquidity asset for at least the first 11 years of her 20 year life and holds more than 7 times more of the high than the low liquidity asset in her first month. Another important source of heterogeneity is participation by agents who care about something other than consumption from their labor income and financial wealth portfolios. Cuoco and Kaniel (2006) show how the existence of a delegated portfolio management industry in which managers receive a symmetric fulcrum fee that depends on performance relative to a benchmark can cause funds to tilt fund portfolios towards the stocks in the benchmark. The result is higher prices and lower Sharpe ratios in equilibrium for these benchmark stocks, which are typically high liquidity stocks. Thus, funds managed on behalf of others may hold large amounts of the high liquidity stocks while agents like our base case agent are the inframarginal investors in the low liquidity stocks.

Another variable of interest is turnover. Using simulations, we find that annual turnover goes from about 4% per annum in the standard case to 38% per annum for the base case agent with a wealth to monthly income ratio of 10. This turnover number is in the ballpark of the monthly turnover numbers reported by Acharya and Pedersen (2002) for their low liquidity portfolios, numbers which ranged from 3.25% up to 4.19%. Labor income drives a wedge between the liquidity premia and the direct cost of trading which implies that labor income causes the agent to trade more often when poor than when rich in utility terms. For an agent with a wealth to monthly income ratio of 1, the direct cost in the base case is 1.03% per annum compared to a liquidity premium of 1.44% in the base case while the direct cost in the case of i.i.d. returns and labor income growth is less than half the 1.38% per annum liquidity premium.

We also consider multiplicative wealth shocks, for which comparative static analysis is easier

because of the much lighter computational burden. It is difficult to calibrate these shocks, since there are a variety of possible reasons for their occurrence. Sources include shocks to the value of the agent’s real estate holdings, proprietary income shocks and health shocks. To make the multiplicative shock analysis more directly comparable to the labor income cases we consider, we calibrate the shock process to the base case labor income process. We find that the liquidity premium exhibits a strong u-shaped pattern as a function of the unconditional correlation between a given month’s wealth shock and the dividend yield at the start of that month with the lowest premium occurring for a correlation close to 0. This u-shaped pattern suggests that the premia for the labor income cases likely would be higher if the correlation between the permanent labor income growth and start-of-month dividend yield was further away from 0 in either direction. When the proportional transaction cost rate is allowed to be state dependent, we find that, consistent with intuition, wealth shocks are especially painful and so generate higher liquidity premia if negative wealth shocks occur when the transaction cost rate is high.

The paper is organized as follows. Section 2 discusses related literature while section 3 describes the investor’s dynamic optimization problem with predictable returns, transaction costs and either labor income or multiplicative wealth shocks. Section 4 calibrates the investor problems considered to the U.S. economy. Section 5 discusses the liquidity premia and turnover results while Section 6 discusses equilibrium issues and market clearing. Section 7 concludes.

## 2 Related Literature

A number of recent empirical papers have examined how expected returns vary with measures of liquidity. Brennan and Subrahmanyam (1996) form 25 portfolios forming quintiles on size and then within each size quintile, forming quintiles on the Kyle (1985) inverse measure of market depth,  $\lambda$ , estimated as in Glosten and Harris (1988). They find a 6.6% per annum spread in average abnormal return from the Fama-French (1993) three-factor model between the low- $\lambda$  and the high- $\lambda$  quintiles. Rather than sorting on a measure of stock illiquidity, Pastor and Stambaugh (2003) form deciles based on covariance of return with a measure of market liquidity and find a spread in abnormal return between the two decile extremes of 7.5% per annum with respect to a four factor model that accounts for sensitivities to the market, size and book-to-market factors of Fama-French (1993) and a momentum factor. Easley, Hvidkjaer and O’Hara (2002) examine how information-based trading affects asset returns and reports that a difference of 10 percentage points in the probability of information-based trading between two stocks leads to a difference in expected returns of 2.5%

per annum. Other papers to examine how expected returns vary with measures of liquidity include Brennan, Chordia and Subrahmanyam (1998), Amihud (2002) and Hasbrouck (2003).

Several theoretical and numerical papers have considered how illiquidity and transaction costs affect asset prices. Early work by Stoll (1978) and Ho and Stoll (1981) examines how a dealer sets the spread given that she faces inventory carrying costs. In a single period setting, Amihud and Mendelson (1986) show how transaction costs can affect expected returns on stocks. In an economy with two classes of agents, Heaton and Lucas (1996) examine numerically how idiosyncratic and uninsurable labor income affects equilibrium expected returns both with and without transaction costs on the riskless and equity assets. In an overlapping generations economy, Vayanos (1998) shows how prices are affected by the presence of transaction costs. In his model, agents have a lifecycle motive for trading and trading behavior is predetermined. Huang (2002) studies an equilibrium model in which agents receive unexpected liquidity shocks and can invest in liquid and illiquid riskless assets. Other papers to examine theoretically how illiquidity and transaction costs affect assets prices include Lo, Mamaysky and Wang (2001), Aiyagari and Gertler (1991) and Acharya and Pedersen (2002). Recent papers also examine multiperiod portfolio choice in the presence of labor income, but few incorporate portfolio rebalancing costs (see Viceira, 1997 and 2001, Cocco, Gomes and Maenhout, 2002, and Gomes and Michaelides, 2002). Two exceptions are Balduzzi and Lynch (1999) and Lynch and Balduzzi (1999) who solve numerically the investor's multi-period problem with transaction costs. However, neither of these papers incorporate labor income or multiplicative wealth shocks and neither focus on liquidity premia, though Balduzzi and Lynch do report utility costs associated with ignoring transaction costs.

### **3 The investor's portfolio allocation problem**

This section lays out the preferences of and constraints faced by the investor. We characterize the optimization problem for a dynamic investor who faces either i.i.d. or predictable returns. We first describe the problem when the agent receives labor income and then when the agent receives a multiplicative wealth shock. We also describe the solution technique for numerically solving the investor's problem and how the liquidity premia and turnover numbers are calculated.

#### **3.1 *Labor income problem***

We consider the portfolio allocation between  $N$  risky assets and a riskless asset. In all the cases we consider, the investor has access to either one or two risky assets. The investor faces transaction

costs that are proportional to the dollar amount traded.

Following Carroll (1996) and (1997), labor income is specified to have both permanent and temporary components:

$$y_t = y_t^P + \epsilon_t, \quad (1)$$

$$g_t \equiv y_t^P - y_{t-1}^P = \bar{g} + b_g d_t + u_{t+1}, \quad (2)$$

where  $y_t$  is log labor income received at  $t$ ,  $y_t^P$  is log permanent labor income at  $t$ ,  $\epsilon_t$  is log temporary labor income at  $t$ ,  $d_t \equiv \ln(1 + D_t)$ , and  $\epsilon_t$  and  $u_{t+1}$  are uncorrelated i.i.d. processes. All the cases reported in the paper switch off the temporary component because unreported results (available from the authors upon request) show that the presence of a temporary component calibrated to data has a negligible impact on liquidity premia. Thus, for all the cases reported,  $y_t = y_t^P$  and  $\epsilon_t = 0$  for all  $t$ .

With labor income, the law of motion for the investor's wealth,  $W$ , is given by

$$W_{t+1} = (W_t + Y_t - c_t)(1 - f_t) \left[ \alpha_t (\mathbf{R}_{t+1} - R_t^f \mathbf{i}_N) + R_t^f \right] \\ \text{for } t = 1, \dots, T - 1, \quad (3)$$

where  $c$  is consumption,  $\alpha$  is an  $N \times 1$  vector of portfolio weights in the  $N$  risky assets,  $\mathbf{R}$  is an  $N \times 1$  vector of returns on the  $N$  risky assets,  $R^f$  is the risk-free rate,  $Y_t$  is labor income received at time- $t$ , and  $f$  is the transactions cost per dollar of portfolio value. At the terminal date  $T$ ,  $c_T = W_T$  so the investor does not receive labor income at the terminal date.

Let  $\hat{\alpha}_t^i$  be the allocation to the  $i$ th risky asset inherited from the previous period. Then

$$\hat{\alpha}_t^i \equiv \frac{\alpha_{t-1}^i (W_{t-1} + Y_{t-1} - c_{t-1})(1 - f_{t-1}) R_t^i}{W_t} = \frac{\alpha_{t-1}^i R_t^i}{\alpha_{t-1}' (\mathbf{R}_t - R_{t-1}^f \mathbf{i}_N) + R_{t-1}^f}. \quad (4)$$

where  $\hat{\alpha}_t$  is the  $N \times 1$  vector of these inherited portfolio weights. We assume that consumption at time  $t$  is obtained by liquidating costlessly the  $i$ th risky asset and the riskless asset in the proportions  $\hat{\alpha}_t^i$  and  $(1 - \hat{\alpha}_t' \mathbf{i}_N)$ . This assumption is not so onerous given the availability of money-market bank accounts and given that equities pay dividends. To the extent that the sum of the risky assets' dividends exceeds the consumption out of the risky asset,  $c$ , a dividend reinvestment plan can be used to costlessly reinvest the excess dividend in the risky asset.

We allow returns to be predictable and assume that there exists a "predictive" variable,  $D$  which affects the conditional mean of the risky assets' return. We assume  $D$  follows a first-order

Markov process. For simplicity, the riskless rate is assumed to be constant, and so  $R_t^f = R^f$  for every  $t$ .

Labor income  $Y_t$  is assumed to be received as the riskless asset. Consequently the vector of inherited risky asset holdings becomes  $\frac{\hat{\alpha}_t}{1+Y_t/W_t}$  after the shock. Since  $Y_t$  is always positive, the shock is like a cash inflow. The investor sees the labor income at  $t$  before both the consumption and allocation decisions at  $t$ . However, neither  $u_t$  nor  $\epsilon_t$  (if non-zero) contain any information about future returns ( $\mathbf{R}_{t+1}, \mathbf{R}_{t+2}, \dots$ ) or about future  $D$  values ( $D_{t+1}, D_{t+2}, \dots$ ).

The transaction cost function  $f_t$  depends on the chosen portfolio weights  $\alpha_t$ , the  $N \times 1$  vector of portfolio weights inherited from the previous period,  $\hat{\alpha}_t$  and labor income  $Y_t$ :

$$f_t = \Phi_t' |\alpha_t - \frac{\hat{\alpha}_t \Gamma_t}{\Gamma_t + \exp\{g_t + \epsilon_t\}}|. \quad (5)$$

where  $Y_{t-1}^P = \exp\{y_{t-1}^P\}$  is permanent labor income at  $t-1$  and  $\Gamma_t$  is defined to be  $\frac{W_t}{Y_{t-1}^P}$ . For a given inherited risky-asset allocation at  $t$   $\hat{\alpha}_t$ , the post-labor income inherited risky-asset allocation is decreasing in  $Y_t$ . This specification accommodates transaction costs on an asset that are proportional to the change in the value of the portfolio holding of that asset, as in Constantinides (1986). In general, the  $N \times 1$  vector  $\Phi$  has  $i$ th element  $\Phi^i$  which gives the proportional cost rate associated with trading the  $i$ th risky asset. However, in all applications considered here, only one asset has a non-zero transaction cost rate and so  $\Phi$  is used to denote that rate. It is straightforward to modify the transaction cost function to accommodate more elaborate transaction cost functions, like, for example, a cost that is the same fraction of portfolio value irrespective of how much of the asset is traded (see Lynch and Tan, 2003).

We also allow the cost parameters  $\Phi_t$  to be random, with distributions that can be state dependent and thus depend on  $D_t$  when returns are predictable. The investor sees the cost parameter realizations at  $t$ ,  $\Phi_t$ , before both the consumption and allocation decisions at  $t$ . The cost parameter realizations for  $t$  do not contain any information about future returns or future  $D$  values.

We consider the optimal portfolio problem of a investor with a finite life of  $T$  periods and utility over intermediate consumption. Preferences are time separable and exhibit constant relative risk aversion (CRRA):

$$E \left[ \sum_{t=1}^T \delta^t \frac{c_t^{1-\gamma}}{1-\gamma} | \Gamma_1, D_1, \hat{\alpha}_1 \right], \quad (6)$$

where  $\gamma$  is the relative-risk-aversion coefficient and  $\delta$  is the time-discount parameter and  $E[. | \Gamma_t, D_t, \hat{\alpha}_t]$  denotes the expectation taken using the conditional distribution given  $\Gamma_t$ ,  $D_t$  and  $\hat{\alpha}_t$ . Note that

the expected lifetime utility depends on the state of the economy at time 1. Further, the inherited portfolio weight for the  $i$ th risky asset  $\hat{\alpha}_1^i$  is a state variable whenever the  $i$ th element of  $\Phi$  is greater than zero, since the value of this inherited portfolio weight determines the transaction costs to be paid at time 1. These preferences have been extensively used in empirical work by Grossman and Shiller (1981), Hansen and Singleton (1982), and many others.

Given this specification of the agent's problem with labor income, the value function at  $t$  is homogenous in  $Y_{t-1}^P$  and has an additional state variable: the ratio of financial wealth at  $t$  to lagged permanent labor income  $\Gamma_t$ . The law of motion for the investor's wealth,  $W$ , can be rewritten as

$$\Gamma_{t+1} = (\Gamma_t - \hat{\kappa}_t + \exp\{g_t + \epsilon_t\})(1 - f_t) \exp\{-g_t\} \left[ \alpha_t(\mathbf{R}_{t+1} - R_t^f \mathbf{i}_N) + R_t^f \right] \quad \text{for } t = 1, \dots, T - 1. \quad (7)$$

where  $\hat{\kappa}_t \equiv \frac{c_t}{Y_{t-1}^P}$ .

Given our parametric assumptions, the Bellman equation faced by the investor is given by

$$\begin{aligned} \frac{a(\Gamma_t, D_t, \hat{\alpha}_t, t)(Y_{t-1}^P)^{1-\gamma}}{1-\gamma} &= E \left[ \max_{\hat{\kappa}(\Gamma_t, D_t, \hat{\alpha}_t, g_t, \epsilon_t, \Phi_t, t), \alpha(\Gamma_t, D_t, \hat{\alpha}_t, g_t, \epsilon_t, \Phi_t, t)} \left\{ \frac{\hat{\kappa}_t^{1-\gamma} (Y_{t-1}^P)^{1-\gamma}}{1-\gamma} \right. \right. \\ &+ \left. \left. \delta \frac{(Y_{t-1}^P)^{1-\gamma}}{1-\gamma} E \left[ a(\Gamma_{t+1}, D_{t+1}, \hat{\alpha}_{t+1}, t+1) (\exp\{g_t\})^{1-\gamma} | \Gamma_t, D_t, \hat{\alpha}_t, g_t, \epsilon_t, \Phi_t \right] \right\} | \Gamma_t, D_t, \hat{\alpha}_t \right], \end{aligned} \quad \text{for } t = 1, \dots, T - 1, \quad (8)$$

where  $\alpha_t \equiv \alpha(\Gamma_t, D_t, \hat{\alpha}_t, g_t, \epsilon_t, \Phi_t, t)$  and  $\hat{\kappa}_t \equiv \hat{\kappa}(\Gamma_t, D_t, \hat{\alpha}_t, g_t, \epsilon_t, \Phi_t, t)$ , both time dependent since the time horizon  $T$  is finite. This form of the value function derives from the CRRA utility specification in eqs. (6). The Bellman (8) is solved by *backward iteration*, starting with  $t = T - 1$  and  $a(\Gamma, D, \hat{\alpha}, T) = \Gamma^{1-\gamma}$ . If returns and permanent labor income growth are not predictable,  $D_t$  is no longer a state variable for the problem and so the  $a$  is no longer a function of  $D_t$ . Moreover,  $\hat{\kappa}$  and  $\alpha$  aren't functions of  $D_t$  either.

### 3.2 Multiplicative wealth shock problem

We now describe the agent's problem when the shock to financial wealth is multiplicative and a stationary variable. This structure makes the wealth shock easy to handle, since its presence has no impact on the number of state variables for the agent's problem. The law of motion of the investor's wealth,  $W$ , is given by

$$W_{t+1} = [W_t(1 + L_t) - c_t](1 - f_t) \left[ \alpha'_t(\mathbf{R}_{t+1} - R_t^f \mathbf{i}_N) + R_t^f \right], \quad \text{for } t = 1, \dots, T - 1, \quad (9)$$

where  $L$  is the wealth shock expressed as the percentage change in wealth as result of the shock. The wealth shock  $L$  is exogenous and assumed to follow a stationary process with age-dependent parameters. Letting  $l_t = \ln(L_t)$  and  $\bar{l}_t(D_t)$  be the age- and state-dependent mean of  $l_t$ , then  $l_t - \bar{l}_t(D_t)$  is assumed to be i.i.d. with volatility  $\sigma_l$ . The dollar wealth shock at  $t$  is  $W_t L_t$ . At the terminal date  $T$ ,  $c_T = W_T$  so the investor does not receive a wealth shock at the terminal date. Dollar transaction costs at  $t$  are  $[W_t(1 + L_t) - c_t]f_t$ , and are paid by costlessly liquidating the  $i$ th risky and the riskless assets in the proportions  $\alpha_t^i$  and  $(1 - \alpha_t' \mathbf{i}_N)$ .

As with the labor income at  $t$ , the wealth shock  $W_t L_t$  is like a cash inflow and is assumed to affect the riskless asset holding. Consequently the vector of inherited risky asset holdings becomes  $\frac{\hat{\alpha}_t}{1+L_t}$  after the shock. When returns are predictable, we allow the distribution of the wealth shock  $L_t$  to be state dependent and thus depend on  $D_t$ . The transaction cost function  $f$  becomes:

$$f_t = \mathbf{\Phi}_t' \left| \alpha_t - \frac{\hat{\alpha}_t}{1 + L_t} \right|. \quad (10)$$

The evolution equation for state variable  $\hat{\alpha}_t$  remains (4). To make the multiplicative shock case as comparable to the labor income case as possible, consumption here is allowed to depend on the current date's wealth shock as well as the transaction cost shock and neither of these shocks contain any information about future returns ( $\mathbf{R}_{t+1}, \mathbf{R}_{t+2}, \dots$ ) or future  $D$  values ( $D_{t+1}, D_{t+2}, \dots$ ).

We define  $R_W$  as the rate of return on wealth, after the wealth shock and net of the transaction costs incurred. Given our parametric assumptions, the Bellman equation faced by the investor is given by

$$\begin{aligned} \frac{a(D_t, \hat{\alpha}_t, t) W_t^{1-\gamma}}{1-\gamma} = \max_{\kappa(D_t, \hat{\alpha}_t, L_t, \Phi_t, t)} & \left\{ \frac{\kappa_t^{1-\gamma} W_t^{1-\gamma}}{1-\gamma} + \delta \frac{(1-\kappa_t)^{1-\gamma} W_t^{1-\gamma}}{1-\gamma} \right. \\ E \left[ \max_{\alpha(D_t, \hat{\alpha}_t, L_t, \Phi_t, t)} & \left\{ E \left[ a(D_{t+1}, \hat{\alpha}_{t+1}, t+1) R_{W,t+1}^{1-\gamma} \mid D_t, \hat{\alpha}_t, L_t, \Phi_t \right] \right\} \mid D_t, \hat{\alpha}_t \right] \Big\} \\ & \text{for } t = 1, \dots, T-1, \end{aligned} \quad (11)$$

where  $\kappa_t \equiv \frac{c_t}{W_t(1+L_t)}$ ,  $\alpha_t \equiv \alpha(D_t, \hat{\alpha}_t, L_t, \Phi_t, t)$  and  $\kappa_t \equiv \kappa(D_t, \hat{\alpha}_t, L_t, \Phi_t, t)$ . As in the labor income case, the Bellman (11) is solved by *backward iteration*, starting with  $t = T - 1$  and  $a(D, \hat{\alpha}, T) = 1$ .

### 3.3 Solution technique

The dynamic programming problems are solved by backward recursion. Irrespective of whether there is one or two risky assets, the state variable  $\hat{\alpha}_1$  is discretized and the value function is linearly interpolated between  $\hat{\alpha}_1$  points. This technique yields an approximate solution that converges to

the actual solution as the  $\hat{\alpha}_1$  grid becomes finer. In all the optimizations, the holdings of both the risky and the riskless assets are constrained to be non-negative. In the two-risky asset case, this restricts the action space with respect to allocation choice to the triangular region characterized by  $\alpha_1 + \alpha_2 \leq 1$ ,  $\alpha_1 \geq 0$  and  $\alpha_2 \geq 0$ . When actions are restricted to this set, implied inherited allocations for any return realization on the assets are again in the same region. We use a finer discretization for the action space of allocations than the state space of inherited allocations. Allocation choices on each asset available to the investor always include the discrete grid  $\{0.000, 0.001, \dots, 0.999, 1.000\}$ .

For the labor income problem, the presence of an additional state variable, the wealth to lagged permanent income ratio, considerably complicates the methodology needed to obtain accurate solutions in a manageable time-frame. Appendix A details the methodology employed.

### 3.4 *Liquidity premia*

Each of the investor problems described above imply a policy function that, in turn, yields a particular level of expected lifetime utility. Specifically, for the labor income problem, the policy functions  $\{\alpha(\Gamma_t, D_t, \hat{\alpha}_t, g_t, \epsilon_t, \Phi_t, t)\}_{t=1}^{T-1}$  and  $\{\hat{\kappa}(\Gamma_t, D_t, \hat{\alpha}_t, g_t, \epsilon_t, \Phi_t, t)\}_{t=1}^{T-1}$  can be substituted into the actual law of motion for investor's wealth (3) to obtain the consumption sequence  $\{c_t = \hat{\kappa}(\Gamma_t, D_t, \hat{\alpha}_t, g_t, \epsilon_t, \Phi_t, t) Y_{t-1}^P\}_{t=1}^T$ . This consumption sequence is then substituted into (6) to obtain the investor's expected lifetime utility. Analogous substitutions can be performed for the multiplicative wealth shock problem to obtain the investor's expected lifetime utility.

Constantinides (1986) found that for a CRRA investor with access to a risky asset whose return is i.i.d., proportional transaction costs produce per-annum liquidity premia that are an order of magnitude smaller than the cost rate. We are interested in determining whether this result is robust to the introduction of the real-world complications like return predictability, state-dependent wealth shocks and state dependent transaction costs. Consequently, our definition of liquidity premium is in line with that adopted by Constantinides (1986). The liquidity premium is defined to be the decrease in the unconditional mean log return on the low liquidity asset that the investor requires to be indifferent between having access to the risky asset without the transaction costs rather than with them. The mean is decreased by subtracting a constant from every state.

As mentioned above, the expected lifetime utility depends on the initial value of the inherited portfolio allocation,  $\hat{\alpha}_1$ , the initial value of the wealth to lagged permanent labor income  $\Gamma_1$  and the initial value of the vector characterizing the state of the economy,  $D_1$ . For simplicity, we

always take the inherited allocation for a given state to be the optimal allocation for the analogous no-transaction-cost problem.

### 3.5 Turnover and Direct Trading Cost

Another variable of interest is turnover. Turnover is calculated for the labor income problem by simulating 1 million paths and applying the optimal policies to each path. Per-annum turnover is defined to be:

$$\frac{12}{240} \sum_{t=1}^{240} \frac{\text{av}(|A_t - \hat{A}_t|)}{\text{av}(\max(A_t, \hat{A}_t))}, \quad (12)$$

where  $\text{av}(\cdot)$  is obtained by taking the average across the simulation paths,  $A_t = [W_t - c_t + Y_t]\alpha_t$  and  $\hat{A}_t = [W_t - c_t + Y_t](\hat{\alpha}_t W_t)/(W_t + Y_t)$ . So  $A_t$  is the dollar chosen risky-asset holdings at time  $t$  and  $\hat{A}_t$  is the dollar effective inherited risky-asset allocation at time  $t$ . The fraction being summed in (12) represents turnover for a given month of life. The denominator must be some measure of the dollar risky-asset holdings for the month. We take the maximum of the dollar chosen risky-asset allocation and the dollar effective inherited risky-asset allocation for the month.<sup>2</sup> This measure of turnover equally weighs the 240 months of the agent's life.<sup>3</sup> Turnover is calculated analogously for the multiplicative wealth shock problem.

The turnover number can be multiplied by the average transaction cost rate to get the direct effect of transaction costs on expected return. The extent to which the liquidity premium exceeds this direct cost can be attributed to some combination of risk premium for trading more when the agent is poor in utility terms, and in the case of a state-dependent cost rate, to the agent trading more when the cost rate is high.

## 4 Calibration

This section describes how the return, labor income, multiplicative wealth shock, transaction cost rate and predictive variable processes are calibrated to data. Parameter value choices are also described.

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<sup>2</sup>We also tried using a measure of holding based on the average of the chosen risky-asset allocation and the effective inherited allocation. The results were virtually identical.

<sup>3</sup>We tried other weighting schemes including one that weighs months according to the average dollar value of holdings over the months. The results were qualitatively similar to the ones we report.

#### 4.1 Return calibration

We use a high and a low liquidity portfolio as the risky assets. The Acharya and Pedersen (2002) data set provides 25 value-weighted portfolios of NYSE and AMEX stocks sorted on ILLIQ, a liquidity measure suggested by Amihud (2002). The high liquidity asset is taken to be the value-weighted portfolio of the most liquid 12 portfolios and the low liquidity asset is taken to be the value-weighted portfolio of the least liquid 13 portfolios. The stock return and riskfree rate series are deflated using monthly CPI inflation. The continuously compounded riskfree rate is estimated to be the mean of the continuously compounded real one-month Treasury-bill rate over this period, which gives a value for  $R^f$  of 0.110 percent. We use the 12-month dividend yield on the value-weighted NYSE as the predictive variable  $D$ .

Define  $\mathbf{R}$  to be an  $N \times 1$  risky-asset return vector and let  $\mathbf{r} \equiv \ln(\mathbf{1} + \mathbf{R})$  and  $d \equiv \ln(1 + D)$ . We estimate a VAR for the two risky-asset returns assuming that  $[\mathbf{r}'d]'$  follows the vector autoregressive model (VAR):

$$\mathbf{r}_{t+1} = \mathbf{a}_r + \mathbf{b}_r d_t + \mathbf{e}_{t+1}, \quad (13)$$

$$d_{t+1} = a_d + b_d d_t + v_{t+1}, \quad (14)$$

where  $\mathbf{a}_r$ ,  $N \times 1$ , and  $a_d$  are intercepts,  $\mathbf{b}_r$ ,  $N \times 1$ , and  $b_d$  are coefficients and  $[\mathbf{e}'v]'$  is an i.i.d. vector of mean-zero, multivariate normal disturbances, with constant covariance matrix  $\Sigma_{\mathbf{e}\mathbf{v},\mathbf{e}\mathbf{v}}$ ; the covariance matrix of  $\mathbf{e}$  is  $\Sigma_{\mathbf{e},\mathbf{e}}$  and the variance of  $v$  is  $\sigma_v^2$ . Similarly, the unconditional covariance matrix  $[\mathbf{r}'d]'$  is  $\Sigma_{\mathbf{r}d,\mathbf{r}d}$ ; the unconditional variance matrices for  $\mathbf{r}$  and  $d$  are  $\Sigma_{r,r}$  and  $\sigma_d^2$  respectively. Without loss of generality, we normalize the mean of  $d$ ,  $\mu_d$ , to be zero and its variance,  $\sigma_d^2$ , to be 1. The specification in (13)-(14) assumes that  $d_t$  is the only state variable needed to forecast  $r_{t+1}$  which is in line with other papers on optimal portfolio selection (e.g., Barberis (2000) and Campbell and Viceira (1999)).

The data VAR is estimated using ordinary least squares (OLS) and discretized using a variation of Tauchen and Hussey's (1991) Gaussian quadrature method; the variation is designed to ensure that  $d$  is the only state variable (see Balduzzi and Lynch (1999) for details). However, following Lynch (2000), this study implements the discretization in a manner that produces exact matches for important moments for portfolio choice.<sup>4</sup> We choose 19 quadrature points for the dividend yield and 3 points for the stock-return innovations since Balduzzi and Lynch (1999) find that the

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<sup>4</sup>In particular, the procedure matches both the conditional mean vector and the covariance matrix for log returns at all grid points of the predictive variables, as well as the unconditional volatilities of the predictive variables and the correlations of log returns with the predictive variables.

resulting approximation is able to capture important dimensions of the return predictability in the data.

Table 1 present data and quadrature VAR parameter values for the high and low liquidity returns. Panel A reports the slope coefficients  $\mathbf{b}_r$  and  $b_d$  as well as unconditional means for  $\mathbf{r}$  and  $d$ . Panel B reports the unconditional covariance matrix for  $[\mathbf{r}'d]'$  and the cross-correlations. Panel C reports the unconditional covariance matrix for  $[\mathbf{e}'v]'$  and the cross-correlations. The quadrature values almost always replicate the data values, which suggests that the discretization is capturing the important features of the data.

#### 4.2 *Transaction cost rate, labor income and multiplicative wealth shock calibrations*

When calibrating the transaction cost rates, it is the cost of trading the individual stocks and not the portfolio itself that is relevant. The transaction cost rate on the low liquidity asset is calibrated to the transaction cost spread between stocks in a low transaction cost portfolio and stocks in a high transaction cost portfolio. Lesmond, Ogden and Trzcinka (1999) form size deciles and then report the average round-trip transaction cost for the individual stocks in each decile. According to Table 3 of Lesmond, Ogden and Trzcinka (1999), the average round-trip cost of trading a stock in the 5 largest portfolios less the average to trade one in the 5 smallest equals 4.01%, and so we take 2% to be the one-way transaction cost rate for the low liquidity asset. This number is likely to be a ballpark figure for the transaction cost spread between the low and high liquidity portfolios and a lower bound for the cost of trading the low liquidity portfolio. In practice, investors face transaction costs on both portfolios, but intuition suggests that the spread in transaction costs across the two portfolios is what is critical for the spread in expected return across the two. Because of this, the cost rate on the low liquidity asset is taken to be this 2% while the cost rate on the high liquidity asset (if available) is always taken to be zero. Keeping the transaction cost rate on one of the risky assets equal to zero keeps the inherited allocation state space one-dimensional, which keeps computation time manageable.

When we allow the cost rate,  $\Phi$ , to be state-dependent, the risky asset returns are always predictable and the conditional expectation of  $\phi$  is linear in the  $d$  value. We always keep  $\Phi$ 's unconditional mean  $\mu_\Phi$  equal to its value in the analogous case when it's a constant. We apply Gaussian quadrature rules to  $\phi \equiv \ln(1 + \Phi)$  in such a way that the volatility of  $\phi$ ,  $\sigma_\phi$ , and the mean of  $\Phi$  match chosen values. In particular, for a given  $\mu_\Phi$ , the unconditional volatility of  $\sigma_\phi$  is always chosen such that  $\frac{\mu_\Phi}{\sigma_\phi}$  equals 2.63. The three values taken by the  $\phi$  shock are always the same and

2.63 is chosen to ensure a large spread in  $\Phi$  but without any negative values.

To calculate a volatility estimate for  $\phi$ , we use a half-spread estimate developed by Hasbrouck (2003) using a Bayesian approach and daily data for all ordinary common equity issues on the CRSP daily database from 1962 to 2002. Hasbrouck averages the daily numbers to obtain half-spread estimates at an annual frequency for each stock. Fixing year, we sort the stocks on half-spread and equally weight the top and the bottom 50% to obtain annual time series estimates for two hypothetical portfolios (liquid portfolio and illiquid portfolio respectively). If the half-spread is greater than 30% for an observation, we set that observation to 30%, analogous to the treatment of outliers in Amihud's ILLIQ measure by Acharya and Pedersen (2002). We construct a third series by subtracting the liquid portfolio half-spread from the illiquid portfolio half-spread, which gives us our proxy for  $\Phi$ .

Turning to the labor income process used in the base base, parameter values are chosen to be the baseline values in Viceira (1997, 2001) who describes these values as consistent with those obtained by Gakidis (1997) based on PSID data for professionals and managers not self-employed under age 45. Liquidity premia given these values are of interest since this is a group that holds stocks, and in particular, low liquidity stocks, as we will see. Viceira's baseline value for the standard deviation of the change in log permanent labor income is 15% per year, and for the mean growth of permanent labor income is 3% per annum. These are used to back out values for the unconditional volatility of  $g$ ,  $\sigma_g$  and the unconditional mean of  $g_t$ ,  $\bar{g}$ . A number of papers (see, for example, Chamberlain and Hirano, 1997 and Carroll and Samwick, 1995) have estimated labor income parameters and a range of values are reported across these studies. However, the Gakidis values seem to lie within this range, which makes them reasonable to use.

We are also interested in calibrating to U.S. data the correlations between start-of-the-month dividend yield, the transaction cost rate and the growth rate of labor income. The dividend yield series that we use is the one described above. But we need proxies for the log growth in permanent labor income,  $g$ , and the transaction cost rate  $\phi$ , since the proxy described above for  $\phi$  is only available at an annual frequency.

Monthly aggregate labor income data is used to compute covariances between permanent labor income growth and both dividend yield and the transaction cost rate. Correlations are then computed using the standard deviation for monthly individual permanent income growth monthly calculated from Viceira (1997, 2001) as described above. It is reasonable to use aggregate data to estimate covariance if the idiosyncratic component of individual labor income growth is uncorre-

lated with these two series. Aggregate labor income data is from the Bureau of Labor Statistics website. We use the Retail Trade income data which is series CEU4200000004, measured at a monthly frequency, and available from January 1972 through until the end of December 2003. The income data are disinflated using a CPI measure, series CPIAUCNS, available from U.S. Department of Labor: Bureau of Labor Statistics. Per capita income values are generated by dividing all income series with a population measure, series POP, available from U.S. Department of Commerce: Census Bureau.

The proxy for  $\phi$  available at a monthly frequency is a less direct measure of the per-trade cost than Hasbrouck's measure which is why we only use it to calculate correlations. The Acharya and Pedersen (2002) data set, in addition to monthly returns on 25 value-weighted portfolios sorted on Amihud's ILLIQ measure, also provides a corresponding normalized ILLIQ for each portfolio, which they argue can be interpreted as a one-way transactions cost rate. We value-weight to obtain monthly series of one-way proportional transactions cost rates for the low and the high liquidity portfolios. Again, we construct a third series by subtracting the high liquidity portfolio cost rate from the low liquidity portfolio cost rate. Data is available from February 1964 to December 1996. We implement the transformation  $\phi = \log(1 + \Phi)$  on this series and the Hasbrouck series.

Gaussian quadrature is used to deliver the joint distributions for  $(g_t, \phi_t, d_{t+1})$  and  $(l_t, \phi_t, d_{t+1})$ . Three grid points are used for the  $g$  shock and for the  $l$  shock. Unconditional moments for  $\phi$  are chosen as described above. A number of labor income cases are considered. In the Base case, the labor income process and its correlation with start-of-the month dividend yield is calibrated to data as described above and the transaction cost rate is constant. To gauge the importance of the business-cycle variation in mean log permanent labor income growth found in the data, we consider a case (the i.i.d. Labor Growth case) with predictable returns and i.i.d. labor income growth that has the same annual mean and volatility as base-case labor income. To assess whether labor income's ability to generate a sizeable liquidity premium depends on returns being predictable, we consider a case (the i.i.d. Return case) in which both returns and labor income growth are i.i.d.. To see how much the availability of the high liquidity asset reduces the liquidity premia, we also consider a case (the One Risky case) exactly like the base case except there is no access to the high liquidity asset. Finally, the effect on liquidity premia of making the transaction cost rate state-dependent as in the data, is assessed by making the transaction cost rate in the one-risky case state dependent. We call this the State-dependent Transaction Cost case.

We calibrate the multiplicative shock process to the labor income growth process that's used in

the base case described above. In our base case, log labor income growth,  $\Delta y_t$ , is calibrated to be  $N(\bar{g} + b_g d_t, \sigma_u^2)$ . Given the wealth income ratio at  $t$ ,  $\Gamma_t = W_t/Y_{t-1}$ , then  $\ln(Y_t/W_t) = \Delta y_t - \ln(\Gamma_t)$  and is distributed  $N(\bar{g} + b_g d_t - \ln(\Gamma_t), \sigma_u^2)$ . Since the wealth shock  $L_t$  is multiplicative, it makes sense to calibrate the distribution of  $\ln(L)$  to that of  $\ln(Y_t/W_t)$ . We allow it's mean to be age- and dividend yield state-dependent. The mean of  $l \equiv \ln(L)$  is obtained by calculating  $(\bar{g} + b_g d_t)$  using the base case parameters which are calibrated to data and using a  $\Gamma$ -age and -dividend profile calculated by taking  $\Gamma$  at age 1 to be 1, simulating and averaging at each age and each dividend yield state. With the correlation between  $l$  and  $d$  fixed at the data value for  $\Delta y$  and  $d$ 's correlation of -0.0372, we then adjust the volatility of  $l$  so as to match the liquidity premium for the base case agent with a  $\Gamma$  at age 1 of 1. Gaussian quadrature is applied to  $l$  and  $\phi$  to obtain their joint distribution, with the conditional expectation of  $l$  linear in the  $d$  value. Three grid points are used for the  $l$  shock. Fixing the unconditional volatility of the wealth shock and the unconditional mean and volatility of the transaction cost rate (if state-dependent), the environment facing the investor depends on: 1) Returns i.i.d. or not. 2) Transaction cost rate state-dependent or not. 3) Contemporaneous correlation between the wealth shock and the transaction cost rate,  $\rho_{l,\phi} \equiv \rho[l_t, \phi_t]$ . 4) Correlation between the wealth shock and the start-of-the-month dividend yield,  $\rho_{l,d} \equiv \rho[l_t, d_t]$ . 5) Correlation between the transaction cost rate and the start-of-the-month dividend yield,  $\rho_{d,\phi} \equiv \rho[d_t, \phi_t]$ . We focus on varying the last 3 to get an idea as to how each of these correlations affects liquidity premia.

Panel A of Table 2 reports the unconditional means and the standard deviations (in percent) of the three Hasbrouck series for the period 1962 to 2002. The volatility of the half-spread difference is 7.08% which is much larger than the 0.76% used in the liquidity premia calculations. This result suggests that we are using a conservative volatility number for  $\phi$  when we allow  $\phi$  to be state-dependent. Interestingly, the mean of 5.44% is also larger than the 2% used in the liquidity premia calculations.

Panel B of Table 2 reports the correlations, and again we focus on the cost rate differential across the low and high liquidity portfolios.<sup>5</sup> We find that the correlation of  $g$  with  $d$  is negative, consistent with the intuition that labor income is lower during recessions. We also find that  $\phi$  is positively correlated with  $d$ , which is consistent with the idea that stocks are more expensive to trade in recessions. The correlation between  $\phi$  and  $g$  is negative. We realize that the series we

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<sup>5</sup>When calculating a correlation, the same data period is used to calculate the standard deviations and the covariance, to ensure the correlation lies between -1 and 1. For a given pair of variables, all dates with data for both are used in the calculation of their covariance.

are using to proxy  $g$  and  $\phi$  are quite noisy and so the reported correlations are likely to be noisy estimates of the true correlations. But these numbers should at least be informative as to the direction of the relations and may even contain some information as to the magnitudes.

### 4.3 *Other parameter choices*

The investor’s risk aversion parameter,  $\gamma$ , is set to six for all the labor income cases described in section 4.2 and for all the multiplicative shock cases considered. This  $\gamma$  choice is motivated by the Mehra and Prescott (1985) argument that the existing evidence from macro and micro studies constitutes an *a priori* justification for restricting the value of  $\gamma$  to be less than ten. The horizon  $T$  of the young investor is 240 periods or 20 years, since the return processes are calibrated to monthly returns. A 20-year horizon is a realistic investment horizon for an investor who retires at time 1. We keep the horizon at 20 years for the labor income and multiplicative wealth shock cases to allow more direct comparisons to the canonical i.i.d. return problem with no labor income. The time preference parameter,  $\delta$  is set equal to the inverse of the riskfree return.

## 5 Liquidity premia and turnover results

This section discusses the liquidity premium, direct trading cost and turnover numbers reported in Table 3 for the labor income cases and the liquidity premium numbers reported in Table 4 and Figures 2 and 3 for the multiplicative wealth shock cases. Table 3 reports annual liquidity premia (in percent) on the low liquidity portfolio in Panel A, the associated direct trading costs in Panel B and turnover values in Panel C, for financial wealth to monthly permanent income ratios ( $\Gamma$ ) of 0, 1, 10, 100, 1000 and  $\infty$ . Each column contains results for one labor income case and the following cases are included: i.i.d. Return; i.i.d. Labor Income Growth; Base; One Risky; State-dependent Transaction Cost; and Liquidity Preferring. All these cases were described in section 4.2 except the Liquidity Preferring case which will be described in the market clearing section below.

### 5.1 *Return predictability*

As the financial wealth to monthly permanent income ratio  $\Gamma$  converges to  $\infty$ , the labor income problem converges to the otherwise identical problem without labor income. Consequently, the  $\Gamma = \infty$  rows in Table 3 report liquidity premia, direct trading costs and turnover numbers for the otherwise identical problem without labor income. Thus, the i.i.d. Return case with  $\Gamma = \infty$  is the canonical allocation problem with i.i.d. returns, a constant transaction cost rate and no wealth

shocks. The  $\Gamma = \infty$  liquidity premium for this i.i.d. Return case in Panel A of Table 3 is 0.08% per annum. Since the transaction cost rate is 2%, this premium is an order of magnitude smaller than the rate, consistent with results in Constantinides (1986). Similarly, the i.i.d. Labor Income Growth case with  $\Gamma = \infty$  is the allocation problem with predictable returns calibrated to data, a constant transaction cost rate and no wealth shocks. The reported premium for this problem is 0.43% per annum which means that return predictability calibrated to that in the data increases the liquidity premium on the low liquidity portfolio by a factor of 5 relative to the canonical case.

The reason for the increase is as follows. The usual motive for trading is to rebalance the portfolio back to the optimal weights after realized risky asset returns alter the portfolio's composition from the optimal weights. Return predictability causes the optimal portfolio weights to move around through time as the agent takes advantage of the time-varying expected returns. This time variation in the optimal weights creates an additional motive for trading that causes higher liquidity premia when returns are predictable.

## 5.2 *i.i.d. Labor income growth*

The i.i.d. Return column in Panel A of Table 3 shows that with i.i.d. returns and a fixed transaction cost rate, the inclusion of i.i.d. labor income growth uncorrelated with returns causes the liquidity premium on the low liquidity portfolio to become 1.42% per annum for an agent with no financial wealth, an almost 18-fold increase relative to the canonical i.i.d. case. For an agent with a wealth to monthly labor income ratio of 10, the liquidity premium is still 0.94% per annum, a more than 11-fold increase. The i.i.d. Labor Income Growth column of Panel A in Table 3 shows that once returns are allowed to be predictable (with the transaction cost rate remaining a constant), i.i.d. labor income growth uncorrelated with returns causes the liquidity premium to be 1.63% and 1.08% for an agent with a wealth to monthly labor income ratio of 0 and 10 respectively. These represent 20-fold and 13-fold increases, respectively, relative to the canonical i.i.d. Return case with no labor income. So the introduction of i.i.d. labor income growth uncorrelated with returns is enough to drastically increase the liquidity premium relative to the canonical problem, especially for agents with wealth to monthly labor income ratios lower than 10. Interestingly, labor income still increases liquidity premia markedly when returns are i.i.d., which means that labor income still causes liquidity premia to increase considerably even if the sample estimates of  $b_g$  are higher than the true values. For a given wealth to monthly labor income ratio, the liquidity premium for i.i.d. returns can be regarded as a lower bound on the premium given the actual return predictability

in the data.

Labor income causes liquidity premia to increase for two reasons. First, labor income is paid in cash and earns the riskless rate unless invested in stock. Thus, labor income distorts portfolio holdings away from the optimal weights causing the agent to rebalance back to the optimal weights. Second, shocks to labor income are permanent and so have large effects on the agent's total wealth. These total wealth shocks can lead to large shifts in the optimal weights in the agent's financial wealth portfolio particularly when the agent has a low wealth income ratio. These shifts in the optimal weights provide a strong motive for the agent to trade, leading to much higher liquidity premia than in the case with no labor income. Adding a temporary component to the monthly shock to labor income has almost no effect on liquidity premia and the implication is that the second of these two channels is much more important than the first. It is also not surprising that return predictability still has an incremental effect, further inflating the premium, in the presence of labor income. As discussed above, return predictability causes the optimal portfolio weights to move around with the stage in the business cycle. This additional variation in the optimal weights can provide a motive for trading over and above that provided by the effects of labor income on the optimal portfolio weights.

### ***5.3 Procyclical labor income growth***

In the Base case, returns are predictable and labor income growth is allowed to be procyclical, as in the data, but the transaction cost rate is constant. The Base case column in Panel A of Table 3 shows that allowing labor income growth to be procyclical when returns are predictable slightly reduces the liquidity premium at very low and very high wealth income ratios. The liquidity premium declines 0.15% to 1.48% when the agent has no financial wealth but this is still an 18-fold increase in the liquidity premium relative to the canonical i.i.d. Return case with no labor income. The premium actually increases slightly to 1.12% per annum at a wealth to monthly labor income of 10.

The reductions in liquidity premia occur because expected returns are countercyclical but labor income growth is procyclical. When returns are predictable, the optimal stock portfolio weights vary positively with expected stock returns. When dividend yield is high, expected returns and the optimal stock portfolio weights are high but labor income growth is low and so labor income's downward pressure on the portfolio weights in stocks is small. Conversely, when dividend yield is low, expected returns and the optimal stock portfolio weights are low but labor income growth is

high and so labor income's downward pressure on the portfolio weights in stocks is large. In both cases, the agent trades less relative to the i.i.d. labor income growth case and so liquidity premia are slightly lower also.

Labor income being procyclical causes liquidity premia to increase at moderate  $\Gamma$  values for the following reason. The hedging demand due to labor income growth being procyclical causes optimal stock holdings to decline (see Lynch and Tan, 2004). The result is that the 100% maximum for total holdings of stocks binds less often. Consequently, the agent trades more relative to the i.i.d. labor income growth case and the liquidity premium increases. We expect this increase to be largest for moderate  $\Gamma$  values: that is, for values not so small that the 100% stock-holding maximum binds almost always even with labor income being procyclical nor so big that the 100% stock-holding maximum rarely binds even when labor income is not procyclical.

An interesting question is the magnitude of the spread in risk-adjusted returns across the low and the high liquidity portfolios as compared to the liquidity premium on the low liquidity portfolio required by the Base case agent. Since the Base case agent is risk averse, we are interested in calculating the spread in volatility-adjusted returns across the two assets. To do this, we find the expected monthly discretely-compounded return on the low liquidity portfolio that would equate the monthly unconditional Sharpe ratio for the low liquidity portfolio to the monthly unconditional Sharpe ratio for the high liquidity portfolio and then we compare this expected return to the low liquidity portfolio's unconditional mean discretely-compounded monthly return in the data. The difference in the two is a measure of the difference in the volatility-adjusted returns across the two assets. Table 1 reports unconditional Sharpe ratios and unconditional mean monthly returns for the low and high liquidity portfolios based on monthly discretely-compounded returns. The unconditional Sharpe ratio for the high liquidity portfolio is 0.1023, while the unconditional Sharpe ratio for the low liquidity portfolio is higher, at 0.1641. Because the low liquidity portfolio's return volatility is higher than that of the high liquidity portfolio, the unconditional mean monthly return for the low liquidity portfolio that equates the two unconditional Sharpe ratios, 0.65%, while lower than the portfolio's unconditional mean monthly return in the data of 0.98%, is still higher than the unconditional mean monthly return on the high liquidity portfolio in the data of 0.55%. So the spread in volatility-adjusted returns across the two portfolios is the difference between 0.98% per month and 0.65% per month, with the low liquidity portfolio having the higher volatility-adjusted return. This spread in the volatility-adjusted return of 0.33% per month, or 3.90% per annum, is much higher than than the liquidity premium of 1.48% per annum required by the Base case agent

with a wealth-income ratio of 0.

Since the volatility-adjusted return spread across the low and high portfolios is larger than the liquidity premium required by the Base case agent with a wealth-income ratio of 0, we are interested in finding out how much the spread must be reduced for this agent to require a liquidity premium equal to the spread. That is, what spread in volatility-adjusted returns across the low and high portfolios can cause this agent to demand a liquidity premium equal to the spread? Since any given return spread generates a liquidity premium required by this agent, we are looking for a fixed point such that the return spread generates a liquidity premium required by this agent that is exactly equal to the volatility-adjusted return spread. We have in mind an iterative procedure. At the second iteration, we reduce the mean return of the low liquidity portfolio by 2.50% per annum to reduce the spread in mean return by 2.50% per annum. This also reduces the volatility-adjusted spread by 2.50% per annum from the 3.90% per annum in the data down to 1.40% per annum, which is lower than the 1.48% liquidity premium demanded by the Base case agent with a zero wealth-income ratio. Confronted with this reduced spread in volatility-adjusted returns, Table 3 shows (see the Fixed Point column) that this agent with a wealth-income ratio of 0 demands a liquidity premium of 1.42% per annum, which is slightly higher but essentially the same as the spread in volatility-adjusted returns of 1.40% per annum. Since the liquidity premium number is essentially the same as the spread in volatility-adjusted returns, we don't iterate further. Thus, we have shown that a spread in volatility-adjusted returns of 1.40% per annum across the two portfolios can generate a liquidity premium of roughly the same magnitude.<sup>6</sup>

#### ***5.4 Only one risky asset available: the low liquidity portfolio***

Comparing the Base case to the One Risky case in Panel A of Table 3, we find that removing access to the high liquidity asset only slightly increases liquidity premia. For a wealth to monthly labor income ratio of 10, the premium increases only 0.01% per annum while the increase is still only 0.06% when the agent has no financial wealth. Removing the high liquidity risky asset would be expected to cause only small increases in liquidity premia if the base case agent holds very little of this asset.

Figure 1 plots average holdings of the two risky assets by the Base-case agent for initial wealth to monthly income values of 0, 1 and 10. Average holdings as a function of age are obtained by

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<sup>6</sup>Even if we ignore the differences in volatility across the two assets and just compare the spread in their expected returns to the liquidity premium required by the Base case agent with a wealth-income ratio of 0, we find that a spread in expected returns across the two assets of 1.02% per annum causes this agent to demand a liquidity premium of 1.05% per annum.

simulating a large number of return and labor income paths, with the unconditional distribution of the dividend yield used to select an initial state for each path. Figure 1 shows that the agent’s average holding of the high liquidity asset is much smaller than the agent’s average holding of the low liquidity portfolio except in the last few months of life for all four initial  $\Gamma$  values. Moreover, the average holding of the high liquidity asset is less than 10% for all but the last 3 years of life for all four initial  $\Gamma$  values.<sup>7</sup> The implications for market clearing of the Base-case agent holding much less of the high liquidity asset than the low liquidity asset are explored in the next section.

### 5.5 *State-dependent transaction cost rate*

The effect on liquidity premia of allowing the transaction cost rate to be state-dependent with the dependence calibrated to data is assessed by examining a case that is identical to the One Risky case except for the state-dependent transaction cost rate. The reason that the high liquidity asset is not made available is the excessive computational burden associated with the two risky asset problem when the cost rate is state dependent. Comparing the State-dependent Transaction Cost column to the One Risky column in Panel A of Table 3, we see that liquidity premia increase even further when the transaction cost rate is allowed to be state-dependent as observed in data. In particular, the per annum premium increases 0.17% for the agent with no financial wealth and 0.11% when the agent’s wealth to monthly income ratio is 10. It seems reasonable to expect similar though slightly smaller increases when the transaction cost rate in the Base case with two risky assets is allowed to be state dependent.

The reason for the increase in the liquidity premia is as follows. In the data, the transaction cost rate is countercyclical which means it’s high when future expected returns are high and so the optimal portfolio weights in stocks are high too. These are exactly the times when the agent trades the most since the effect of labor income (because it’s paid in cash) is to lower the portfolio weights in stocks.

### 5.6 *Multiplicative wealth shocks*

Table 4 reports per annum liquidity premia on the low liquidity portfolio in the presence of multiplicative wealth shocks for an agent with a risk aversion of 6. Panel A tabulates liquidity premia for cases with a constant transaction cost rate while Panel B tabulates them for cases with a state-dependent transaction cost rate. Subpanels vary the wealth shock dynamics. Recall that to make

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<sup>7</sup>The Base-case agent’s risky asset allocations wiggle in last few months of life because the agent does not receive labor income at the terminal date.

the multiplicative shock analysis more directly comparable to the labor income cases we consider, the multiplicative shock process is calibrated to the Base case labor income process.

Panel A indicates that when the transaction cost rate is a constant the liquidity premium exhibits a strong u-shaped pattern as a function of the unconditional correlation between a given month's wealth shock and the dividend yield at the start of that month  $\rho_{l,d}$ . The table reports premia for  $\rho_{l,d}$  of -0.5, 0, 0.5 and the data value of -0.0372. The liquidity premium is lowest for the data value. To ascertain exactly where the minimum liquidity premium value occurs, we plot the liquidity premium as a function of  $\rho_{l,d}$  when the transaction cost rate is a constant in Figure 2. The u-shaped pattern suggested by Table 4 is confirmed by the graph with the lowest premium occurring for a correlation of about -0.05, which is close to the data value. Panel B of Table 4 shows that this u-shaped pattern persists when the transaction cost rate is state-dependent, irrespective of the correlation between the cost rate and either start-of-month dividend yield or permanent labor income growth. The implication of the u-shaped pattern for the liquidity premium as a function of  $\rho_{l,d}$  is that the premia for the labor income cases likely would be higher if the correlation between the permanent labor income growth and start-of-month dividend yield was further away from 0 in either direction than the data correlation (obtained using Retail Trade as the aggregate labor income series).

The intuition for the large premium when the correlation is negative is as follows. The implied hedging demand with respect to future labor income is negative when this correlation is negative since return shocks are negatively correlated with dividend yield shocks. The resulting lower average holding means a higher premium for the same amount of trading. Turning to the large premium when this wealth shock - dividend yield correlation is positive, the dividend yield at the start of a month is positively related to the expected stock return over that month so a positive wealth shock - dividend yield correlation implies a positive correlation between the wealth shock and expected stock return over the month. Since the wealth shock is paid in cash just like labor income, a higher wealth shock causes the inherited allocation to the risky assets to decrease more. So a positive unconditional relation between the wealth shock and conditional expected risky-asset return means that the inherited allocation to the risky assets is low because of the wealth shock precisely when the investor wants to hold the risky assets because of their high conditional expected returns. The result is a particularly large liquidity premium.

When the proportional transaction cost rate is allowed to be state dependent, intuition suggests that wealth shocks are especially painful if negative wealth shocks occur when the transaction cost

rate is high. Panel B of Table 4 confirms this intuition with the liquidity premium always decreasing in the unconditional correlation between wealth shocks and the transaction cost rate, holding all else equal.

Another question of interest is how sensitive the liquidity premium is to changes in the volatility of the wealth shock. Intuition suggests the relation should be increasing and monotonic and Figure 3, which plots the liquidity premium as a function of the shock volatility, confirms this. Figure 3 plots liquidity premia for shock volatilities as high as 30% per annum which generate premia as high as 2.21%. When the shock volatility is zero, the problem does not collapse to the Base-case problem with an infinite wealth-income ratio because of the non-zero mean of the multiplicative shock. The liquidity premium is 0.63% per annum which is slightly higher than the 0.43% in the Base labor income case with  $\Gamma = \infty$ .

### 5.7 *Turnover and direct trading costs*

Recall that turnover of the low liquidity portfolio per annum and the associated direct cost per annum are reported in Panels C and B respectively of Table 3 for all the labor income cases considered in the paper. Panel C shows that annual turnover goes from about 4% per annum in the canonical case with i.i.d. returns and no labor income to 38% per annum for the Base-case agent with a wealth to monthly income ratio of 10 and to 51% per annum for the Base-case agent with a wealth to monthly income ratio of 1. These turnover numbers in the presence of labor income are in the ballpark of the monthly turnover numbers reported by Acharya and Pedersen (2002) for their low liquidity portfolios, numbers which ranged from 3.25% up to 4.19%. Consistent with return predictability alone increasing liquidity premia considerably and labor income alone increasing them even more, turnover increases to 19% per annum when returns are predictable and to 32.02% when the agent receives labor income whose growth is i.i.d. and the agent's wealth to monthly labor income ratio is 1.

The turnover number can be multiplied by the average cost to get the direct effect of transaction costs on expected return. The extent to which the liquidity premium exceeds this direct cost can be attributed to some combination of risk premium for trading more when the agent is poor in utility terms, and in the case of a state-dependent cost rate, to the agent trading more when the cost rate is high. Not surprisingly, the direct cost is close to the liquidity premium when there is no labor income and the cost rate is constant, irrespective of whether returns are predictable or not. Labor income drives a wedge between the liquidity premia and the direct costs. For the Base-case agent

with a wealth to monthly income ratio of 1, Panel B reports a direct cost of 1.03% per annum while the liquidity premium in Panel A is 1.44%. For the case of i.i.d. returns and labor income growth and the same wealth income ratio of 1, the direct cost reported in Panel B is less than half the 1.38% per annum liquidity premium reported in Panel A.

The reason for the wedge is the following. Labor income causes the 100% upper bound on total stock holdings to bind often, which means that the agent is more likely to sell stock after a negative shock to permanent labor income than to buy stock after a positive shock. So the agent is trading more after negative labor income shocks which tend to be times when the agent is poor in utility terms. Return predictability ameliorates this asymmetry because the 100% upper bound is likely not to bind when last month's dividend yield state was low.

## 6 *Equilibrium issues and market clearing*

Market clearing is an important consideration when thinking about how prices and liquidity premia are determined in equilibrium. This section discusses how heterogeneity in labor income and risk aversion and heterogeneity induced by delegated portfolio management can allow all assets to be held and net trade each month to sum to zero.

### 6.1 *All assets must be held*

In the U.S. economy, the high liquidity asset has a larger market capitalization than the low liquidity asset, so market clearing is not possible at observed prices if all agents in the economy are identical to the Base case agent. However, it is likely that there is considerable heterogeneity across agents regarding their risk aversions and the income processes they receive. Some agents, like our Base case agent, invest large fractions of their portfolios in the low liquidity asset and small fractions in the high liquidity asset, while other agents invest small fractions of their portfolio in the low liquidity asset and large fractions in the high liquidity asset. They do so because they find the low liquidity asset less attractive as compared to the high liquidity asset than agents like our Base case agent. Aggregating across agents gives the aggregate values of the low and high liquidity portfolios. The aggregate value for the low liquidity asset can be much lower than for the high liquidity asset because most of the wealth in the economy is held by agents who hold small fractions of their portfolios in the low liquidity asset. We focus on an agent who invests a large positive fraction of her portfolio in the low liquidity asset because this is an agent who is definitely a marginal or inframarginal investor in the low liquidity asset.

For this heterogeneity argument to be convincing, we need to demonstrate that agents with plausible preference parameters and receiving plausible labor income streams holds much larger fractions of their portfolios in the high than the low liquidity asset, at least for considerable parts of their lives. Holdings for one such liquidity-preferring agent, the Moderate Liquidity Preferring agent, are reported in Figure 4. When her initial wealth to monthly income ratios is between zero and 1, this agent holds more of the high than the low liquidity asset for at least the first 5 years of her 20 year life. At wealth to monthly income ratios as high as 10, this agent still holds more of the high than the low liquidity asset for some period of time early in life. This Moderate Liquidity Preferring agent has risk aversion of 12 which is higher than that for the agent in the other cases but certainly is not unreasonably high. The permanent labor income growth process for this Liquidity Preferring case is calibrated to that for young collage educated service workers in Gakidis (1997) who uses PSID data. The volatility of its log is 30% per annum which matches the number in Gakidis and its mean is 4% which lies between the 3% per annum for the Base case agent and the 7.42% per annum number implied by Gakidis's results. Panel A of Table 3 reports liquidity premia for this Moderate Liquidity Preferring agent and these premia are higher than those for the Base-case agent, holding wealth-income ratio fixed.

Holdings for another such liquidity-preferring agent, the Extreme Liquidity Preferring agent are reported in Figure 5. This Extreme Liquidity Preferring agent also has risk aversion of 12 and receives the same permanent labor income growth process as the Moderate Liquidity Preferring agent except that its conditional contemporaneous correlation with the low liquidity portfolio's return is 30% and with high liquidity portfolio's return is -20% (both correlations are 0 for the Moderate Liquidity Preferring agent). When her initial wealth to monthly income ratio is between zero and 1, this agent holds more of the high than the low liquidity asset for at least the first 11 years of her 20 year life and holds more than 7 times more of the high than the low liquidity asset in her first month. And in the last 9 years of life, the agent never holds more than 3.5 times more of the low than the high liquidity asset. At wealth to monthly income ratios as high as 10, this agent still holds more of the high than the low liquidity asset for the first 7 years of life and holds more than twice the high than the low liquidity asset in her first month. Panel A of Table 3 reports liquidity premia for this Extreme Liquidity Preferring agent and these premia are higher than those for both the Base-case and Moderate Liquidity Preferring agents, holding wealth-income ratio fixed.

So far we have taken as given that the agent lives for 20 years, even though most agents have

working lives that are much longer than this. An agent who works from age 22 to age 65 has 43 years of working life. Holding the labor income and return processes fixed, the portfolio allocation and consumption rules we obtain for our 20-year agent can also be regarded as the portfolio allocation and consumption rules for the last 20 years of life for a 43-year agent. This agent's portfolio allocation rules for the first 23 years of her life are likely to look much more like her rule with 20 years till the terminal date than her rule with 1 month till the terminal date. But both liquidity-preferring agents tend to hold more of the high liquidity asset relative to the low liquidity asset as they get younger, which suggests that for either agent, the 43-year version would on average allocate a much larger fraction of her portfolio to the high liquidity asset as a multiple of the fraction allocated to the low liquidity asset than the 20-year version. We can't verify this directly due to computational constraints, but in unreported results we find that the 20-year version of either on average allocates a much larger fraction of her portfolio to the high liquidity asset as a multiple of the fraction allocated to the low liquidity asset than the 10-year version. So the intuition described above works going from a 10-year to a 20-year liquidity-preferring agent which makes it likely that it also works going from a 20-year to a 43-year liquidity-preferring agent.

There is another important source of heterogeneity that can help markets clear despite the presence of agents who hold more of the low liquidity than the high liquidity stocks. Heterogeneity may also come from participation by agents who care about something other than consumption from their labor income and financial wealth portfolios. In particular, Cuoco and Kaniel (2006) show how the existence of a delegated portfolio management industry in which managers receive a symmetric fulcrum fee that depends on performance relative to a benchmark can cause funds to tilt fund portfolios towards the stocks in the benchmark. The result is higher prices and lower Sharpe ratios in equilibrium for these benchmark stocks, which are typically high liquidity stocks. Thus, funds managed on behalf of others may hold disproportionate amounts of the high liquidity stocks while agents like our base case agent are the inframarginal investors in the low liquidity stocks. Moreover, funds managed on behalf of others are likely to be playing a big role in market clearing in the U.S. because the delegated portfolio management industry holds such a large fraction of U.S. equities. As an illustration, 22% of U.S. corporate equities were held by U.S. mutual funds as at December 31, 2003 (see page 59 of the ICI Mutual Fund Fact Book, 2004). And the total fraction of U.S. equity held by the delegated portfolio management industry is likely to be much higher than this number once pension fund holdings are included.

## 6.2 Net trades must sum to zero

Labor income is able to generate large increases in turnover and the question arises as to whether it's reasonable to think that this trading is sufficiently unsynchronized to allow the trades to sum to zero each period. What matters is the extent to which shocks to labor income are idiosyncratic rather than systematic. It is important to realize that this distinction is not the same as the distinction between permanent and temporary shocks to labor income. To get an idea of how much trading can be generated by the idiosyncratic component, we decompose the variance of permanent individual labor income shocks into a systematic component and an idiosyncratic component and show that almost all the variance is due to the idiosyncratic component. We ignore temporary shocks since their presence has a negligible impact on liquidity premia and their inclusion only makes the fraction due to the idiosyncratic component even larger.

To implement the decomposition, we compare aggregate and individual labor income growth volatility in the data. The wedge between the two can tell us something about the relative magnitudes of the idiosyncratic and systematic components. Specifically we can decompose individual labor income growth into a systematic component and an idiosyncratic component:

$$\Delta y^i = \Delta y^s + \Delta y^r, \quad (15)$$

where  $\Delta y^i$  is the log of individual labor income growth,  $\Delta y^s$  is the systematic component and  $\Delta y^r$  is the idiosyncratic component. Since almost by definition the two components are orthogonal, it follows that:

$$\sigma_{\Delta y^i}^2 = \sigma_{\Delta y^s}^2 + \sigma_{\Delta y^r}^2, \quad (16)$$

where  $\sigma_{\Delta y^i}^2$  is the variance of  $\Delta y^i$ ,  $\sigma_{\Delta y^s}^2$  is the variance of  $\Delta y^s$ ,  $\sigma_{\Delta y^r}^2$  is the variance of  $\Delta y^r$ . Assuming a large number of agents and sufficiently low cross-correlations between agents' idiosyncratic components, it is reasonable to use the variance of log aggregate labor income growth,  $\sigma_{\Delta y^a}^2$ , as a proxy for the average variance of the systematic component. The variance of growth in per capita Retail Trade income is 0.000165 per month and we can use it as an estimate of  $\sigma_{\Delta y^a}^2$ . Given the variance of the permanent component of individual labor income growth that we use in the Base case of 0.001875 per month and ignoring the temporary component, we can calculate an estimate of the contribution of the idiosyncratic component to the total variation of individual labor income growth as follows:

$$\frac{\sigma_{\Delta y^r}^2}{\sigma_{\Delta y^i}^2} = 1 - \frac{\sigma_{\Delta y^a}^2}{\sigma_{\Delta y^i}^2} = 0.9107, \quad (17)$$

which is a very large fraction.

As described earlier, the variance of the permanent component of individual labor income growth is taken from Gakidis and is representative of the numbers that are obtained using data for U.S. households. The implication is that because the idiosyncratic component constitutes such a large fraction of the variation in individual labor income growth, the idiosyncratic component of labor income can generate rebalancing demands that offset across agents and allow markets to clear. While this evidence is by no means conclusive, it is suggestive that markets can clear with agents rebalancing in response to labor income receipts. Moreover, to the extent that the idiosyncratic components are negatively correlated across agents due to economic growth being negatively correlated across geographic regions, rebalancing in response to the idiosyncratic component can also offset synchronized trading due to the systematic component and return predictability.

## 7 Conclusions

The seminal work of Constantinides (1986) documents how, when the risky return is calibrated to the U.S. market return, the impact of transaction costs on per-annum liquidity premia is an order of magnitude smaller than the cost rate itself. A number of recent papers have formed portfolios sorted on liquidity measures and found a spread in expected per-annum return that is definitely not an order of magnitude smaller than the transaction cost spread: the expected per-annum return spread is found to be around 6-7% per annum. Our paper bridges the gap between Constantinides' theoretical result and the empirical magnitude of the liquidity premium by examining dynamic portfolio choice with transaction costs in a variety of more elaborate settings that move the problem closer to the one solved by real-world investors. In particular, we allow returns to be predictable and we introduce wealth shocks, mainly labor income but also stationary multiplicative. With predictable returns, we also allow the wealth shocks and transaction costs to be state dependent.

We find that adding these real world complications to the canonical problem can cause transactions costs to produce per-annum liquidity premia that are no longer an order of magnitude smaller than the rate, but are instead the same order of magnitude. For example, the presence of predictable returns and i.i.d. labor income growth uncorrelated with returns, both calibrated to data, causes the liquidity premium for an agent with a wealth to monthly labor income ratio of 0 or 10 to be 1.63% per annum and 1.08% per annum respectively; these are 20-fold and 13-fold increases, respectively, relative to that in the standard i.i.d. return case with no labor income. And allowing labor income growth to exhibit the predictability observed in U.S. data causes very small

reductions in these premia even for very low wealth-income ratios. We conclude that the effect of proportional transaction costs on the standard consumption and portfolio allocation problem with i.i.d. returns can be materially altered by reasonable perturbations that bring the problem closer to the one investors are actually solving.

Clearly, our paper is but a first step toward bridging the gap between the theoretical literature to date and the empirical work finding large spreads in expected returns for portfolios formed on the basis of liquidity. One important limitation of our analysis is that it is partial equilibrium. Hence, it says nothing about how transaction costs affect equilibrium prices by limiting the ability of agents to risk share. More work is needed to understand how transaction costs effect prices and returns in a general equilibrium setting.

## Appendix A: Solution Technique for the Labor Income Problem

This appendix sketches the numerical procedures associated with computing liquidity premia in a dynamic savings and portfolio choice problem with predictable or i.i.d. returns and labor income.

There are three key elements to the implementation. The first is to endogenize the discrete state representation of the value function to bound error propagation at each iteration. The second is to resort to extrapolation only when the problem on hand is economically sufficiently close (in a sense to be made clear below) to a problem for which the functional form for the value function is known. The third is to exploit a natural sense in which the algorithm can be parallelized across computational units to obtain linear reductions in run-time.

The concern that gives rise to the above elements is that the wealth to lagged permanent labor income (wealth-income, henceforth) ratio state is unbounded on the non-negative side of the real line. To represent the value function on this dimension, this range is partitioned into three disjoint, non-degenerate intervals. A different algorithm is applied to obtain an approximation on each interval. At each iteration, the lower intermediate boundary point is chosen to be the smallest value of wealth-income ratio such that an agent without the labor income for all periods to terminal date requires no more than 10% extra wealth to be equally happy as an otherwise identical agent with the labor income at that wealth-income ratio. The upper intermediate boundary point is chosen to satisfy the same definition at 1%. Over the lower-end interval, the value function is approximated as a piecewise linear form or piecewise shape-preserving monotone cubic hermite interpolant of Fritsch and Carlson (1980) for the wealth-income ratio state and as a piecewise linear form for the inherited allocation state. Over the higher-end interval, the value function is taken to be that of the otherwise identical problem without the labor income. Over the middle interval, the value function is approximated as a function of the form,  $V(W) = a \times (W - b)^c + d$ , where  $a, b, c, d$  are constants in  $\mathbb{R}$ , to match the function and the first derivative values at the upper and the lower intermediate boundary points. Further, at any given iteration and at any given grid node of the discretized state space, the objective function has to be jointly solved for consumption and portfolio policies subject to the short sales constraints on the T-bill and the risky assets. A recursive golden section algorithm is used to optimize the consumption policy, defined as the fraction of last period's permanent labor income consumed, accurate to the fourth decimal digit and the portfolio policy, defined as the fraction of wealth invested in each of the risky assets, accurate to the third decimal digit.

We suggest a dynamic gridding algorithm to bound errors on policy functions at each iteration.

This algorithm takes the value function representation for the previous iteration as given, computes the intermediate boundary points for the iteration at hand and continues to add points in the lower-end interval in a particular way until policies at no point on a representative grid for the next iteration differ by more than prespecified magnitudes across increasingly denser grids for the iteration on hand.

The steps of this algorithm in detail are:

- 1) take value function representation for the previous iteration,  $V_{t+1}(\cdot)$ , as given
- 2) initialize a set of grid nodes,  $X_t = \{\Gamma_{t,1}, \Gamma_{t,2}, \Gamma_{t,3}, \dots, \Gamma_{t,n}\}$ , in increasing order, where  $\Gamma_{t,1} = 0$ ,  $\Gamma_{t,n} = LP_{t,10\%}$ ,  $n$  is a constant, and  $LP_{t,10\%}$  is the lower intermediate boundary point for iteration  $t$  with the defining parameter set to 10%
- 3) define  $Y_t = \{(\Gamma_{t,1} + \Gamma_{t,2})/2, (\Gamma_{t,2} + \Gamma_{t,3})/2, \dots, (\Gamma_{t,n-1} + \Gamma_{t,n})/2\}$ , in increasing order
- 4) evaluate  $V_t(X_t)$  using the value function representation,  $V_{t+1}(\cdot)$
- 5) evaluate  $V_t(Y_t)$  using the value function representation,  $V_{t+1}(\cdot)$
- 6) evaluate the lower intermediate point,  $LP_{t-1,10\%}$  for iteration  $t - 1$  using  $V_t$  as represented by  $X_t \cup Y_t$
- 7) define  $A_{t-1} = \{(i - 1) \times (LP_{t-1,10\%})/q\}_{i=1}^{q+1}$ , where  $q \geq 2$  is a constant.
- 8) evaluate  $V_{t-1}(A_{t-1})$  using  $V_t$  as represented by  $X_t$
- 9) evaluate  $V_{t-1}(A_{t-1})$  using  $V_t$  as represented by  $X_t \cup Y_t$
- 10) define  $Z_{t-1} \subseteq A_{t-1} = \{x \in A_{t-1} \mid \text{and policy functions at } x \text{ using representations in (8) and (9) differ more than prespecified magnitudes}\}$
- 11) if  $Z_{t-1} = \emptyset$ , accept  $X_t \cup Y_t$  as sufficient representation for  $V_t$ , exit subroutine.
- 12) compute the range of wealth-income,  $\Gamma_t$ , range the system can possibly assume by starting at any point in  $Z$  under any realization of the return and the labor income shocks and under any allowed policy. This range is generically of the form  $[0 \ h_t]$ , for some  $h_t > 0$ .
- 13) update  $X_t$  as  $X_t \cup Y_t$ , in increasing order,
- 14) update  $Y_t$  as  $\{(X_{t,1} + X_{t,2})/2, (X_{t,2} + X_{t,3})/2, \dots, (X_{t,k-1} + X_{t,k})/2\}$ ; where  $k$  is the smallest positive integer satisfying  $X_{t,k} \geq \min(h_t, LP_t)$ .
- 15) return to step 5 and repeat.

Step (10) ensures that the maximal absolute scaled deviation in consumption policy (defined as the fraction of last period's permanent component of labor income consumed),  $|(\hat{\kappa}_{coarse} - \hat{\kappa}_{dense})/\hat{\kappa}_{dense}|$  is bounded from above by  $10^{-3}$  and the maximal absolute deviation in portfolio policy (defined as the fraction of wealth invested in the risky asset),  $|\alpha_{coarse} - \alpha_{dense}|$ , is bounded

from above by  $10^{-2}$  for each of the risky assets.  $q$  and  $n$  are set to 50. The initialization in step 2 is designed to take advantage of the previous iteration's ( $V_{t+2}$ ) representation.

This algorithm, given the choice of the interpolant produces a set of grid nodes which then, can represent the value function at hand sufficiently well before one goes on to the next iteration. For the i.i.d. Return case without the temporary shocks to labor income under the parametrization given in Table 3, choice of a piecewise linear interpolant leads, on average (over the life cycle), to approximately 400 grid nodes, and the choice of the piecewise shape-preserving monotone cubic hermite interpolant leads, on average (over the life cycle), to approximately 140 grid nodes. For the predictability cases, we always use the shape-preserving monotone cubic hermite interpolant. For the i.i.d. Labor Growth case with  $\rho_{g,d} = 0$  and other parameters as given in table 3, this algorithm leads, on average (over the life cycle), to approximately 230 grid nodes.

For any distinct parametric specification and return generating process, this dynamic gridding scheme is run without the transactions costs and each node on the resultant grid for the wealth-income ratio state is augmented with the 51-node uniform inherited allocation grid on  $[0, 1]$  to get the state discretization for the transactions cost problem. This procedure implies that the joint state for the i.i.d. Labor Growth case is on average (over the life cycle), represented with  $19 \times 230 \times 51 \cong 222,000$  grid nodes.

To determine the reduction in unconditional mean returns required to offset elimination of the transactions costs and keep the agent's expected utility the same, a standard bisection algorithm is used. This algorithm is set to produce liquidity premia accurate to the fourth decimal digit.

While reducing the unconditional mean return in the no transactions cost problem to calculate liquidity premia, the gridding scheme obtained for the otherwise similar case with the returns calibrated to the data is used. For the i.i.d. Return case without the temporary shocks in table 3 and for the no financial wealth case, we rerun the gridding scheme under the return processes with the unconditional mean reduced exactly in the magnitude of the liquidity premium for this case, and recompute liquidity premium. The liquidity premium is unchanged to reported precision.

To ensure that results are robust to the parameters of the solution algorithm, for the i.i.d. Return case without the temporary shocks in table 3, we change the defining parameter for the lower boundary point from 10% to 9%, 8%, 7%, and 6%. The maximal  $|(\hat{\kappa}_{10\%} - \hat{\kappa}_{x\%})/\hat{\kappa}_{x\%}|$ , for an equally spaced 100-node grid on  $[0, LP_{t,10\%}]$  is less than  $10^{-3}$  for all  $t = 1, \dots, 240$  and  $x = 6, 7, 8, 9$ . Similarly, the maximal  $|\alpha_{10\%} - \alpha_{x\%}|$  is less than  $10^{-2}$  for each of the risky assets. Fixing the defining parameter for the lower boundary point at 10%, we change the precision on

the consumption policy,  $\hat{\kappa}$ , from  $10^{-4}$  to  $10^{-5}$  and  $10^{-6}$ . We again note that the maximal absolute scaled consumption policies is less than than  $10^{-3}$  across the life cycle.

Finally, to keep run-time manageable, state space at each iteration is decomposed into disjoint subsets. Each subset is optimized by a particular peripheral computational unit, the value function values are aggregated by a central aggregator unit which in turn passes the interpolant representations back to the peripheral units. This parallelization scheme leads to linear reductions in run-time.

## References

- Acharya, V.V., Pedersen, L. H., 2002. Asset pricing with liquidity risk. Unpublished working paper. New York University, NY.
- Aiyagari, S.R., Gertler, M., 1991. Asset returns with transactions costs and uninsured individual risk: a stage III exercise. *Journal of Monetary Economics* 27, 309-31.
- Amihud, Y., 2002. Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets* 5, 31-56.
- Amihud, Y., Mendelson, H., 1986. Asset pricing and the bid-ask spread. *Journal of Financial Economics* 17, 223-49.
- Balduzzi, P., Lynch, A., 1999. Transaction costs and predictability: some utility cost calculations. *Journal of Financial Economics* 52, 47-78.
- Barberis, N., 2000. Investing for the long run when returns are predictable. *Journal of Finance* 55, 225-64.
- Brennan, M., Chordia, T. and Subrahmanyam, A., 1998. Alternative factor specifications, security characteristics and the cross-section of expected returns, *Journal of Financial Economics* 49, 345-374.
- Brennan, M., Subrahmanyam, A., 1996. Market microstructure and asset pricing: on the compensation for illiquidity in stock returns. *Journal of Financial Economics* 41, 441-64.
- Campbell, J., Viceira, L., 1999. Consumption and portfolio decisions when expected returns are time varying. *Quarterly Journal of Economics* 114, 433-495.
- Carroll, C., 1996. Buffer-stock saving: Some theory, Working paper, John Hopkins University.
- Carroll, C., 1997. Buffer-stock saving and the life cycle/permanent income hypothesis. *Quarterly Journal of Economics* 112, 1-55.
- Carroll, C., Samwick, A., 1997. The nature of precautionary wealth. *Journal of Monetary Economics* 40, 41-71.

- Cocco, J., Gomes, F., Maenhout, P., 2002. Consumption and portfolio choice over the lifecycle. Working paper, London Business School.
- Chamberlain, G., Hirano, K., 1997. Predictive distributions based on longitudinal earnings data. Working paper, Harvard University.
- Constantinides, G., 1986. Capital market equilibrium with transaction costs. *Journal of Political Economy* 94, 842-62.
- Cuoco, D., Kaniel, R., 2006. Equilibrium Prices in the Presence of Delegated Portfolio Management. Unpublished working paper. Duke University.
- Easley, D., Hvidkjaer, S., O'Hara, M., 2002. Is information risk a determinant of asset prices?. Forthcoming *Journal of Finance*.
- Fama, E., French, K., 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3-56.
- Fritsch, F. N., Carlson, R. E., 1980. Monotone Piecewise Cubic Interpolation. *SIAM Journal of Numerical Analysis* 17, 238-246
- Gakidis, H., 1997. Earnings uncertainty and life-cycle portfolio choice. Working paper, MIT.
- Gomes, F., Michaelides, A., 2002. Portfolio choice with habit formation: A life-cycle model with uninsurable labor income risk. Working paper, London School of Economics.
- Glosten, L., Harris, L., 1988. Estimating the components of the bid-ask spread. *Journal of Financial Economics* 21, 123-42.
- Grossman, S., Shiller, R., 1981. The determinants of the variability of stock market prices. *American Economic Review* 71, 222-227.
- Hansen, L., Singleton, K., 1982. Generalized instrumental variables estimation of nonlinear rational expectations models. *Econometrica* 50, 1269-1289.
- Hasbrouck, J. 2003. Trading costs and returns for U.S. equities: evidence from daily data. Unpublished working paper. New York University, NY.

- Ho, T. and Stoll, H., 1981, Optimal Dealer Pricing Under Transactions and Return Uncertainty, *Journal of Financial Economics* 9, 47-73.
- Heaton, J., Lucas, D., 1996. Evaluating the effects of incomplete markets on risk sharing and asset pricing. *Journal of Political Economy* 104, 443-487.
- Huang, M., 2002. Liquidity shocks and equilibrium liquidity premia. *Journal of Economic Theory*, forthcoming.
- Investment Company Institute, 2004. *Mutual Fund Fact Book*. Investment Company Institute.
- Kyle, A., 1985. Continuous auctions and insider trading. *Econometrica* 53, 1315-35.
- Lesmond, D., Ogden, J., Trzcinka, T., 1999. A new measure of total transaction costs. *Review of Financial Studies* 12, 1113-41.
- Lettau, M., Ludvigson, S., 2001. Resurrecting the (C)CAPM: A Cross-Sectional Test When Risk Premia Are Time-Varying. *Journal of Political Economy*, 109(6), 1238-1287.
- Lo, A., Mamaysky, H., Wang, J., 2001. Asset prices and trading volume under fixed transactions costs. Unpublished working paper. MIT and Yale University.
- Lynch A. W., 2000. Portfolio choice and equity characteristics: characterizing the hedging demands induced by return predictability. *Journal of Financial Economics* 62, 67-130.
- Lynch A. W., Balduzzi, P., 2000. Predictability and Transaction Costs: The Impact on Rebalancing Rules and Behavior. *Journal of Finance* 55, 2285-2310.
- Lynch A.W., Tan, S., 2003. Multiple risky assets, transaction costs and return predictability: implications for portfolio choice. Unpublished working paper. New York University, NY.
- Lynch A.W., Tan, S., 2004. Labor Income Dynamics at Business-cycle Frequencies: Implications for Portfolio Choice. Unpublished working paper. New York University, NY.
- Mehra, R., Prescott, E., 1985. The equity premium: a puzzle. *Journal of Monetary Economics* 15, 145-161.

Pástor, L., Stambaugh, R., 2003. Liquidity risk and expected stock returns. *Journal of Political Economy* 111, 642-685.

Stoll, H., 1978, The Supply of Dealer Services in Securities Markets, *Journal of Finance* 33, 1133-1151.

Tauchen, G., Hussey, R., 1991. Quadrature-based methods for obtaining approximate solutions to nonlinear asset pricing models. *Econometrica* 59, 317-96.

Vayanos, D., 1998. Transaction costs and asset prices: a dynamic equilibrium model. *Review of Financial Studies* 11, 1-58.

Viceira, L., 1997. Optimal portfolio choice for long-horizon investors with non-tradable labor income. Working paper, Harvard University.

Viceira, L., 2001. Optimal portfolio choice for long-horizon investors with non-tradable labor income. *Journal of Finance* 56, 433-470.

Table 1. SAMPLE STATISTICS, VAR COEFFICIENTS AND QUADRATURE APPROXIMATION: LOW AND HIGH LIQUIDITY PORTFOLIOS. The table reports moments and parameters for the high and low liquidity portfolios estimated from U.S. data and calculated for the quadrature approximation based on the VAR that uses log dividend yield as the only state variable. The t-stats for the slope coefficient estimates are also provided. The t-stats are computed using exactly identified GMM with 3 and 12 Newey-West lags. The VAR is described in section 4.1. The Acharya & Pedersen (2002) data set provides 25 value-weighted portfolios sorted on ILLIQ, a liquidity measure suggested by Amihud (2002). The high liquidity portfolio is the value-weighted portfolio of the most liquid 12 portfolios and the low liquidity portfolio is the value-weighted portfolio of the least liquid 13 portfolios. The data period is from February 1964 to December 1996. Panel A reports unconditional means, VAR slopes and VAR  $R^2$ s for the data and the quadrature approximation:  $b$  is the vector of VAR slopes and  $R^2$  denotes the regression  $R^2$ . Panel B reports the unconditional covariance matrix for the data and for the quadrature approximation. Panels C reports the conditional covariance matrices for the data VAR and the quadrature VAR. All results are for continuously compounded returns except as noted in the table. Returns are expressed per month and in percent.

Data								Quadrature		
<i>Panel A: Unconditional sample moments and VAR coefficients</i>										
Asset/Variable	Discretely-compounded		Uncond. Mean	$b$	t-stats		$R^2$	Uncond. Mean	$b$	$R^2$
	Uncond. Mean	Sharpe Ratio			3mth-lag	12mth-lag				
High Liquidity	0.55	0.1023	0.45	0.37	1.66	1.68	0.73	0.45	0.37	0.73
Low Liquidity	0.98	0.1641	0.84	0.58	2.07	2.10	1.19	0.84	0.58	1.19
Dividend Yield			0.00	0.98	79.53	86.14	95.67	0.00	0.96	92.58
Asset/Variable	High Liquidity	Low Liquidity	Dividend Yield					High Liquidity	Low Liquidity	Dividend Yield
<i>Panel B: Unconditional standard deviations, covariances (above diagonal), and correlations (below)</i>										
High Liquidity	4.34	20.42	-0.49					4.34	20.42	-0.75
Low Liquidity	0.88	5.33	-0.36					0.88	5.33	-0.65
Dividend Yield	-0.11	-0.07	1.00					-0.17	-0.12	1.00
<i>Panel C: Conditional standard deviations, covariances (above diagonal), and correlations (below) for the VAR with Dividend Yield as Predictor</i>										
High Liquidity	4.32	20.20	-0.85					4.32	20.20	-1.11
Low Liquidity	0.88	5.30	-0.92					0.88	5.30	-1.21
Dividend Yield	-0.94	-0.84	0.21					-0.94	-0.84	0.27

Table 2. EMPIRICAL ESTIMATES OF MEANS, STANDARD DEVIATIONS, AND CORRELATIONS OF WEALTH SHOCKS, TRANSACTION COSTS AND DIVIDEND YIELD. The table presents empirical means (in percent), standard deviations (in percent) and correlations of wealth shocks, proportional transaction costs and dividend yield, where the wealth shock is taken to be the change in the permanent income. Hasbrouck (2003), employs a bayesian approach to estimating half-spreads using daily data from all ordinary common equity issues on the CRSP daily database from 1962 to 2002 and averages the daily numbers to obtain half-spread estimates at an annual frequency for each stock. Fixing year, we sort the stocks on half-spread and equally weight the top and the bottom 50 % to obtain annual time series estimates for two hypothetical portfolios, *liquid portfolio* and *illiquid portfolio*, respectively. If half spread is greater than 30% for an observation, we set that observation to 30% analogous to a treatment in Acharya and Pedersen (2002). We construct a third time series, *Illiquid - Liquid*, by subtracting the liquid portfolio half spread from the illiquid portfolio half spread. In Panel A, we report the unconditional means and the standard deviations (in percent) of these series. The Acharya & Pedersen (2002) data set provides 25 value-weighted portfolios sorted on ILLIQ, a liquidity measure suggested by Amihud (2002). The low liquidity portfolio is the value-weighted portfolio of the least liquid 13 portfolios. The high liquidity portfolio is the value-weighted portfolio of the most liquid 12 portfolios. Acharya & Pedersen (2002) data set also provides the corresponding normalized ILLIQ which can be interpreted as a round trip transaction cost rate at the same monthly frequency. We halve this normalized variable and value weight to obtain one way proportional transaction cost rates for the low and the high liquidity portfolios. We also construct a third time series, *Low - High*, by subtracting the transaction cost rate of the high liquidity portfolio from that of the low liquidity portfolio. We correlate these time series, with the start-of-month dividend yield series,  $d$ , (12 month dividend yield on the value weighted NYSE index) on monthly data. The data period for this correlation is from February 1964 to December 1996. To obtain correlations between transaction costs and individual labor income growth, we first covary the cost and aggregate income growth series (total US Retail Trade Income) and divide this covariance with the appropriate monthly volatility of *individual* income (see section 4.2) and volatility of transaction costs. The data period for this correlation is from January 1972 to December 1996. We follow the same approach to obtain the correlation between the dividend yield and individual income growth. The data period for this correlation is from January 1972 to December 2003. Retail Trade Income is inflation and population adjusted CEU4200000004 from BLS. data.  $\Phi$  denotes the proportional transaction cost percentage,  $\rho$  denotes unconditional correlation,  $\mu$  denotes unconditional mean and  $\sigma$  denotes unconditional standard deviation. Further, we have that  $\phi = \log(1 + \Phi)$ .

<i>Panel A: Means and Standard Deviations</i>		
$\Phi$ Series	$\mu_{\Phi}$	$\sigma_{\Phi}$
Liquid	0.39	0.40
Illiquid	5.84	7.24
Illiquid-Liquid	5.44	7.08

<i>Panel B: Correlations</i>		
$\rho_{g,d}$	-3.72%	
$\Phi$ Series	$\rho_{\phi,d}$	$\rho_{\phi,g}$
High Liquidity	13.49%	-3.79%
Low Liquidity	11.40%	-1.55%
Low-High	11.16%	-1.46%

Table 3. PER ANNUM LIQUIDITY PREMIA, TURNOVER AND DIRECT TRADING COSTS ON THE LOW LIQUIDITY PORTFOLIO IN THE PRESENCE OF LABOR INCOME. The table reports annual liquidity premia (in percent) on the low liquidity portfolio as defined in Constantinides (1986) (Panel A), the associated direct trading costs (Panel B) and the turnover values (Panel C), for a number of labor income cases and for financial wealth to monthly permanent income ratios ( $\Gamma$ ) of 0, 1, 10, 100, 1000 and  $\infty$ . The return distribution in all cases is calibrated to data. Risk aversion is 6 in all cases but *Moderate* and *Extreme Liquidity Preferring*. All cases except *One Risky* and *State-dependent Transaction Cost* assume that the investor has access to the low liquidity portfolio, the high liquidity portfolio well as a riskless asset. Except for the *State-dependent Transaction Cost* case, the low liquidity portfolio trades at a constant 2% proportional transaction cost rate while the high liquidity portfolio trades costlessly. The labor income growth process in all cases but *Moderate* and *Extreme Liquidity Preferring* is calibrated to data with the annual growth rate of income set to 3% and annual standard deviation of log income growth set to 15%. In the *i.i.d. Return* case, returns and labor income growth are i.i.d.. In the *i.i.d. Labor Income Growth* case, returns are predictable but labor income growth is i.i.d.. In the Base case, returns and labor income growth are both predictable with  $\rho_{g,d} = 3.72\%$ . The *One Risky* case is the same as the Base case except that the investor does not have access to the high liquidity portfolio. The *State-dependent Transaction Cost* case is the same as the *One Risky* case except that the transaction cost rate is state dependent with  $\rho_{phi,d} = 11.16\%$ ;  $\rho_{phi,g} = 1.46\%$ ,  $\mu_{\Phi} = 2\%$  and  $\sigma_{\Phi} = 0.76\%$ . In the *Moderate Liquidity Preferring* case, returns are predictable but labor income growth is i.i.d. with a mean of 4% per annum and an annual standard deviation for its log of 30%, and risk aversion is 12. The *Extreme Liquidity Preferring* case is the same as the *Moderate Liquidity Preferring* case except that the correlation between shock to income and low liquidity return is 30% and shock to income and high liquidity return is -20% rather than 0%. The *Fixed Point* case, is the same as the *Base* case except that we calibrate the unconditional means differently. We first equate the sharpe ratios of the low and high liquidity portfolios by adjusting the mean of the log returns of the low liquidity portfolio and then add 1.4% per annum to the log returns of the low liquidity portfolio. Gaussian quadrature is used to deliver the joint distribution for  $(g_t, \phi_t, d_{t+1})$ . Three grid points are used for the g shock. The liquidity premium is defined to be the decrease in the unconditional mean log return on the low liquidity asset that the investor requires to be indifferent between having access to the risky asset without the transaction costs rather than with them. The mean is decreased by subtracting a constant from every state. Turnover per annum is defined to be  $12/240 \times \sum_{t=1}^{240} av(|A_t - \hat{A}_t|)/av(max(A_t, \hat{A}_t))$ , where  $av(\cdot)$  is taking the average across the 1,000,000 simulation paths under the optimal policies and where  $A_t$  is the dollar chosen low-liquidity asset holdings and  $\hat{A}_t$  is the dollar effective low-liquidity asset holdings at time t. Direct trading cost per annum is defined to be  $2\% \times turnover$ . The initial fraction in the low liquidity portfolio is set to the optimum in the analogous no transactions cost case. Initial dividend yield values come from the unconditional dividend yield distribution. The Acharya & Pedersen (2002) data set provides 25 value-weighted portfolios sorted on ILLIQ, a liquidity measure suggested by Amihud (2002). The low liquidity portfolio is the value-weighted portfolio of the least liquid 13 portfolios. The high liquidity portfolio is the value-weighted portfolio of the most liquid 13 portfolios. The data period is from February 1964 to December 1996.  $\Phi$  denotes the proportional transaction cost rate on the low liquidity portfolio,  $G$  denotes the multiplicative monthly permanent income shock,  $d$  denotes log start-of-month dividend yield,  $\rho$  denotes unconditional correlation,  $\mu$  denotes unconditional mean and  $\sigma$  denotes unconditional standard deviation. Further, we have that  $\phi = \log(1 + \Phi)$  and  $g = \log(G)$ .

$\Gamma$	<i>i.i.d. Return</i>	<i>i.i.d. Labor Income Growth</i>	<i>Base</i>	<i>One Risky</i>	<i>State-dep. Trans. Costs</i>	<i>Moderate Liquidity Preferring</i>	<i>Extreme Liquidity Preferring</i>	<i>Fixed Point</i>
<i>Panel A: Liquidity Premia Per Annum</i>								
0	1.42	1.63	1.48	1.54	1.71	1.53	1.67	1.42
1	1.38	1.56	1.44	1.50	1.63	1.48	1.59	1.36
10	0.94	1.08	1.12	1.13	1.24	1.22	1.31	1.06
100	0.27	0.54	0.53	0.58	0.68	0.57	0.62	0.48
1000	0.11	0.50	0.47	0.48	0.51	0.53	0.54	0.42
$\infty$	0.08	0.43	0.43	0.46	0.34	0.49	0.49	0.38
<i>Panel B: Direct Cost Per Annum</i>								
0	0.65	1.04	1.06	1.18	0.92	0.97	1.15	1.05
1	0.64	1.01	1.03	1.14	0.90	0.95	1.10	0.96
10	0.46	0.71	0.76	0.85	0.69	0.76	0.84	0.69
100	0.23	0.42	0.40	0.44	0.48	0.43	0.46	0.35
1000	0.10	0.39	0.40	0.42	0.44	0.40	0.41	0.32
$\infty$	0.08	0.39	0.39	0.41	0.42	0.36	0.36	0.31
<i>Panel C: Turnover Per Annum</i>								
0	32.58	52.14	53.14	58.95	46.09	48.64	57.32	52.59
1	32.02	50.38	51.40	56.87	45.03	47.57	55.12	47.83
10	23.12	35.49	38.14	42.28	34.62	37.88	41.83	34.27
100	11.44	20.84	20.13	22.15	23.80	21.73	22.76	17.41
1000	4.94	19.74	19.93	20.78	22.12	19.84	20.28	16.01
$\infty$	3.90	19.49	19.49	20.54	21.17	18.05	18.05	15.47

Table 4. PER ANNUM LIQUIDITY PREMIA ON THE LOW LIQUIDITY PORTFOLIO IN THE PRESENCE OF MULTIPLICATIVE WEALTH SHOCKS FOR RISK AVERSION OF 6. The table reports annual liquidity premia (in percent) on the low liquidity portfolio as defined in Constantinides (1986) when a portfolio of high liquidity stocks (high liquidity portfolio) is also available. Panel A tabulates liquidity premia for cases with a constant transaction cost rate while Panel B tabulates them for cases with a state-dependent transaction cost rate whose unconditional mean is always fixed at 2%. Subpanels vary the wealth shock dynamics. The liquidity premium is defined to be the decrease in the unconditional mean log return on the low liquidity asset that the investor requires to be indifferent between having access to the risky asset without the transaction costs rather than with them. The mean is decreased by subtracting a constant from every state. The multiplicative shock process to the labor income growth process that's used in the base case described above. The distribution of the log wealth shock  $l$  is calibrated to that of  $\ln(Y_t/W_t)$  in the *Base* labor income case which is distributed  $N(\bar{g} + b_g d_t - \ln(\Gamma_t), \sigma_u^2)$ . We allow  $l$ 's mean to be age-and dividend yield state-dependent. The mean is obtained by calculating  $(\bar{g} + b_g d_t)$  using the *Base* case parameters which are calibrated to data and using a  $\Gamma$ -age and -dividend profile calculated by taking  $\Gamma$  at age 1 to be 1, simulating and averaging at each age and each dividend yield state. Then we adjust the volatility of  $l$  so as to match the liquidity premium for the base case agent with a  $\Gamma$  at age 1 of 1. Gaussian quadrature is applied to  $l$  and  $\phi$  to obtain their joint distribution, with the conditional expectation of  $l$  linear in the  $d$  value. Three grid points are used for the  $l$  shock. Returns can be predictable or i.i.d. and in either case are calibrated to U.S. data using a quadrature approximation. The initial fraction in the low liquidity portfolio is set to the optimum in the analogous no transactions cost case. Initial dividend yield values come from the unconditional dividend yield distribution. The Acharya & Pedersen (2002) data set provides 25 value-weighted portfolios sorted on ILLIQ, a liquidity measure suggested by Amihud (2002). The low liquidity portfolio is the value-weighted portfolio of the least liquid 13 portfolios. The high liquidity portfolio is the value-weighted portfolio of the most liquid 13 portfolios. The data period is from February 1964 to December 1996.  $\Phi$  denotes the proportional transaction cost percentage on the low liquidity portfolio,  $L$  denotes the monthly percentage wealth shock,  $d$  denotes start-of-month log dividend yield,  $\rho$  denotes unconditional correlation,  $\mu$  denotes unconditional mean and  $\sigma$  denotes unconditional standard deviation. Further, we have that  $\phi = \log(1 + \Phi)$  and  $l = \log(L)$ . The risk aversion parameter,  $\gamma$ , is set to 6.

<i>Panel A:</i>		$\Phi = 2\%$	
$\rho_{l,d}$		<i>i.i.d.</i>	<i>Predictable</i>
-0.5			4.24
-0.0372			1.44
0		1.46	1.71
0.5			3.62
<i>Panel B:</i>		$\mu_\Phi = 2\% \sigma_\phi = 0.76\%$	
		$\rho_{\phi,l} = -0.5$	$\rho_{\phi,l} = 0$
		<i>i.i.d.</i>	<i>Predictable</i>
$\rho_{\phi,d}$	$\rho_{l,d}$	<i>i.i.d.</i>	<i>Predictable</i>
	-0.5		4.61
-0.5	0		2.39
	0.5		4.50
	-0.5		4.99
0	0	1.12	2.33
	0.5		3.71
	-0.5		5.84
0.5	0		2.47
	0.5		3.19
		$\rho_{\phi,l} = 0$	$\rho_{\phi,l} = 0.5$
		<i>i.i.d.</i>	<i>Predictable</i>
		<i>i.i.d.</i>	<i>Predictable</i>
			4.53
			2.07
			4.25
			4.86
		1.07	1.97
			3.30
			5.46
			2.09
			2.82
			4.99
			1.82
			2.48

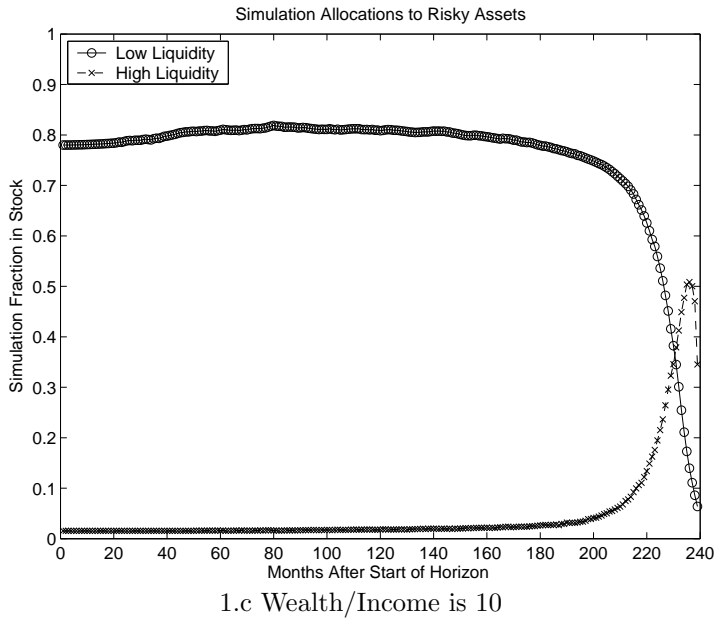
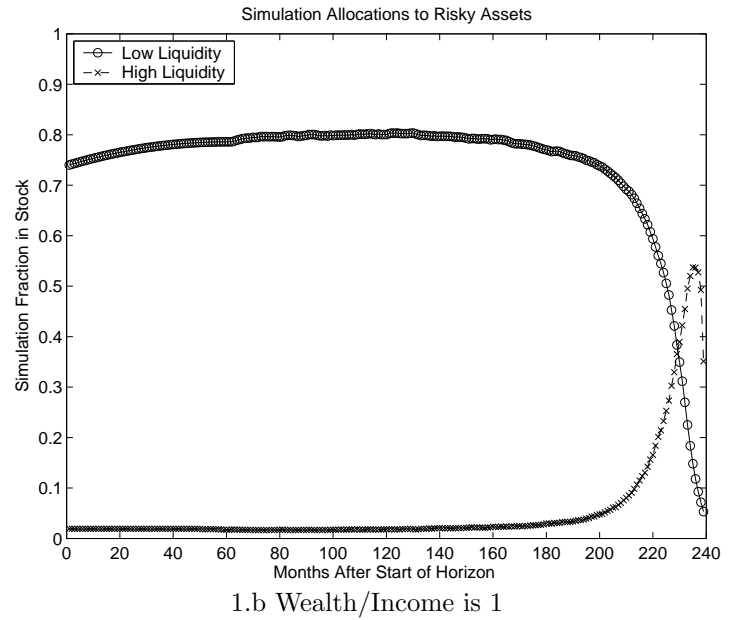
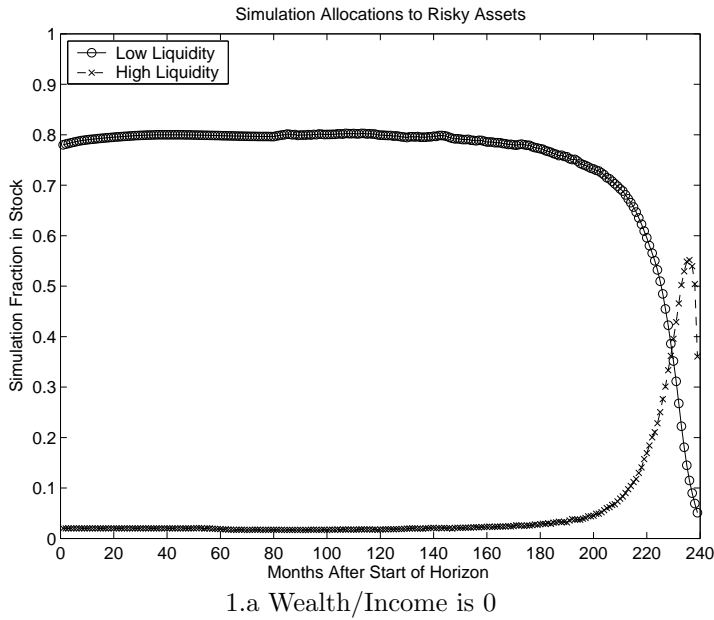


FIGURE 1. SIMULATION ALLOCATION RESULTS FOR THE LOW AND HIGH LIQUIDITY PORTFOLIOS: BASE CASE. The figures report simulation allocation fractions for the Low and High Liquidity Portfolios for the *Base* case when the initial wealth to monthly permanent income ratios of 0, 1 and 10. The investor has access to the low liquidity portfolio, the high liquidity portfolio has well as a riskless asset. The low liquidity portfolio trades with a constant 2% proportional transactions cost while the high liquidity portfolio trades costlessly. The marginal and conditional distributions of returns and labor income growth are calibrated as given in sections 4.1 and 6.1. *Base* case assumes returns and labor income growth are both predictable and that risk aversion is 6. The annual growth rate of income and the annual standard deviation of income growth to 3% and 15% respectively. Simulations set the initial fraction in the low liquidity portfolio to the optimum in the analogous no transactions cost case. Initial dividend yield values come from the unconditional dividend yield distribution. 1 Million simulations are performed. The Acharya & Pedersen (2002) data set provides 25 value-weighted portfolios sorted on ILLIQ, a liquidity measure suggested by Amihud (2002). The low liquidity portfolio is the value-weighted portfolio of the least liquid 13 portfolios. The high liquidity portfolio is the value-weighted portfolio of the most liquid 13 portfolios. The data period is from February 1964 to December 1996.

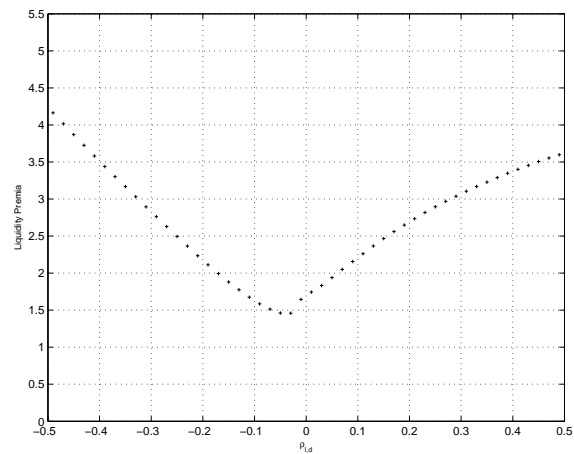


FIGURE 2. LIQUIDITY PREMIA AS A FUNCTION OF THE CORRELATION BETWEEN THE MULTIPLICATIVE WEALTH SHOCKS AND DIVIDEND YIELD FOR RISK AVERSION OF 6. The figure reports annual liquidity premia in percent on the low liquidity portfolio, against the unconditional correlation of the monthly log wealth shock with the log start-of-month dividend yield,  $\rho_{l,d}$ , when  $\Phi = 2\%$ . The liquidity premium is defined to be the decrease in the unconditional mean log return on the low liquidity asset that the investor requires to be indifferent between having access to the risky asset without the transaction costs rather than with them. The mean is decreased by subtracting a constant from every state. Except for its correlation with the log start-of-month dividend yield, the multiplicative shock process is calibrated to the labor income growth process that's used in the *Base* case described in section 4.2., in particular the distribution of the log wealth shock  $l$  is calibrated to that of  $\ln(Y_t/W_t)$  in the *Base* labor income case which is distributed  $N(\bar{g} + b_g d_t - \ln(\Gamma_t), \sigma_u^2)$ , with adjusted  $b_g$  and  $\sigma_u$  to accommodate changing  $\rho_{l,d}$ . We allow  $l$ 's mean to be age- and dividend yield state-dependent. The mean is obtained by calculating  $(\bar{g} + b_g d_t)$  using the adjusted  $b_g$  and using a  $\Gamma$ -age and -dividend profile calculated by taking  $\Gamma$  at age 1 to be 1, simulating the *Base* case and averaging at each age and each dividend yield state. We adjust the volatility of  $l$  so as to match the liquidity premium for the base case agent with a  $\Gamma$  at age 1 of 1 when  $\rho_{l,d} = -3.72\%$ . Gaussian quadrature is applied to  $l$  and  $\phi$  to obtain their joint distribution, with the conditional expectation of  $l$  linear in the  $d$  value. Three grid points are used for the  $l$  shock. The investor has access to the low liquidity portfolio, the high liquidity portfolio as well as a riskless asset. The low liquidity portfolio trades with a constant 2% proportional transactions cost while the high liquidity portfolio trades costlessly. Returns are predictable and are calibrated to data by a quadrature approximation. The Acharya & Pedersen (2002) data set provides 25 value-weighted portfolios sorted on ILLIQ, a liquidity measure suggested by Amihud (2002). The low liquidity portfolio is the value-weighted portfolio of the least liquid 13 portfolios. Data period is from February 1964 to December 1996. Let  $\Phi$  denote the proportional transaction cost rate,  $L$  denote the monthly percentage wealth shock,  $d$  denote the log dividend yield,  $\rho$  denote unconditional correlation,  $\mu$  denote unconditional mean and  $\sigma$  denote unconditional standard deviation. Further, define  $\phi = \log(1 + \Phi)$  and  $l = \log(L)$ .

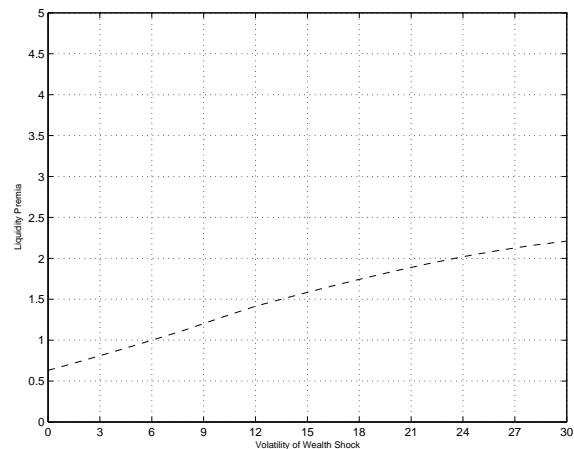
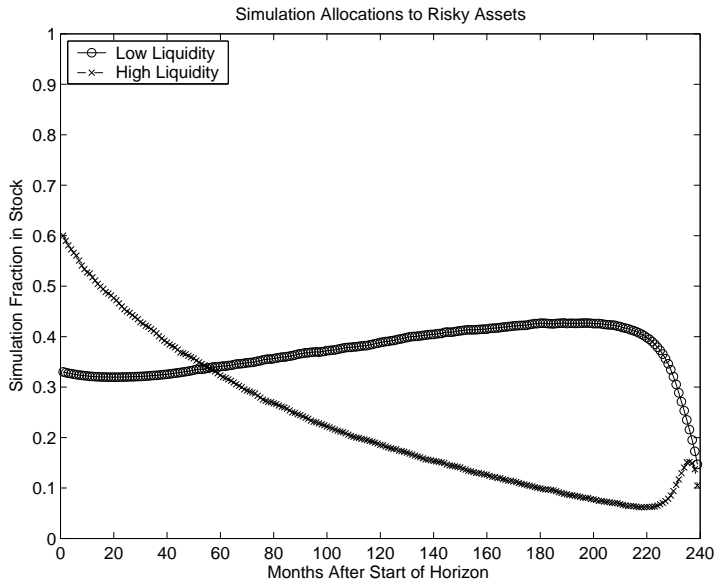
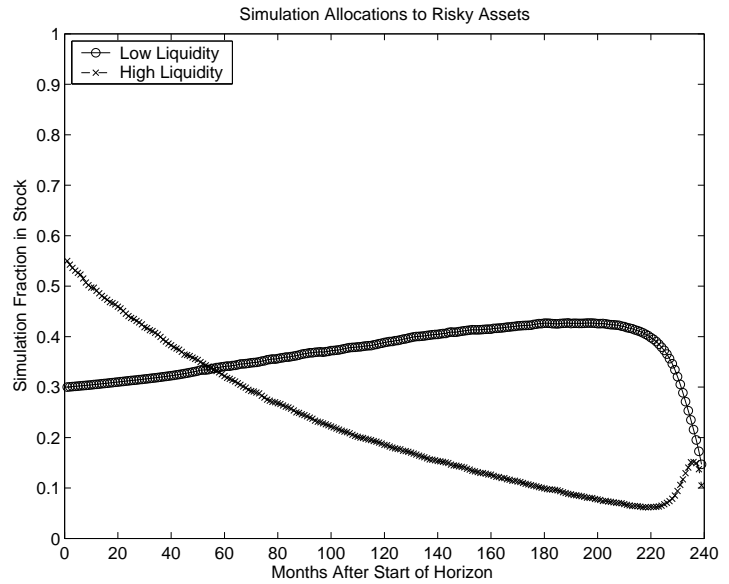


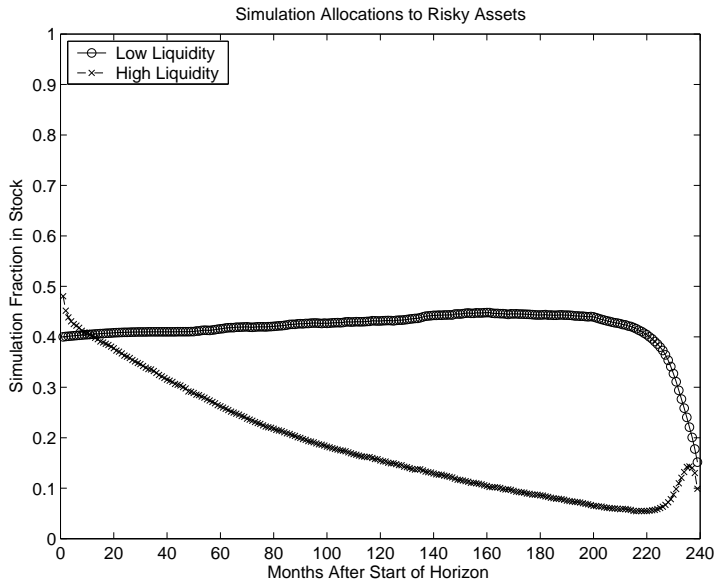
FIGURE 3. LIQUIDITY PREMIA AS A FUNCTION OF THE VOLATILITY OF THE MULTIPLICATIVE WEALTH SHOCKS FOR RISK AVERSION OF 6. The figure reports annual liquidity premia in percent on the low liquidity portfolio, against the unconditional standard deviation of the monthly log wealth shock  $\sigma_u$ , when  $\Phi = 2\%$ . The liquidity premium is defined to be the decrease in the unconditional mean log return on the low liquidity asset that the investor requires to be indifferent between having access to the risky asset without the transaction costs rather than with them. The mean is decreased by subtracting a constant from every state. Except for its unconditional standard deviation, the multiplicative shock process is calibrated to the labor income growth process that's used in the *Base* case described in section 4.2., in particular the distribution of the log wealth shock  $l$  is calibrated to that of  $\ln(Y_t/W_t)$  in the *Base* labor income case which is distributed  $N(\bar{g} + b_g d_t - \ln(\Gamma_t), \sigma_u^2)$ , with adjusted  $b_g$  and  $\sigma_u$  to accommodate changing  $\sigma_u$ . We allow  $l$ 's mean to be age- and dividend yield state-dependent. The mean is obtained by calculating  $(\bar{g} + b_g d_t)$  using the adjusted  $b_g$  and using a  $\Gamma$ -age and -dividend profile calculated by taking  $\Gamma$  at age 1 to be 1, simulating the *Base* case and averaging at each age and each dividend yield state. Gaussian quadrature is applied to  $l$  and  $\phi$  to obtain their joint distribution, with the conditional expectation of  $l$  linear in the  $d$  value. Three grid points are used for the  $l$  shock. The investor has access to the low liquidity portfolio, the high liquidity portfolio as well as a riskless asset. The low liquidity portfolio trades with a constant 2% proportional transactions cost while the high liquidity portfolio trades costlessly. Returns are predictable and are calibrated to data by a quadrature approximation. The Acharya & Pedersen (2002) data set provides 25 value-weighted portfolios sorted on ILLIQ, a liquidity measure suggested by Amihud (2002). The low liquidity portfolio is the value-weighted portfolio of the least liquid 13 portfolios. Data period is from February 1964 to December 1996. Let  $\Phi$  denote the proportional transaction cost rate,  $L$  denote the monthly percentage wealth shock,  $d$  denote the log dividend yield,  $\rho$  denote unconditional correlation,  $\mu$  denote unconditional mean and  $\sigma$  denote unconditional standard deviation. Further, define  $\phi = \log(1 + \Phi)$  and  $l = \log(L)$ .



4.a Wealth/Income is 0



4.b Wealth/Income is 1



4.c Wealth/Income is 10

FIGURE 4. SIMULATION ALLOCATION RESULTS FOR THE LOW AND HIGH LIQUIDITY PORTFOLIOS: MODERATE LIQUIDITY PREFERRING CASE. The figures report simulation allocation fractions for the Low and High Liquidity Portfolios for the *Moderate Liquidity Preferring* case when the initial wealth to monthly permanent income ratios of 0, 1 and 10. The investor has access to the low liquidity portfolio, the high liquidity portfolio as well as a riskless asset. The low liquidity portfolio trades with a constant 2% proportional transactions cost while the high liquidity portfolio trades costlessly. The marginal and conditional distributions of returns and the marginal distribution of labor income growth are calibrated as given in sections 4.1 and 6.1, respectively. *Moderate Liquidity Preferring* case assumes returns are predictable but labor income growth is i.i.d. and that risk aversion is 12. The annual growth rate of income and the annual standard deviation of income growth are 4% and 30% respectively. Simulations set the initial fraction in the low liquidity portfolio to the optimum in the analogous no transactions cost case. Initial dividend yield values come from the unconditional dividend yield distribution. 1 Million simulations are performed. The Acharya & Pedersen (2002) data set provides 25 value-weighted portfolios sorted on ILLIQ, a liquidity measure suggested by Amihud (2002). The low liquidity portfolio is the value-weighted portfolio of the least liquid 13 portfolios. The high liquidity portfolio is the value-weighted portfolio of the most liquid 13 portfolios. The data period is from February 1964 to December 1996.

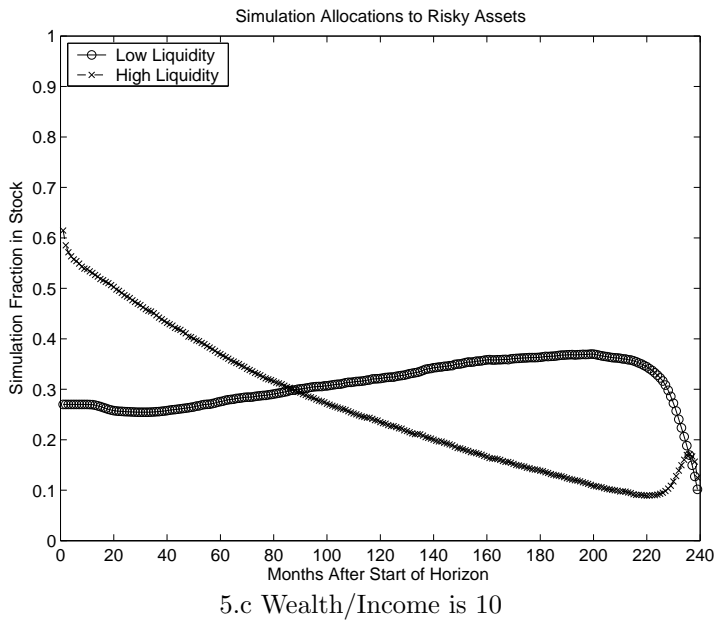
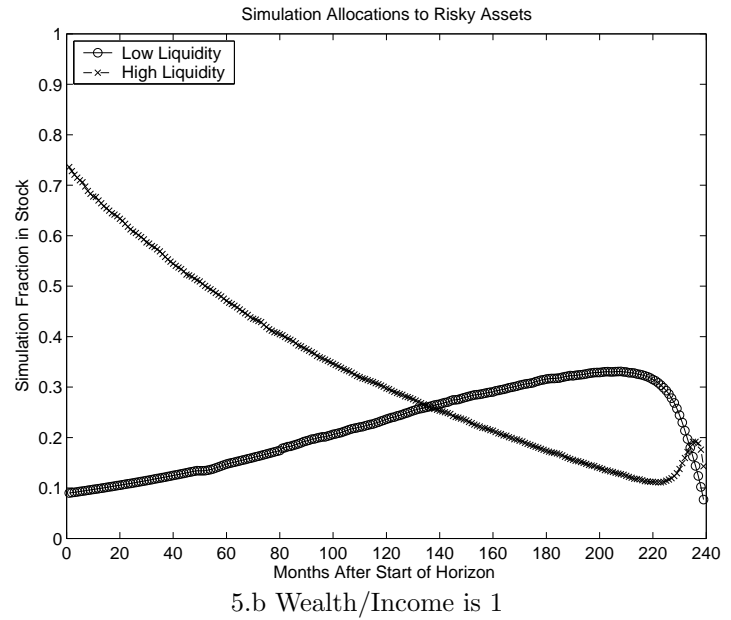
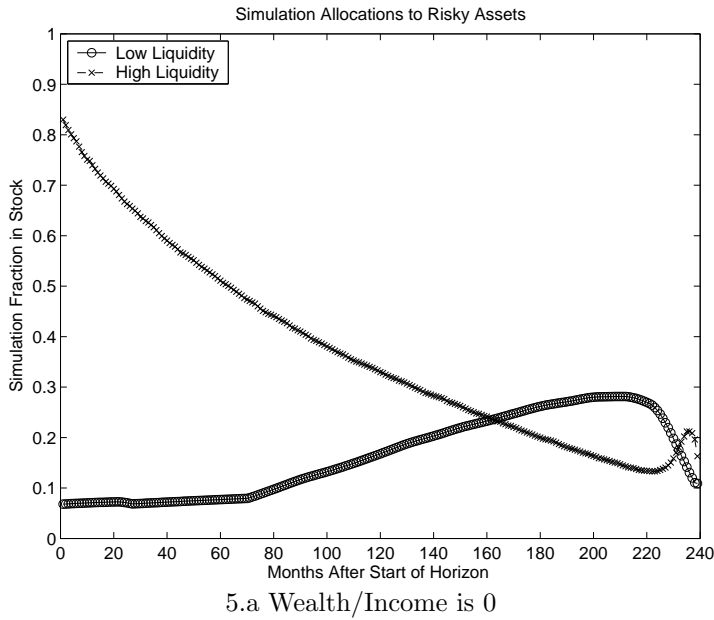


FIGURE 5. SIMULATION ALLOCATION RESULTS FOR THE LOW AND HIGH LIQUIDITY PORTFOLIOS: EXTREME LIQUIDITY PREFERRING CASE. CORRELATION BETWEEN SHOCK TO INCOME AND LOW LIQUIDITY RETURN IS 30% AND SHOCK TO INCOME AND HIGH LIQUIDITY RETURN IS -20%. The figures report simulation allocation fractions for the Low and High Liquidity Portfolios for the *Extreme Liquidity Preferring* case when the initial wealth to monthly permanent income ratios of 0, 1 and 10. The investor has access to the low liquidity portfolio, the high liquidity portfolio as well as a riskless asset. The low liquidity portfolio trades with a constant 2% proportional transactions cost while the high liquidity portfolio trades costlessly. The marginal and conditional distributions of returns and labor income growth is calibrated as described in section 4.1 and 6.1 respectively. *Extreme Liquidity Preferring* case assumes returns are predictable but and labor income growth is i.i.d. and that risk aversion is 12. The annual growth rate of income and the annual standard deviation of income growth to 4% and 30% respectively. Simulations set the initial fraction in the low liquidity portfolio to the optimum in the analogous no transactions cost case. Initial dividend yield values come from the unconditional dividend yield distribution. 1 Million simulations are performed. The Acharya & Pedersen (2002) data set provides 25 value-weighted portfolios sorted on ILLIQ, a liquidity measure suggested by Amihud (2002). The low liquidity portfolio is the value-weighted portfolio of the least liquid 13 portfolios. The high liquidity portfolio is the value-weighted portfolio of the most liquid 13 portfolios. The data period is from February 1964 to December 1996.