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The Macroeconomics of Shadow Banking

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ABSTRACT

We build a macrofinance model of shadow banking—the transformation of risky assets into securities that are money-like in quiet times but become illiquid when uncertainty spikes. Shadow banking economizes on scarce collateral, expanding liquidity provision, boosting asset prices and growth, but also building up fragility. A rise in uncertainty raises shadow banking spreads, forcing financial institutions to switch to collateral-intensive funding. Shadow banking collapses, liquidity provision shrinks, liquidity premia and discount rates rise, asset prices and investment fall. The model generates slow recoveries, collateral runs, and flight-to-quality effects, and it sheds light on Large-Scale Asset Purchases, Operation Twist, and other interventions.

RECENT ECONOMIC PERFORMANCE HAS BEEN the story of a boom, a bust, and a slow recovery. The rise and fall of shadow banking plays a central role in that story. In the boom years, shadow banking transformed risky loans into short-term money-like instruments held by households, firms, and institutional investors. These instruments traded at low spreads over traditional money-like instruments such as Treasury bills, indicating a high level of liquidity. This liquidity evaporated, however, with the onset of the financial crisis, when spreads opened up and shadow banking all but shut down, causing both liquidity and credit to contract sharply.[1] Shadow banking can thus be interpreted as fragile liquidity

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[1] Bernanke (2013) writes that "Shadow banking . . . was an important source of instability during the crisis . . . . Shadow banking includes vehicles for credit intermediation, maturity transformation, liquidity provision . . . . In the run-up to the crisis, the shadow banking sector involved a high degree of maturity transformation and leverage. Illiquid loans to households and businesses were securitized, and the tranches of the securitizations with the highest credit ratings were funded by

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transformation: it extends credit to riskier borrowers and provides liquidity to investors, liquidity that is as good as any other during quiet times but that disappears when the environment becomes more uncertain. Under this view, shadow banking presents us with a trade-off between stability and growth. In this paper, we build a dynamic macrofinance model of shadow banking as fragile liquidity transformation. We show how it boosts asset prices and economic growth while at the same time exposing the economy to changes in uncertainty. We also show how it builds financial and economic fragility, how it sets up slow recoveries, and how a number of policy interventions interact with these effects.

The model works as follows. Investors use liquid securities to take advantage of high-value opportunities that require them to trade quickly and in large amounts. Intermediaries create liquid securities by tranching assets. The top tranche is safe, which makes it fully insensitive to any private information about asset values and allows investors to trade it without fear of adverse selection (Gorton and Pennacchi (1990)). A safe security is thus always liquid. By contrast, the bottom residual tranche is risky, which makes it sensitive to private information and hence illiquid. Its role is to provide a cushion for the liquid securities.

The middle security tranche takes a loss only if a large shock called a crash hits. This loss is limited and rare enough to make the security insensitive to private information and thus liquid most of the time. However, there is a small probability that a crash becomes much more likely, in which case it becomes profitable to trade the security based on private information. The presence of privately informed trading creates adverse selection, causing the security to become illiquid. A security with limited crash exposure is therefore liquid most of the time, but not always. We call it fragile-liquid.

In sum, intermediaries can issue securities that are liquid most of the time by limiting their crash exposure and securities that are liquid all of the time by making them safe. The overall amount of liquid securities is thus constrained by the value of intermediaries' assets in a crash, that is, by their collateral value. Since fragile-liquid securities have a higher crash exposure than always-liquid securities, they require less collateral, which enables intermediaries to provide investors with a lot more liquidity overall. Investors can thus have a lot of liquidity most of the time, or a little liquidity all of the time.

We call the safe, always-liquid security "money." Examples include traditional bank deposits, government money market funds, and general collateral repurchase agreements. We call the fragile-liquid security "shadow money." Examples include large uninsured deposits, prime money market funds, private-label repurchase agreements, financial-backed commercial paper, and asset-backed commercial paper (ABCP), and other forms of short-term wholesale

very short-term debt, such as asset-backed commercial paper and repurchase agreements (repos). The short-term funding was in turn provided by institutions, such as money market funds, whose investors expected payment in full on demand . . . . When investors lost confidence in the quality of the assets . . . they ran. Their flight created serious funding pressures throughout the financial system . . . and inflicted serious damage on the broader economy."

funding. We interpret shadow banking as the process of creating shadow money.[2]

Shadow banking expands liquidity provision and raises asset prices in times of low uncertainty. Intuitively, investors are willing to rely on shadow money for their liquidity needs as long as it is likely to remain liquid. This is the case when a crash is unlikely, that is, when uncertainty is low. Low uncertainty thus results in a low spread between shadow money and money. The low spread makes shadow money an attractive source of funding for intermediaries. Its low collateral requirement enables them to make liquidity more abundant. Abundant liquidity allows investors to deploy their wealth when it is most valuable, which lowers their required return on savings and boosts asset prices. The prices of riskier assets rise the most because their low collateral values make them more reliant on shadow money funding. A boom in investment and growth ensues, but fragility builds up over time as the investment is concentrated in riskier assets.

A period of low uncertainty (e.g., the "Great Moderation" of the 1990s and early 2000s) thus induces a shadow banking boom similar to the one that preceded the 2008 financial crisis: spreads are narrow, shadow banking securities crowd out traditional money-like instruments, liquidity is abundant, and asset prices are high. The shadow banking boom in turn induces an economic boom: investment and growth are high, especially in riskier sectors. Consistent with this dynamic, the shadow banking boom that preceded the crisis led to a large expansion in residential and commercial real estate loans, as well as in auto, student, and credit card loans, all of which contributed to employment and economic growth. Moreover, the credit expansion was heavily concentrated among riskier borrowers (Mian and Sufi (2009)).

A rise in uncertainty brings the shadow banking boom to an end. Households are less willing to hold shadow money because its liquidity might evaporate. The spread between shadow money and money opens up, as did the spreads on shadow banking instruments in the summer of 2007. Intermediaries respond by sharply contracting shadow money (e.g., the collapse of the ABCP market) and switching to money. Since money requires a lot more collateral, intermediaries must also issue a larger illiquid residual tranche (equity). The supply of liquidity shrinks, more so given the low collateral value of the assets created during the boom. The liquidity contraction raises discount rates and lowers asset prices, and as a result investment falls and growth turns negative. In short, the liquidity cycle drives the macrocycle.

While uncertainty remains high, intermediaries invest only in safe, high collateral-value assets that they can fund primarily with money. Over time, this "collateral mining" makes the economy's capital stock safer, which allows

[2] Pozsar (2014) shows that the shadow banking system met the large and growing demand for highly liquid instruments of institutions such as asset managers and corporations whose holdings of such instruments tripled in size from \$2 trillion to \$6 trillion between 1997 and 2013. Sunderam (2014) further shows that ABCP issuance responds strongly to changes in liquidity premia.

liquidity provision to expand.[3] Yet growth remains low because safe assets are relatively less productive. Thus, it is liquidity transformation—funding risky productive assets with liquid securities—rather than liquidity per se that drives growth. In fact, growth remains low even after uncertainty recedes because it takes time to return to a productive capital mix and restart the cycle. Shadow banking booms thus lead to both severe busts and slow recoveries.

Beyond the recent crisis, the link between liquidity transformation in the financial sector and economic fragility has been documented for a broad set of countries and historical periods (Schularick and Taylor (e.g., 2012)). In particular, Krishnamurthy and Muir (2015) show that the increase in credit spreads at the onset of a crisis is particularly informative about its severity. In our model this increase in spreads reveals the amount of fragility built up during the boom and hence also predicts the magnitude of the subsequent contraction. The data also point to a trade-off between financial stability and economic growth: Rancière, Tornell, and Westermann (2008) show that countries that experience occasional crises tend to grow faster than countries with stable financial conditions. Such a trade-off is a key aspect of our framework.

We begin the paper with a simplified static model that we use to develop the notion of fragile liquidity and the trade-off between the size and stability of the liquidity supply. We then embed this trade-off in a dynamic framework with time-varying uncertainty and long-lived capital. Dynamics allow us to show how fragile liquidity affects asset prices and collateral values, and how the liquidity and macrocycles interact.

We model uncertainty in the dynamic framework as a time-varying probability of a crash that is the outcome of a learning process. It drifts down in quiet times, producing periods of low uncertainty like the Great Moderation, but jumps up after a crash as investors update their beliefs. The jump is largest from moderately low levels, similar to a "Minsky moment" (Minsky (1986)). Uncertainty also varies without crashes, due to news.

Our dynamic framework generates endogenous amplification via collateral runs (margin spirals in Brunnermeier and Pedersen (2009)). These are episodes during which falling collateral values reinforce falling asset prices. Collateral values in our model depend on prices because assets are long-lived. A collateral run occurs when prices become more exposed to crashes, which causes collateral values to fall. This happens at the end of a shadow banking boom, when exposure to uncertainty shocks rises as shadow banking starts to contract. With collateral values falling, intermediaries become more constrained, liquidity contracts further, asset prices fall more, and so on.

[3] The shift toward safety after the 2008 financial crisis took several forms such as the sharp and persistent tightening of securitization and lending standards (Becker and Ivashina (2014)) and the large increase in financial institutions' holdings of government-backed assets in place of private loans (Krishnamurthy and Vissing-Jorgensen (2015)). For instance, between 2007 and 2013, private mortgage originations declined from 60% to 20% of total originations, banks' risk-weighted assets ratios decreased from 76% to 67%, and their liquid assets ratios increased from 15% to 28% (Council (2015)).

Once uncertainty reaches a very high level and shadow banking shuts down, the economy is no longer exposed to uncertainty shocks. This causes collateral values to recover. Interestingly, this means that when uncertainty starts to come down and shadow banking picks up, exposure to uncertainty rises, causing collateral values at first to decline. This keeps liquidity tight and discount rates high, further contributing to the slow recovery. We call this novel mechanism the collateral decelerator.

Our framework also produces strong flight-to-quality effects—the tendency for safe assets to appreciate as overall asset prices decline. In our model this happens at the end of a shadow banking boom, when the liquidity supply is most exposed to uncertainty shocks. The flight from shadow money to money drives down the required return on money relative to all other securities. Intermediaries respond by bidding up the prices of safe assets, whose high collateral values can be used to back a lot of money. Importantly, flight to quality makes safe assets a hedge for risky assets on intermediary balance sheets, increasing overall collateral values.

We use our framework to shed light on several recent policy interventions. We first look at Large-Scale Asset Purchases (LSAP), whereby the government (an "intermediary" with lump-sum taxation power) purchases risky assets and sells safe assets when a crash hits. We show that by providing collateral when it is most needed, LSAP supports asset prices and by extension economic activity. The possibility of LSAP also boosts asset prices ex ante, amplifying booms.

Our second policy application is "Operation Twist," whereby the central bank buys long-term government bonds and sells zero-duration floating-rate bonds (e.g., reserves). In contrast to LSAP, Operation Twist is generally counterproductive. The reason is that, while long-term government bonds appreciate in a crisis as a result of flight to quality, floating-rate bonds always trade at par. Therefore, long-term government bonds have higher collateral values than floating-rate bonds. By swapping them, the central bank reduces the overall collateral value of intermediary balance sheets. Importantly, Operation Twist causes the yields of long-term government bonds to decline, but this is because the premium for collateral is higher. Therefore, the effectiveness of unconventional monetary policy cannot be judged solely by the response of government bond yields.

Our paper belongs to the macrofinance literature, an important strand of which focuses on the scarcity of net worth in the financial sector (e.g., Bernanke and Gertler (1989), He and Krishnamurthy (2013), Gârleanu and Pedersen (2011), Brunnermeier and Sannikov (2014), Rampini and Viswanathan (2012), Adrian and Boyarchenko (2012), Di Tella (2017)). In these papers net worth is the key state variable. A related strand of the literature emphasizes the role of collateral constraints (e.g., Kiyotaki and Moore (1997), Geanakoplos (2003), Gertler and Kiyotaki (2010), Gertler and Kiyotaki (2015), Gorton and Ordoñez (2014), Maggiori (2013)). In these papers net worth is again scarce and external financing is restricted by collateral. In this sense collateral is a substitute for net worth.

In our framework net worth plays no role. Intermediaries are constrained only in how many liquid securities they can issue. This distinction matters: in our setting a high level of intermediary equity is a sign of a constrained financial sector, high discount rates, and low investment, instead of the opposite. Implications for policy also differ: the most effective interventions increase aggregate collateral rather than inject capital into financial institutions. While we abstract from net worth–type frictions in the paper in order to highlight the novel aspects of our framework, we introduce such a friction in the Internet Appendix, where we show that it amplifies our main results.[4]

The link between liquidity creation and fragility has been studied in the banking literature (e.g., Diamond and Dybvig (1983), Allen and Gale (1998), Holmström and Tirole (1998)). To be clear, there are no bank runs or multiple equilibria in our framework. Liquidity is created by making securities informationally insensitive as in Gorton and Pennacchi (1990) and Dang, Gorton, and Holmström (2012). As in Kiyotaki and Moore (2012) and Caballero and Farhi (2013), we study the role of liquidity in a macroeconomic framework.[5] Our paper contributes to this literature by focusing on the trade-off between the size and stability of the liquidity supply and its implications for the macroeconomy.

Shadow banking has also been viewed through the lenses of behavioral bias (Gennaioli, Shleifer, and Vishny (2013)) and regulatory arbitrage (Acharya, Schnabl, and Suarez (2013), Harris, Opp, and Opp (2014)), in contrast to our emphasis on liquidity transformation and its importance for growth. Consistent with this liquidity transformation view, Shin (2012) shows that, alongside domestic shadow banks, European banks facilitated the expansion of credit in the United States by financing relatively risky loans with short-term wholesale funding. Chernenko and Sunderam (2014), Ivashina, Scharfstein, and Stein (2015), and Benmelech, Meisenzahl, and Ramcharan (2017) show that contractions in shadow banking sharply reduce the supply of credit to the economy.

The rest of the paper is organized as follows: Section I presents a simplified static model. Section II presents the full dynamic model. Section III presents numerical results. Section IV analyzes policy interventions. Section V concludes.

I. Static Model

We begin with a simplified static model that we use to develop the notion of fragile liquidity and the trade-off between the size and stability of the liquidity supply. This trade-off is at the core of our framework.

There are three dates spanning a short period: an initial date 0, an interim trading date 1, and a payoff date 2. There is a unit mass of risk-neutral investors who are subject to liquidity events in the spirit of Diamond and Dybvig (1983).

[4] The Internet Appendix is available in the online version of the article on the *Journal of Finance* website.

[5] In a related class of models, Eisfeldt (2004), Kurlat (2013), and Bigio (2015) study dynamic adverse selection.

Intuitively, a liquidity event is a valuable opportunity that requires trading. We model it as a shock to the marginal utility of consumption on date 1. Specifically, investors maximize

$$U_0 = \max\ E_0\left[z_1 C_1 + C_2\right], \tag{1}$$

where $z_1 \in \{1, \psi\}$, $\psi > 1$, is the liquidity-event shock, with $z_1 = \psi$ signifying a liquidity event. Liquidity events are privately observed and independent across investors; they arrive with probability h, and hence a fraction h of investors experience a liquidity event. Liquidity events generate gains from trade: investors who experience a liquidity event ($z_1 = \psi$) consume as much as they can on date 1 by promising to give up consumption on date 2. Investors who do not experience a liquidity event ($z_1 = 1$) are willing to take the other side and give up consumption on date 1 for consumption on date 2 at a one-for-one rate.[6]

Following Hart and Moore (1994), investors have limited commitment, so they cannot make credible promises without having assets to back them. Investors are endowed with assets that pay off Y_2 units of consumption on date 2, where

$$Y_2 = \begin{cases} 1 + \mu_Y, & \text{prob.} \quad 1 - \lambda_0 \\ 1 - \kappa_Y, & \text{prob.} \quad \lambda_0. \end{cases} \tag{2}$$

We interpret the low state as a rare crash (i.e., λ_0 is small). We normalize $E_0[Y_2] = 1$ by setting $\mu_Y = \frac{\lambda_0}{1-\lambda_0}\kappa_Y$, which makes changes in λ_0 a mean-preserving spread. We can therefore interpret λ_0 as a measure of uncertainty. Since there is no consumption on date 0, we take assets as the numeraire and normalize their price to one.

At the interim date 1, just before investors trade, there is a shock to the information environment. Specifically, investors learn the updated probability of a crash $\lambda_1 \in \{\lambda^L, \lambda^H\}$, with $\lambda^H > \lambda^L$. We consider the natural case in which asset payoffs become more uncertain when a crash becomes more likely, that is $\lambda^H(1 - \lambda^H) > \lambda^L(1 - \lambda^L)$. By the law of iterated expectations, the probability of this high interim-uncertainty state is $p_H(\lambda_0) = \frac{\lambda_0 - \lambda^L}{\lambda^H - \lambda^L}$, which is increasing in overall uncertainty λ_0.

The interim uncertainty shock λ_1 impacts liquidity-event trading by changing the potential for adverse selection in asset markets. We provide a formal description of how adverse selection arises in Appendix A.1; we summarize it here.

Investors can hire fund managers who have access to a private signal that reveals whether a crash will take place. The signal has a fixed cost f, and it generates profits from trading claims that are exposed to crashes. If the expected trading profit exceeds the cost, informed fund managers trade alongside liquidity-event investors. This creates asymmetric information and adverse selection. Adverse selection leads to costly fire sales as investors cannot sell their

[6] The usual interpretation is that negative consumption stands for supplying labor.

assets for their full present value under public information. Based on the idea that such costs are especially high for liquidity-event investors who must sell their assets quickly and in large amounts, we make the following assumption.[7]

ASSUMPTION 1 (Liquidity): *In a liquidity event, investors trade only claims that they can sell for their present value under public information. We call these liquid claims.*

Assumption 1 implies that liquidity-event consumption is constrained by the supply of liquid claims. Consequently, there is value in tranching assets in a way that maximizes this supply. This is done by competitive firms called intermediaries. Intermediaries buy assets on date 0 and use them as collateral to issue securities that pay off from the underlying assets' payoff on date 2. Each security x is defined by its crash return, $1 - \kappa_x$, which specifies how much collateral is pledged to the security in a crash. We call κ_x the crash exposure of security x and we show next that it determines whether the security is liquid in a given state on date 1.

To trade at present value and be liquid, a security must be designed in a way that deters private information acquisition (Gorton and Pennacchi (1990)). For this to hold, the expected profit from trading the security based on the private signal must be lower than the signal's cost f. This trading profit can be expressed as

$$\pi_1 \propto \lambda_1 (1 - \lambda_1)(\alpha + \kappa_x), \tag{3}$$

where α is a constant that controls fund managers' ability to take leverage (see Appendix A.1). The trading profit is increasing in crash exposure, κ_x, because the private signal predicts crashes. Hence, a liquid security must have a sufficiently low crash exposure. Given κ_x, the trading profit is also increasing in interim uncertainty because more uncertainty makes the private signal more informative. Therefore, a security that has a sufficiently low crash exposure to be liquid when $\lambda_1 = \lambda^L$ can become illiquid when $\lambda_1 = \lambda^H$. We call such a security fragile-liquid. To remain liquid at any level of interim uncertainty, the security must have even lower crash exposure.

In sum, each security has one of three liquidity profiles: illiquid, fragile-liquid, and always-liquid. Each requires progressively lower crash exposure, that is, progressively more collateral backing. Since collateral is limited by the low value of assets in a crash, $1 - \kappa_Y$, intermediaries issue only the securities with the highest crash exposure within each liquidity profile. This leads to the following result (the proof is in Appendix A.2).

[7] Consistent with our notion of liquidity, Gorton and Pennacchi (1990, p. 50) describe a liquid security as follows: "A liquid security has the characteristic that it can be traded by uninformed agents without loss to insiders." While we focus on the case of perfect liquidity for simplicity, Holmström (2015) argues that it describes the way money markets operate in practice.

PROPOSITION 1 (Securities): *Intermediaries optimally issue the following three securities:*

(i) money, m, with crash exposure $\kappa_m = 0$ is liquid for any $\lambda_1 \in \{\lambda^L, \lambda^H\}$ (always-liquid),
(ii) shadow money, s, with crash exposure $\kappa_s = \overline{\kappa}$ is liquid only if $\lambda_1 = \lambda^L$ (fragile-liquid), and
(iii) equity, e, with crash exposure $\kappa_e = 1$ is illiquid,

where $0 < \overline{\kappa} < 1$ under appropriate parameter restrictions on α and f.

The first security, money, has zero crash exposure and this makes it liquid in all states. The third security, equity, gets wiped out in a crash and this makes it illiquid. Equity is the residual tranche; its role is to provide a cushion for the remaining securities.

Shadow money lies in between. Its crash exposure, $\overline{\kappa}$, makes it just safe enough to be liquid when interim uncertainty λ_1 is low but not when it is high. Recall that interim uncertainty is more likely to be high when overall uncertainty λ_0 is high. Thus, shadow money is more likely to become illiquid when overall uncertainty is high.[8]

In equilibrium, investors choose their holdings of money, m_0, and shadow money, s_0, (equity is the residual) to maximize expected utility (1). In Appendix A.3, we show that we can write this problem simply as

$$\max_{m_0, s_0 \geq 0} \; E_0\left[h(\psi - 1)C_1 + Y_2\right] \tag{4}$$

subject to $m_0 + s_0 \leq 1$, the liquidity constraint

$$C_1 \leq \begin{cases} m_0 + s_0 & \text{if} \quad \lambda_1 = \lambda^L, \quad \text{prob.} \quad 1 - p_H(\lambda_0) \\ m_0 & \text{if} \quad \lambda_1 = \lambda^H, \quad \text{prob.} \quad p_H(\lambda_0), \end{cases} \tag{5}$$

and the collateral constraint

$$m_0 + s_0(1 - \overline{\kappa}) \leq 1 - \kappa_Y. \tag{6}$$

The objective (4) says that investors consume their endowment at marginal utility one, earning an additional net benefit of $\psi - 1$ for the part of that endowment that they get to consume in a liquidity event whose probability is h. The expectation is over the aggregate state $\{\lambda_1, Y_2\}$. The liquidity constraint (5) says that consuming in a liquidity event requires selling liquid securities. Money is always liquid but shadow money becomes illiquid if interim uncertainty is high (i.e., if $\lambda_1 = \lambda^H$). Finally, the collateral constraint (6) says that asset payoffs must be sufficient to pay off the issued securities in a crash. Figure 1 illustrates the economy's resulting balance sheet.

[8] Empirically, Nagel (2012) shows that market liquidity tends to evaporate when uncertainty spikes.

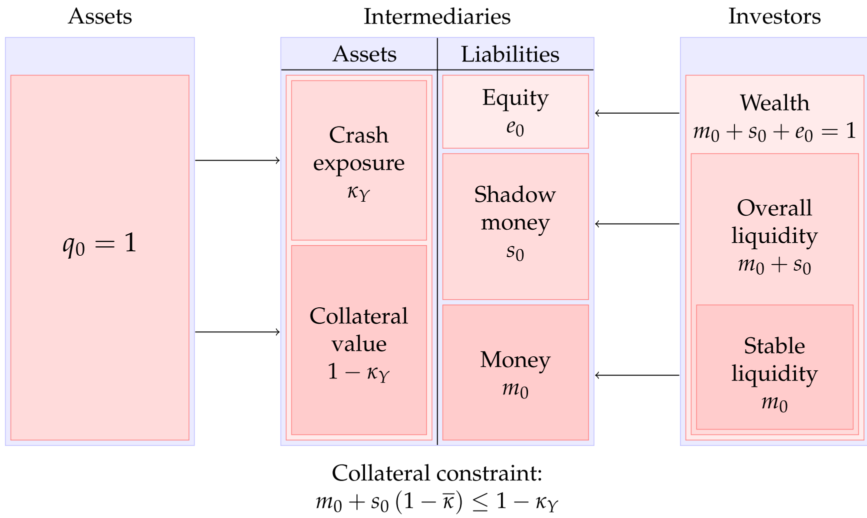

Figure 1. The economy's balance sheet. This figure represents the economy's balance sheet. The left side represents real assets (claims to output/capital), whose value is normalized to one in the static model. Intermediaries hold assets and issue money, shadow money, and equity against them, subject to the collateral constraint (6). Investors hold these securities. In the dynamic model all quantities receive a t subscript, the value of assets becomes the value of the aggregate capital stock, $q_t^a k_t^a + q_t^b k_t^b$, and collateral values become endogenous, $1-\kappa_{A,t}$. (Color figure can be viewed at wileyonlinelibrary.com)

The constrained optimization problem (4) to (6) shows the key trade-off in our framework. From (4), investors value liquidity because it allows them to transfer consumption to high marginal utility states. From (5) and (6), they can have a lot of liquidity most of the time by issuing shadow money, or a little liquidity all of the time by issuing money. They cannot have both because collateral is scarce.

Investors weigh the liquidity advantage of money, which depends on the probability that shadow money becomes illiquid, $p_H(\lambda_0)$, against the collateral advantage of shadow money, which depends on its ability to absorb losses in a crash, $\overline{\kappa}$. Focusing on the case $\overline{\kappa} \leq \kappa_Y$, if $p_H(\lambda_0) \leq \overline{\kappa}$, only shadow money is issued until it uses up all collateral: $m_0 = 0$ and $s_0 = \frac{1-\kappa_Y}{1-\overline{\kappa}}$ (the proof is in Appendix A.3). If instead $p_H(\lambda_0) > \overline{\kappa}$, only money is issued: $m_0 = 1-\kappa_Y$ and $s_0 = 0$. Therefore, since $p_H(\lambda_0)$ is increasing in λ_0, when uncertainty is low, shadow money crowds out money and the liquidity supply is large but fragile, whereas when uncertainty is high, the liquidity supply is small but stable.

II. Dynamic Model

We now present our full dynamic model. Motivated by the microfoundations we developed in the static model, here we take shadow money and the notion of fragile liquidity as given and introduce two additional ingredients. The first is fluctuations in uncertainty, which induce fluctuations in liquidity premia

Table I
Summary of Variables

This table contains descriptions of the variables in the dynamic model in Section II.

Description	Variable
Investor:	
Consumption outside a liquidity event	dc_t
Consumption in a liquidity event	dC_t
Upper bound on liquidity event consumption	$d\overline{C}_t$
Wealth	W_t
Holdings of money, shadow money, and equity (% of wealth)	m_t, s_t, e_t
Technology:	
Compensated crash process	dZ_t
Uncertainty (perceived crash intensity)	λ_t
Capital stock	k_t^a, k_t^b
Investment rate	ι_t^a, ι_t^b
Risky capital share	χ_t
Output	Y_t
Asset markets:	
Price of assets	q_t^a, q_t^b
Drift and volatility of asset prices	$\mu_{q,t}^a, \mu_{q,t}^b, \sigma_{q,t}^a, \sigma_{q,t}^b$
Asset price crash exposure (% of price)	$\kappa_{q,t}^a, \kappa_{q,t}^b$
Asset collateral values (% of market value)	$1-\kappa_t^a, 1-\kappa_t^b$
Price-weighted risky capital share	χ_t^q
Aggregate collateral values (% of wealth)	$1-\kappa_{A,t}$
Collateral premium	θ_t
Security markets:	
Returns on money, shadow money, and equity	dr_t^m, dr_t^s, dr_t^e
Expected return on money, shadow money, equity, and wealth	$\mu_{m,t}, \mu_{s,t}, \mu_{e,t}, \mu_{W,t}$

and liquidity provision. The second is long-lived capital and investment, which allows us to endogenize asset prices and collateral values, as well as to examine the implications of our framework for the real side of the economy. Table I summarizes the model's variables.

A. Investors

The dynamic model is set in continuous time $t \geq 0$. As in the static model, a unit mass of risk-neutral investors are subject to liquidity events. Investors are infinitely lived and discount the future at the rate ρ:

$$V_0 = \max\ E_0\left[\int_0^{\infty} e^{-\rho t} W_t\left(\psi dC_t + dc_t\right)\right], \tag{7}$$

where dC_t is consumption in a liquidity event, which gives marginal utility $\psi > 1$, and dc_t is consumption outside a liquidity event, which gives marginal utility one. Both are expressed as a fraction of wealth W_t.

As before, liquidity events arrive with instantaneous probability h. We now further impose a distribution on their size. Formally, $dC_t \leq d\overline{C}_t$, where $d\overline{C}_t$ is a jump process that is i.i.d. across investors and over time. Its intensity is h and its distribution conditional on a jump, $F^{\psi}(\cdot)$, is exponential with mean $1/\eta$ (i.e., $F^{\psi}(x) = 1 - e^{-\eta x}$). The assumption that the size of liquidity events is stochastic makes each additional dollar of liquid holdings less valuable ex ante because it is less likely to be used (η controls the satiation rate). The resulting concavity in the demand for liquid securities produces a decreasing relationship between liquid holdings and liquidity premia.

B. Capital Accumulation

We replace the endowment in the static model with two types of capital that cost the same to produce but have different risk-return profiles. Specifically, type a capital is more productive but riskier, while type b capital is less productive but safer. Risk again refers to exposure to a crash shock, dZ_t, which we describe below.

Let k_t^a and k_t^b be the stocks of a and b capital. Output accrues at a rate $Y_t = y^a k_t^a + y^b k_t^b$, with $y^a > y^b$ reflecting that a is more productive than b. Capital evolves according to

$$dk_t^i = \mu_0 \left(k_t^a + k_t^b\right) dt + k_t^i \left[\phi\left(\iota_t^i\right) - \delta\right] dt - k_t^i \kappa_k^i dZ_t, \qquad i = a, b. \tag{8}$$

The first term is a level inflow of new capital that captures exogenous sources of growth. It accrues to the aggregate capital stock but not inside investors' portfolios.[9] In the second term, ι_t^i is the investment rate, $\phi(\cdot)$ is a concave function capturing adjustment costs, δ is depreciation, and κ_k^i is crash exposure, with $\kappa_k^a > \kappa_k^b$ reflecting that a is riskier than b.

With heterogeneous capital, the composition of the economy's capital stock becomes a state variable. We capture it with the risky capital share

$$\chi_t \equiv k_t^a / \left(k_t^a + k_t^b\right). \tag{9}$$

Intuitively, the drift of χ_t depends on the difference in investment rates between the two types of capital, $\phi(\iota_t^a) - \phi(\iota_t^b)$ (see equation (C1)), which depends in turn on their prices. Hence, asset prices influence the evolution of the economy's capital stock, which feeds back into asset prices through its effect on aggregate collateral values. Capital heterogeneity thus interacts with developments in financial markets to produce cycles.

C. Uncertainty

We model the crash shock dZ_t as a compensated (i.e., mean-zero) Poisson process with time-varying intensity λ_t. As in the static model, the compensation implies that changes in λ_t are a mean-preserving spread in output, and hence it remains a measure of uncertainty.

[9] For example, it can be interpreted as productivity or population growth embodied in vintages of new capital as in Gârleanu, Panageas, and Yu (2012). This term is not important for our qualitative results. It helps to ensure that there is always a positive amount of each type of capital.

We model λ_t as the outcome of a learning problem. A latent true crash intensity $\tilde{\lambda}_t$ follows a two-state Markov chain $\tilde{\lambda}_t \in \{\lambda^L, \lambda^H\}$ with unconditional mean $\overline{\lambda}$ and overall transition rate between states φ. Agents learn about $\tilde{\lambda}_t$ from the occurrence of crashes (or a lack thereof) and from a Brownian signal with precision ν that captures exogenous news about the economy. The explicit formulation of this signal is presented in Appendix B.1, where we show that Bayesian learning implies the following dynamics for the filtered crash intensity $\lambda_t = E_t[\tilde{\lambda}_t]$:

$$d\lambda_t = \varphi\left(\overline{\lambda} - \lambda_t\right)dt + \Sigma_t\left(\nu dB_t + \frac{1}{\lambda_t}dZ_t\right), \tag{10}$$

where $\Sigma_t \equiv (\lambda^H - \lambda_t)(\lambda_t - \lambda^L)$ is the conditional variance of $\tilde{\lambda}_t$ and dB_t conveys the Brownian news signal. Intuitively, absent shocks, λ_t drifts toward $\overline{\lambda}$, while shocks lead to more updating when $\tilde{\lambda}_t$ is less precisely estimated (when Σ_t is large).

In addition to arising from a natural learning problem, the uncertainty process (10) has three empirically motivated properties. First, λ_t jumps up when a crash hits, and hence uncertainty is higher after a crash.[10] Second, λ_t then drifts down, making crashes less likely after a long quiet period like the Great Moderation. And third, λ_t jumps most from moderately low levels (note the $1/\lambda_t$ loading on dZ_t), a type of Bayesian Minsky moment (Minsky (1986)).[11]

D. Intermediaries

As in the static model, competitive intermediaries buy assets and issue securities against them. Intermediaries are long-lived and maximize the present value of future profits.

An asset is a claim to one unit of capital of either type. Its price is an endogenous function of the two state variables, $q_t^i = q^i(\lambda_t, \chi_t)$. Applying Itô's Lemma to this function gives a law of motion of the form

$$dq_t^i/q_t^i = \mu_{q,t}^i dt + \sigma_{q,t}^i dB_t - \kappa_{q,t}^i dZ_t, \qquad i = a, b. \tag{11}$$

We solve for asset prices and their implicit dynamics in equilibrium. Intermediaries also set investment, which is pinned down by asset prices as in standard q-theory:

$$1 = q_t^i \phi'\left(\iota_t^i\right), \qquad i = a, b. \tag{12}$$

Since ϕ is concave, higher asset prices imply greater investment. This channel transmits variation in asset prices to economic growth.

As before, intermediaries tranche the assets they buy into securities, which become short-lived in continuous time. We denote security x's return process

[10] Reinhart and Reinhart (2010) find that half of all financial crises are followed by severe aftershocks.

[11] The jump reaches maximum size at the point $\sqrt{\lambda^L\lambda^H}$, which is less than $\frac{1}{2}(\lambda^L + \lambda^H)$.

by

$$dr^x_t = \mu_{x,t}dt + \sigma_{x,t}dB_t - \kappa_{x,t}dZ_t, \tag{13}$$

where $\mu_{x,t}$ is its expected return. We rely on the microfoundations we developed in the static model and take the issued securities and their liquidity profiles as given.

ASSUMPTION 2 (Securities): *Intermediaries issue the following three securities:*

(i) money m with $\kappa_{m,t} = \sigma_{m,t} = 0$ is liquid with probability one (always-liquid),
(ii) shadow money s with $\kappa_{s,t} = \overline{\kappa}$ and $\sigma_{s,t} = 0$ is liquid with probability $1 - p_H(\lambda_t)$, where $p'_H(\lambda_t) > 0$ (fragile-liquid), and
(iii) equity e with $\kappa_{e,t} = 1$ and $|\sigma_{e,t}| > 0$ is illiquid.

Note that equity now also bears the assets' exposure to the Brownian news signal dB_t.

E. Equilibrium

We first solve the representative investor's problem, which gives securities' expected returns as a function of issuance. We then solve for equilibrium issuance and asset prices. For details and a complete characterization of equilibrium, see Appendix B.

E.1. Security Expected Returns

We can express the representative investor's problem recursively as follows:

$$\rho V_t dt = \max_{m_t, s_t, dC_t, dc_t} E_t\left[W_t\left(\psi dC_t + dc_t\right)\right] + E_t\left[dV_t\right] \tag{14}$$

subject to $dC_t \leq d\overline{C}_t$, the dynamic budget constraint

$$\frac{dW_t}{W_t} = dr^e_t + m_t(dr^m_t - dr^e_t) + s_t(dr^s_t - dr^e_t) - dC_t - dc_t, \tag{15}$$

and the liquidity constraint

$$dC_t \leq \begin{cases} m_t + s_t & \text{prob.} \quad 1 - p_H\left(\lambda_t\right) \\ m_t & \text{prob.} \quad p_H\left(\lambda_t\right). \end{cases} \tag{16}$$

Risk-neutrality implies that the investor's marginal value of wealth is equal to one ($V_{W,t} = 1$) and that her consumption outside of a liquidity event is perfectly elastic. In a liquidity event, she consumes as much as she can. She stops when she reaches the event size $d\overline{C}_t$ or runs out of liquid securities, whichever comes first. We can use this to simplify her problem as follows (the extra steps are in Appendix B.2):

$$\rho = \max_{m_t, s_t} h(\psi - 1)\left[[1 - p_H(\lambda_t)] \int_0^\infty \min\{x, m_t + s_t\} dF^\psi(x) + p_H(\lambda_t) \int_0^\infty \min\{x, m_t\} dF^\psi(x) \right] + \mu_{W,t}. \tag{17}$$

This equation is analogous to (4) in the static model. The investor's rate of time preference ρ equals her expected net gain from liquidity-event consumption plus the expected return on her portfolio, $\mu_{W,t} \equiv \mu_{e,t} + m_t(\mu_{m,t} - \mu_{e,t}) + s_t(\mu_{s,t} - \mu_{e,t})$. The expected net gain from liquidity-event consumption equals the intensity h of liquidity events, times the net gain $\psi - 1$ per unit of liquidity-event consumption, times expected liquidity-event consumption (in brackets). Expected liquidity-event consumption is probability-weighted across the states in which shadow money is liquid and illiquid, and integrated over the exponential distribution of the size of liquidity events, $F^\psi(x) = 1 - e^{-\eta x}$. Solving (17), we obtain three equilibrium conditions.

PROPOSITION 2 (Security expected returns): *The expected returns on money ($\mu_{m,t}$), shadow money ($\mu_{s,t}$), and equity ($\mu_{e,t}$) satisfy*

$$\mu_{e,t} - \mu_{m,t} = h(\psi - 1)\left(\left[1 - p_H(\lambda_t)\right] e^{-\eta(m_t + s_t)} + p_H(\lambda_t) e^{-\eta m_t}\right). \tag{18}$$

$$\mu_{s,t} - \mu_{m,t} = h(\psi - 1)\, p_H(\lambda_t) e^{-\eta m_t}. \tag{19}$$

The expected return on the representative investor's portfolio ($\mu_{W,t}$) satisfies

$$\mu_{W,t} = \left[\rho - \frac{h}{\eta}(\psi - 1)\right] + \frac{1}{\eta}\left(\mu_{e,t} - \mu_{m,t}\right). \tag{20}$$

Proposition 2 relates liquid security holdings and expected returns. Equation (18) is the spread between equity and money. We call it the liquidity premium. It equals the marginal value of having a dollar of always-liquid securities (money) instead of a dollar of illiquid securities (equity). This value equals the net utility from consuming in a liquidity event ($\psi - 1$) times the joint probability that a liquidity event takes place (h) and its size exceeds the investor's state-contingent liquid holdings (exponential terms in parentheses). The event size must exceed the investor's liquid holdings for the marginal dollar of liquid holdings to be useful. This is more likely when liquid holdings are low, and hence the liquidity premium is decreasing in liquid holdings.

Equation (19) is the spread between shadow money and money. It equals the marginal value of liquid holdings in the state in which shadow money becomes illiquid times its probability, $p_H(\lambda_t)$. Since shadow money is more likely to become illiquid when uncertainty is high ($p'_H(\lambda_t) > 0$), the shadow money–money spread is increasing in uncertainty.

Equation (20) shows that the expected return on the investor's portfolio, $\mu_{W,t}$, is low when the liquidity premium $\mu_{e,t} - \mu_{m,t}$ is low. A low liquidity premium lowers the cost of transferring consumption to a liquidity event, inducing investors to save more. This lowers the expected return on their portfolios, which equals the economy's aggregate discount rate in equilibrium. Thus, abundant liquidity lowers discount rates.

E.2. Security Issuance

Although intermediaries are long-lived, the zero-profit condition and the absence of net-worth frictions imply that they maximize profits at each point in time. They do so by jointly minimizing funding costs and maximizing the return on their assets. In this section, we focus on the funding side of their problem and explain how it fits inside their overall problem (see Appendix B.3 for proofs and derivations). Given the collateral value of their assets, intermediaries minimize funding costs by solving

$$\min_{m_t, s_t \geq 0} E_t\left[dr_t^e + m_t\left(dr_t^m - dr_t^e\right) + s_t\left(dr_t^s - dr_t^e\right)\right] \tag{21}$$

subject to $m_t + s_t \leq 1$ and the collateral constraint

$$m_t + s_t(1 - \overline{\kappa}) \leq 1 - \kappa_{A,t}, \qquad [\theta_t] \tag{22}$$

where $1 - \kappa_{A,t}$ is the value of assets in the case of a crash per dollar of current market value, that is, their collateral value (we analyze it below). The issued amounts of money, m_t, and shadow money, s_t, are also per dollar of current assets.

As in the static model (see (6)), intermediaries must have enough collateral to pay off their securities in a crash. We refer to the Lagrange multiplier on the collateral constraint, θ_t, as the collateral premium. It measures the amount by which an increase in collateral values lowers intermediaries' funding costs. Through funding costs, the collateral premium transmits changes in collateral scarcity to discount rates and asset prices.

To minimize funding costs, intermediaries can issue more money and less equity, more shadow money and less equity, or both, subject to the collateral constraint (22). Issuing more money and less equity lowers funding costs by the spread between equity and money, $\mu_{e,t} - \mu_{m,t}$, and tightens the constraint by θ_t. Issuing more shadow money and less equity lowers funding costs by the spread between equity and shadow money, $\mu_{e,t} - \mu_{s,t}$, and tightens the constraint by $\theta_t(1 - \overline{\kappa})$. Intermediaries therefore issue more money and less shadow money when the ratio between the equity-money spread and the equity-shadow money spread is greater than $1/(1 - \overline{\kappa})$, the "collateral multiplier" of shadow money. At an interior optimum, the two are equal. Combining this policy with Proposition 2, we have the following result.

PROPOSITION 3 (Equilibrium security issuance): *Let* $\mathcal{M}_t \equiv \frac{1}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{1-p_H(\lambda_t)}{p_H(\lambda_t)}\right)$. *Then in equilibrium, security issuance follows*

(i) $m_t = \max\{0, 1 - \frac{\kappa_{A,t}}{\overline{\kappa}}\}$ *and* $s_t = \min\{\frac{1-\kappa_{A,t}}{1-\overline{\kappa}}, \frac{\kappa_{A,t}}{\overline{\kappa}}\}$ *if* $\mathcal{M}_t > \min\{\frac{\kappa_{A,t}}{\overline{\kappa}}, \frac{1-\kappa_{A,t}}{1-\overline{\kappa}}\}$,

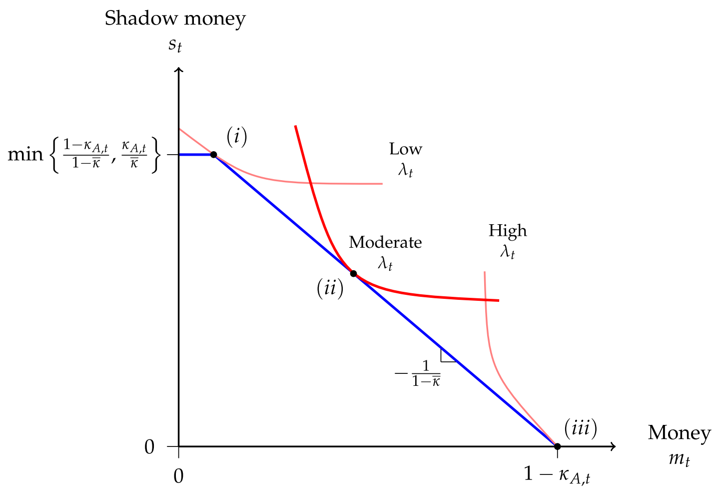

Figure 2. Equilibrium security issuance. This figure illustrates equilibrium issuance of money m_t and shadow money s_t in the dynamic model as given in Proposition 3. The blue line is the intermediary's liquidity provision frontier implied by the collateral constraint (22). The red lines are investor indifference curves at three different levels of uncertainty λ_t. Equilibrium issuance is shown in black circles and labeled according to the three cases in Proposition 3. (Color figure can be viewed at wileyonlinelibrary.com)

(ii) $m_t = 1 - \kappa_{A,t} - (1-\bar{\kappa})\mathcal{M}_t$ *and* $s_t = \mathcal{M}_t$ *if* $0 \le \mathcal{M}_t \le \min\{\frac{\kappa_{A,t}}{\bar{\kappa}}, \frac{1-\kappa_{A,t}}{1-\bar{\kappa}}\}$, *and*

(iii) $m_t = 1 - \kappa_{A,t}$ *and* $s_t = 0$ *if* $\mathcal{M}_t < 0$.

Equilibrium issuance depends on the quantity $\mathcal{M}_t$, which measures the profitability of the first dollar of shadow money. It increases with the collateral multiplier of shadow money, $1/(1-\bar{\kappa})$, because it makes shadow money cheaper to produce. It decreases with uncertainty, λ_t, because it reduces demand for shadow money.

The three cases of Proposition 3 are illustrated in Figure 2. As in the static model, when uncertainty λ_t is low, shadow money is very profitable ($\mathcal{M}_t$ is high) and it crowds out money (case (i)). When uncertainty is moderate, we get an interior optimum (case (ii)). This is due to the concavity of liquidity demand. Finally, when uncertainty is high, shadow money becomes unprofitable ($\mathcal{M}_t < 0$) and disappears from intermediaries' balance sheets (case (iii)).

E.3. Collateral Values

The collateral value of an asset is the fraction of its market value that remains after a crash (equivalently, its gross crash return). It is important because it constrains the amount of liquid securities that the asset can back. Unlike in the static model, collateral values here are endogenous and forward-looking because assets are long-lived.

Formally, the aggregate collateral value of intermediaries' assets, $1-\kappa_{A,t}$, is a value-weighted average of the collateral values of the two assets,

$$1-\kappa_{A,t} = \chi_t^q\left(1-\kappa_t^a\right)+\left(1-\chi_t^q\right)\left(1-\kappa_t^b\right), \tag{23}$$

where $\chi_t^q \equiv q_t^a\chi_t/[q_t^a\chi_t + q_t^b(1-\chi_t)]$ is the price-weighted risky asset share and $1-\kappa_t^i$ is the collateral value of asset $i=a,b$. In turn, the collateral value of asset i depends on the impact of crashes on its cash flows, κ_k^i, and price, $\kappa_{q,t}^i$,

$$1-\kappa_t^i \equiv \left(1-\kappa_k^i\right)\left(1-\kappa_{q,t}^i\right), \qquad i=a,b. \tag{24}$$

When asset prices become more exposed to crashes ($\kappa_{q,t}^i$ rises), collateral values fall. Falling collateral values reduce the amount of liquid securities intermediaries can issue, raising funding costs. Higher funding costs cause asset prices to fall. As we show in Section III.E below, this leads to endogenous amplification in the form of collateral runs.

E.4. Asset Prices

Intermediaries can scale up their balance sheets by issuing more securities and buying more assets. Profit maximization implies that asset prices must satisfy the two partial differential equations

$$q_t^i = \frac{y^i-\iota_t^i}{\left(\mu_{W,t}-\theta_t\left[\left(1-\kappa_t^i\right)-\left(1-\kappa_{A,t}\right)\right]\right)-\left[\mu_{q,t}^i+\kappa_k^i\kappa_{q,t}^i\lambda_t+\phi\left(\iota_t^i\right)-\delta\right]} \tag{25}$$

for $i=a,b$. The solution to (25) implicitly defines the coefficients in the evolution equation for asset prices (11), including the drift and crash exposure of prices, $\mu_{q,t}^i$ and $\kappa_{q,t}^i$.

Prices have the familiar form of net cash flows divided by a discount rate minus a growth rate. The net cash flow tends to be higher for a because $y^a > y^b$. The growth rate (bottom right) consists of price growth, physical growth, and depreciation.

Discount rates (bottom left) feature a common component and an asset-specific component. The common component is the expected return on investors' wealth, $\mu_{W,t}$ (see Proposition 2), which in equilibrium equals the aggregate discount rate as intermediaries pass through their cost of capital to asset prices. The asset-specific component depends on the amount by which an asset's collateral value, $1-\kappa_t^i$, exceeds the aggregate collateral value, $1-\kappa_{A,t}$. Each dollar of additional collateral value lowers the asset's discount rate by the collateral premium θ_t. Since asset a's cash flows have higher crash exposure than b's ($\kappa_k^a > \kappa_k^b$), the cash-flow component of a's collateral value in (24) is lower than b's. As a result, a's discount rate tends to be higher than b's. If the collateral premium θ_t is high enough, a's price can be lower than b's even though a has higher cash flows than b.

As we show in Section III.C, the dynamics of asset prices amplify these effects. The reason is that the collateral premium increases after a crash as

Table II
Benchmark Parameters

This table contains benchmark values for the model parameters used to produce results for the dynamic model. The investment cost function is parameterized as $\phi(\iota) = 1/\gamma(\sqrt{1+2\gamma\iota} - 1)$. We use the specification implied by the static model for the probability that shadow money becomes illiquid (i.e., $p_H(\lambda) = (\lambda - \lambda^L)/(\lambda^H - \lambda^L)$).

Description	Parameter	Value
Technology:		
Asset cash flows	y^a, y^b	0.138, 0.1
Depreciation rate	δ	0.1
Exogenous aggregate growth	μ_0	0.01
Adjustment cost parameter	γ	3
Asset crash exposures	κ_k^a, κ_k^b	0.5, 0
Information sensitivity constraint:		
Crash exposure limit for fragile liquid securities	$\overline{\kappa}$	0.7
Uncertainty:		
Low/high uncertainty states	λ^L, λ^H	0.005, 1
Average uncertainty	$\overline{\lambda}$	0.0245
Uncertainty rate of mean reversion	φ	0.5
Uncertainty news signal precision	$1/\sigma$	0.1
Preferences and liquidity events:		
Liquidity event frequency	h	0.28
Liquidity event marginal utility	ψ	5
Average size of liquidity event	$1/\eta$	0.33
Subjective discounting parameter	ρ	0.37

higher uncertainty causes demand for shadow money to fall and demand for the collateral-intensive money to rise. The increase in the collateral premium causes the price of asset a to fall and the price of asset b to rise. As a result of these ex post differences in prices, the collateral value of asset a is even lower and the collateral value of asset b is even higher ex ante.

III. Results

In this section, we present results from the dynamic model. We use projection methods to solve for asset prices $q^i(\lambda, \chi)$, $i = a, b$ (details are in Appendix C). Given asset prices, the model is solved in closed form.

A. Parameter Values

Table II lists our benchmark parameter values. Our approach is to use broadly plausible numbers that showcase the qualitative features of our model. We provide further details in Appendix D and show robustness to alternative choices in the Internet Appendix.

We use estimates from the quantitative macroliterature for the production side of the economy (see He and Krishnamurthy (2014) for references). We use estimates from the rare disasters literature (e.g., Barro (2006)) for the uncertainty process λ_t and the cash flow risk of asset a. We model asset b as perfectly safe; it has no cash flow risk ($\kappa_k^b = 0$) and its productivity equals depreciation.

We set $\overline{\kappa}$, the crash exposure of shadow money, to 0.7, which is in line with losses on Lehman Brothers' commercial paper during the financial crisis (Helwege et al. (2010)). We use the specification implied by the static model for the probability that shadow money becomes illiquid: $p_H(\lambda_t) = \frac{\lambda_t - \lambda^L}{\lambda^H - \lambda^L}$. This gives a steady-state probability of 1.95%.

The literature offers less guidance for parameterizing liquidity events. We set h so that the annual probability of a liquidity event is 24%. We set the average size of liquidity event opportunities to one-third of net worth ($1/\eta = 1/3$). We set ψ, the marginal utility of liquidity-event consumption, to 5. By analogy to standard models with risk aversion of 10, these liquidity events are comparable to idiosyncratic shocks that temporarily reduce consumption by about 15% once every four years or so.[12]

We can gauge the plausibility of these numbers by their implications for liquidity premia. Our steady-state liquidity premium is about 6%. An empirical counterpart is the beta-adjusted return on a fully illiquid security in excess of a fully liquid one like T-bills. Estimates of this number are found in the asset pricing literature. Baker, Bradley, and Taliaferro (2014) give 6.9% for U.S. stocks. Frazzini and Pedersen (2014) give between 1.2% and 8% for corporate bonds. We view these numbers as a high upper bound.

The monetary economics literature looks at the rates of return on very safe securities with different levels of liquidity. Krishnamurthy and Vissing-Jorgensen (2012a) estimate an average spread of 0.73% between Treasuries and Baa corporate bonds and of 1.44% between on-the-run and off-the-run Treasuries. These spreads exhibit dramatic spikes in periods of high uncertainty, and as such correspond most closely to our model's shadow money-money spread, which is about 1% in steady state.

B. Security Markets

Figure 3 shows equilibrium issuance and expected returns in security markets. Along the horizontal axis in each panel is the uncertainty state variable λ_t, which ranges from λ^L to λ^H. It is useful to keep in mind that the steady state for λ_t is at the low end, at 0.0245, while the 99$^{\text{th}}$ percentile is at 0.5038. Each panel contains two lines that hold the risky capital share state variable χ_t fixed at one of two levels: 0.95, the point toward which χ_t tends when uncertainty is low, and 0.75, the point toward which χ_t tends when uncertainty is high.

The top row of panels in Figure 3 shows the issuance of money, shadow money, and equity, which follows Proposition 3. Shadow money (top center) dominates intermediaries' balance sheets at low levels of uncertainty. This is a shadow banking boom. Since crashes here are unlikely, investors view money

[12] Schmidt (2014) finds evidence for comparably large idiosyncratic shocks in U.S. household data.

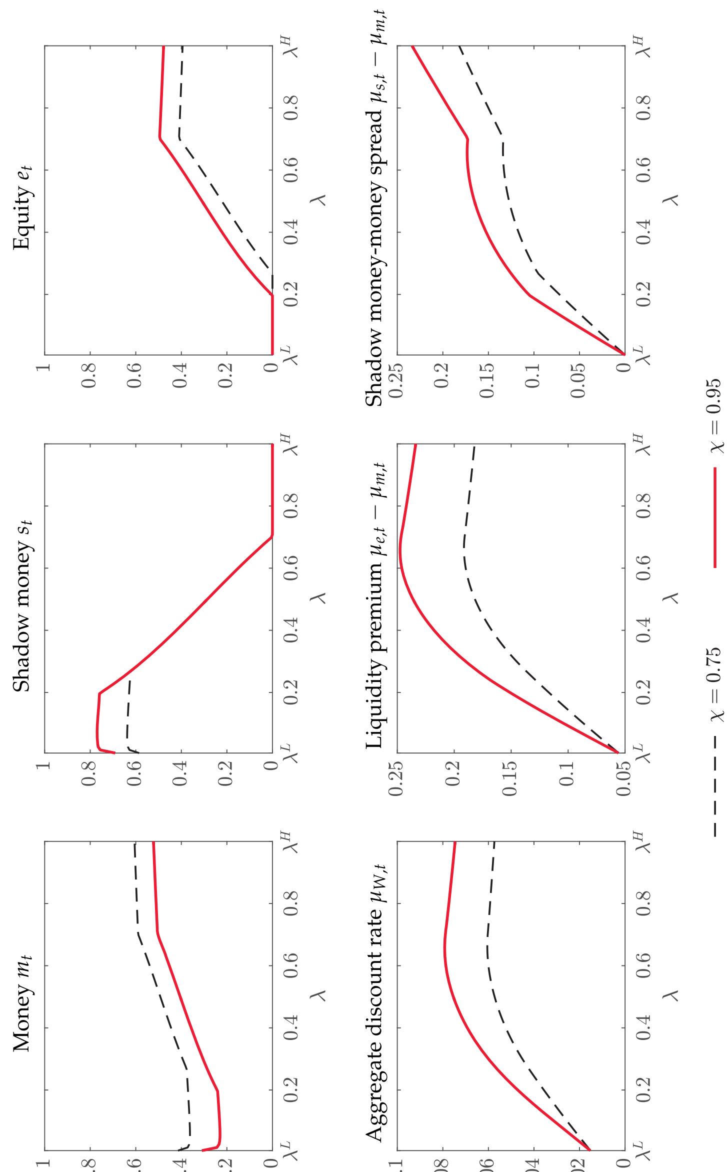

Figure 3. Security issuance and expected returns. This figure shows money m_t, shadow money s_t, equity e_t, the aggregate discount rate $\mu_{W,t}$, the liquidity premium $\mu_{e,t} - \mu_{s,t}$, and the shadow money-money spread $\mu_{s,t} - \mu_{m,t}$ in equilibrium under the benchmark parameters in Table II. Each quantity is plotted against uncertainty λ_t while holding the risky capital share χ_t fixed at a low level of 0.75 (dashed black lines) and a high level of 0.95 (solid red lines). These levels are near the steady states for χ_t under high and low uncertainty, respectively. (Color figure can be viewed at wileyonlinelibrary.com)

and shadow money as close substitutes and the shadow money–money spread (bottom right) approaches zero, consistent with Proposition 2. Empirically, the spreads on a variety of shadow banking instruments over T-bills were very low during the boom before the financial crisis.

Intermediaries are eager to supply shadow money whenever investors are willing to hold it because it allows them to produce $1/(1-\overline{\kappa})$ times more liquidity than money (see the collateral constraint (22)). This is why shadow money crowds out money when uncertainty is low (top left panel of Figure 3).

By taking on some crash exposure ($\overline{\kappa} > 0$), shadow money also allows intermediaries to reduce their equity, which is an expensive source of funding because it is illiquid. Shadow banking booms are thus associated with high leverage and high liquidity provision, resulting in a low liquidity premium and a low aggregate discount rate (bottom left and center panels), consistent with Proposition 2.[13] This liquidity, however, is fragile.

A rise in uncertainty brings the shadow banking boom to an end. Investors are no longer willing to hold shadow money because its liquidity is now more likely to evaporate. The shadow money-money spread opens up, recalling the widening of spreads in the summer of 2007, when uncertainty about spillovers from mortgage markets grew.[14]

Intermediaries respond to the shift in investors' demand by contracting shadow money and expanding money (see Proposition 3). Shadow banking shuts down altogether if uncertainty rises high enough (case (iii) of Proposition 3). The ABCP market suffered a similar collapse in the crisis (Acharya, Schnabl, and Suarez (2013)).

The shift from shadow money to money requires issuing more equity to provide the necessary cushion. This can be seen by rewriting the collateral constraint (22) in "haircut form" as $e_t \geq \kappa_{A,t} - s_t\overline{\kappa}$. As equity replaces shadow money in bad (i.e., high-uncertainty) times, leverage is procyclical. These leverage dynamics fit the observed behavior of repo- and commercial paper–dependent institutions such as hedge funds and broker-dealers.[15]

Because intermediaries cannot fully offset the contraction of shadow money with money, the overall supply of liquidity shrinks, raising the liquidity premium and the aggregate discount rate $\mu_{W,t}$. Discount rates stabilize only when shadow banking shuts down and the liquidity contraction is complete.

Both boom and bust are more pronounced when the risky capital share χ_t is high. A high risky capital share lowers collateral values (see (23)) and hence

[13] For empirical evidence, see Baron and Xiong (2017), who show that high credit growth is associated with low equity risk premia.

[14] Kacperczyk and Schnabl (2013) and Acharya and Mora (2015) date the opening up of spreads to July 2007 when two Bear Stearns hedge funds failed. We interpret this episode as an uncertainty shock. The Financial Crisis Inquiry Commission (2011)) calls it "a canary in the mineshaft."

[15] He, Khang, and Krishnamurthy (2010), Adrian and Shin (2010), Ang, Gorovyy, and van Inwegen (2011), Adrian, Etula, and Muir (2014), and He et al. (2017) document countercyclical leverage for commercial banks and procyclical leverage for hedge funds and broker-dealers.

makes collateral more scarce. This leads to more shadow banking in quiet times, but since shadow banking is sensitive to uncertainty, it also leads to a larger liquidity crunch when uncertainty rises.

C. Asset Markets and the Macroeconomy

The uncertainty exposure of the liquidity supply spills over to asset prices through discount rates as implied by (25). First, as uncertainty rises and liquidity contracts, the aggregate discount rate $\mu_{W,t}$ rises (Proposition 2), and this causes asset prices to fall. Second, discount rates also depend on the collateral premium θ_t. In particular, the discount rate of asset a, whose collateral value is low, is increasing in the collateral premium, while the discount rate of asset b, whose collateral value is high, is decreasing in the collateral premium.

To understand the behavior of asset prices, we must therefore understand the behavior of the collateral premium. To illustrate, when security issuance is at an interior optimum (case (ii) of Proposition 3; see (B33)), the collateral premium satisfies

$$\theta_t \propto e^{-\eta(1-\kappa_{A,t})} p_H(\lambda_t)^{\overline{\kappa}} [1 - p_H(\lambda_t)]^{1-\overline{\kappa}}. \tag{26}$$

Since in this region $p_H(\lambda_t) \leq \overline{\kappa}$, the collateral premium is increasing in uncertainty. Intuitively, as uncertainty rises, the flight from shadow money to money makes collateral more scarce, driving up its price. This effect is illustrated in the bottom left panel of Figure 4, which shows that the collateral premium rises steeply at low uncertainty when shadow banking contracts, and flattens out once shadow banking has shut down.

The rise in the collateral premium reinforces the rise in the aggregate discount rate to drive down the price of the risky asset a (top left panel). When uncertainty is low, intermediaries can fund asset a cheaply with shadow money and hence bid up its price. When uncertainty rises, however, this funding dries up and the price of asset a falls.

By contrast, the rise in the collateral premium counteracts and indeed overcomes the rise in the aggregate discount rate to drive up the price of asset b (top center panel). Consistent with this result, long-term safe bonds appreciated during the 2008 financial crisis. This phenomenon is often called flight to quality.[16] It arises here as a result of the contraction in liquidity at the end of a shadow banking boom, which drives up the value of collateral and by extension the prices of collateral-rich assets.[17]

[16] See Krishnamurthy (2010), McCauley and McGuire (2009), and Beber, Brandt, and Kavajecz (2009) on flight to quality in U.S. Treasuries, the U.S. dollar, and European sovereign debt.

[17] Flight to quality occurs whenever the collateral value of asset b is high enough. For the price of asset b to rise, its discount rate, $\mu_{W,t} - \theta_t[1 - \kappa_t^b - (1 - \kappa_{A,t})]$, must fall (see (25)). Using Proposition 2, the aggregate discount rate $\mu_{W,t}$ rises at a rate $1/\eta$ with the liquidity premium, $\mu_{e,t} - \mu_{m,t}$. In the interior optimum case, the liquidity premium coincides with the collateral premium, $\mu_{e,t} - \mu_{m,t} = \theta_t$ (see (B23)). Therefore, the price of asset b rises as long as $1 - \kappa_t^b - (1 - \kappa_{A,t}) > 1/\eta$.

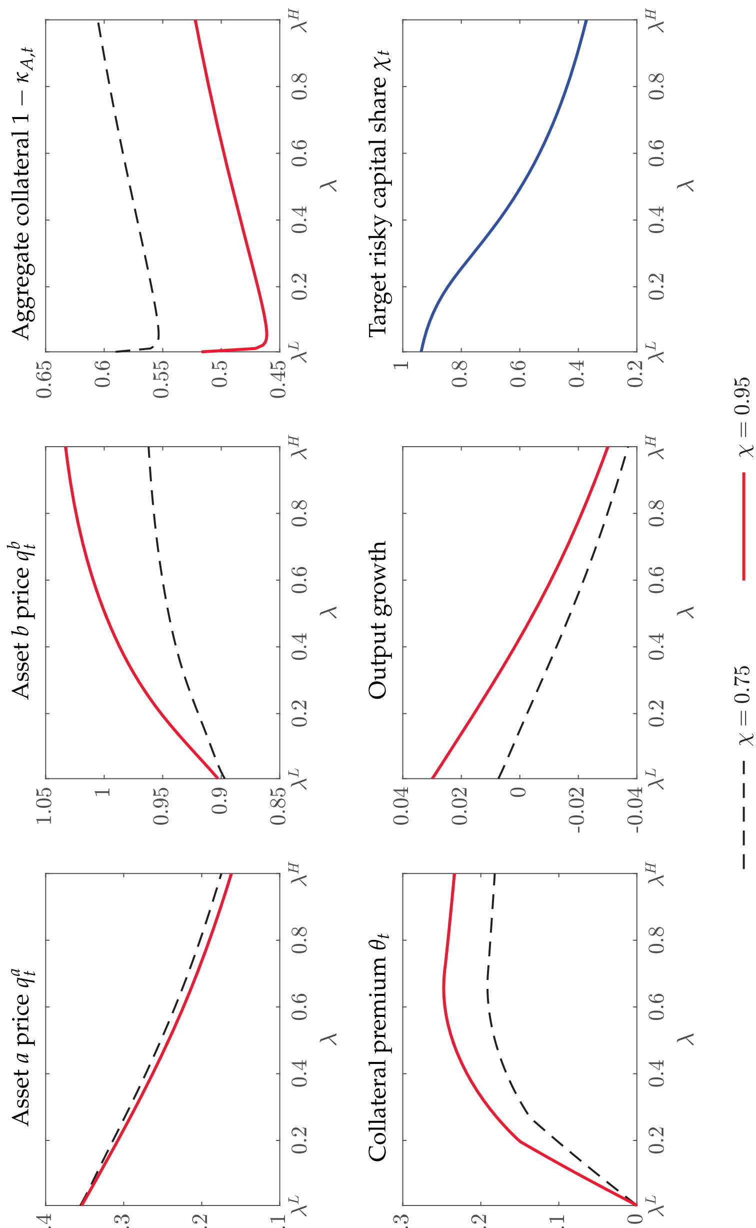

Figure 4. Asset prices, collateral, and economic activity. This figure shows asset prices q_t^a and q_t^b, the collateral premium θ_t, aggregate collateral value $1-\kappa_{A,t}$, output growth, and the target risky capital share in equilibrium under the benchmark parameters in Table II. Output growth is the percentage drift of output. The target risky capital share is the value of χ_t for each value of λ_t such that χ_t has zero drift, $E_t[d\chi_t]=0$. Each quantity is plotted against uncertainty λ_t. All plots except the steady-state risky capital share hold χ_t fixed at a low level of 0.75 (dashed black lines) and a high level of 0.95 (solid red lines). (Color figure can be viewed at wileyonlinelibrary.com)

The rise in the collateral premium is steeper when the risky capital share χ_t is high and hence aggregate collateral values $1 - \kappa_{A,t}$ are low (top right panel). From (26), low collateral values push up the collateral premium and make it more sensitive to uncertainty. When uncertainty rises and intermediaries shift from shadow money to money, liquidity provision becomes more collateral-intensive. It must therefore contract by more when collateral values are low, and this causes the collateral premium to rise faster. The end result is that asset prices are also more sensitive to uncertainty when the risky capital share is high. In particular, the flight to quality in asset b is stronger.

From the top right panel of Figure 4, aggregate collateral values exhibit a pronounced U-shape. We explain why this happens in Section III.E, below. Here we note that the decline in collateral values on the left side of the U-shape reinforces the direct effect of higher uncertainty on the collateral premium in (26). As uncertainty increases from a low level, the contraction of shadow banking causes demand for collateral to rise, while the drop in collateral values simultaneously causes the supply of collateral to fall. Higher demand and lower supply combine to accelerate the increase in the collateral premium, further amplifying the sensitivity of asset prices to uncertainty.

The last two panels of Figure 4 show the real effects of the sensitivity of asset prices to uncertainty induced by shadow banking. In the bottom center plot, output growth declines sharply and turns negative as uncertainty rises. This occurs because productive investment falls with the price of asset a (see (12)). The output growth plot further shows that economic expansions coincide with periods of low uncertainty and a risky but productive capital mix, that is, with shadow banking booms. In other words, the key to growth in the model is liquidity transformation, funding risky assets with liquid securities. This is what shadow banking does.

The bottom right panel of Figure 4 plots the economy's target risky capital share: the value of χ_t where it has zero drift for a given level of λ_t (the drift can be seen in (C1)). This target share is high when uncertainty is low and low when uncertainty is high. This relationship illustrates the feedback between the liquidity and macrocycles. At low uncertainty, shadow banking spurs investment in risky capital. As the risky capital share rises, aggregate collateral values drift down, making the economy even more reliant on shadow banking. This builds fragility. When uncertainty rises, low collateral values make the liquidity contraction more acute and the economic downturn more severe. As the price of the risky asset falls and the price of the safe asset rises, investment shifts toward safety and the target risky capital share declines. This collateral mining phase sets up a slow recovery during which output growth remains low even after uncertainty recedes.

D. The Economy with and without Shadow Banking

In this section, we compare our benchmark economy to one without shadow banking (implemented with $\overline{\kappa} = 0$). We think of it as a policy that bans fragile liquidity creation, perhaps in the interest of financial stability. The results are shown in Figure 5.

Without shadow money, intermediaries use all available collateral to issue money (the economy is always in case (iii) of Proposition 3). This is why in Figure 5, money is higher without shadow banking, particularly at low uncertainty where shadow money no longer crowds it out (top left panel). Backstopping the extra money is extra equity (top center panel). The economy without shadow banking thus features low leverage and a safe liability mix.

Without shadow banking, the supply of liquid securities becomes insensitive to uncertainty (top row). The stable liquidity supply leads to stable discount rates (top right panel), asset prices, and collateral values (bottom row). In short, the economy without shadow banking is highly stable.

Discount rates are generally lower with shadow banking. Shadow banking expands liquidity provision when uncertainty is low, which is most of the time, and this lowers discount rates. By contrast, when uncertainty is high, shadow banking raises discount rates. The reason is that by exposing asset prices to uncertainty, shadow banking lowers collateral values, which forces liquidity provision to contract and discount rates to rise. This effect dominates when uncertainty is high.

The higher discount rates cause the price of the risky asset a to be lower without shadow banking, implying lower levels of investment and economic growth. Yet the price of the safe asset b is higher. This is due to a higher collateral premium. The collateral premium is higher because money requires a lot of collateral, making it scarcer even though aggregate collateral values are actually higher. Thus, without shadow banking the rate of return on safe assets is depressed.

In the Internet Appendix, we analyze liquidity requirements like those adopted under Basel III and show that they restrict the issuance of shadow money. As in this section, this reduces the economy's exposure to uncertainty and increases stability. Also as in this section, it lowers overall asset prices and reduces growth during booms.

E. Uncertainty News and Collateral Runs

We trace out the economy's response to the two aggregate shocks, uncertainty news and crashes. Figure 6 presents results for the uncertainty news shock. As mentioned earlier, we think of this shock as recalling the events of the summer of 2007 after the two Bear Stearns hedge funds failed. We initialize both state variables in steady state and shock uncertainty λ_t up to 0.06 (from 0.0245). We plot the expected path of each quantity net of its steady-state trend. For comparison, we also show the responses in the economy without shadow banking (see Section III.D).

The rise in uncertainty causes the shadow money–money spread, the collateral premium, and the aggregate discount rate to rise. The price of the risky asset a falls and the price of the safe asset b rises. Investment in asset a falls,

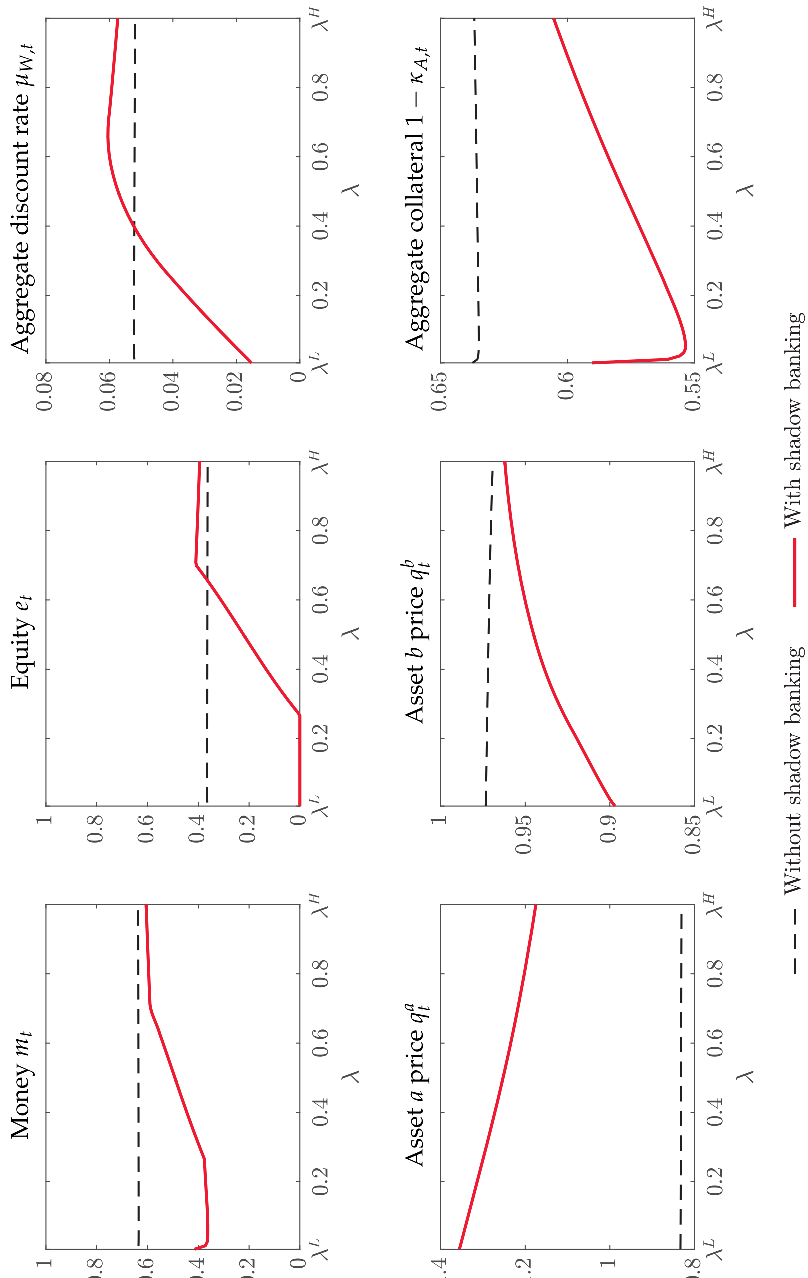

Figure 5. The economy with and without shadow banking. This figure shows money m_t, equity e_t, the aggregate discount rate $\mu_{W,t}$, asset prices q_t^a and q_t^b, and aggregate collateral $1-\kappa_{A,t}$ in equilibrium in economies without shadow banking (dashed black lines) and with shadow banking (solid red lines) under the benchmark parameters in Table II. The economy without shadow banking uses $\overline{\kappa}=0$. Each quantity is plotted against uncertainty λ_t while holding the risky capital share χ_t fixed at 0.75. (Color figure can be viewed at wileyonlinelibrary.com)

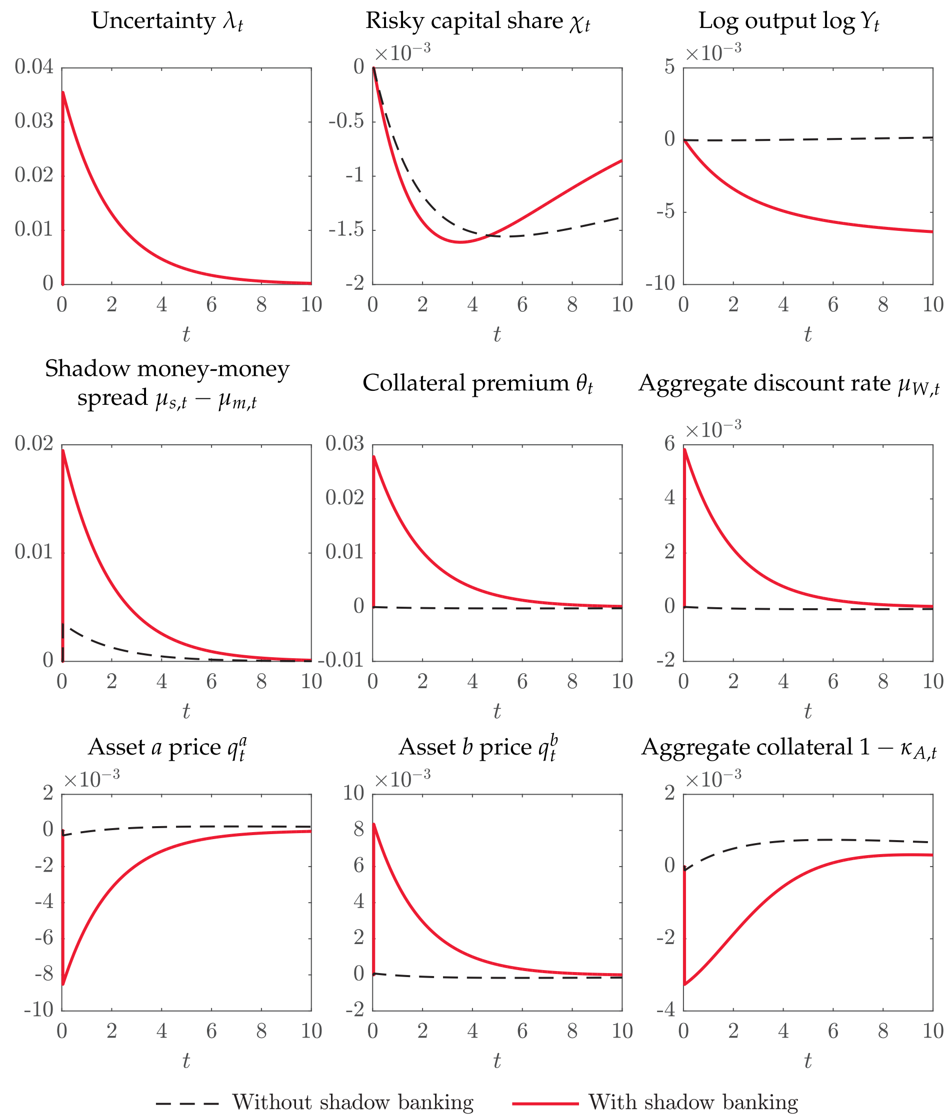

Figure 6. Uncertainty shock response. This figure shows impulse responses to an uncertainty shock to λ_t from its steady state of 0.0245 to 0.06 under the benchmark parameters in Table II. The initial risky capital share χ_t is in steady state. Responses are relative to trend. Solid red lines are for the economy with shadow banking. Dashed black lines are for the economy without shadow banking ($\overline{\kappa} = 0$). Log output is the cumulative sum of the expected growth rate of output. (Color figure can be viewed at wileyonlinelibrary.com)

causing χ_t to drift down.[18] As the uncertainty shock dissipates, χ_t eventually starts to drift back up. The drop in investment leads to lower growth, and output ends up permanently below trend. By contrast, there is no drop in output in the economy without shadow banking.

The uncertainty news shock causes collateral values to fall (bottom right panel). This is the left side of the U-shape in Figure 4. As we saw in Section II.E.3, collateral values fall when asset prices become more exposed to crashes. Crashes impact asset prices through discount rates. When a crash hits, uncertainty jumps as investors revise their beliefs (see (10)). The jump is largest at moderately low levels of uncertainty (this is the Minsky moment). It feeds into discount rates to the extent that the economy's liquidity supply is exposed to uncertainty. This is the case when shadow banking starts contracting at the end of a shadow banking boom. Therefore, as uncertainty rises from a low level, asset prices become more exposed to crashes, causing collateral values to fall.

As collateral values fall, intermediaries are forced to further contract liquidity provision, amplifying the drop in asset prices. This is known as a collateral run.[19] It arises here as a result of fragile liquidity transformation, that is, as a result of shadow banking.

F. Crashes and the Slow Recovery

Figure 7 presents impulse responses for the crash shock, which recalls the peak of the 2008 financial crisis around the collapse of Lehman Brothers. The crash shock causes the risky capital share χ_t to drop since it impacts only a capital. It also causes uncertainty to shoot up as agents perceive future crashes to be more likely. It is this jump in uncertainty and not the destruction of capital itself that has a negative impact on asset prices (recall that asset prices are defined per unit of capital).[20]

As with the uncertainty news shock in Figure 6, the shadow money-money spread, the collateral premium, and the aggregate discount rate rise sharply. The price of asset a again falls and the price of asset b rises, albeit by much larger amounts since the increase in uncertainty is more dramatic. The risky capital share drifts down as intermediaries mine for collateral. This delays the eventual recovery.

[18] There is a smaller drift in χ_t in the economy without shadow banking due to its nonlinearity (see (C1)).

[19] For evidence of collateral runs in the repo market during the 2008 financial crisis, see Gorton and Metrick (2012), Krishnamurthy, Nagel, and Orlov (2014), and Copeland, Martin, and Walker (2014).

[20] In fact, the destruction of capital has a dampening effect on asset prices because it lowers the risky capital share, pushing it closer to its target at high uncertainty (see Figure 4). We also note that the initial drop in χ_t is larger in the no-shadow banking economy because its steady-state value of χ_t is closer to one-half, the point at which a given drop in asset a impacts χ_t the most (see equation (C1)).

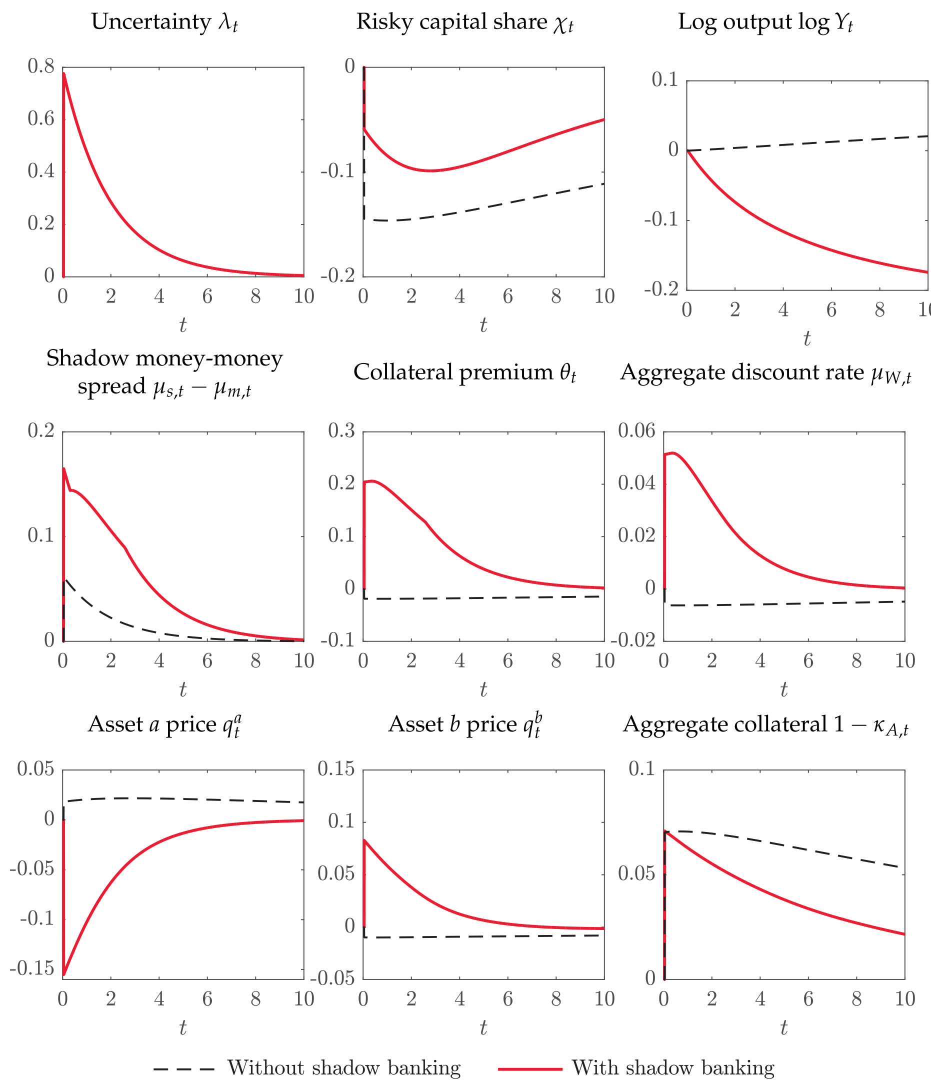

Figure 7. Crash response. This figure shows impulse responses to a crash shock when uncertainty λ_t and the risky capital share χ_t are in steady state. Responses are relative to trend. Solid red lines are for the economy with shadow banking, solved using the parameters in Table II. Dashed black lines are for the economy without shadow banking, solved using $\overline{\kappa} = 0$. Log output is the cumulative sum of the expected growth rate of output. (Color figure can be viewed at wileyonlinelibrary.com)

The "crash-rally" of the safe asset implies that it acts as an endogenous hedge on intermediary balance sheets. As such, it increases aggregate collateral values and enables greater liquidity provision ex ante. This result has important implications for monetary policy, as we show in Section IV.B.

Unlike uncertainty news, crashes cause aggregate collateral values to rise. This happens for two reasons. The first is the drop in the price-weighted risky capital share χ_t^q in (23) resulting from the destruction of a capital and the drop in its price. The second is that the collapse of shadow banking makes discount rates less exposed to further shocks, causing collateral values to increase (this is the right side of the U-shape in collateral values in Figure 4).

Once they go up, collateral values start to fall. This happens despite the fact that intermediaries are mining for collateral (χ_t drifts down). The reason is that as uncertainty declines, shadow banking picks up, causing discount rates to once again become exposed to uncertainty. This depresses collateral values, delaying the recovery of asset prices. We call this novel mechanism the collateral decelerator; it contributes to the slow recovery.

When uncertainty drops sufficiently, collateral values bounce back in a reverse replay of the collateral run. The risky capital share drifts back up, completing the cycle. Output, however, ends up permanently below trend, and this is ignoring the direct cash flow effect of the crash.

IV. Policy Interventions

In the aftermath of the 2008 financial crisis, central banks have experimented with a variety of policy interventions. Among them, LSAP and the Federal Reserve's Maturity Extension Program (i.e., "Operation Twist") have received special scrutiny due to their large size and a lack of consensus on the mechanisms through which they affect financial markets and the broader economy. In this section, we study the effects of these interventions inside our model. Our aim is to shed light on their interaction with the fragile liquidity mechanism at the heart of our paper. These policies also further illustrate how this mechanism works.[21]

A. Large-Scale Asset Purchases

In November 2008 the Federal Reserve began purchasing agency debt and mortgage-backed securities, and in May 2009 the European Central Bank began purchasing covered bonds. Their goal was to support prices and reduce spreads in the markets for assets that were perceived as risky, and in doing so to expand the supply of credit to the economy.[22]

[21] Our analysis of the interaction between financial intermediation and monetary policy complements work by Adrian and Shin (2009), Gertler and Karadi (2011), Ashcraft, Gârleanu, and Pedersen (2011), Kiyotaki and Moore (2012), Brunnermeier and Sannikov (2016), Caballero and Farhi (2013), and Drechsler, Savov, and Schnabl (2017).

[22] From the Fed's announcement on November 25, 2008, "Spreads of rates on GSE debt and on GSE-guaranteed mortgages have widened appreciably of late. This action is being taken to reduce the cost and increase the availability of credit for the purchase of houses, which in turn should support housing markets and foster improved conditions in financial markets more generally" (available at https://www.federalreserve.gov/newsevents/press/monetary/20081125b.htm.)

We model LSAP interventions as follows. In an LSAP, the central bank (an intermediary without a collateral constraint) buys some amount of the risky asset a and sells an equal dollar amount of the safe asset b immediately after a crash. It covers any subsequent cash flow mismatch with lump-sum taxes. The LSAP is unwound at a random future date. For simplicity, we model LSAP as a one-off intervention (this could reflect fiscal constraints). Appendix E.1 provides the details.

Figure 8 shows results for an LSAP in which the central bank buys 20% of the stock of asset a, which it is expected to sell after 10 years. The top two panels look at the announcement effect of LSAP by comparing prices post-LSAP with prices when it does not get implemented. The panels show that the price of asset a rises and the price of asset b falls.[23] By increasing the supply of safe assets after a crash, the central bank relieves collateral scarcity and allows liquidity provision to expand. This lowers the aggregate discount rate and the collateral premium, raising the price of asset a and lowering the price of asset b. This result is consistent with the evidence in Krishnamurthy and Vissing-Jorgensen (2011) that LSAP reduced the spread between mortgage-backed securities and Treasuries.

The middle panels of Figure 8 show the ex ante effects of LSAP by comparing prices across economies with and without the possibility of LSAP. When an LSAP intervention is expected, collateral values are higher ex ante. This raises the price of asset a and lowers the price of asset b. Thus, anticipated future LSAP interventions amplify shadow banking booms. This mechanism echoes concerns raised by Rajan (2005).

The bottom panels of Figure 8 look at a "taper shock": news that LSAP will be unwound sooner than expected. This shock recalls events from the summer of 2013 when discussion of LSAP withdrawal led to large price drops across asset markets (see Fischer (2015)). In our model, the taper shock largely reverses the effect of LSAP. This highlights the importance of central bank credibility for unconventional monetary policy.

B. *Operation Twist*

In September 2011, the Fed began purchasing long-term Treasury bonds and selling short-term Treasury bonds. This program, colloquially called "Operation Twist," was predicated on the idea that risky productive assets are exposed to duration risk just like long-term Treasury bonds, so that reducing the supply of long-term Treasury bonds might free up balance-sheet capacity for risky assets.[24] As we show below, in our economy the opposite happens. In the presence

[23] The price of asset a can fall if uncertainty and the risky capital share are low. The reason is the central bank's limited capacity—a mistimed intervention implies that there will be no LSAP at the next crash.

[24] From the Federal Reserve's FAQ about the Maturity Extension Program (Operation Twist), "By reducing the supply of longer-term Treasury securities in the market, this action should put downward pressure on longer-term interest rates, including rates on financial assets that

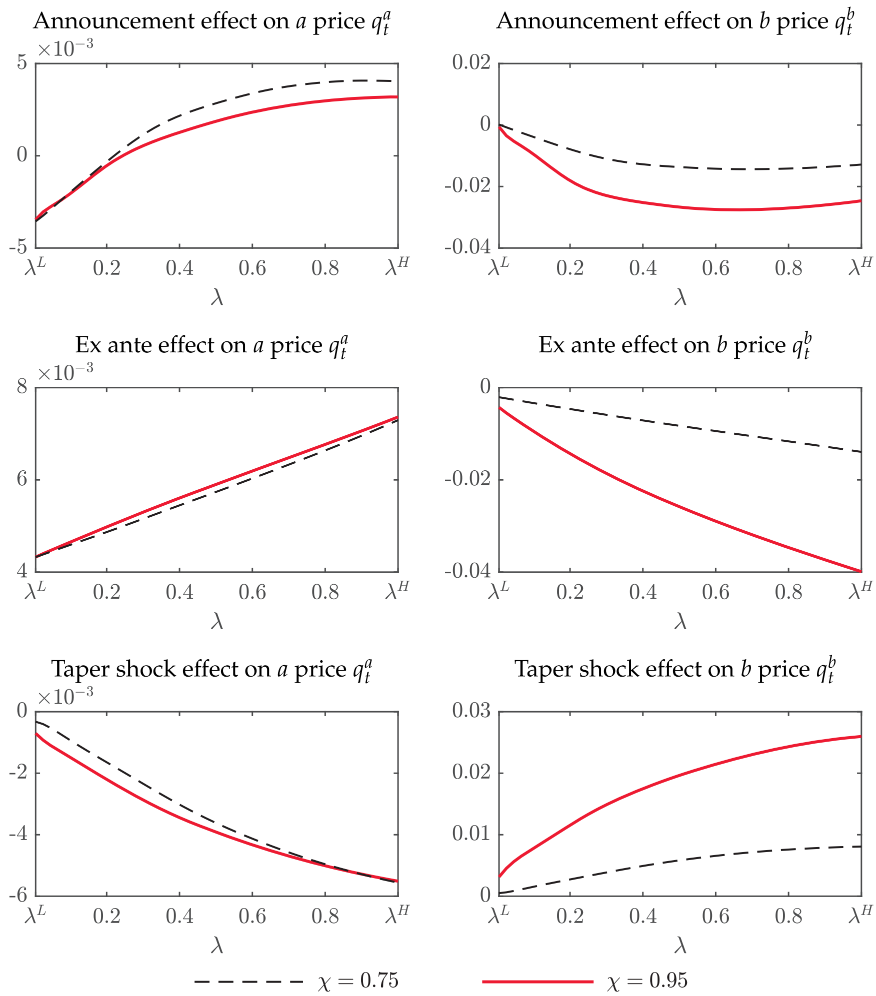

Figure 8. Large Scale Asset Purchases (LSAP). This figure shows the percentage change in asset prices around an LSAP intervention (announcement effect), between economies with and without the possibility of LSAP (ex ante effect), and around a taper shock. In an LSAP intervention, the central bank buys 20% of the stock of asset a and sells an equal dollar amount of asset b at postannouncement prices. LSAP interventions occur with probability one immediately after a crash. The announcement effect compares post-LSAP prices with prices when an LSAP fails to occur (it is still expected to occur at the next crash). The taper shock is an increase in the intensity of LSAP withdrawal from 0.1 to 1. See Section E.1 and Appendix E.1 for details. (Color figure can be viewed at wileyonlinelibrary.com)

of flight to quality, long-term government bonds increase aggregate collateral values, acting as complements rather than substitutes for risky assets.

investors consider to be close substitutes for longer-term Treasury securities" (available at https://www.federalreserve.gov/monetarypolicy/maturityextensionprogram-faqs.htm).

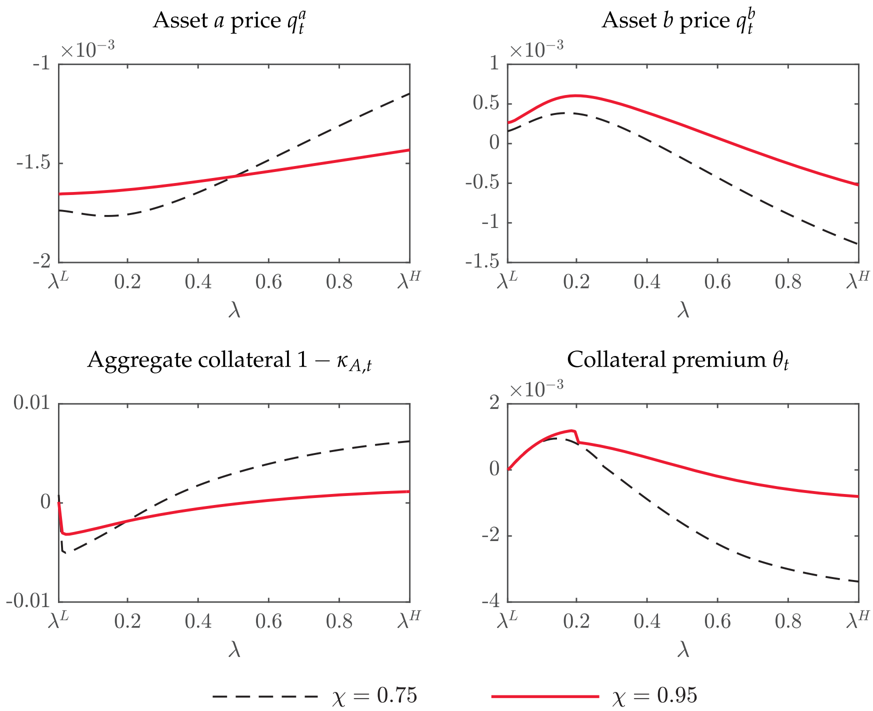

Figure 9. Operation Twist. This figure shows the percentage change in asset prices q_t^a and q_t^b, and the change in aggregate collateral $1-\kappa_{A,t}$ and the collateral premium θ_t around an Operation Twist intervention. In an Operation Twist intervention, the central bank purchases the economy's stock of asset b funded with floating-rate (zero-duration) risk-free assets. The effects are computed by comparing quantities just before and just after an unanticipated Operation Twist intervention takes place. We set the rate of issuance of government bonds to 7%. See Section IV.B and Appendix E.2 for details. (Color figure can be viewed at wileyonlinelibrary.com)

We model Operation Twist as an intervention that reduces the duration of safe bonds on intermediary balance sheets. We map the safe asset b to government bonds by assuming that the private sector cannot create them but that the government issues them at a constant rate. In an Operation Twist intervention, the central bank buys these long-term government bonds and sells safe zero-duration floating-rate bonds of equal dollar value (e.g., T-bills or reserves). The floating-rate bonds have the same yield as money so they trade at par. The details are in Appendix E.2. For simplicity, we consider a one-off unanticipated intervention in which the central bank buys the whole stock of long-term government debt.

The top left panel of Figure 9 shows that Operation Twist reduces the price of the risky asset a. The mechanism is as follows. Because of the strong flight-to-quality effect (i.e., $\kappa_{q,t}^b < 0$; see Section III.C), the long-term safe bond acts as a hedge for asset a on intermediary balance sheets, increasing aggregate collateral values in (23). By contrast, floating-rate bonds always trade at par.

Thus, when the central bank swaps floating-rate bonds for long-term bonds, aggregate collateral values fall (bottom left panel). With less collateral, intermediaries become more constrained and liquidity provision contracts, causing aggregate discount rates to rise and the price of asset a to fall. Since asset a is the economy's productive asset, Operation Twist ultimately reduces growth.

The price of long-term government bonds goes up under Operation Twist. This is due to a higher collateral premium (lower right panel). Therefore, the effectiveness of Operation Twist cannot be judged by the response of long-term government bond yields, which go down even though the policy has a contractionary effect.

Consistent with our framework, Krishnamurthy and Vissing-Jorgensen (2012b) show that the liquidity premium on long-term Treasury bonds was substantially higher than T-bills at the time of Operation Twist. This implies that Operation Twist contracted rather than expanded the supply of liquidity. Consistent with such a contractionary effect, Krishnamurthy and Vissing-Jorgensen (2013) find that Operation Twist increased mortgage spreads while LSAP decreased them.

The insight that long-term safe assets can act as complements to risky assets also has implications for regulations such as the Volcker rule and the recently adopted SEC rules for money market funds.[25] Both policies effectively segregate risky from safe assets on intermediary balance sheets. In our setting this can have the unintended effect of wasting collateral and ultimately reducing economic activity.

V. Conclusion

We present a macrofinance model in which liquidity transformation in the financial sector drives the macrocycle. The key mechanism is that while investors demand liquid securities, producing them requires collateral. Since collateral is scarce, intermediaries optimally produce securities that require less collateral but have fragile liquidity: they are money-like most of the time but cease to be liquid when uncertainty spikes. Fragile liquidity allows intermediaries to fund risky assets with liquid securities. This process of fragile liquidity transformation characterizes shadow banking.

Our model shows how shadow banking as fragile liquidity transformation boosts asset prices and creates growth in good times at the expense of bad times. As such, it provides a framework for analyzing the trade-off between financial stability and growth and the impact of policy interventions.

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[25] The Volcker rule prevents banks from engaging in proprietary trading. Observers have argued that the distinction between market-making and proprietary trading is difficult as market-making typically involves holding a substantial inventory of risky assets. SEC rule 2014-143 mandates that only funds with more than 99.5% of their portfolio in cash or government securities can issue shares with a stable net asset value.

Appendix A: Static Model Proofs and Derivations

A.1. Private Information Trading

On date 1, investors decide whether to hire a competitive firm called a fund manager (there is no incentive to hire a fund manager on date 0 because there is no liquidity-event trading on that date). If she is hired, the fund manager acquires the private signal for a fixed cost f which she passes on to the investor as a management fee. For simplicity, we assume that each fund manager focuses on a particular security.

Managers trade by exchanging their investors' securities for units of consumption and vice versa. We assume they can buy or sell up to $\omega(\kappa_x)$ units of security x, where

$$\omega(\kappa_x) = 1 + \alpha/\kappa_x \tag{A1}$$

for some $\alpha \geq 0$. This specification is useful for the following reasons. First, there has to be a limit to how much fund managers can trade for the information cost, which is a fixed cost, to matter for profits. The case of a constant trading limit is nested by $\alpha = 0$. Second, the case $\alpha > 0$ is convenient for making always-liquid securities require zero crash exposure, which is the simplest case to work with though it is not required for our model. We will see why this is the case in the proof of Proposition 1 below. We note here that it is intuitive that managers can trade securities with lower crash exposure more aggressively.

Trading on date 1 works as follows. Each seller (liquidity-event investors and informed managers if the private signal reveals a crash) meets a potential buyer (nonliquidity-event investors and informed manager if the private signal reveals no crash). Uninformed sellers make a take-it-or-leave-it offer to sell their security for its present value given the public signal λ_1. If the offer is accepted by uninformed buyers, the security trades and is liquid. If it is not accepted, the security does not trade and is illiquid. This trading protocol implements the notion of liquidity in Assumption 1.

A.2. Proof of Proposition 1

PROOF: Let the payoff of security x be as follows:

$$r_2^x = \begin{cases} 1 + \mu_x & \text{if} \quad Y_2 = 1 + \mu_Y \\ 1 - \kappa_x & \text{if} \quad Y_2 = 1 - \kappa_Y. \end{cases} \tag{A2}$$

The yield μ_x can be decomposed into a liquidity premium and a crash compensation component, $\mu_x = \mu_{x,0} + \mu_{x,1}$. The liquidity premium $\mu_{x,0}$ arises from the liquidity services the security provides on date 1 to the date-0 investor. We thus assume that it is paid by the date-0 investor when security payoffs are settled on date 2. The crash compensation $\mu_{x,1}$ is such that investors break even on average between crashes and no crashes (i.e., $E_0[r_2^x - \mu_{x,0}] = 1$). Hence,

$$\mu_{x,1} = \frac{\lambda_0}{1-\lambda_0}\kappa_x. \tag{A3}$$

This component gets incorporated into the present value of the security when it trades on date 1. This present value, which depends on the public signal λ_1, is given by

$$pv\,(x|\,\lambda_1) = (1-\lambda_1)(1+\mu_{x,1}) + \lambda_1(1-\kappa_x). \tag{A4}$$

Fund managers sell the security if the private signal indicates a crash and buy it if it does not. Their trading profits per unit traded are given by

$$\tilde{\pi}_1\,(x|\,Y_2) = \begin{cases} 1+\mu_{x,1} - pv\,(x|\,\lambda_1) & \text{if } Y_2 = 1+\mu_Y \\ pv\,(x|\,\lambda_1) - (1-\kappa_x) & \text{if } Y_2 = 1-\kappa_Y. \end{cases} \tag{A5}$$

The trading limit $\omega(\kappa_x)$ is always binding. The manager's expected profit is thus

$$\pi_1 \equiv E[\omega(\kappa_x)\tilde{\pi}_1(x|Y_2)|\lambda_1] = \frac{2}{1-\lambda_0}\lambda_1(1-\lambda_1)(\alpha+\kappa_x). \tag{A6}$$

(Equation (3) in the text is based on (A6).) The manager's net expected profit is $\pi_1 - f$. There will be no private information acquisition on date 1 if the net profit is (weakly) negative. For this to hold for any $\lambda_1 \in \{\lambda^L, \lambda^H\}$, we need

$$\kappa_x \le \underline{\kappa} \;\equiv\; \frac{f}{2}\left[\frac{1-\lambda_0}{\lambda^H\left(1-\lambda^H\right)}\right] - \alpha. \tag{A7}$$

This follows from the assumption that interim uncertainty is higher when $\lambda_1 = \lambda^H$ than when $\lambda_1 = \lambda^L$ (i.e., $\lambda^L(1-\lambda^L) < \lambda^H(1-\lambda^H)$). On the other hand, for there to be no information acquisition when $\lambda_1 = \lambda_L$, it is enough to have

$$\kappa_x \le \overline{\kappa} \;\equiv\; \frac{f}{2}\left[\frac{1-\lambda_0}{\lambda^L\left(1-\lambda^L\right)}\right] - \alpha. \tag{A8}$$

We note that $\underline{\kappa} < \overline{\kappa}$ for any α. Thus, the cutoff crash exposure that triggers private information acquisition is always lower when interim uncertainty is high ($\lambda_1 = \lambda^H$). In other words, high interim uncertainty is associated with more private information acquisition.

The parameter α allows us to set $\underline{\kappa} = 0$. This implies that being always-liquid requires zero crash exposure, which is simple and convenient. Intuitively, the profitability π_1 of trading a security falls as crash exposure declines. Assuming that trading limits $\omega(\kappa_x)$ increase at the same time as in (A1) makes total profits fall more slowly (they still fall as can be seen in (A6)). The slower fall in profits pushes the crash exposure cutoff for acquiring private information, $\underline{\kappa}$, toward zero. To specifically obtain $\underline{\kappa} = 0$, we set $\alpha = \frac{f}{2}[\frac{1-\lambda_0}{\lambda^H(1-\lambda^H)}]$. This then implies

$$\overline{\kappa} = \frac{f}{2}(1-\lambda_0)\left[\frac{1}{\lambda^L(1-\lambda^L)} - \frac{1}{\lambda^H(1-\lambda^H)}\right] > 0. \tag{A9}$$

For $\overline{\kappa} < 1$, we need $f < \frac{2}{1-\lambda_0}[\frac{1}{\lambda^L(1-\lambda^L)} - \frac{1}{\lambda^H(1-\lambda^H)}]^{-1}$. This number is positive so there is a solution with $f > 0$.

In what follows, let $\kappa_1 \in \{\underline{\kappa}, \overline{\kappa}\}$ with $\kappa_1 = \underline{\kappa}$ if $\lambda_1 = \lambda^H$ and $\kappa_1 = \overline{\kappa}$ if $\lambda_1 = \lambda^L$.

The results above show that if $\kappa_x \leq \kappa_1$, there will be no private information in the market for security x. In this case buyers know that sellers are not informed, and so they accept an offer to buy the security for its present value under public information, $pv(x|\lambda_1)$. This means that securities with $\kappa_x \leq \kappa_1$ are liquid.

We now show that this is not true if $\kappa_x > \kappa_1$. This is because, in the presence of private information, buyers update about the value of the security from the fact that they are matched with a seller. To show this, let $p_I(\lambda_1) > 0$ be the probability that the seller is informed. Then the value to the buyer is

$$p_I(\lambda_1)(1-\kappa_x) + \left[1 - p_I(\lambda_1)\right] pv(x|\lambda_1), \tag{A10}$$

because an informed investor would not sell unless her private signal revealed a crash. Since $p_I(\lambda_1) > 0$ and $1 - \kappa_x < pv(x|\lambda_1)$, this value is less than $pv(x|\lambda_1)$. Hence, the buyer will not accept a take-it-or-leave-it offer at $pv(x|\lambda_1)$ and the security is illiquid.

We note that order flow does not reveal the private signal to the buyer because individual investors and their fund managers have undetermined mass. Thus, a large sell order could come from a large liquidity-event trader or a small sell order could come from a small fund manager.

So far we have shown that, under appropriate parameter restrictions, always-liquid securities require $\kappa_x = 0$ and fragile-liquid securities require $\kappa_x \leq \overline{\kappa} < 1$. We now define money m with $\kappa_m = 0$. It has the highest crash exposure for an always-liquid security. Next, we define shadow money s with $\kappa_s = \overline{\kappa}$. It has the highest crash exposure for a fragile-liquid security. And finally, we define equity e with $\kappa_e = 1$. It has the highest crash exposure for an illiquid security. We confirm that it is indeed optimal to issue securities with the highest crash exposure for their liquidity profile as part of the proof of Proposition A.1. below. ■

A.3. Equilibrium

We first show that (4) follows from (1). The expectation in (1) is taken over both the idiosyncratic shock z_1 and the aggregate shocks λ_1 and Y_2. To get (4), we integrate over z_1, which is independent of all other shocks:

$$\begin{aligned} E_0\left[z_1 C_1 + C_2\right] &= E_0\left[(z_1 - 1)C_1 + (C_1 + C_2)\right] \\ &= E_0\left[h(\psi - 1)C_1 + (C_1 + C_2)\right]. \end{aligned} \tag{A11}$$

To get (4), we have to show $E_0[C_1 + C_2] = E_0[Y_2]$, that is, expected total consumption equals the payoff of the endowment. The argument is as follows. From Assumption 1, investors trade a security only if its market value equals its present value. Since there is no discounting between dates 1 and 2, a security's present value equals its expected payoff. Thus, investors trade a dollar of date-1 consumption for what they expect to be a dollar of date-2 consumption. As a result, expected total consumption is invariant to interim trading and so it equals the expected payoff of the endowment, $E_0[C_1 + C_2] = E_0[Y_2]$. Combining this with (A11) establishes (4).

Next, we make a simplifying assumption that allows us to write the liquidity constraint as in (5). From the proof of Proposition 1, we know that shadow money appreciates slightly when $\lambda_1 = \lambda^L$ since $\lambda^L < \lambda_0$. This appreciation is of the order of the maturity of the security. In particular, it vanishes in continuous time where it is of order dt. For this reason, and for simplicity, we assume that the appreciation in shadow money cannot be used for liquidity-event consumption. Instead it gets consumed after the liquidity event. This implies that each initial dollar of shadow money provides one dollar of liquidity-event consumption as written in (5). With this simplification, the results from the static model carry over to the dynamic model unchanged.

PROPOSITION A.1. (Equilibrium security issuance): *Suppose that $\overline{\kappa} \leq \kappa_Y$. Then in equilibrium money and shadow money issuance, m_0 and s_0, are as follows:*

(i) if $p_H(\lambda_0) \leq \overline{\kappa}$, then $m_0 = 0$ and $s_0 = \frac{1-\kappa_Y}{1-\overline{\kappa}}$, and
(ii) if $p_H(\lambda_0) > \overline{\kappa}$, then $m_0 = 1 - \kappa_Y$ and $s_0 = 0$.

PROOF OF PROPOSITION A.1: Substituting the constraints into the objective and simplifying, we solve

$$\max_{0 \leq s_0 \leq 1} \; h(\psi - 1)(1 - \kappa_Y - (1 - \overline{\kappa})s_0 + [1 - p_H(\lambda_0)]s_0) + E_0[Y_2]. \tag{A12}$$

We get $s_0 = (1 - \kappa_Y)/(1 - \overline{\kappa})$ if $\overline{\kappa} \leq \kappa_Y$ and $p_H(\lambda_0) \leq \overline{\kappa}$. Substituting into the collateral constraint (6), this implies $m_0 = 0$. If instead $p_H(\lambda_0) > \overline{\kappa}$, $s_0 = 0$ and hence $m_0 = 1 - \kappa_Y$.

For completeness, we also state the solution when $\overline{\kappa} > \kappa_Y$ and $p_H(\lambda_0) \leq \overline{\kappa}$. In this case $m_0 + s_0 \leq 1$ binds and we get $s_0 = \kappa_Y/\overline{\kappa}$ and $m_0 = 1 - \kappa_Y/\overline{\kappa}$ (see also case (i.a) in the proof of Proposition 3).

To show that shadow-money is optimally issued, consider reducing its crash exposure: $0 < \kappa_s < \overline{\kappa}$. This tightens the collateral constraint (6) but does not relax the liquidity constraint (5), and hence it is not optimal. The same argument applies to equity. It would also apply to money if it were not the unique always-liquid security, which automatically makes it the optimally issued security. ∎

Appendix B: Dynamic Model Proofs and Derivations

B.1. Optimal Filtering

In this section, we derive the optimal filter (10). Let $\mathcal{F}_t$ represent investors' information filtration. Investors form beliefs

$$\lambda_t = E\left[\tilde{\lambda}_t \middle| \mathcal{F}_t\right]. \tag{B1}$$

They learn about $\tilde{\lambda}$ from crash realizations and from uncertainty news. The solution to the filtering problem with Markov switching is analyzed in Wonham (1965).

The exogenous news signal is represented by the process $(\tilde{\lambda}_t - \lambda_t)dt + \sigma d\tilde{B}_t$. The crash realization process is the uncompensated Poisson process $d\tilde{J}_t$, which must coincide with the observed crash realization process dJ_t. The innovation to the household filtration $\mathcal{F}_t$ can therefore be represented as the 2×1 signal

$$d\zeta_t = \begin{bmatrix} \left(\tilde{\lambda}_t - \lambda_t\right) dt + \sigma d\tilde{B}_t \\ d\tilde{J}_t - \lambda_t dt \end{bmatrix}. \tag{B2}$$

We seek an innovations representation of the form $d\lambda_t = a_t dt + b_t' d\xi_t$. Note that

$$d\lambda_t = E\left[\tilde{\lambda}_{t+dt} \middle| \mathcal{F}_t, d\xi_t\right] - E\left[\tilde{\lambda}_t \middle| \mathcal{F}_t\right] \tag{B3}$$

$$= E\left[\tilde{\lambda}_t + d\tilde{\lambda}_t \middle| \mathcal{F}_t, d\xi_t\right] - E\left[\tilde{\lambda}_t \middle| \mathcal{F}_t\right] \tag{B4}$$

$$= \left[\left(\lambda^H - \lambda_t\right)\varphi^L - \left(\lambda_t - \lambda^L\right)\varphi^H\right] dt + E\left[\tilde{\lambda}_t \middle| \mathcal{F}_t, d\xi_t\right] - E\left[\tilde{\lambda}_t \middle| \mathcal{F}_t\right], \tag{B5}$$

where φ^H and φ^L are the transition rates from the high and low states, respectively. The last line follows from the fact that the crash and news innovations are uncorrelated with the switching process for $\tilde{\lambda}$. The innovation representation is therefore the conditional mean of the population regression

$$\tilde{\lambda}_t - \lambda_t = \left[\left(\lambda^H - \lambda_t\right)\varphi^L - \left(\lambda_t - \lambda^L\right)\varphi^H\right] dt + b_t' d\xi_t + d\epsilon_t. \tag{B6}$$

The orthogonality condition for $d\epsilon_t$ gives

$$b_t = E\left[d\xi_t d\xi_t' \middle| \mathcal{F}_t\right]^{-1} E\left[d\xi_t\left(\tilde{\lambda}_t - \lambda_t\right) \middle| \mathcal{F}_t\right] \tag{B7}$$

$$= \begin{bmatrix} \sigma^2 dt & 0 \\ 0 & \lambda_t dt \end{bmatrix}^{-1} \begin{bmatrix} \left(\lambda^H - \lambda_t\right)\left(\lambda_t - \lambda^L\right) dt \\ \left(\lambda^H - \lambda_t\right)\left(\lambda_t - \lambda^L\right) dt \end{bmatrix} \tag{B8}$$

$$= \begin{bmatrix} 1/\sigma^2 \\ 1/\lambda_t \end{bmatrix}\left(\lambda^H - \lambda_t\right)\left(\lambda_t - \lambda^L\right). \tag{B9}$$

Therefore, we can write the dynamics of the perceived crash intensity as

$$d\lambda_t = [(\lambda^H - \lambda_t)\varphi^L - (\lambda_t - \lambda^L)\varphi^H]dt + (\lambda^H - \lambda_t)(\lambda_t - \lambda^L)\left[\begin{array}{cc} \frac{1}{\sigma^2} & \frac{1}{\lambda_t} \end{array}\right] d\xi_t \tag{B10}$$

$$= [(\lambda^H - \lambda_t)\varphi^L - (\lambda_t - \lambda^L)\varphi^H]dt + (\lambda^H - \lambda_t)(\lambda_t - \lambda^L)\left(\frac{1}{\sigma}dB_t + \frac{1}{\lambda_t}dZ_t\right), \tag{B11}$$

where $dB_t = \frac{1}{\sigma}(\tilde{\lambda}_t - \lambda_t)dt + d\tilde{B}_t$ is a Brownian motion and $dZ_t = d\tilde{J}_t - \lambda_t dt = dJ_t - \lambda_t dt$ is a compensated Poisson jump process with intensity λ_t, both adapted to $\mathcal{F}_t$. Letting $\varphi \equiv \varphi^L + \varphi^H$, $\overline{\lambda} = (\frac{\varphi^L}{\varphi^L+\varphi^H})\lambda^H + (\frac{\varphi^H}{\varphi^L+\varphi^H})\lambda^L$, $\nu \equiv 1/\sigma$, and $\Sigma_t \equiv (\lambda^H - \lambda_t)(\lambda_t - \lambda^L)$ and rearranging (B11) gives (10).

B.2. Investor's Problem

We first show how to get (17) from (14). Taking first-order conditions with respect to dc_t in (14) gives $V_{W,t} = 1$. But V_t is also homogeneous in wealth because of the homogeneity of preferences, and hence $V_t = W_t$. Substituting this, using the fact that liquidity-event consumption is always constrained, and evaluating the expectation gives (17). This equation is the analog to (4) from the static model.

PROOF OF PROPOSITION 2: Expanding (14),

$$\rho = \max_{m_t, s_t} h(\psi - 1)\left[[1 - p_H(\lambda_t)]\left(\int_0^{m_t+s_t} \eta x e^{-\eta x}dx + (m_t + s_t)e^{-\eta(m_t+s_t)}\right)\right.$$
$$\left. + p_H(\lambda_t)\left(\int_0^{m_t} \eta x e^{-\eta x}dx + m_t e^{-\eta m_t}\right)\right] + \mu_{W,t} \tag{B12}$$

$$= \max_{m_t, s_t} \frac{h}{\eta}(\psi - 1)\left[[1 - p_H(\lambda_t)]\left(1 - e^{-\eta(m_t+s_t)}\right) + p_H(\lambda_t)\left(1 - e^{-\eta m_t}\right)\right] + \mu_{W,t}. \tag{B13}$$

Taking first-order conditions for m_t and s_t gives (18) and (19). Plugging (18) into the maximized objective gives (20). ■

B.3. Intermediary Problem

We first state the overall problem of the representative intermediary to show how the liabilities and assets sides of the problem are connected.

Intermediaries maximize the value of existing equity V_t, which is discounted at the required rate of return on equity. Dropping time subscripts for convenience,

$$\mu_e V dt = \max_{m,s,e,k^a,k^b,\iota^a,\iota^b} \left[(y^a - \iota^a) k^a + \left(y^b - \iota^b\right) k^b\right] dt + E\left[dA\right]$$
$$- E\left[Amdr^m + Asdr^s + (Ae - V)dr^e\right] + E\left[dV\right], \tag{B14}$$

where $A = q^a k^a + q^b k^b$ is the value of assets. The return on existing equity is equal to the current flow of expected profits plus the expected change in the value of future profits. The current flow of expected profits equals cash flows net of investment plus the change in the value of assets minus payments on money, shadow money, and newly issued equity $(Ae - V)$. Since intermediaries are competitive, $V = 0$. Substituting and writing the full Lagrangian,

$$0 = \max_{m,s,e,k^a,k^b,\iota^a,\iota^b} \left[(y^a - \iota^a)k^a + \left(y^b - \iota^b\right) k^b\right] dt + A\left[E\left[\frac{dA}{A}\right] - (m\mu_m + s\mu_s\right.$$
$$\left. + e\mu_e)dt\right] + A\left[\theta\left(1 - \kappa_A - \left[m + s\left(1 - \overline{\kappa}\right)\right]\right) + \theta^m m + \theta^s s + \theta^e e\right] dt, \tag{B15}$$

where $1 - \kappa_A$ is the collateral value of assets (calculated below) and θ^m, θ^s, and θ^e are the multipliers on the nonnegativity constraints $m, s, e \geq 0$.

We can nest the liabilities-side problem inside the overall problem by rewriting (B15):

$$0 = \max_{k^a,k^b,\iota^a,\iota^b} \left\{\left[(y^a - \iota^a)k^a + \left(y^b - \iota^b\right) k^b\right] dt + E\left[dA\right] + A\left[\max_{m,s,e} -(m\mu_m + s\mu_s\right.\right.$$
$$\left.\left. + e\mu_e) + \theta\left(1 - \kappa_A - \left[m + s\left(1 - \overline{\kappa}\right)\right]\right) + \theta^m m + \theta^s s + \theta^e e\right] dt\right\}. \tag{B16}$$

The inner maximization problem is the one we analyze in Section II.E.2. To solve the outer problem, let $\chi^q = q^a k^a / (q^a k^a + q^b k^b)$. Then, applying Itô's Lemma, we have

$$\frac{dA}{A} = \chi^q \frac{d\left(q^a k^a\right)}{q^a k^a} + \left(1 - \chi^q\right) \frac{d\left(q^b k^b\right)}{q^b k^b} \tag{B17}$$

$$\frac{d\left(q^i k^i\right)}{q^i k^i} = \left[\phi\left(\iota^i\right) - \delta + \mu_q^i + \kappa_k^i \kappa_q^i \lambda\right] dt + \sigma_q^i dB - \left[1 - \left(1 - \kappa_k^i\right)\left(1 - \kappa_q^i\right)\right] dZ \tag{B18}$$

for $i = a, b$ (recall that the level inflow term $\mu_0(k^a + k^b)$ does not accrue inside investors' portfolios.) Let

$$\mu_A = \chi^q \left[\frac{y^a - \iota^a}{q^a} + \phi\left(\iota^a\right) - \delta + \mu_q^a + \kappa_k^a \kappa_q^a \lambda\right] \tag{B19}$$

$$+ \left(1 - \chi^q\right)\left[\frac{y^b - \iota^b}{q^b} + \phi\left(\iota^b\right) - \delta + \mu_q^b + \kappa_k^b \kappa_q^b \lambda\right]$$

$$\sigma_A = \chi^q \sigma_q^A + (1-\chi^q)\,\sigma_q^b \tag{B20}$$

$$1-\kappa_A = \chi^q(1-\kappa_k^a)\left(1-\kappa_q^a\right) + (1-\chi^q)\left(1-\kappa_k^b\right)\left(1-\kappa_q^b\right). \tag{B21}$$

Note that μ_A is defined as the drift of assets plus the net cash flow (dividend) yield. Note also that we need $e\sigma_e = \sigma_A$, but this can always be achieved with arbitrarily large σ_e so Brownian risk plays no further role in the solution to the intermediary's problem.

Using complementary slackness on the nonnegativity constraints, the asset-side optimality conditions are

$$\begin{aligned} 0 = {} & \frac{y^i-\iota^i}{q^i} + \phi\left(\iota^i\right) - \delta + \mu_q^i + \kappa_k^i\kappa_q^i\lambda - (m\mu_m + s\mu_s + e\mu_e) \\ & + \theta\left[m + s\,(1-\overline{\kappa}) - \left(1-\kappa_k^i\right)\left(1-\kappa_q^i\right)\right], \qquad i = a, b. \end{aligned} \tag{B22}$$

Substituting $\mu_W = m\mu_m + s\mu_s + e\mu_e$ and $m + s(1-\overline{\kappa}) = 1-\kappa_A$, and solving for q^i, gives (25). The optimality conditions for investment give (12).

B.4. Proof of Proposition 3

PROOF: We note first that the zero-profit condition together with nonnegative profits in a crash (from the collateral constraint) imply that profits are nonnegative in normal times. Hence, intermediaries do not violate limited commitment in any state.

Dropping time subscripts for convenience, the first-order conditions for m and s are

$$\mu_e - \mu_m = \theta - \theta^m + \theta^e \tag{B23}$$

$$\mu_e - \mu_s = \theta\,(1-\overline{\kappa}) - \theta^s + \theta^e. \tag{B24}$$

We first show that $\theta > 0$. Suppose not, $\theta = 0$. Since $\mu_e - \mu_m > 0$ (see Proposition 2), we must have $\theta^e > 0$, so $e = 0$. Subtracting (B24) from (B23) then gives $\mu_s - \mu_m = \theta^s - \theta^m$. Since $\mu_s - \mu_m > 0$ (see Proposition 2), we must have $\theta^s > 0$, so $s = 0$. But if $s = e = 0$, then we must have $m = 1$, which is not feasible. Thus, $\theta > 0$.

This leaves four cases. We label them (i.a), (i.b), (ii), and (iii):

Case (i.a): $\theta^m > 0$. In this case $m = 0$ and, from the collateral constraint, $s = (1-\kappa_A)/(1-\overline{\kappa})$, which leaves $e = 1-(1-\kappa_A)/(1-\overline{\kappa})$. This case clearly requires $\kappa_A \geq \overline{\kappa}$. It follows that $\theta^e = \theta^s = 0$. Substituting (18) and (19) into (B23) and (B24),

$$h(\psi-1)\left[\left(1-p_H(\lambda)\right)e^{-\eta\frac{1-\kappa_A}{1-\overline{\kappa}}} + p_H(\lambda)\right] = \theta - \theta_0^m \tag{B25}$$

$$h(\psi-1)\left(1-p_H(\lambda)\right)e^{-\eta\frac{1-\kappa_A}{1-\overline{\kappa}}} = \theta\,(1-\overline{\kappa}). \tag{B26}$$

The solution is

$$\theta = h(\psi - 1)\left(1 - p_H(\lambda)\right)\frac{e^{-\eta\frac{1-\kappa_A}{1-\overline{\kappa}}}}{1-\overline{\kappa}} \tag{B27}$$

$$\theta_0^m = h(\psi - 1)\left[\left(1 - p_H(\lambda)\right)\frac{\overline{\kappa}}{1-\overline{\kappa}}e^{-\eta\frac{1-\kappa_A}{1-\overline{\kappa}}} - p_H(\lambda)\right]. \tag{B28}$$

For $\theta^m > 0$, we need $\mathcal{M} > \frac{1-\kappa_A}{1-\overline{\kappa}}$, where $\mathcal{M} = \frac{1}{\eta}\log(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{1-p_H(\lambda)}{p_H(\lambda)})$.

Case (i.b): $\theta^e > 0$. In this case $e = 0$ so $m + s = 1$ and, from the collateral constraint, $m = 1 - \kappa_A/\overline{\kappa}$ and $s = \kappa_A/\overline{\kappa}$. This case clearly requires $\kappa_A < \overline{\kappa}$. It follows that $\theta^m = \theta^s = 0$. Substituting (18) and (19) into (B23) and (B24),

$$h(\psi - 1)\left[\left(1 - p_H(\lambda)\right)e^{-\eta} + p_H(\lambda)e^{-\eta(1-\frac{\kappa_A}{\overline{\kappa}})}\right] = \theta + \theta^e \tag{B29}$$

$$h(\psi - 1)\left(1 - p_H(\lambda)\right)e^{-\eta} = \theta\left(1-\overline{\kappa}\right) + \theta^e. \tag{B30}$$

The solution is

$$\theta = h(\psi - 1)p_H(\lambda)\frac{e^{-\eta(1-\frac{\kappa_A}{\overline{\kappa}})}}{\overline{\kappa}} \tag{B31}$$

$$\theta_0^e = h(\psi - 1)\left[\left(1 - p_H(\lambda)\right)e^{-\eta} - p_H(\lambda)\frac{1-\overline{\kappa}}{\overline{\kappa}}e^{-\eta(1-\frac{\kappa_A}{\overline{\kappa}})}\right]. \tag{B32}$$

For $\theta^e > 0$, we need $\mathcal{M} > \frac{\kappa_A}{\overline{\kappa}}$. Combining cases (i.a) and (i.b) gives case (i) of Proposition 3.

Case (ii): $\theta^m = \theta^s = \theta^e = 0$. Substituting (18) and (19) into (B23) and (B24) and combining gives $s = \mathcal{M}$. Plugging into the collateral constraint, $m = 1 - \kappa_A - (1-\overline{\kappa})\mathcal{M}$ and so $e = 1 - m - s = \kappa_A - \overline{\kappa}\mathcal{M}$. For $m, s, e \geq 0$, we need $0 \leq \mathcal{M} \leq \min\{\frac{\kappa_A}{\overline{\kappa}}, \frac{1-\kappa_A}{1-\overline{\kappa}}\}$. Finally, using (19) and (B24),

$$\theta = h(\psi - 1)e^{-\eta(1-\kappa_A)}\left(\frac{1-p_H(\lambda)}{1-\overline{\kappa}}\right)^{1-\overline{\kappa}}\left(\frac{p_H(\lambda)}{\overline{\kappa}}\right)^{\overline{\kappa}}. \tag{B33}$$

Case (iii): $\theta^s > 0$. In this case $s = 0$, so from the collateral constraint $m = 1 - \kappa_A$ and $e = \kappa_A$. It follows that $\theta^m = \theta^e = 0$. Substituting (18) and (19) into (B23) and (B24),

$$h(\psi - 1)e^{-\eta(1-\kappa_A)} = \theta \tag{B34}$$

$$h(\psi - 1)\left(1 - p_H(\lambda)\right)e^{-\eta(1-\kappa_A)} = \theta\left(1-\overline{\kappa}\right) - \theta_0^s. \tag{B35}$$

The solution is

$$\theta = h(\psi - 1)e^{-\eta(1-\kappa_A)}, \;\; \theta_0^s = h(\psi - 1)\left(p_H(\lambda) - \overline{\kappa}\right)e^{-\eta(1-\kappa_A)}. \tag{B36}$$

For $\theta^s > 0$, we need $p_H(\lambda) > \overline{\kappa}$, which is equivalent to $\mathcal{M} < 0$. ■

B.5. Full Equilibrium Characterization

Equilibrium issuance follows Proposition 3. Given issuance, equilibrium security yields are as in Proposition 2. Equilibrium asset prices solve the partial differential equations (25). The consumption-wealth ratio of investors who get a liquidity event is $dC_t = \min\{m_t + s_t, d\overline{C}_t\}$ with probability $1 - p_H(\lambda_t)$ and $dC_t = \min\{m_t, d\overline{C}_t\}$ with probability $p_H(\lambda_t)$. The goods market clears by the elastic consumption of investors who do not experience a liquidity event.

Appendix C: Numerical Solution

We use $\phi(\iota_t) = 1/\gamma(\sqrt{1 + 2\gamma\iota_t} - 1)$ for the investment cost function (see Appendix D for discussion). We apply Itô's Lemma to (9) to get the law of motion for χ:

$$\begin{aligned} d\chi &= \mu_0(1 - 2\chi)dt + \chi(1-\chi)\left[\frac{1}{\varphi}\left(q^a - q^b\right) + \left(\kappa_k^a - \kappa_k^b\right)\lambda\right]dt \\ &\quad - \frac{\chi(1-\chi)\left(\kappa_k^a - \kappa_k^b\right)}{\chi\left(1-\kappa_k^a\right) + (1-\chi)\left(1-\kappa_k^b\right)}dJ. \end{aligned} \tag{C1}$$

(Recall that dJ is the uncompensated crash process, that is, $dZ = dJ - \lambda dt$). We next apply Itô's Lemma to $q^i = q(\lambda, \chi)$, $i = a, b$, to get the dynamics of prices (11):

$$\begin{aligned} \frac{dq_t^i}{q_t^i} &= \left(\frac{q_\lambda^i}{q^i}\left[\varphi\left(\overline{\lambda} - \lambda\right) - \Sigma\right] + \frac{1}{2}\nu^2\Sigma^2\frac{q_{\lambda\lambda}^i}{q^i} + \frac{q_\chi^i}{q^i}\chi(1-\chi)\left[\mu_0\left(\frac{1}{\chi} - \frac{1}{1-\chi}\right)\right.\right. \\ &\quad \left.\left. + \frac{1}{\gamma}\left(q^a - q^b\right) + \left(\kappa_k^a - \kappa_k^b\right)\lambda\right]\right)dt + \nu\frac{q_\lambda^i}{q^i}\Sigma dB - \left[1 - \frac{q^i\left(\lambda^+, \chi^+\right)}{q^i}\right]dJ. \end{aligned} \tag{C2}$$

for $i = a, b$, where λ^+ and χ^+ are the jump-to points of λ and χ.

Our solution method follows Judd (1998, Chapter 11). We first conjecture asset price functions $q^a(\lambda, \chi)$ and $q^b(\lambda, \chi)$ expressed as bivariate Chebyshev polynomials of order N (we use $N = 10$). We calculate the derivatives of these functions as well as the state variable dynamics, collateral values, security returns, capital structure, and investment using the relevant expressions. We then plug these quantities into the partial differential equations (25) and project the resulting residuals onto the complete set of Chebyshev polynomials up to order

N. We use the built-in Matlab routine `fsolve` to find the coefficients of the asset price polynomials that make the projected residuals equal to zero.

Appendix D: Parameter Values: Additional Details

We provide further background for our parameter choices. Consistent with the numbers in He and Krishnamurthy (2014) for aggregate capital, we set the productivity of a capital y^a to 0.138 and the productivity of b capital y^b to 0.1, which also equals the depreciation rate ($\delta = 0.1$). We use a cost function that implies moderate quadratic investment adjustment costs, namely, $\phi(\iota) = 1/\gamma(\sqrt{1+2\gamma\iota} - 1)$ with $\gamma = 3$. We also set the exogenous component of aggregate growth μ_0 to 0.01.

We set κ_k^a, the crash exposure of risky capital, to 0.5. This implies that when the risky capital share is $\chi = 0.75$ or 0.95, crashes reduce output by 40% and 46%, respectively. For comparison, Barro (2006) reports an average rare disaster in output of 35%, with one in six exceeding 45%. Disaster models further rely on risk aversion, which amplifies the impact of these large shocks.[26]

For $\overline{\kappa}$, we look at the bankruptcy of Lehman Brothers. Lehman's bonds sold for 10 cents on the dollar (implying a 90-cent write-down) at auction following bankruptcy (Helwege et al. (2010)). The ultimate recovery rate on its commercial paper (three years later) was 50 cents on the dollar (Moody's Investors Service (2013)). Our benchmark number is the average of these two.

Turning to uncertainty λ, we set the low and high bounds to $\lambda^L = 0.005$ and $\lambda^H = 1$ (which imply annual crash probabilities of 0.5% and 63%). We set the transition intensities to $p^L = 0.01$ and $p^H = 0.5$. These numbers imply an unconditional crash probability $\overline{\lambda} = 2.45\%$, consistent with Barro (2006), and persistence parameter φ of 0.5.[27] The crash probability jumps to 55% from steady state in the wake of a crash, which is near the observed 50% frequency of severe aftershocks to crises in Reinhart and Reinhart (2010). We also set the precision of the exogenous signal about λ to $\nu = 0.1$.

Finally, we set ρ to 0.37, which implies that households have an overall discount rate of about 1.6% when their savings are fully liquid.

Appendix E: Policy Interventions

E.1. Large-Scale Asset Purchases

This appendix provides details for the implementation of LSAP. Let $\zeta_t \in \{\zeta_E, \zeta_F\}$ denote the state of the central bank's balance sheet, which is either

[26] In our model, some agents experience a spike in marginal utility during a crash, but everyone else is unaffected and willing to work to supply consumption. Thus, there is no sense in which the marginal utility of the representative agent increases as it does in a model with risk aversion.

[27] Risk-neutral crash probabilities in the disaster literature are much larger at about 20% (Gabaix (2012)).

"empty" ($\zeta_t = \zeta_E$) or "full" ($\zeta_t = \zeta_F$). An empty balance sheet is filled with probability $\beta_{LSAP}(\lambda_t, \chi_t)$ immediately after a crash, in which case it becomes full. The size of the program is given as a state-contingent fraction $\alpha(\lambda_t, \chi_t)$ of the outstanding supply of asset a. A full balance sheet is emptied ($\zeta_t = \zeta_E$) with intensity $\beta_{REVERSAL}(\lambda_t, \chi_t)$. The case $\beta_{REVERSAL}(\lambda_t, \chi_t) = 0$ nests a permanent LSAP. We focus on one-off LSAP interventions, and hence following a reversal the economy reverts to the baseline no-LSAP economy.

We can write asset prices in the form $q_t^i = q^i(\lambda_t, \chi_t, \zeta_t)$ for $i = a, b$. Their drifts and collateral values in the case $\zeta_t = \zeta_E$ are given by

$$\mu^i_{q,\zeta_E} = \mu^i_{q,0} - \lambda\beta_{LSAP}\left(\kappa^i_{q,LSAP} - \kappa^i_{q,No\,LSAP}\right) \tag{E1}$$

$$1 - \kappa^i_{q,\zeta_E} = \min\left\{1 - \kappa^i_{q,LSAP}, 1 - \kappa^i_{q,No\,LSAP}\right\} \tag{E2}$$

for $i = a, b$, where μ^i_{q,ζ_0} is given in (C2), $1 - \kappa^i_{q,LSAP} = \frac{q^i(\lambda_+, \chi_+ - \alpha, \zeta_F)}{q(\lambda, \chi, \zeta_E)}$ (LSAP shifts χ^+ to $\chi^+ - \alpha$), and $1 - \kappa^i_{q,No\,LSAP} = \frac{q^i(\lambda_+, \chi_+, \zeta_E)}{q^i(\lambda, \chi, \zeta_E)}$. These modified dynamics enter into the partial differential equations for asset prices (25). Under $\zeta_t = \zeta_F$, we instead have

$$\mu^i_{q,\zeta_F} = \mu^i_{q,0} - \beta_{REVERSAL}\kappa^i_{q,REVERSAL} \tag{E3}$$

for $i = a, b$ with $1 - \kappa_{q,REVERSAL} = \frac{q^i(\lambda, \chi + \alpha, \zeta_E)}{q^i(\lambda, \chi, \zeta_F)}$. For simplicity, we do not impose a separate collateral constraint with respect to the reversal shock. This has the effect of understating the impact of policy withdrawal. We also note that the policy's entry and exit are generally not of equal size as the economy drifts in the meantime. In this case one can think of the central bank's balance sheet as retaining a residual position.

We measure the announcement effect of an LSAP program with the ratio of prices following a crash with and without the intervention, $\frac{1-\kappa^i_{q,LSAP}}{1-\kappa^i_{q,No\,LSAP}} - 1$. We measure the ex ante effect as the ratio of prices in an LSAP economy before an LSAP and our baseline economy, $\frac{q^i(\lambda, \chi, \zeta_E|\beta_{LSAP})}{q^i(\lambda, \chi, \zeta_E|0)} - 1$. We measure the effect of a tapering shock that raises $\beta_{REVERSAL}$ from $\beta^L_{REVERSAL}$ to $\beta^H_{REVERSAL}$ as $\frac{q^i(\lambda, \chi, \zeta_F|\beta^H_{REVERSAL})}{q^i(\lambda, \chi, \zeta_F|\beta^L_{REVERSAL})} - 1$.

E.2. Operation Twist

We map the safe capital k_t^b to government bonds by assuming that the private sector cannot create it but that the government issues it at constant rate $\phi(\iota^b)$.[28] To model a change in bond maturity within a given economy, we allow the central bank to issue zero-duration floating-rate bonds k^{fb}. Floating bonds pay

[28] Specifically, private investment ι_t^b equals zero in (25). Existing long bonds depreciate on the balance sheet but the government controls their supply by setting issuance to $dk_t^b/k_t^b = [\phi(\iota^b) - \delta]dt$.

the yield of money μ_m and trade at par in equilibrium. Long bonds pay the fixed coupon y^b as in the baseline model.

In an Operation Twist intervention, the central bank buys long-term bonds by selling floating-rate bonds of equal dollar amount at postannouncement prices (again using lump-sum taxes to absorb any future cash flow mismatch). Operation Twist thus changes the composition of government debt in the hands of intermediaries, while keeping its value constant. This resembles how the Fed implemented its Maturity Extension Program.

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.