Online Appendix for
“The Macroeconomics of Shadow Banking”

Alan Moreira∗  Alexi Savov†

April 29, 2016

Abstract

This document contains additional results for the paper “The Macroeconomics of Shadow Banking.” These results include robustness to alternative parameter values, evidence on the composition of the liquidity supply over the 2003–2015 cycle, and extensions for analyzing liquidity requirements and equity issuance costs.

∗Yale University School of Management, alan.moreira@yale.edu.
†New York University Stern School of Business and NBER, asavov@stern.nyu.edu.
1 Robustness to alternative parameter values

To provide further support for our results, we explore robustness to alternative parameter values. The benchmark parameters are listed in Table 1 in the paper. We focus on the parameters that are unique to our setting, those governing crash exposure, the information sensitivity constraint, and liquidity events.

Whenever we change a given parameter, we adjust the subjective discounting parameter $\rho$ to keep the overall discount rate in the case where wealth is fully liquid constant at 1.6%. Doing so allows us to compare across economies without changing the rate at which investors discount the future in the absence of frictions.

The results of the model under alternative parameters are presented in Figure A.1. We keep the capital mix $\chi$ at 0.95, which is close to its unconditional steady state. The first column of panels shows shadow money issuance and the second and third column show the prices of the two assets.

In the first row of panels we vary the crash exposure of the risky asset $\kappa^{a}_k$, which controls the exogenous component of the supply of collateral in the economy. A higher $\kappa^{a}_k$ (less collateral) leads to more shadow money issuance at low uncertainty. This occurs because shadow money is ideally suited to producing liquidity with limited collateral and because investors are only willing to hold shadow money at low uncertainty. Less collateral means less overall liquidity, however, which brings down asset prices, in particular the price of the risky asset $a$. The price of the safe asset $b$ actually rises as the collateral it carries becomes more scarce. Higher $\kappa^{a}_k$ also increases the uncertainty exposure of asset prices. Overall, $\kappa^{a}_k$ illustrates the important role of collateral scarcity in the model.

The second row of panels in Figure A.1 varies the tightness of the information sensitivity constraint on fragile-liquid securities, which is set by the crash exposure bound $\bar{\kappa}$. This parameter affects the collateral advantage of shadow money. When $\bar{\kappa}$ is high, shadow money can produce a lot of liquidity with only a little collateral. This means that at low uncertainty a small amount of shadow money can cover the crash exposure of the balance sheet, leaving more collateral for money. It also means that as uncertainty rises shadow banking takes longer to shut down. Both effects expand liquidity provision, which leads to a higher price for the risky asset $a$ and a lower price for the safe asset $b$.

Next, we change $\psi$, the marginal utility of investors in a liquidity event. Since $\psi$ enters all expressions in tandem with the intensity of liquidity events $h$ as the term $h (\psi - 1)$, this experiment is also equivalent to changing $h$. When $\psi$ is high, liquidity is very valuable at the margin, and so discount rates become more sensitive to changes in liquidity. This is why even though shadow money issuance is relatively unaffected, asset prices show large
changes. In particular, higher $\psi$ makes the price of asset $a$ decline much more steeply with uncertainty. By contrast, the price of asset $b$ becomes more steeply increasing in uncertainty, that is flight to quality intensifies.

Lastly, we change $\eta$ which controls both the size of liquidity events and the rate at which demand for liquidity becomes satiated. When $\eta$ is high, liquidity demand becomes satiated quickly, and so investors prefer low but smooth liquidity across states. This reduces the comparative advantage of shadow money, which expands liquidity only in normal times. Shadow money issuance therefore falls. This makes the liquidity supply and discount rates less sensitive to uncertainty. Since we have fixed discount rates at full liquidity by adjusting $\rho$, discount rates are lower overall.

Overall, Figure A.1 shows that the model’s results hold across a range of parameters.

2 The liquidity supply during the 2003–2015 cycle

Our framework makes predictions about the evolution of the liquidity supply. During a low-uncertainty period, its composition shifts from money to shadow money. Then when uncertainty rises as in a crisis, it shifts back quickly from shadow money to money. Backstopping the additional money is a higher amount of illiquid equity.

In this section we provide some simple estimates of the dynamics of shadow money and money during the 2003–2015 cycle. We leave a more comprehensive empirical analysis for future work.

Our approach is the following. Figure 5.2.1 in the Financial Stability Oversight Council’s 2015 annual report (FSOC, 2015) shows aggregate time series data on retail deposits and wholesale funding issued by the banking system. We reproduce this figure in the top left panel of Figure A.2. Retail deposits and wholesale funding give us aggregate proxies for money (retail deposits) and shadow money (wholesale funding). Retail deposits are mostly insured and considered stable as a source of funding, in contrast to wholesale funding instruments. Of course, the mapping is not perfect; it is broadly consistent with Bai, Krishnamurthy and Weymuller (2013).

While large and important ($22$ trillion in 2015), these aggregates do not account for some components of the liquidity supply such as money market funds, foreign banks, credit unions, and captive financial companies. Foreign banks and captive financial companies mostly issue shadow money-like instruments, credit unions mostly issue money, and money market funds issue a mix of both. Adding these additional institutions is non-trivial because many of their assets and liabilities are held and issued within the financial
system.\(^1\) We do not attempt to net them out here but instead focus on the institutions covered in the figure from the FSOC report.

Retail deposits in the FSOC report are for all FDIC-insured institutions (excluding, for example, credit unions) while wholesale funding also includes a “system-wide measure of repurchase agreements”. We take this to include banks, bank holding companies, and security brokers and dealers. Thus, we construct our measure of total balance sheet size using data from the Financial Accounts of the United States by adding the total financial assets of (i) U.S.-chartered depository institutions (series FL764090005.Q), (ii) security brokers and dealers (series FL664090005.Q), and (iii) holding companies (series FL734090005.Q). The three series are shown in the top right Panel of Figure A.2. We then construct equity as follows:

\[
\text{equity} = \text{total assets} - \text{wholesale funding} - \text{retail deposits}. \tag{1}
\]

We then report money, shadow money and equity in dollars and in shares in the bottom panels of Figure A.2.

The pattern in the data is strong and consistent with our framework. In the boom years before the 2008 financial crisis, the share of shadow money grew, peaking at 57% in 2007. The share of money trended down, reaching a low of 32% around the same time. During this time, equity was low at around 10%.

These trends reversed sharply in 2008. Shadow banking went from 57% to 39% within a year and eventually reached 35% in 2015. Equity made up most of the adjustment in the short run. Its share rose from 10% to 25% within a year and slowly trended down to 21% in the end of 2015. Money rose steadily starting in 2008, going from 32% to 44%. Overall liquidity contracted from 90% at the peak of the cycle to 72% within a year, a 20% contraction in the liquidity supply. Liquidity slowly recovered reaching 78% in the end of our sample, but remained still 14% below the peak it reached in 2007.

### 3 Liquidity requirements

The Basel III framework adopts two kinds of liquidity requirements, the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR) (BIS, 2013, 2014). Liquidity requirements limit the liquidity mismatch between an intermediary’s assets and liabilities. The standard rationale for liquidity requirements is that they mitigate fire sales

\(^1\)For example, money market funds hold wholesale funding instruments issued by both domestic and foreign banks.
Our framework features a type of fire sale in the form of collateral runs. In this section we explore the effectiveness of liquidity requirements in arresting collateral runs, as well as their overall impact on asset prices and the economy.

Since assets in our model are themselves illiquid, a liquidity requirement amounts to limiting the issuance of liquid securities, \( m_t + s_t \leq \bar{I} \), where \( \bar{I} < 1 \). The solution largely follows the baseline model. The details are in Section 3.1 below.

Figure A.3 compares an economy with a 15\% liquidity requirement \((1 - \bar{I} = 0.15)\) with our baseline economy. The requirement binds at low uncertainty when intermediaries normally lever up with shadow money. The result is a large drop in shadow money and an increase in money. The liquidity requirement thus increases the stability of intermediaries’ funding by inducing a shift from shadow money to money.

The rise in money cannot make up for the drop in shadow money because money requires more collateral. Overall liquidity thus shrinks, pushing up discount rates at low uncertainty (right panel). Since this is where the economy spends most of its time, asset prices are significantly lower.

Asset prices are also more stable under a liquidity requirement due to the greater stability of the liquidity supply. Stable asset prices in turn mean higher collateral values (lower right panel). Liquidity requirements are thus effective at arresting collateral runs, weighing in on the side of stability in the tradeoff between stability and growth.

### 3.1 Liquidity requirements proofs and derivations

The equilibrium security issuance with a liquidity requirement is characterized by

**Proposition 1** (Equilibrium issuance with a liquidity requirement). Let \( \bar{I} \) be the liquidity requirement. Equilibrium issuance follows:

i. If \( 1 - \kappa_{A,t} \geq \bar{I} \), then \( m_t = \bar{I} \) and \( s_t = 0 \).

ii. If \( 1 - \kappa_{A,t} < \bar{I} \),
   a. \( M_t > \frac{\kappa_{A,t} - (1 - \bar{I})}{\bar{K}} \), and \( \kappa_{A,t} < \left( 1 - \bar{I} \right) + \bar{K} \), then \( m_t = \bar{I} - \frac{\kappa_{A,t} - (1 - \bar{I})}{\bar{K}} \), and \( s_t = \frac{\kappa_{A,t} - (1 - \bar{I})}{\bar{K}} \).
   b. \( M_t > \frac{1 - \kappa_{A,t}}{1 - \bar{K}} \) and \( \kappa_{A,t} \geq \left( 1 - \bar{I} \right) + \bar{K} \), then \( m_t = 0 \) and \( s_t = \frac{1 - \kappa_{A,t}}{1 - \bar{K}} \).

iii. If \( 1 - \kappa_{A,t} < \bar{I} \) and \( M_t < 0 \), then \( m_t = 1 - \kappa_{A,t} \) and \( s_t = 0 \).

iv. If \( 1 - \kappa_{A,t} < \bar{I} \) and \( 0 \leq M_t \leq \min \left\{ \frac{\kappa_{A,t} - (1 - \bar{I})}{\bar{K}}, \frac{1 - \kappa_{A,t}}{1 - \bar{K}} \right\} \), then \( m_t = 1 - \kappa_{A,t} - (1 - \bar{K}) M_t \) and \( s_t = M_t \),

where \( M_t = \frac{1}{\eta} \log \left( \frac{\bar{K}}{1 - \bar{K}} \frac{1 - p_H(\lambda_t)}{p_H(\lambda_t)} \right) \) as before.
Proof of Proposition 1. The proof follows that of Proposition 3 in the paper, replacing the equity non-negativity constraint with the liquidity requirement \( e_t \leq 1 - \bar{I} \). Only case (i) is new. It arises when asset risk is so low that \( 1 - \bar{I} \) worth of equity covers all asset risk and the rest of the balance sheet can be funded with money. \( \square \)

4 Equity issuance costs

In the paper we abstract from frictions in the primary market for intermediary equity. Such frictions have been studied in the literature as an important contributor to the severity of the financial crisis (He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014). In this section we examine the effects of such a friction in our framework.

When equity issuance is subject to a friction, past equity becomes a state variable. To keep the total number of state variables unchanged, we remove capital heterogeneity by making asset \( b \) very costly to produce (high \( \phi^b \)) and focusing on the case where asset \( a \) dominates the economy (\( \chi = 1 \)). We can still price asset \( b \) at the margin.

To model equity issuance frictions, we assume that intermediary \( i \)'s equity share of liabilities \( e^i \) is fixed in the short run, and that it can be changed at a rate \( \alpha^i \) (i.e. \( de^i_t = \alpha^i_t dt \)) subject to a quadratic cost \( \zeta (\alpha^i)^2 \) per dollar of assets. We also impose a bound on the rate at which equity can be repurchased: \( \alpha^i_t \geq -\tau e^i_t \). The details are in Section 4.1 below.

Results for the extended model with equity issuance frictions are presented in Figure A.4. The red lines are from our baseline economy (i.e. \( \zeta = 0 \)). The black dashed and blue dash-dotted lines are from the economy with equity issuance costs, holding equity (now a state variable) fixed at a low level of 0 (near the steady state at low uncertainty) and a high level of 0.2 (near the steady state at high uncertainty). As the economy spends most of its time at low uncertainty, we focus on the black dashed lines.

With costly equity issuance, money and shadow money are less sensitive to uncertainty (top left and center panels). Intermediaries can retire shadow money only slowly since it takes time to issue new equity. They are also reluctant to de-lever since uncertainty reverts relatively quickly. Both effects leave shadow money high and money low when uncertainty spikes.

Since demand for shadow money is low at high uncertainty, discount rates rise higher than in the baseline economy (top right panel). This is especially true at low levels of equity when little money can be produced. Conversely, at low levels of uncertainty discount rates are especially high when equity is high as it prevents maximizing liquidity provision through shadow banking.
The suboptimal capital mix and higher funding costs result in a lower price for asset \( a \) (bottom left panel). Asset \( a \)'s price is also more sensitive to uncertainty, especially when equity is low. This reflects the steeper discount rates. Equity issuance frictions thus make asset prices and the economy more exposed to uncertainty and hence more unstable.

This instability is also reflected in the price of asset \( b \), which is higher due to a higher collateral premium, as well as more steeply increasing in uncertainty. In other words, flight to quality is stronger. The collateral run is also stronger, as reflected in aggregate collateral whose \( U \) shape is more pronounced (bottom right panel).

In sum, Figure A.4 shows that equity issuance frictions increase exposure to uncertainty by preventing intermediaries from quickly adjusting their funding mix in response to shocks. Thus, they act as an amplifier of the mechanisms in our model.

### 4.1 Equity issuance costs proofs and derivations

Here we derive the solution to the extended model with costly equity issuance. We work with just one type of capital, which eliminates the capital mix state variable \( \chi \) and keeps the total number of state variables equal to two (equity and uncertainty).

We model costly equity issuance as follows. Intermediary \( i \) has equity share of the balance sheet \( e_i \), which is fixed in the short run. Intermediaries are thus free to issue equity each period as long as its share of the balance sheet remains stable. The idea is that changing the equity share quickly might be perceived to convey information, resulting in price impact and effectively making equity issuance costly.

Over time, intermediary \( i \) can adjust its equity at rate \( \alpha^i \):

\[
de^i = \alpha^i dt. \tag{2}
\]

In doing so, intermediary \( i \) faces a quadratic cost

\[
\zeta A^i \left( \alpha^i \right)^2, \tag{3}
\]

where \( A^i \) is the size of the balance sheet. We further impose that intermediaries cannot repurchase equity at a rate faster than \( \tau \) percent per year (this helps with convergence at the zero-equity low-uncertainty boundary):

\[
\alpha^i \geq -\tau e^i \tag{4}
\]

Given this specification, the intermediary problem is homogeneous in the size of the
balance sheet.\footnote{There is no cost to quickly scaling up the balance sheet as long as the liability mix remains unchanged. This allows the market for assets to remain competitive.} The intermediary thus maximizes net profits minus issuance costs per dollar of assets. The resulting Lagrangian is

\[
\mu_e v^i dt = \max_{m^i, s^i, \alpha^i, \tau^i} \left( y - i^i \right) dt + E \left[ \frac{dA_i}{A_i} \right] - \left( m^i \mu_m + s^i \mu_s + e^i \mu_e \right) dt
\]

\[
- \xi \alpha^i dt + E \left[ d\alpha^i \right] + \theta^i \left( \alpha^i + \tau e^i \right) dt.
\]

Since the equity share \( e^i \) is fixed in the short run, it is optimal to issue enough shadow money to absorb any remaining risk and issue money with the remaining collateral. So the intermediary capital structure can be written as

\[
s^i = \max \left\{ 0, \frac{\kappa_A - e^i}{\bar{k}} \right\} \quad \text{and} \quad m^i = 1 - e^i - s^i.
\]

For this we need \( \kappa_A \leq \bar{k} \), which holds at all points of the state space with our parameters.\footnote{If \( \kappa_A - e^i > \bar{k} \) for any \( e^i \), there is no feasible capital structure with only \( e^i \) worth of equity. In this case, the intermediary is forced to sell the assets to other intermediaries or households and ceases to intermediate. This never happens for our chosen parameters.} Note that the intermediary cannot affect \( \kappa_A \) as there is only one type of capital traded.

Given this capital structure, the net profits per dollar of assets can be written as:

\[
\left[ H(e, \lambda) + e^i K(e, \lambda) \right] dt = \max_{m^i, s^i, \alpha^i, \tau^i} \left( y - i^i \right) dt + E \left[ \frac{d(qk)}{qk} \right] - \left( e^i \mu_e + m^i \mu_m + s^i \mu_s \right) dt,
\]

where

\[
K(e, \lambda) = \begin{cases} - (\mu_e - \mu_m) + \frac{1}{\bar{k}} (\mu_s - \mu_m) & \kappa_A \geq e^i \\ - (\mu_e - \mu_m) & \kappa_A < e^i \end{cases}
\]

\[
H(e, \lambda) = \begin{cases} \frac{y - i^i}{q} + \frac{1}{dt} E \left[ \frac{d(qk)}{qk} \right] - \mu_m \left( 1 - \frac{\kappa_A}{\bar{k}} \right) - \mu_s \frac{\kappa_A}{\bar{k}} & \kappa_A \geq e^i \\ \frac{y - i^i}{q} + \frac{1}{dt} E \left[ \frac{d(qk)}{qk} \right] - \mu_m & \kappa_A < e^i \end{cases}
\]

We now conjecture that the intermediary’s per-dollar value function is of the form
$v^i = e^{i}\nu^1(e, \lambda) + \nu^2(e, \lambda)$ and verify that this satisfies (5). We have

$$E_t \left[ dv^i \right] = v_1 \dot{a}^i dt + \left( e^{i}\nu^1_\lambda + \nu^2_\lambda \right) \alpha^i dt + \left( e^{i}\nu^1_\lambda + \nu^2_\lambda \right) \mu^i dt$$

$$+ \frac{1}{2} \left( e^{i}\nu^1_{\lambda^2} dt + \nu^2_{\lambda^2} \right) \sigma^2_\lambda dt + \lambda \left( e^{i}\nu^1 (e, \lambda^+) + \nu^2 (e, \lambda^+) - e^{i}\nu^1 - \nu^2 \right) dt,$$

where $\mu^i_\lambda$, $\sigma^i_\lambda$, and $\lambda^+$ are the drift, volatility, and jump-to value of $\lambda$ from (9).

Substituting into (5) and collecting terms in $e^i$, we get two PDEs for $\nu^1$ and $\nu^2$:

$$\mu e^{\nu^1} = \mathcal{K}(e, \lambda) + \tau e^{\nu^1} \alpha + \nu^1_\lambda \mu^i + \frac{1}{2} \nu^1_{\lambda^2} \sigma^2_\lambda + \lambda \left( v^1_+ - v \right)$$

(11)

$$\mu e^{\nu^2} = \mathcal{H}(e, \lambda) + \theta \alpha^i - \zeta (\alpha^i)^2 + \nu^1_\chi \alpha + \nu^2_\mu^i + \frac{1}{2} \nu^2_{\lambda^2} \sigma^2_\lambda + \lambda \left( v^2_+ - v^2 \right).$$

(12)

The optimality condition for $\alpha^i$ implies $\alpha^i = \left( v^1 + \theta^i \right) / (2\zeta)$, and because intermediaries are symmetric $\alpha = \alpha^i$. The multiplier is $\theta^i = \max \{ 0, -2\zeta \tau e - v^1 \}$. Substituting into the PDE for $\nu^1$, we get

$$\mu e^{\nu^1} = \mathcal{K}(e, \lambda) + \tau e^{\nu^1} \alpha + \frac{1}{2} \nu^1_\chi \left( v^1 + \theta^i \right) + \nu^1_\lambda \mu^i + \frac{1}{2} \nu^1_{\lambda^2} \sigma^2_\lambda + \lambda \left( v^1_+ - v \right).$$

(13)

Note that $\nu^1$ is decoupled from $\nu^2$ and is sufficient to characterize equity issuance. The solution of (13) characterizes the dynamics of aggregate equity issuance, with $\nu^1 > 0$ implying intermediaries are raising equity. Note that $\mathcal{K}(e, \lambda)$ is a function of funding costs, which we know from Proposition 2 in the paper is a function of aggregate collateral values and uncertainty. So we solve $\nu^1$ jointly with asset prices, which as before must satisfy equation (24) in the paper. The aggregate discount rate and can computed by applying the capital structure choice under sticky equity (equation (6)) to Proposition 2 in the paper. Similarly, the expected price change is adjusted to incorporate the dynamics of equity:

$$\mu q = \frac{q_\lambda}{q} \mu^i + \frac{1}{2} \frac{q_{\lambda^2}}{q} \sigma^2_\lambda \left( \theta^i + v^1 \right) - \kappa q^i.$$
today’s assets do not affect tomorrow’s, this problem can be written simply as

\[
\max_{A^i} A^i \left( \frac{y - i^i}{q} \right) dt + A^i \left[ E \left[ \frac{dA^i}{A^i} \right] - \left( m^i \mu_m + s^i \mu_s + e^i \mu_e \right) dt - \zeta (a^i)^2 dt \right],
\]

(15)

where \( m^i, s^i, a^i \) and \( i^i \) are their optimal values. The optimality condition for \( A^i \) gives a PDE for asset prices:

\[
q^i = \frac{y^i - i^i}{\mu_W - \left[ \mu_q^i - \frac{1}{\Sigma} (v^1 + \theta^a)^2 + \kappa^i \kappa_q \lambda + \phi (i^i) - \delta \right]}.
\]

(16)

To solve the model we conjecture functions \( q^i (\lambda, e) \) and \( v^1 (\lambda, e) \), calculate their derivatives and plug them into their respective PDEs to verify the conjectures. We use \( \zeta = 3 \) and \( \tau = 1 \) to illustrate the extended model.
References

Figure A.1: Alternative parameters

This figure shows shadow money $s$ and asset prices $q^a$ and $q^b$ for alternative parameter specifications. Each row varies one parameter, setting all others to their values in Table 1 in the paper. The capital mix is fixed at $\chi = 0.95$. We also adjust $\rho$ to keep the discount rate at full liquidity equal to 1.6% (i.e., $\rho + h/\eta (\psi - 1) (e^{-\eta} - 1) = 0.016$).
Figure A.2: The liquidity supply during the 2003–2015 cycle

Data on retail deposits (l.i) and wholesale funding (l.ii) of the U.S banking sector is from FSOC (2015) (see Chart 5.2.1). Data on total asset of the U.S. banking sector is from the U.S. Financial Accounts, where we use the following series (a.i) U.S.-chartered depository institutions (series FL764090005.Q), (a.ii) Security Brokers and Dealers (series FL664090005.Q), and (a.iii) Bank Holding Companies (series FL734090005.Q). Equity is the residual of total assets (a.i+a.ii+a.iii) net of liquid liabilities (l.i+l.ii).

Total assets by institution type ($ trillion)  Liquid liabilities ($ trillion)

Liquidity supply ($ trillion)  Liquidity supply (%)

[Charts and graphs showing data trends from 2004 to 2014]
Figure A.3: Liquidity requirements

This figure shows money $m$, shadow money $s$, the aggregate discount rate $\mu_W$, asset prices $q^a$ and $q^b$, and aggregate collateral $1 - \kappa_A$ in equilibrium in economies with a liquidity requirement (solid red lines) and without a liquidity requirement (dashed black lines). The economy with a liquidity requirement is explained in Appendix 3. The liquidity requirement is set to $1 - \tilde{l} = 0.15$. Each quantity is plotted against uncertainty $\lambda$ while holding the capital mix $\chi$ fixed at 0.75.
Figure A.4: Equity issuance costs

This figure shows money $m$, equity $e$, the aggregate discount rate $\mu_W$, asset prices $q^a$ and $q^b$, and aggregate collateral $1 - \kappa_A$ in the baseline economy (solid red lines) and an economy with equity issuance costs (see Appendix 4). For the economy with equity issuance costs, we plot each quantity while holding equity fixed at 0 (black dashed lines) and 0.2 (blue dash-dot lines). These values are near the steady states of equity at low and high levels of uncertainty, respectively. We use $\gamma = 3$ (issuance cost) and $\tau = 1$ (upper bound on repurchases). (See Appendix 4 for details.)