

Foundations of Finance

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1. a. The (intrinsic) price of a stock can be determined using the dividend growth model:

$$Price = E_1 \times (1-b) / [k-b \times ROE] = [5 \times (1-0.5) / [0.1-0.5 \times 16]] = \$125.$$
- b. Recall from the formula above, that a higher required rate of return implies a lower intrinsic price of the stock. On the other hand, as the growth rate (which is a function of ROE) decreases, the price of the stock decreases. Thus, a drop in the observed equilibrium stock price could indicate that either the required rate of return is higher than originally expected, and/or the ROE on funds plowed back is less than originally estimated.
- c. To solve for the expected return on equity, use the dividend growth model as follows:

$$50 = [5 \times (1-0.5) / [0.1-0.5 \times ROE]]$$
 Solving for ROE, you get: $50 = 2.5 / [0.1-0.5 \times ROE]$.
 So, $5 - 25 \times ROE = 2.5$. Therefore, $ROE = 0.1$ or 10%

Note: when the ROE equals the required rate of return, plowing back earnings neither increases or decreases the price of the stock. In addition, the price-earnings ratio (P_0/E_1) is now equal to the inverse of the expected return on equity.

2. a. $P_0 = D_1 / (k-g) = 5 / (.14-.06) = 62.5$
 b. The retention ratio $b=3/8$. $g = b \times ROE \Rightarrow .06 = 3/8 \times ROE \Rightarrow ROE = .16$
 c. If the firm were valued as a perpetuity, it would be worth $E_1/k = 8/.14 = 57.14$.
 The excess, $62.50-57.14 = 5.36$, is due to future growth.
3. See BKM solutions manual.
4. See BKM solutions manual.
5. See BKM solutions manual.
6. a. Use your financial calculator to get $P_A=859.53$; $P_B=1,116.54$; $P_C=816.30$; $P_D=258.42$.
 b. As we discussed in class, the total holding one-year return will be 7% on all bonds, although the components (current yield and capital gains/losses) will vary.
 c. One year from now, use your financial calculator to get
 $P_A=869.70$; $P_B=1,114.69$; $P_C=873.44$; $P_D=276.51$. As stated in the question, the after-tax rates of return are computed as simple returns, except we subtract the tax on capital gains and coupon income (zeros have only capital gains):
 (A) $[(869.70-859.53)(1-.30) + 50(1-.35)]/859.53 = 4.61\%$
 (B) $[(1,114.69-1,116.54)(.7) + 80(.65)]/1,116.54 = 4.54\%$
 (C) $[(873.44-816.30)(.7)]/816.30 = 4.9\%$
 (D) $[(276.51-258.42)(.7)]/258.42 = 4.9\%$
- d. At $y = 6\%$, the end-of-year price of bond A is 931.98, so the after-tax return is
 (A) $[(931.98-859.53)(1-.30) + 50(1-.35)]/859.53 = 9.68\%$

7. a. $(1 - 1/1.07^{20})/0.07 = 10.6M$
 b. Using the formula for the duration of annuity: $D = 1.07/0.07 - 20/(1.07^{20} - 1) = 8.32$ years
 c. Let x be the fraction in bond C and $(1-x)$ in bond D. Then $3x + (1-x)20 = 8.32$
 $\Rightarrow x = .69$. So 69%(10.6M) = 7.3M in C and 3.3M in D
 d. Not unique, you could form an immunized portfolio out of any set of bonds as long as one of the bonds has a duration less than 8.32 years and one has a duration greater than 8.32 years. Use of zeroes minimizes the reinvestment and rebalancing requirements.
 e. Immunization will ensure that the nominal CF requirements are met. In the present situation, this is an appropriate requirement.

8. Real Time Exercise

Using collected data: (Your answer will differ from the below depending on when you collected your data. But you should follow Lines 1-10 to determine whether your answer is correct.)

	<i>U.K.</i>	<i>Japan</i>
1 1-year spot rate (%)	3.828	0.020
2 2-year yield of a zero (%)	4.009	0.050
3 Expected 1-yr Rate (in 1 year)	3.828	0.020
4 Implied forward rate	4.190	0.080
Today:		
5 Buy 2-yr zero (100 par value)	92.440	99.900
6 Enter into 1 year repo	<u>-92.440</u>	<u>-99.900</u>
7 Total cash outlay today	0.000	0.000
Expected for End of Year 1:		
8 Sell 2-yr zero after 1 year	96.313	99.980
9 Complete the Repo	<u>-95.978</u>	<u>-99.920</u>
10 Net proceeds	0.335	0.060

Explanation:

- Line 1: collected from Bloomberg
 Line 2: collected from Bloomberg
 Line 3: the assumption in the exercise (hence same as Line 1)
 Line 4: using the definition of the forward rate between year 1 and year 2, and Lines 1,2
 Line 5: 100 discounted over two years at the rate in Line 2
 Line 6: Sell a two-year zero on a one year repo (the rate for repurchase in a year is in Line 1)
 Line 7: Summarizes cash outflow (=0)
 Line 8: 100 discounted at the rate in Line 3
 Line 9: The cost of funds in Line 6 is [Line 6 \times (1 + (Line 1 / 100))]
 Line 10: Projected profit (not guaranteed)

Remarks:

Note that in both countries you can expect to capture the liquidity premium, because the forward rate, f_2 , is higher than the expected spot rate, $E[r_2]$, but U.K. produces the higher premium. (The short-end of the yield curve in Japan is relatively less upward sloping.) You expect higher profits in the UK, because the bond market there, according to the data, is dominated to a larger extent by the one-year-horizon investors who may need money in one year. These investors are requiring a higher premium to hold the longer maturity 2-year zero, because of the interest rate uncertainty in one year's time. In Japan, on the other hand, the data indicates that investors are more willing to hold longer maturity bonds, not planning to sell them within a year.

However, this is not a riskless transaction (not an arbitrage). The net proceeds at the end of one year will depend on the price at which you will be able to sell the remaining 1 year of the 2-year zero.

If your expectations are correct, the net proceeds will be as shown. But if 1-year interest rates increase so that the actual spot rate is higher next year than you expect (and therefore in Line 8 you will actually have lower values), the transaction could lose money. This is exactly the "liquidity problem" that the one-year-horizon investors fear (they know they may need money, i.e. liquidity, in one year, but if they buy the two-year zero and next year rates increase, the value of their zero will decrease, and these investors will face a liquidity squeeze).

If you follow the above strategy you may indeed sometime lose money, but on average if you form your expectations correctly, you will be making money. Your average reward is the compensation for the risk you bear. Individual investors may not be able to afford occasional losses. However, financial institutions performing such transactions repeatedly, will make money on average.

Answers to Suggested Problems

- S1. a. Price = $90 \times APVF(7.5\%, 10) + 1,000 \times PVIF(7.5\%, 10) = 1,102.96$.
 b. The time line for the coupon payments (in bold) is:

<i>0</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>10</i>
	90	90	90	90	90					
					*507					**711
						90	90	90	90	90
										***518

where:

*507 = $90 \times AFVF(6\%, 5)$

**711 = $507 \times FVIF(7\%, 5)$

***518 = $90 \times AFVF(7\%, 5)$

So at the end of year 10, the total investment is $711 + 518 + 1,000 = 2,229$.
 The realized compound yield, i.e. the realized EAR, is r ,
 where $2,229 = 1,102.96 \times FVIF(r, 10)$: $r = 7.29\%$

- S2. Recall: the formula relating change in price to duration is as follows:

$$\frac{\Delta P}{P} = -D \times \frac{\Delta y}{(1+y)}$$

- a. Using the formula, we can calculate the new price of a bond, for a given change in yield. For the 20 year 8 percent coupon bond with a 10.3 duration, the change in price divided by the price (the proportional change) will equal:

$$- 10.3 \times \frac{.0001}{(1.08)} = -.000954$$

Thus, the price of the bond drops by .095%.

- b. For the 30 year 17% coupon bond, with a duration of 10.1, the proportional change is equal to:

$$- 10.1 \times \frac{.0001}{(1.08)} = -.000935$$

Thus the price of the second bond drops by .093% < .095% due to the shorter duration of the second bond.

- c. Clearly, the price for the longer duration bond (but shorter maturity) exhibits a larger percentage change when the yield to maturity changes by one basis point. In our case, the 30 year bond has a shorter duration than the 20 year bond (because it has a much higher coupon). Thus, it is not correct in general to say that longer maturity bonds have greater “interest rate risk”.