Question 1. (Equity Premium Puzzle). Consider the Mehra-Prescott model as described in the notes. The (gross) growth rate of dividends \( x' \equiv y'/y \) follows a symmetric 2-state Markov chain with states
\[
\begin{align*}
x_1 &= 1 + 0.018 - 0.036 \\
x_2 &= 1 + 0.018 + 0.036
\end{align*}
\]
and symmetric transition probabilities
\[
\begin{pmatrix}
\pi_{11} & \pi_{12} \\
\pi_{21} & \pi_{22}
\end{pmatrix} = \begin{pmatrix}
0.43 & 0.57 \\
0.57 & 0.43
\end{pmatrix}
\]
where
\[
\pi_{ij} = \Pr(x_{t+1} = x_j|x_t = x_i), \quad i, j = 1, 2
\]
The Bellman equation for the household is
\[
V(w, x, y) = \max_{s', B'} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + 0.96 \sum_{x'} V(w', x', y') \pi(x', x) \right\}
\]
where \( w \) denotes household wealth and the maximization on the right hand side is subject to a budget constraint
\[
c + p(x, y)s' + q(x, y)B' \leq w
\]
and the laws of motion
\[
\begin{align*}
w' &= [p(x', y') + y']s' + B' \\
y' &= x'y
\end{align*}
\]
• Let \( \sigma = 1.5 \). Solve for equilibrium bond prices \( q(x, y) \) and equity prices \( p(x, y) \). Starting from an initial distribution of \( \pi_0 = \pi = (0.5, 0.5) \), iterate on the Markov chain for dividend growth for \( t = 1, ..., 100 \). Calculate the realized bond prices, bond returns and equity prices and equity returns. Plot each of these variables and describe the equilibrium stochastic processes you observe.
• Compute the equity premium as defined by Mehra and Prescott.
• Repeat the previous exercises for \( \sigma = 5, 10, 20 \). Explain how your answers differ as you increase relative risk aversion. Give economic intuition for your findings.
Question 2. (Incomplete Markets). Consider an Aiyagari model where the typical household has Bellman equation

\[
V(k,n) = \max_{k' \geq 0} \left\{ \frac{c^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}\{V(k',n')|n]\right\}, \quad 0 < \beta < 1
\]

where the maximization on the right hand side is subject to the budget constraint

\[
c + k' \leq (1 + r - \delta)k + wn, \quad 0 < \delta < 1
\]

Log labor supply follows an exogenous AR(1) process

\[
\log(n') = \rho \log(n) + \sigma(1 - \rho^2)^{1/2}\varepsilon', \quad 0 < \rho < 1 \text{ and } \sigma > 0
\]

where the innovations \(\varepsilon'\) are standard normal random variables.

The typical firm has production function

\[
F(K,N) = K^\alpha N^{1-\alpha}, \quad 0 < \alpha < 1
\]

The parameters values are

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>time discount factor</td>
<td>0.96</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>capital’s share in national output</td>
<td>0.36</td>
</tr>
<tr>
<td>(\delta)</td>
<td>depreciation rate of physical capital</td>
<td>0.08</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>coefficient of relative risk aversion</td>
<td>3.00</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>standard deviation of log labor supply</td>
<td>0.40</td>
</tr>
<tr>
<td>(\rho)</td>
<td>autocorrelation of log labor supply</td>
<td>0.60</td>
</tr>
</tbody>
</table>

- Define a stationary recursive competitive equilibrium for this economy.
- Calibrate a 2-state Markov chain representation for \(\log(n)\). That is, choose parameters of the 2-state Markov chain so that the mean, variance, and autocorrelation of the Markov chain equal their counterparts for the AR(1) process.
- Solve for a stationary recursive competitive equilibrium when log labor supply follows the calibrated 2-state Markov chain.
- Compute an approximation to the policy function \(k' = g(k,n)\). Plot \(g(k,n_H)\) and \(g(k,n_L)\) where \(n_L < n_H\) are the two calibrated states of the Markov chain.
- Compute the equilibrium invariant distribution and plot each of \(\mu(k,n_H)\) and \(\mu(k,n_L)\). Describe the results you find.
- Compute the equilibrium interest rate (i.e., \(r - \delta\)) and savings rate (i.e., \(\delta K/F(K,N)\)) for this incomplete markets economy. How do they compare to the corresponding values for the complete markets benchmark? [Hint: what is the interest rate in the steady state of the neoclassical growth model].