Problem Set #2

Due 6 September, 2004

This problem set is designed to introduce you to solving simple dynamic models on a computer.

Consider the social planning problem of maximizing utility

\[
E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [U(c_t) + V(l_t)] \right\}, \quad 0 < \beta < 1
\]

subject to a resource constraint

\[c_t + i_t = z_t F(k_t, n_t)\]

a constraint on the time endowment

\[n_t + \ell_t = 1\]

and the following law of motion for capital accumulation

\[k_{t+1} = (1 - \delta)k_t + i_t - \phi \left( \frac{i_t}{k_t} \right) k_t, \quad 0 < \delta < 1\]

where \(\phi\) is a strictly increasing strictly convex cost of adjustment function with the properties \(\phi(\delta) = \phi'(\delta) = 0\) and where \(\delta\) is the depreciation rate of physical capital. Finally, let log technology follow an AR(1),

\[\log(z_{t+1}) = \rho \log(z_t) + \varepsilon_{t+1}, \quad 0 < \rho < 1\]

where \(\varepsilon_{t+1}\) is Gaussian white noise with initial realization \(z_0\) given.

Other than the costs of capital adjustment, this is just the same model as in the notes.

**Question 1.** Derive first order conditions for the planner’s problem and show how these characterize optimal choices of consumption, leisure, employment, and investment.

**Question 2.** Let the unconditional mean of the stochastic technology shock be \(\bar{z} = 1\).

Show how to characterize the non-stochastic steady state of the model. Now let the functional forms be

\[
\begin{align*}
U(c) &= \log(c) \\
V(\ell) &= \log(\ell) \\
F(k, n) &= k^\theta n^{1-\theta} \\
\phi \left( \frac{i}{k} \right) &= \frac{1}{2} \left( \frac{i}{k} - \delta \right)^2
\end{align*}
\]

and let the parameters of the model be given by

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>time discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>(\theta)</td>
<td>capital’s share in national output</td>
<td>0.33</td>
</tr>
<tr>
<td>(\delta)</td>
<td>depreciation rate of physical capital</td>
<td>0.04</td>
</tr>
<tr>
<td>(\rho)</td>
<td>serial correlation of technology shock</td>
<td>0.95</td>
</tr>
</tbody>
</table>
Use a programming language like Matlab to solve for the non-stochastic steady state with these functional forms and parameter values. Matlab is available through the Department’s Citrix server

(http://hearn.ecom.unimelb.edu.au/Citrix/MetaFrameXP/default/login.asp)

**Question 3.** Log-linearize the model around the non-stochastic steady state. That is, if the model is written in the form

\[
\begin{align*}
0 &= AX_t + BX_{t-1} + CY_t + DZ_t \\
0 &= E_t\{FX_{t+1} + GX_t + HX_{t-1} + JY_{t+1} + KY_t + LZ_{t+1} + MZ_t\} \\
Z_{t+1} &= NZ_t + \varepsilon_{t+1}
\end{align*}
\]

provide explicit solutions for each of the coefficients, \(A, B, C, ..., N\). In these equations, \(X_t\) contains the endogenous state variables, \(Y_t\) contains the control variables, and \(Z_t\) contains the exogenous state variables. As part of your answer, you will need to explain exactly which variables from the model are in each of \(X_t, Y_t\) and \(Z_t\).

**Question 4.** Guess that a solution takes the form

\[
\begin{align*}
X_t &= PX_{t-1} + QZ_t \\
Y_t &= RX_{t-1} + S Z_t
\end{align*}
\]

for unknown coefficient matrices \(P, Q, R, S\). Show that solving this model reduces to solving a quadratic equation in \(P\). Solve for the value of \(P\) using the answers from Questions 2 and 3, then recover the \(Q, R, S\) values. Again, this is easy to do in Matlab.

**Question 5.** Use your answers to simulate the effect of a one-time shock to the level of productivity. That is, set the value of \(\varepsilon_0 = 1\) and \(\varepsilon_t = 0\) for \(t \geq 1\) and trace out the effects on productivity, consumption, employment, investment etc. Graph your answers for \(t = 0, 1, ..., 50\).

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\[23\:August\:2004\]