Long-run Economic Growth

Part I: Production and Solow’s Growth Model

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Agenda

• Two classes on ‘long-run economic growth’
  – the aggregate production function, a fundamental tool (today)
  – Robert Solow’s growth model
  – growth accounting (next class)
  – productivity and institutions

• To start with, what do we mean by ‘long-run growth’?
  – in contrast with business cycle fluctuations
  – a key conceptual distinction in macroeconomics
US real GDP

Source: Department of Commerce, BEA.
Trend

\[ y = 2.02 + 0.036x \]

Source: Department of Commerce, BEA.
Business cycle fluctuations

annual percentage change in real GDP

Source: Department of Commerce, BEA.
Long-run economic growth

• Questions
  – why does a country’s income per capita grow over time?
  – why is there so much cross-country variation in income per capita?

• Data
  – Penn World Tables, careful cross-country measurements
## Cross-country GDP differences

<table>
<thead>
<tr>
<th>Country</th>
<th>GDP per person (2000 dollars at ‘PPP’)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>10,938</td>
</tr>
<tr>
<td>Brazil</td>
<td>7,204</td>
</tr>
<tr>
<td>China</td>
<td>5,332</td>
</tr>
<tr>
<td>Egypt</td>
<td>4,759</td>
</tr>
<tr>
<td>France</td>
<td>26,168</td>
</tr>
<tr>
<td>India</td>
<td>2,990</td>
</tr>
<tr>
<td>Ireland</td>
<td>28,956</td>
</tr>
<tr>
<td>Japan</td>
<td>24,661</td>
</tr>
<tr>
<td>Korea</td>
<td>18,423</td>
</tr>
<tr>
<td>Mexico</td>
<td>8,165</td>
</tr>
<tr>
<td>United States</td>
<td>36,098</td>
</tr>
<tr>
<td>Zimbabwe</td>
<td>2,438</td>
</tr>
</tbody>
</table>

Source: Penn World Tables, 6.2
GDP per person

in 2000 dollars at ‘PPP’

Source: Penn World Tables, 6.2
Production functions

GDP

Capital and Labor

Productivity ("Black Box")

Output

Inputs
Production functions

- Relate output to inputs, e.g., capital and labor
- Mathematical version

\[ Y = AK^\alpha L^{1-\alpha} \]

('Cobb-Douglas')

- Definitions

\[ Y = \text{quantity of output} \]
\[ K = \text{quantity of physical capital used (plant and equipment)} \]
\[ L = \text{quantity of labor used} \]
\[ A = \text{‘total factor productivity,’ i.e., everything left out} \]
\[ \alpha = 0.33 \text{ in US data} \]
Properties

• More inputs give more output
  – positive marginal products of capital and labor
    \[
    \frac{\partial Y}{\partial K} > 0, \quad \frac{\partial Y}{\partial L} > 0
    \]

• Output effect of additional inputs falls
  – diminishing marginal products
    \[
    \frac{\partial^2 Y}{\partial K^2} < 0, \quad \frac{\partial^2 Y}{\partial L^2} < 0
    \]

• If we double all inputs, we double output
  – constant returns to scale
    \[
    \lambda Y = AF(\lambda K, \lambda L), \quad \lambda > 0
    \]
Production function

\[ Y = AF(K, L), \text{ holding labor fixed} \]
Capital

• Meaning: physical capital, plant and equipment

• Why does it change?
  – depreciation
  – destruction
  – new investment

• Mathematical version

\[ K_{t+1} - K_t = I_t - \delta K_t \]

for simplicity, a constant rate of depreciation, \( \delta \)

• Adjustments for quality?
Capital per worker

Source: Penn World Tables, 6.2
Labor

• Meaning: units of work effort

• Why does it change?
  – population growth
  – fraction of population employed, hours worked

• Adjustments for quality?
  – skills? education? other factors?
  – if call this $H$ for human capital, then
    \[
    Y = AF(K, HL) \\
    = AK^\alpha(HL)^{1-\alpha} \\
    = (AH^{1-\alpha})K^\alpha L^{1-\alpha}
    \]
    (‘augmented production function’)

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Employed as fraction of population

Source: Penn World Tables, 6.2

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Human capital?

Source: Barro-Lee educational attainments database
Productivity

• Standard number
  – average product of labor, $\frac{Y}{L}$

• Our preferred number
  – total factor productivity, $A = \frac{Y}{F(K,L)}$

• How do we measure it?
  – use the production function to solve for $A$

$$F(K, L) = K^\alpha L^{1-\alpha} \Rightarrow A = \frac{Y}{L} \div \left( \frac{K}{L} \right)^\alpha$$

• US example
  – $\frac{Y}{L} = 67, 865$ and $\frac{K}{L} = 177, 007$ and $\alpha = \frac{1}{3}$ so $A = 1, 208$
## Simple productivity calculations

<table>
<thead>
<tr>
<th>Country</th>
<th>GDP/person</th>
<th>GDP/worker</th>
<th>Capital/worker</th>
<th>TFP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>10,170</td>
<td>24,284</td>
<td>57,858</td>
<td>627</td>
</tr>
<tr>
<td>Brazil</td>
<td>7,204</td>
<td>15,461</td>
<td>37,603</td>
<td>461</td>
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<tr>
<td>China</td>
<td>4,969</td>
<td>8,283</td>
<td>18,015</td>
<td>315</td>
</tr>
<tr>
<td>Egypt</td>
<td>4,759</td>
<td>12,051</td>
<td>9,593</td>
<td>567</td>
</tr>
<tr>
<td>France</td>
<td>25,663</td>
<td>56,908</td>
<td>166,636</td>
<td>1,034</td>
</tr>
<tr>
<td>India</td>
<td>2,990</td>
<td>6,724</td>
<td>7,891</td>
<td>337</td>
</tr>
<tr>
<td>Ireland</td>
<td>28,248</td>
<td>65,924</td>
<td>131,131</td>
<td>1,297</td>
</tr>
<tr>
<td>Japan</td>
<td>24,036</td>
<td>45,030</td>
<td>180,283</td>
<td>797</td>
</tr>
<tr>
<td>Korea</td>
<td>17,596</td>
<td>33,783</td>
<td>113,711</td>
<td>697</td>
</tr>
<tr>
<td>Mexico</td>
<td>7,938</td>
<td>18,627</td>
<td>47,089</td>
<td>515</td>
</tr>
<tr>
<td>United States</td>
<td>36,098</td>
<td>67,865</td>
<td>177,007</td>
<td>1,208</td>
</tr>
<tr>
<td>Zimbabwe</td>
<td>2,438</td>
<td>5,416</td>
<td>14,064</td>
<td>224</td>
</tr>
</tbody>
</table>

Source: Penn World Tables, 6.2
What have we learned so far?

• links output to inputs and productivity

\[ Y = AK^\alpha L^{1-\alpha} \]

• Capital input, \( K \)
  – plant and equipment
  – a consequence of investment, \( I \)

• Labor input, \( L \)
  – population growth
  – participation and hours
  – skills etc, \( H \)

• TFP, \( A \), can be inferred from data on output and inputs
Rest of this class

• Robert Solow’s growth model
  – pieces of the model
  – dynamics
  – ‘steady state’

• Main implication: productivity is key
  – long-run growth due to productivity increases, not capital investment
  – cross-country variation in income per capita due to variation in productivity
Solow’s growth model

• Just four equations
  – production function
    \[ Y = AF(K, L) = AK^\alpha L^{1-\alpha} \]
  – national income accounting
    \[ C + I = Y \]
  – constant savings rate \( s \)
    \[ S = I = sY \]
  – capital accumulation
    \[ K_{t+1} - K_t = I_t - \delta K_t \]
Capital accumulation dynamics

• Put it all together to get

\[ K_{t+1} - K_t = sAK_t^\alpha L_t^{1-\alpha} - \delta K_t \]

• Suppose constant labor force, \( L_t = L \). Let \( k = K/L \), \( y = Y/L \), etc. Then

\[ k_{t+1} - k_t = sA k_t^\alpha - \delta k_t \]

• Given parameters \((s, A, \alpha, \delta)\) and initial capital \( k_0 \), see how \( k_t \) moves through time
  
  – capital increasing if investment is bigger than depreciation
  
  – capital decreasing if investment is smaller than depreciation

• A picture might help . . .
Savings versus depreciation

\[ y = A k^\alpha \]

\[ s A k^\alpha \]

\[ \delta k \]
‘Steady state’

• If new investment equals depreciation, capital per worker is constant

\[ 0 = sA k^\alpha - \delta k \]

• Solving this we get

\[ \bar{k} = \left( \frac{sA}{\delta} \right)^{\frac{1}{1-\alpha}} \]

• This is the ‘steady state’ level of capital per worker
  – higher when \( s, A, \alpha \) are higher
  – lower when \( \delta \) is higher

• Capital stock ‘gravitates’ to this steady state value

• To see why, let’s look at that picture again . . .
Gravitation to ‘steady state’
Convergence hypothesis

- Capital per worker **converges** to steady state

\[
\bar{k} = \left( \frac{sA}{\delta} \right)^{\frac{1}{1-\alpha}}
\]

- Therefore output per worker converges to steady state too

\[
\bar{y} = A\bar{k}^\alpha = A^{\frac{1}{1-\alpha}} \left( \frac{s}{\delta} \right)^{\frac{\alpha}{1-\alpha}}
\]

- Implication
  - capital investment does not explain long run growth of output per worker (that we see in data)
  - higher savings rate \( s \) increases long run level of output, not growth rate
Higher savings rate

![Graph showing the relationship between higher saving per worker and depreciation per worker.](image)
A historical lesson

- Soviet economic growth very impressive in late 1950s – early 1960s
  - 8-9% per year, roughly three times faster than US
  - concern about strategic implications of Soviet industrial ‘might’

- But ‘perspiration not inspiration’
  - too much reliance on physical capital accumulation
  - generates growth in short run
  - but cannot generate sustained long run growth
  - diminishing returns
But what about long-run growth

• In this theory, capital investment alone cannot generate long run growth
  – why? diminishing returns

• What can generate long run growth in output per worker?
  – growth in total factor productivity $A$
  – if $A$ grows at 2%, then in the long run output per worker will also grow at 2%

• But where does TFP come from?
  – in the Solow model, TFP is a ‘black box’
  – guess what we will talk about next?
Long run growth across countries

in 2000 dollars at ‘PPP’

Source: Penn World Tables, 6.2
Levels of output across countries

GDP per capita as percentage of US

Source: United Nations
Levels of output

- Solow model predicts output per worker

\[ \bar{y} = A^{\frac{1}{1-\alpha}} \left( \frac{s}{\delta} \right)^{\frac{\alpha}{1-\alpha}} \]

- cross-country variation in \( \bar{y} \) explained by four parameters (\( \alpha, s, \delta, A \))
- we can measure \( (\alpha, s, \delta) \)
- they do not vary enough across countries to explain enormous variation in \( \bar{y} \)
- again, leaves TFP as explanation
What have we learned today?

• Production
  – production function links output to inputs and productivity
  – ‘total factor productivity’ can be inferred from data on output and inputs

• Solow model
  – countries with higher savings rates have higher levels of output per worker
  – but capital investment cannot generate long run growth in output per worker
  – need productivity growth if output per worker is to grow in long run
  – need cross-country variation in productivity levels to explain enormous disparities in data