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Global Economy
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Cagan's Model of Hyperinflation

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On first glance, hyperinflations seem like pure chaos. In a classic 1956 article, the Chicago economist Phillip Cagan developed a simple model which does surprisingly well at accounting for the behavior of inflation and the demand for money even the midst of such dramatic events. Cagan was interested in whether 'momentum' effects in the dynamics of inflation expectations could exacerbate inflation that originally gets started because of a government's decision to 'monetize' a fiscal deficit (that is, to finance a deficit by printing money).

The model

Cagan's model consists of two equations, one which describes individuals' demand for money and another which describes the evolution of inflation expectations over time. Specifically, Cagan supposed that the *velocity* of money is increasing in the nominal interest rate i. Since bonds and money are both safe assets (in nominal terms), the opportunity cost of holding money is the nominal interest foregone. Therefore, if the nominal interest rate increases, money should turn over more quickly (velocity should rise) as individuals substitute away from money towards bonds.

To make things straightforward, Cagan specified a log-linear form. Let m and p denote the log of money and the price level so that $m - p$ represents the log of real money. From the exchange equation relating money, velocity and expenditure, $MV = PC$ or

$$m + v = p + c$$

Cagan's specification for velocity was

$$v(i) = \alpha i$$

where $\alpha > 0$. Since the nominal interest rate is the real interest rate plus expected inflation, $i_t = r_t + \pi_t^e$ (the 'Fisher equation'), we can write

$$m_t - p_t = c_t - \alpha r_t - \alpha \pi_t^e$$

Since Cagan is interested in episodes of hyperinflation — that is periods where nominal values are changing very rapidly but where real values are much more stable — he also simplified calculations by assuming that real consumption and the real interest rate are constant, $c_t = c$ and $r_t = r$. In this case, the money demand equation involves a tedious constant which we can get rid of by setting $c = 0$ and $r = 0$. Cagan's money demand equation is therefore just

$$m_t - p_t = -\alpha \pi_t^e \tag{1}$$

In a hyperinflation, nominal interest rate dynamics and expectation inflation dynamics are basically the same, so individuals' demand for real balances is declining in expected inflation.

The second part of the Cagan model is that he assumed *adaptive expectations*, meaning that expected inflation is a weighted average of current inflation $p_t - p_{t-1}$ and past expectations of inflation

$$\pi_t^e = \lambda \pi_{t-1}^e + (1-\lambda)(p_t - p_{t-1}) \tag{2}$$

where $0 < \lambda < 1$. If λ is close to one, then individuals expectations are slow to update, they place a lot of weight on past expectations and little weight on current expectations. But if λ is close to zero, individuals place a lot of weight on current experience.

Solving the model

A solution to the model is an equation giving the evolution of prices over time in terms of (i) the past behavior of prices, (ii) the monetary policy of the government, and (iii) the parameters of the model. Specifically, this means we are looking for an equation of the form

$$p_t = \beta_1 p_{t-1} + \beta_2 m_t + \beta_3 m_{t-1}$$

for some as-yet unknown coefficients $\beta_1, \beta_2, \beta_3$ and where p_t and π_t^e both satisfy equations (2)-(1).

We solve the model as follows: start by 'inverting' money demand equation (1) to get at

$$\pi_t^e = -\frac{1}{\alpha}(m_t - p_t)$$

and similarly at date $t-1$ we have

$$\pi_{t-1}^e = -\frac{1}{\alpha}(m_{t-1} - p_{t-1})$$

Now plug these into the inflation expectations equation (2) to get

$$-\frac{1}{\alpha}(m_t - p_t) = -\frac{\lambda}{\alpha}(m_{t-1} - p_{t-1}) + (1-\lambda)(p_t - p_{t-1})$$

Simplifying

$$m_t - p_t = \lambda(m_{t-1} - p_{t-1}) - \alpha(1-\lambda)(p_t - p_{t-1})$$

Now solve this for p_t so that it can be put in the form of equation (3) above. Doing the algebra gives

$$p_t = \frac{\lambda - \alpha(1-\lambda)}{1-\alpha(1-\lambda)} p_{t-1} + \frac{1}{1-\alpha(1-\lambda)} m_t - \frac{\lambda}{1-\alpha(1-\lambda)} m_{t-1} \tag{4}$$

so we have the coefficients

$$\begin{aligned} \beta_1 &= \frac{\lambda - \alpha(1-\lambda)}{1-\alpha(1-\lambda)} \\ \beta_2 &= \frac{1}{1-\alpha(1-\lambda)} \\ \beta_3 &= -\frac{\lambda}{1-\alpha(1-\lambda)} \end{aligned}$$

Equation (4) is a linear equation and we could estimate its parameters with regression methods so that we could study inflation and expected inflation over time.

Implications

The most important properties of the solution are governed by the coefficient β_1. If $-1 < \beta_1 < 1$, (that is $|\beta_1| < 1$), then the inflation dynamics of the system are said to be '*dynamically stable*,' meaning that if the government stabilizes the money supply process m_t, then the price dynamics will stabilize too. In this case, once a government gets control of the money supply process, inflation will eventually come under control too. But if β_1 is too large, then even a stable monetary process may lead to hyperinflations driven purely by 'momentum' — by individuals extrapolating from past inflation behavior.

Cagan estimated the coefficients in (4) for monthly data on European hyperinflations in the 1920s and 1940s and found that in the cases of the European hyperinflations of the 1920s, in most episodes inflation dynamics were driven purely by fundamentals ($|\beta_1| < 1$) and not by momentum effects. But in the case of Germany (1922-1923) and Russia (1921-1924), he found $\beta_1 = 3.17$ and $\beta_1 = 5.92$, respectively, indicating that in these episodes inflation had a significant momentum component too.

One of the important messages that economists take away from Cagan's paper is the need (i) for fiscal discipline and/or an independent central bank, to prevent monetized deficits that can allow a hyperinflation to get started, and (ii) the need for individuals' inflation expectations to be 'anchored' — and thereby relatively unlikely to lead to a momentum-driven inflation break-out. Of course, part of the trick to anchoring inflation expectations is for government policy to be *credibly* anti-inflation.