Before attempting this problem set, you will probably need to read over the lecture notes on the ‘Sources of Economic Growth’ and on ‘International Trade’.

A. France and the US

France and the US are among the most successful economies in the world, but a closer look suggests some differences. In France, output per capita is lower, but since a smaller fraction of the population works and each worker works fewer hours, output per hour worked is not much different. Our goal is to explain these differences.

Consider the data for 2000:

<table>
<thead>
<tr>
<th>Country</th>
<th>GDP</th>
<th>Capital</th>
<th>Employment</th>
<th>Hours</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>1,351</td>
<td>3,852</td>
<td>27.50</td>
<td>1500</td>
<td>59.30</td>
</tr>
<tr>
<td>US</td>
<td>9,169</td>
<td>19,600</td>
<td>142.08</td>
<td>1827</td>
<td>285.00</td>
</tr>
</tbody>
</table>

GDP and capital are reported in billions of 2000 US dollars while employment and population are in millions. Hours is the average number of hours per year worked by an employed person. Total hours worked can be computed as the product of employment (the number of people working) and hours (the number of hours per worker).

1. Compute output per capita, output per worker, and output per hour worked. How do they differ? (5 points).

2. What are the primary sources of the difference in output per capita? Suggestion: use hours worked as your measure of the quantity of labor $L$. (15 points).

3. Why do you think hours per worker and the ratio of employment to population are lower in France than in the US? (5 points).

Answer.

1. As we know from class, GDP per capita, per worker and per hour worked are simply the ratios of GDP to population, number of workers, and total hours worked, respectively. The numbers are:
<table>
<thead>
<tr>
<th>Country</th>
<th>GDP Per Capita</th>
<th>GDP Per Worker</th>
<th>GDP Per Hour Worked</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>22,782</td>
<td>49,127</td>
<td>32.75</td>
</tr>
<tr>
<td>US</td>
<td>32,172</td>
<td>64,534</td>
<td>35.32</td>
</tr>
<tr>
<td>France/US</td>
<td>0.708</td>
<td>0.761</td>
<td>0.927</td>
</tr>
</tbody>
</table>

Note that the ratios (the bottom line) increase as we move to the right: much of the difference in GDP per person is due to differences in the amount of work.

2. A natural version of the production function in this case is

$$ Y = AK^\alpha (hL)^{1-\alpha} $$

where $h$ is average hours worked and $L$ is the number of workers (employment). Total hours is $hL$. We find $A$ by substituting in for all the other numbers: $A = Y/[K^\alpha (hL)^{1-\alpha}]$. How do we use this to make sense of output per capita? If $POP$ is the population, then

$$ \frac{Y}{POP} = \frac{(Y/L)(L/POP)}{A(K/L)^\alpha h^{1-\alpha}} $$

This isn’t the only possible line of attack on this problem, but it’s a reasonable one. It tells us we can attribute differences in per capita GDP to differences in: participation ($L/POP$), total factor productivity ($A$), capital per worker ($K/L$), and average hours per worker ($h$). The math implies

$$ \frac{(Y/POP)_F}{(Y/POP)_{US}} = \left[ \frac{(L/POP)_F}{(L/POP)_{US}} \right] \left[ \frac{A_F}{A_{US}} \right] \left[ \frac{(K/L)_F}{(K/L)_{US}} \right]^\alpha \left[ \frac{h_F}{h_{US}} \right]^{1-\alpha} $$

$$ = \frac{.464}{.499} \frac{0.0722}{0.0836} \left[ \frac{140,073}{137,941} \right]^{1/3} \left[ \frac{1500}{1827} \right]^{2/3} $$

$$ = (0.930)(0.864)(1.005)(0.877) $$

$$ = 0.708 $$

Output per person in France is only about 70% of that in the US. Where does the difference in GDP per person come from? In order of importance: TFP, hours worked, and participation. Capital per worker goes the other way – it’s higher in France. Obviously there are many caveats to this analysis.

3. Evidently labor market outcomes are very different in the two countries. One conjecture is that differences in the institutions and legislation that regulates the functioning of labor markets play a major role.

B. Korea

Since the end of the Korean War, Korea has been one of the most successful economies in the world. What are the sources of its success? The basic data are:
GDP per capita is expressed in 2000 US dollars. Population and employment are in millions of heads. Capital is in billions of dollars. Education is the average number of years of school completed among people older than 15.

1. Compute the (average continuously compounded) growth rate of GDP per worker. (5 points).

2. Use our growth accounting methodology to allocate growth in output per worker to TFP, capital per worker, and human capital. What factors are most important? (15 points).


Answer.

1. GDP per worker is GDP per capita times the population then divided by the number of workers: 4,576 in 1961 and 36,851 in 2000. Its continuously compounded growth rate is

\[ \gamma = \log(36,851/4,576)/39 = 5.35\%. \]

Refer to the discussion of growth rates in the notes on the ‘Sources of Economic Growth’ if you’re not sure why this works.

2. We use the production function in output per worker form,

\[ \frac{Y}{L} = A\left(\frac{K}{L}\right)^{1/3}H^{2/3}. \]

TFP is then \( A = \left(\frac{Y}{L}\right)/\left[\left(\frac{K}{L}\right)^{1/3}H^{2/3}\right]. \) The necessary data for growth accounting is

<table>
<thead>
<tr>
<th>Year</th>
<th>Y/L</th>
<th>A</th>
<th>K/L</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>1961</td>
<td>4,576</td>
<td>769</td>
<td>11,676</td>
<td>4.25</td>
</tr>
<tr>
<td>2000</td>
<td>36,851</td>
<td>1,600</td>
<td>103,947</td>
<td>10.84</td>
</tr>
</tbody>
</table>

The third row (‘growth rate’) is computed as we described in question 1: the logarithm of the ratio of the 2000 number to the 1961 number, divided by 39 (the number of years between 1961 and 2000). The next row (‘contribution to growth’) modifies these growth rates as they
occur in the formula: the growth rate of $K/L$ gets multiplied by 1/3 (its exponent in the production function) and the growth rate of $H$ gets multiplied by 2/3 (its exponent).

Where does growth in output per worker come from? The 5.35% average growth in output per worker has three components of comparable size: TFP (1.88), capital per worker (1.87), and human capital (1.60).

3. Per capita GDP grew by 5.98% a year. The difference of 0.63% reflects growth in the ratio of workers to population, which rose sharply in Korea over this period.

C. China

China’s remarkable economic growth and large population have created one of the world’s largest economies. In 2000, GDP in China was half the size of US GDP, but growing twice as fast. (The comparison is based on PPP-adjusted data from the Penn World Tables.) The question is how this is likely to change over the next 20 years. Will China become the world’s largest economy?

Your mission is to estimate the economic magnitudes of the US and China in 2020 using the Solow model and some inspired estimates of the parameters. In 2000, the economies had the following characteristics:

<table>
<thead>
<tr>
<th>Country</th>
<th>GDP</th>
<th>Capital</th>
<th>Employment</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>4,717</td>
<td>7,020</td>
<td>764.0</td>
</tr>
<tr>
<td>US</td>
<td>9,169</td>
<td>19,600</td>
<td>142.1</td>
</tr>
</tbody>
</table>

GDP and capital are reported in billions of 2000 US dollars, employment and population in millions. Your economic consultants tell you that the parameter values are

<table>
<thead>
<tr>
<th>Country</th>
<th>Saving</th>
<th>Depreciation</th>
<th>Empl. Growth</th>
<th>TFP Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>22%</td>
<td>6%</td>
<td>1.0%</td>
<td>4%</td>
</tr>
<tr>
<td>US</td>
<td>20%</td>
<td>6%</td>
<td>0.5%</td>
<td>2%</td>
</tr>
</tbody>
</table>

1. Compute TFP for each country in 2000 using the basic production function, $Y = AK^\alpha L^{1-\alpha}$. (5 points).

2. Compute time paths for GDP in both countries starting in 2000 and ending in 2020. How large is China relative to the US in 2020? (15 points).

3. Comment (briefly) on the strengths and weaknesses of your analysis. What parameter values are you least certain about? What features of the world does the model miss? (5 points).
Answer.

1. A variant of the usual calculation: $A = Y/(K^\alpha L^{1-\alpha})$. The numbers are 2.948 for China and 12.488 for the US.

2. This is a dynamic simulation much like the one described in the notes on the ‘Solow Model’. The major difference is the addition of growth in TFP.

We simulate the model like this. At each date, we compute employment $L$ and productivity $A$ from their values at the previous date by using their (assumed constant) growth rates:

\[
L_{t+1} = (1 + n) L_t \\
A_{t+1} = (1 + a) A_t
\]

where $n$ and $a$ are the growth rates of employment and TFP, respectively. Capital evolves as in the Solow model:

\[
K_{t+1} = sA_t K_t^{\alpha} L_t^{1-\alpha} + (1 - \delta) K_t
\]

All we need are the parameters, which are given to us in the problem. In the case of China, the capital stock in 2001 is

\[
K_{2001} = sA_{2000} K_{2000}^{1/3} L_{2000}^{2/3} + (1 - \delta) K_{2000} \\
= .22 \times 2.948 \times 7,020^{1/3} \times 764^{2/3} + .94 \times 7,020 \\
= 7,636.54
\]

The level of TFP in 2001 is

\[
A_{2001} = A_{2000}(1 + a) = 2.948 \times 1.04 = 3.06592
\]

The number of workers in 2001 is

\[
L_{2001} = L_{2000}(1 + n) = 764 \times 1.01 = 771.64
\]

Therefore GDP in 2001 is

\[
Y_{2001} = A_{2001} K_{2001}^{1/3} L_{2001}^{2/3} \\
= 3.06592 \times 7,636.54^{1/3} \times 771.64^{2/3} \\
= 5,079
\]
### Problem Set #2 Answers

<table>
<thead>
<tr>
<th>Year</th>
<th>China</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>4,717</td>
<td>9,169</td>
</tr>
<tr>
<td>2001</td>
<td>5,079</td>
<td>9,487</td>
</tr>
<tr>
<td>2002</td>
<td>5,466</td>
<td>9,817</td>
</tr>
<tr>
<td>2003</td>
<td>5,880</td>
<td>10,159</td>
</tr>
<tr>
<td>2004</td>
<td>6,323</td>
<td>10,512</td>
</tr>
<tr>
<td>2005</td>
<td>6,797</td>
<td>10,879</td>
</tr>
<tr>
<td>2006</td>
<td>7,305</td>
<td>11,258</td>
</tr>
<tr>
<td>2007</td>
<td>7,848</td>
<td>11,651</td>
</tr>
<tr>
<td>2008</td>
<td>8,430</td>
<td>12,058</td>
</tr>
<tr>
<td>2009</td>
<td>9,052</td>
<td>12,479</td>
</tr>
<tr>
<td>2010</td>
<td>9,718</td>
<td>12,916</td>
</tr>
<tr>
<td>2011</td>
<td>10,432</td>
<td>13,368</td>
</tr>
<tr>
<td>2012</td>
<td>11,195</td>
<td>13,836</td>
</tr>
<tr>
<td>2013</td>
<td>12,013</td>
<td>14,320</td>
</tr>
<tr>
<td>2014</td>
<td>12,889</td>
<td>14,822</td>
</tr>
<tr>
<td>2015</td>
<td>13,827</td>
<td>15,342</td>
</tr>
<tr>
<td>2016</td>
<td>14,831</td>
<td>15,880</td>
</tr>
<tr>
<td>2017</td>
<td>15,906</td>
<td>16,437</td>
</tr>
<tr>
<td>2018</td>
<td>17,057</td>
<td>17,014</td>
</tr>
<tr>
<td>2019</td>
<td>18,290</td>
<td>17,611</td>
</tr>
<tr>
<td>2020</td>
<td>19,611</td>
<td>18,230</td>
</tr>
</tbody>
</table>

In this scenario, China’s GDP overcomes the US’s in 2018 and is slightly larger in 2020. US workers would still be more productive, though: GDP per worker is projected to be 116,108 for the US and 21,036 for China.

3. All of the parameters could be questioned — take your pick. For example, in a related exercise described in the Goldman Sachs report entitled ‘Dreaming of BRICs’ it is argued that the rate of TFP growth will fall as China approaches the world’s ‘technology frontier’. Another issue is foreign-financed capital formation. As you recall, in the context of the Solow model capital can grow faster only if the domestic saving rate increases. In reality, this is not true. Foreign residents can lend their savings to domestic companies, providing a further source of financing for capital expenditures. In the exercise we somewhat dealt with this by using numbers for the parameters $s$ which are actually closer to the average investment rates, rather than the saving rates. Still, you you might imagine that changes in the climate for foreign investment could lead to more rapid accumulation of capital than the model allows for.

### D. International trade with lots of symmetry

Suppose we have two countries that each can produce manufactures $M$ or services $S$ with labor. Their productivities in the two activities are reported below.
\[\begin{array}{c|c|c}
\text{Manufactures} & \text{Services} \\
\hline
\text{Home} & A_M = 10 & A_S = 20 \\
\text{Foreign} & A^*_M = 20 & A^*_S = 10 \\
\end{array}\]

Table 1: Productivity levels.

Assume that product and labor markets are competitive in both countries and that both countries have the same sized labor force \( L = L^* = 100 \).

Finally, suppose that households in the two countries have the demand curves

\[
C_M = \frac{1}{2} \frac{wL}{p_M}, \quad C_S = \frac{1}{2} \frac{wL}{p_S}
\]

\[
C^*_M = \frac{1}{2} \frac{w^*L}{p^*_M}, \quad C^*_S = \frac{1}{2} \frac{w^*L}{p^*_S}
\]

These demand curves imply that consumers in each country spend half of their income on each good.

1. Suppose there is no trade. What is the relative price of manufactures to services in each country? Where are manufactures cheap to produce? Where are services cheap to produce? Which country has a comparative advantage in which good? (5 points)

2. Continuing with the case of no trade, we want to study an 'equilibrium' where firms are maximizing their profits and where consumer’s demand equals supply. Calculate equilibrium prices \( p_M \) and \( p_S \), consumption \( C_M \) and \( C_S \), and employment \( L_M \) and \( L_S \) for each sector. Calculate the same variables for the foreign country.

[Hint: you need to pick a unit of account for each country. Use \( w = 1 \) for the home country and \( w^* = 1 \) for the foreign country]. (10 points)

3. Now suppose that the two countries engage in free trade (so there are perfectly integrated markets for both manufactures and services). Suppose that each country specializes in the good in which it has a comparative advantage. Calculate the international relative price of manufactures to services. Then calculate equilibrium consumption and employment in each sector for both countries. Explain how these change relative to the case of no trade. Also explain who imports and exports what goods. Are consumers better off with trade or not? Why or why not?

[Hint: you need to pick a unit of account for this integrated global economy. Measure everything relative to home country income by setting \( w = 1 \)]. (10 points)
Answer.

1. Profit maximization by producers of manufactures and services in the home country leads to 

\[ p_M A_M = w = p_S A_S \]

and so the relative price of manufactures to services is 

\[ \frac{p_M}{p_S} = \frac{A_S}{A_M} = \frac{2}{1} = 2 \]

Similarly, profit maximization by producers of manufactures and services in the foreign country leads to 

\[ p_M^* A_M^* = w^* = p_S^* A_S^* \]

and so the relative price of manufactures to services is 

\[ \frac{p_M^*}{p_S^*} = \frac{A_S^*}{A_M^*} = \frac{1}{2} = 0.5 \]

This tells us that manufactures are relatively cheap to produce in the foreign country but expensive in the home country. And services are relatively cheap to produce in the home country but expensive in the foreign country. (Equivalently, the opportunity cost of producing manufactures is high at home but low in the foreign country). Therefore, the home country has a comparative advantage in services and the foreign country has a comparative advantage in manufactures.

2. We can choose \( w = 1 \) at home and \( w^* = 1 \) abroad. Therefore profit maximization implies that the home prices are \( p_S = 1/A_S = 1/2 \) and \( p_M = 1/A_M = 1 \). Similarly, prices abroad are \( p_S^* = 1/A_S^* = 1 \) and \( p_M^* = 1/A_M^* = 1/2 \). Now plug these prices into the demand functions for manufactures and services at home to get 

\[ C_M = \frac{1}{2} \frac{wL}{p_M} = \frac{1}{2} \times \frac{1 \times 1}{1} = 0.5 \]

\[ C_S = \frac{1}{2} \frac{wL}{p_S} = \frac{1}{2} \times \frac{1 \times 1}{1/2} = 1.0 \]

Now using market clearing and the production functions in each sector

\[ \begin{align*}
0.5 &= C_M = Y_M = A_M L_M &= 1 \times L_M \\
1.0 &= C_S = Y_S = A_S L_S &= 2 \times L_S
\end{align*} \]

So we have \( L_M = 0.5 \) and \( L_S = 0.5 \). Repeating the symmetric calculations for the foreign country, we have 

\[ C_M^* = \frac{1}{2} \frac{w^*L^*}{p_M^*} = \frac{1}{2} \times \frac{1 \times 1}{1/2} = 1.0 \]

\[ C_S^* = \frac{1}{2} \frac{w^*L^*}{p_S^*} = \frac{1}{2} \times \frac{1 \times 1}{1} = 0.5 \]
And again using market clearing and the production functions in each sector

\begin{align*}
1.0 &= C^*_M = Y^*_M = A^*_M L^*_M = 2 \times L^*_M \\
0.5 &= C^*_S = Y^*_S = A^*_S L^*_S = 1 \times L^*_S
\end{align*}

So we also have $L^*_M = 0.5$ and $L^*_S = 0.5$.

3. With free trade, we have one integrated world economy where each country faces the same product prices $p_M$ and $p_S$ but have different wages $w$ and $w^*$. We can only pick one unit of account, so set $w = 1$. Each country specializes in its comparative advantage, so home produces services and foreign produces manufactures. Therefore at home $Y_S = A_S L = 2$ while $Y_M = 0$ and abroad $Y^*_S = 0$ while $Y^*_M = A^*_M \bar{L}^* = 2$.

Profit maximization by service producers at home leads to $p_S = w/A_S = 1/2$ while profit maximization by manufacturers abroad leads to $p_M = w^*/A^*_M = w^*/2$. We still need to solve for $w^*$. Market clearing in manufactures requires

$$C_M + C^*_M = Y^*_M$$

since home produces $Y_M = 0$. Using the demand functions for each country this is the same as

$$\frac{1}{2} \frac{w_L}{p_M} + \frac{1}{2} \frac{w^*_L}{p_M^*} = Y^*_M$$

Now plug in all the things we already know to get

$$\frac{1}{2} \times \frac{1 \times 1}{w^*/2} + \frac{1}{2} \times \frac{w^* \times 1}{w^*/2} = 2$$

This is one equation in one unknown, $w^*$. Simplifying gives

$$\frac{1}{w^*} + 1 = 2$$

Therefore $w^* = 1$. (This is not so surprising, the two countries are mirror images of each other, so it seems natural that their wages are the same). So using our earlier calculation, the price of manufactures is $p_M = w^*/A^*_M = 1/2$. Now we have all the prices, we can use the demand functions to get consumption of each good in each country. At home

$$C_M = \frac{1}{2} \frac{w_L}{p_M} = \frac{1}{2} \times \frac{1 \times 1}{1/2} = 1.0$$

$$C_S = \frac{1}{2} \frac{w_L}{p_S} = \frac{1}{2} \times \frac{1 \times 1}{1/2} = 1.0$$

And similarly for foreign

$$C^*_M = \frac{1}{2} \frac{w^*_L}{p_M} = \frac{1}{2} \times \frac{1 \times 1}{1/2} = 1.0$$

$$C^*_S = \frac{1}{2} \frac{w^*_L}{p_S} = \frac{1}{2} \times \frac{1 \times 1}{1/2} = 1.0$$
So home consumption of manufactures increases from 0.5 to 1.0 and likewise foreign consumption of services increases from 0.5 to 1.0. Home imports manufactures and exports services while foreign does the reverse. Both countries are better off.