Idiosyncratic risk and the equity premium: evidence from the consumer expenditure survey

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Abstract

This paper investigates whether uninsured idiosyncratic risk accounts for the equity premium. Following Mankiw (J. Financial Econ. 17 (1986) 211), the paper develops an equilibrium factor model in which risk premia depend on the covariance between an asset’s return and certain moments of the cross-sectional distribution for consumption growth. Cross-sectional consumption factors are constructed using data from the Consumer Expenditure Survey, but they do not appear to be promising candidates for explaining the equity premium. The cross-sectional factors are weakly correlated with stock returns and generate equity premia of 2 percent or less for preference specifications with low degrees of risk aversion. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Standard representative agent models imply a number of well-known asset pricing puzzles. For example, they generate risk-free rates that are too high on average (Weil, 1989), excess returns on stocks that are too low (Mehra and Prescott, 1985), and they suggest that risk-adjusted excess returns are predictable (Hansen and Singleton, 1982). In addition, in many cases model discount factors are insufficiently volatile (Hansen and Jagannathan, 1991) and too weakly correlated with stock returns (Cochrane and Hansen, 1992).

These puzzles may be related to the fact that representative agent models focus on variation in aggregate consumption and abstract from idiosyncratic variation. This would be innocuous if consumers were able to fully insure against idiosyncratic risk, but tests by Cochrane (1991), Mace (1991), Nelson (1994), and others reject full consumption insurance.

Motivated by both sets of results, a number of economists have begun to explore the asset pricing implications of models with heterogeneous consumers and uninsurable idiosyncratic risk. One especially important paper is by Constantinides and Duffie (1996), who work out the theoretical conditions under which such a model could resolve the equity premium puzzle. Their model contains two key ingredients. The first is that idiosyncratic shocks must be persistent. Otherwise consumers could smooth over idiosyncratic shocks by borrowing and lending or maintaining buffer stocks of securities, and in that case the resulting equilibrium would closely approximate a complete markets allocation (e.g., see Telmer, 1993; Lucas, 1994). Persistence is necessary for overcoming self-insurance.

The second key ingredient is that the conditional variance of idiosyncratic shocks must vary inversely with stock returns, so that equities are unattractive as a store of precautionary wealth. Otherwise, holding equities would be a good way to self-insure, and idiosyncratic risk would enhance demand for stocks.

Heaton and Lucas (1996) and Storesletten et al. (1997) use data from the Panel Study of Income Dynamics to investigate whether idiosyncratic shocks have these properties. On the degree of persistence, they find that estimates are mixed and depend on auxiliary modelling assumptions. For example, Heaton and Lucas specify a univariate model for idiosyncratic income and estimate that shocks have a half-life of only 1.4 years. On the other hand, Storesletten et al. posit an unobserved components model in which idiosyncratic income consists of the sum of a persistent (strongly autocorrelated) and a transitory (white noise) component. Their estimates suggest that the persistent component is more strongly autocorrelated, with a half-life ranging from 2.8 to 9.8 years depending on the treatment of household fixed effects. Indeed, in some cases they are unable to reject a unit root in idiosyncratic income, which is the specification assumed by Constantinides and Duffie. The range of estimates is rather broad, and the ability to self-insure depends sensitively on this feature of the data. In the absence of security market frictions, it would be easy to self-insure against a shock with a half-life of 1.4 or 2.8 years, but much more difficult to self-insure against a shock with a half-life of a decade or more.
Thus, whether idiosyncratic shocks are sufficiently persistent remains an open question.\(^1\)

On the second key ingredient, both papers report estimates from PSID data that suggest a weak inverse relation between aggregate GDP growth and the conditional variance of household income. Income inequality does increase in recessions, but the increase appears to be minor. Both papers conjecture that the failure to detect a stronger, more significant relation may be due to data constraints, and that income inequality may actually vary much more over the business cycle. Indeed, by combining PSID and NIPA data, Storesletten et al. find some evidence of a stronger relation. In any case, this feature of the data is also difficult to pin down.\(^2\)

In this paper, I also investigate whether idiosyncratic risk explains the equity premium, but I use methods that bypass some of the problems confronted by Heaton and Lucas and Storesletten et al. The paper circumvents these problems by conditioning directly on household consumption data. Indeed, the analysis makes no assumptions at all about income shocks. The consumption-based approach has at least three virtues. First, it does not require estimation of the degree of shock persistence or modelling how idiosyncratic income risk varies over the business cycle. Second, it allows more heterogeneity in the kind of idiosyncratic risks that households face. Third, and perhaps most importantly, it recognizes that households have superior information about idiosyncratic risks.

In settings with incomplete markets, private information is the essence of the matter. It is quite plausible, for example, that households have private information vis a vis one another, and they surely have superior information relative to economists who try to model their behavior. Because households observe state variables that we do not, reconstructing their decision rules is likely to be extremely difficult. But the effects of idiosyncratic shocks on asset prices depend on how they affect household consumption, and decision rules for consumption incorporate a household’s private information. Thus, by conditioning directly on consumption, assumptions about household information or the nature of idiosyncratic income shocks are unnecessary, and conclusions are robust to potential specification errors in those parts of more complete models.

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\(^1\)The imprecision of persistence estimates follows from the same sort of issues that arise in the literature on aggregate dynamics. One concerns the fact that the PSID goes back to 1968, and because of its short time span contains relatively little time-series variation useful for identifying persistence. Another concerns the fact that persistence estimates often differ across univariate and multi-shock representations. A third concerns the fact that persistence estimates are often sensitive to assumptions about deterministic components. In the aggregate literature, this manifests itself as a sensitivity to mean shifts or trend breaks; here it manifests itself as a sensitivity to assumptions about fixed effects. Indeed, Laibson et al. (1998) use PSID data to estimate an unobserved components model similar to the one studied by Storesletten et al., but they treat household fixed effects in a different manner and find persistence estimates close to those reported by Heaton and Lucas. The point here is not that one specification is necessarily superior to another, but that persistence estimates are sensitive to auxiliary modelling assumptions.

\(^2\)These papers also address a different feature of the data than the one emphasized by Constantinides and Duffie, who focused on the correlation between stock returns and the conditional variance of household consumption.
Following Mankiw (1986), the paper develops an equilibrium factor model in which risk premia are related to the covariance between an asset's return and moments of the cross-sectional distribution for consumption growth. Data from the Consumer Expenditure Survey (CEX) are used to construct a time series of the relevant cross-sectional moments, and risk factors are computed by estimating their covariance with excess returns on the value-weighted NYSE portfolio. Cochrane (1997) conjectures that cross-sectional factors are likely to be too weakly correlated with returns to explain the equity premium, especially when consumers have low degrees of risk aversion. The results confirm his conjecture: cross-sectional factors are indeed weakly correlated with stock returns, and they generate equity premia of 2 percent or less when the coefficient of relative risk aversion is below 5.

This is one of a growing number of papers that uses household consumption data to assess the empirical validity of consumption-based asset pricing models. Altug and Miller (1990), Mankiw and Zeldes (1991), and Jacobs (1999) use PSID data to investigate the effects of aggregation bias and idiosyncratic risk in empirical asset pricing models. One shortcoming of the PSID, however, is that data are available only for food consumption, and there are legitimate concerns about whether this is adequate for studying problems related to intertemporal substitution and self-insurance. The CEX provides a much broader measure of consumption expenditures and is generally regarded as a superior data source. Brav and Geczy (1996) and Attanasio et al. (1998) use data from the CEX and UK Family Expenditure Survey, respectively, to investigate aggregation bias, but their empirical methods do not encompass incomplete markets models. They construct consumption measures for representative stockholders, but continue to assume complete risk-sharing (perfect consumption correlation) within this group. Vissing-Jorgenson (1998) uses CEX data to investigate the effects of both aggregation bias and idiosyncratic risk. She focuses mostly on log-linear pricing conditions that are analogous to the first-order factor models discussed below, estimating versions for both representative stockholder and incomplete markets models. She finds that the results are very similar across specifications, which is a sign that higher-order moments of the cross section do not matter for pricing equities.\(^3\) Thus, our results are quite similar.

The remainder of the discussion is organized as follows. Section 2 describes an equilibrium factor model that I use to organize the empirical analysis. Section 3 describes the data used to construct empirical proxies for cross-sectional factors, and Section 4 summarizes the basic results. Section 5 considers whether the results are robust to measurement error, and Section 6 concludes.

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\(^3\)In the representative agent version of her model, the approximate log-linear pricing kernel depends on the growth in mean consumption, whereas in the incomplete markets model it depends on mean consumption growth. The difference between the two depends on higher-order moments of the cross section. The fact that the two specifications yield similar results means that the higher-order moments do not matter much for pricing equities.
2. A cross-sectional consumption-based factor model

This section adapts Mankiw’s (1986) model to make it more amenable to empirical analysis. The resulting representation can be interpreted as an approximate equilibrium factor model for returns, where the factors are moments of the cross-sectional distribution of consumption growth.

In this model, preferences are assumed to be identical across consumers, and heterogeneity is introduced by subjecting agents to different histories of idiosyncratic income shocks. In addition, consumers are assumed to have private information about their history and circumstances, and this makes full insurance impossible. Because there are missing insurance markets, the behavior of agents who are identical ex ante may differ ex post. In other words, consumers are assumed to be conditionally heterogeneous but unconditionally homogeneous. An outside observer armed with knowledge of a consumer’s history and circumstances might be able to predict how his behavior would differ from others, but in the absence of any such conditioning information all agents look alike.

Following Mankiw and Constantinides and Duffie, I also assume that financial transactions are costless and that there are no borrowing or short sales constraints. Thus, the paper focuses exclusively on the role of non-diversifiable idiosyncratic risk and abstracts from the role of security market frictions. Storesletten et al. study a model with borrowing constraints as well as non-diversifiable idiosyncratic risk, but find that the constraints bind very infrequently in equilibrium. The model studied here well approximates their framework. Heaton and Lucas study a sequence of models in which transactions costs and borrowing constraints become increasingly important. The analysis in this paper encompasses their frictionless trading model, but not the others.

2.1. No-arbitrage conditions

In this setting, consumers can make arbitrage profits unless the following conditions are satisfied:

\[ E(R_{ft} M(g_{it}) | H_{it-1}) = 1, \]

(1)

\[ E(R_{xt} M(g_{it}) | H_{it-1}) = 0, \]

(2)

where the subscript \( i \) indexes agents and \( t \) indexes time. The variable \( R_{ft} \) is the gross real return on risk-free bonds, \( R_{xt} \) is the excess real return on stocks, \( M(g_{it}) \) is a stochastic discount factor that depends on log consumption growth, \( g_{it} = \ln(c_{it}/c_{i0-1}) \), and \( H_{it-1} \) represents household \( i \)'s information set at time \( t-1 \). If the price of a risk-free bond is known one period in advance, it follows

\[ E(R_{ft} M(g_{it}) | H_{it}) = 1, \]

(3)

\[ E(R_{xt} M(g_{it}) | H_{it}) = 0, \]

(4)

\[ E(R_{ft} M(g_{it}) | H_{it-1}) = 1, \]

(5)

\[ E(R_{xt} M(g_{it}) | H_{it-1}) = 0, \]

(6)

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(6)
from (1) that household discount factors are equated in expectation. But in
the absence of complete insurance markets discount factors need not be equated
state-by-state.

Household information sets, $H_{it-1}$, almost surely contain information that is not
available to econometricians,\(^6\) but by exploiting the law of iterated expectations we
can derive asset pricing conditions that are valid relative to information possessed by
outside observers. Among other things, the law of iterated expectations implies that
(1) and (2) also hold unconditionally,

$$E_R f_t M(g_{it}) = 1,$$  \(3\)

$$E_R x_t M(g_{it}) = 0.$$  \(4\)

Although household decisions depend on information that outsiders cannot see, their behavior should still conform to these conditions on average.

Eqs. (3) and (4) are valid for all households who participate in securities markets,
and in principle they could be investigated directly by exploiting panel data on
consumption. The Consumer Expenditure Survey is more in the nature of a repeated
cross section,\(^7\) however, so we must work out the cross-sectional implications
instead.

To do so, notice that since the IMRS of any participating household is a valid
discount factor, the average IMRS across households is also a valid discount factor.
Therefore the no-arbitrage conditions also imply

$$E_R f_t \left[ (1/N) \sum_i M(g_{it}) \right] = 1,$$  \(5\)

$$E_R x_t \left[ (1/N) \sum_i M(g_{it}) \right] = 0,$$  \(6\)

provided that we sum across households that participate in securities markets.
Conditions (3) and (4) imply (5) and (6), but not conversely. Verification of the latter
provides ambiguous support for the former, but refutation of (5) and (6) implies
refutation of (3) and (4).

To derive a representation in which cross-sectional moments appear as pricing
factors, I approximate $M(g_{it})$ with a third-order polynomial in $g_{it}$.\(^8\) Let

$$\mu_{1t} = (1/N) \sum_i g_{it}$$  \(7\)

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\(^6\) Indeed, if that were not the case, it would be hard to rationalize missing insurance markets.

\(^7\) There is a panel element, but it is limited.

\(^8\) One could also consider higher-order approximations, which would result in pricing models in which
moments of order 4, 5, 6 and so on appeared as factors. In this paper, I stop at a third-order approximation
because measurement error makes it difficult to estimate high-order moments. This is also the reason why I
work with an approximate rather than exact discount factor. The exact discount factor depends on all the
moments of the cross section and is more sensitive to measurement error than the approximate discount
factor.
represent the cross-sectional average of log consumption growth at time $t$. Expanding $M(g_{it})$ around $\mu_{1t}$ yields

$$M(g_{it}) = M(\mu_{1t}) + M'(\mu_{1t})(g_{it} - \mu_{1t}) + (1/2)M''(\mu_{1t})(g_{it} - \mu_{1t})^2$$

$$+ (1/6)M'''(\mu_{1t})(g_{it} - \mu_{1t})^3.$$  \hfill (8)

Next, take the cross-sectional average at each date,

$$\left[(1/N) \sum_i M(g_{it})\right] = M(\mu_{1t}) + (1/2)M''(\mu_{1t})\mu_{2t} + (1/6)M'''(\mu_{1t})\mu_{3t},$$  \hfill (9)

where $\mu_{2t}$ and $\mu_{3t}$ are the second- and third-order cross-sectional moments at time $t$,

$$\mu_{2t} = (1/N) \sum_i (g_{it} - \mu_{1t})^2;$$  \hfill (10)

$$\mu_{3t} = (1/N) \sum_i (g_{it} - \mu_{1t})^3.$$  \hfill (11)

Substituting the average discount factor back into Eqs. (5) and (6) yields the following (approximate) no-arbitrage conditions:

$$E[R_{ft}] = [M(\mu_{1t}) + (1/2)M''(\mu_{1t})\mu_{2t} + (1/6)M'''(\mu_{1t})\mu_{3t}] = 1;$$  \hfill (12)

$$E[R_{xt}] = [(1/N) \sum_i M(g_{it})] - \frac{\text{cov}[R_{xt}, M(\mu_{1t}) + (1/2)M''(\mu_{1t})\mu_{2t} + (1/6)M'''(\mu_{1t})\mu_{3t}]}{E[M(\mu_{1t}) + (1/2)M''(\mu_{1t})\mu_{2t} + (1/6)M'''(\mu_{1t})\mu_{3t}]} = 0.$$  \hfill (13)

Finally, solving for the mean risk-free rate and equity premium yields

$$E[R_{ft}] = \frac{1}{E[M(\mu_{1t}) + (1/2)M''(\mu_{1t})\mu_{2t} + (1/6)M'''(\mu_{1t})\mu_{3t}]].$$  \hfill (14)

$$E[R_{xt}] = \frac{-\text{cov}[R_{xt}, M(\mu_{1t}) + (1/2)M''(\mu_{1t})\mu_{2t} + (1/6)M'''(\mu_{1t})\mu_{3t}]}{E[M(\mu_{1t}) + (1/2)M''(\mu_{1t})\mu_{2t} + (1/6)M'''(\mu_{1t})\mu_{3t}]}.$$  \hfill (15)

Relative to representative agent models, the key difference concerns the presence of higher-order factors, $\mu_{2t}$ and $\mu_{3t}$.

2.2. Preferences

To develop intuition and derive an empirical approximating model, it is useful to specialize the model by specifying a form for preferences. Thus, suppose preferences are separable over time and that period utility is isoelastic,

$$U_{it} = E_{it} \sum_{j=0}^{\infty} \beta^j \frac{c_{it+j}^{1-\gamma}}{1-\gamma}.$$  \hfill (16)
The parameter $\beta$ is the subjective discount factor, and $\alpha$ is the coefficient of relative risk aversion. The intertemporal marginal rate of substitution is $M(g_t) = \beta \exp(-\alpha g_t)$.\(^9\)

For this specification, the mean risk-free rate is

$$ ER_{ft} = \frac{1}{\beta E[\exp(-\alpha \mu_{t1})(1 + (\alpha^2/2)\mu_{2t} - (\alpha^3/6)\mu_{3t})]} $$

(17)

Idiosyncratic variation helps resolve the risk-free rate puzzle by reinforcing the motive for precautionary savings. According to this expression it does so in three ways. First, because marginal utility is convex, demand for safe assets is an increasing function of the average cross-sectional variance, $E\mu_{2t}$. Similarly, because risk averse consumers care more about adverse shocks than favorable ones, negative skewness ($E\mu_{3t} < 0$) also enhances demand for safe assets. Finally, if average consumption growth is low when the higher moments are especially adverse, so that $\text{cov}[\exp(-\alpha \mu_{t1}), (1 + (\alpha^2/2)\mu_{2t} - (\alpha^3/6)\mu_{3t})] > 0$, consumers engage in even more precautionary saving, and this further reduces the mean risk-free rate.

Similarly, the mean excess return is

$$ ER_{xt} = -\frac{\text{cov}[R_{xt}, \exp(-\alpha \mu_{t1})(1 + (\alpha^2/2)\mu_{2t} - (\alpha^3/6)\mu_{3t})]}{E[\exp(-\alpha \mu_{t1})(1 + (\alpha^2/2)\mu_{2t} - (\alpha^3/6)\mu_{3t})]} $$

(18)

To resolve the equity premium puzzle, idiosyncratic risk must also reinforce a household’s aversion to holding equities. In order to do so, equities must be unattractive as a store of precautionary balances. For example, if $\mu_{2t}$ and $\mu_{3t}$ were independent of $R_{xt}$ and $\mu_{1t}$, idiosyncratic variation would have no effect on the equity premium. In that case, the higher-order terms would factor out of (18), and the equity premium would be

$$ ER_{xt} = -\frac{\text{cov}[R_{xt}, \exp(-\alpha \mu_{t1})]}{E[\exp(-\alpha \mu_{t1})]} $$

(19)

which is approximately the same as in an aggregate consumption-based model. Heuristically, if the higher-order factors were unrelated to the return on equities, stocks and bonds would both be safe for the purpose self-insurance, and idiosyncratic risk would reinforce demand for both assets. On average, their returns would fall by the same amount, and the equity premium would remain the same.

To resolve the equity premium puzzle, stock returns must covary negatively with cross-sectional consumption variance and/or positively with its skewness. If the magnitude of idiosyncratic variation or the risk of an adverse shock were especially

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\(^9\)A similar expression can be derived for a version of Epstein and Zin (1989, 1991) preferences. In their model, utility has a recursive representation, $U_t = (c_t^{1-\rho} + \beta E_t U_{t+1}^{1-\rho} (1-\delta)^{(1-\rho)/\rho})^{1/(1-\rho)}$, where $\alpha$ governs the degree of risk aversion and $1/\rho$ the elasticity of intertemporal substitution. The intertemporal marginal rate of substitution depends on future utility, but if expected consumption growth is constant, the discount factor simplifies to $M(g_t) = \beta^\rho \exp(-\alpha g_t)$, where $\beta^\rho = \beta [E \exp(\eta_t)]^{1-\rho}(1-\delta)/(1-\rho)$. Notice that except for a change in the rate of time preference, this has the same form as the power utility model. This affects the expression for the risk-free rate but not for the equity premium. By varying $\rho$, $\beta^\rho$ can be tuned to match the mean risk-free rate, but the model’s implications for the equity premium are the same as those in the text (see Kocherlakota, 1990).
great when stock returns are low, equities would be unattractive as a store of precautionary wealth, and consumers would shy away from them unless compensated for bearing the risk.

3. Background on household consumption data

The empirical strategy is to use data on household consumption to form time-series observations on the cross-sectional variates $\mu_1t$, $\mu_2t$, and $\mu_3t$, and then to estimate the time-averaged moments in the equity premium equation,

$$E \left[ \exp(-\alpha \mu_{1t}) \left( 1 + \frac{\alpha^2}{2} \mu_{2t} - \frac{\alpha^3}{6} \mu_{3t} \right) \right]$$

(20)

and

$$\text{cov} \left[ R_{xt}, \exp(-\alpha \mu_{1t}) \left( 1 + \frac{\alpha^2}{2} \mu_{2t} - \frac{\alpha^3}{6} \mu_{3t} \right) \right].$$

(21)

The test of the model is whether cross-sectional factors can account for the equity premium when $\alpha$ is set at a plausible value. This section describes the data used to construct cross-sectional consumption factors, and the next considers whether they help resolve the puzzle.

3.1. The Consumer Expenditure Survey

Household consumption is measured using data from the Consumer Expenditure Survey (CEX). The CEX is conducted on an ongoing basis by the Bureau of Labor Statistics, who use it primarily to revise the market basket for the Consumer Price Index. In the academic literature, CEX data have been used by (among others) Deaton and Paxson (1994), Attanasio and Weber (1995), Attanasio and Davis (1996), Brav and Geczy (1996), and Vissing-Jorgenson (1998) to study a variety of issues related to inequality, consumption smoothing, consumption insurance, and asset pricing.

The BLS has been conducting the Survey on a quarterly basis since 1980. In each quarter, they interview roughly 5000 households, asking them to report how much they spent on a variety of goods and services in each of the 3 prior months. The 5000 interviews are split more or less evenly over the 3 months of the quarter. In each month, I identify the households being interviewed and sum their expenditures over the prior 3 months. This delivers an overlapping monthly series of quarterly consumption expenditures for each household. I also construct an overlapping monthly series of quarterly stock and bond returns to match the timing of the consumption data. This implies a moving average structure under the null, and the estimators discussed below take this into account. Data are available through the end of 1994, for a total of 180 months or 60 non-overlapping quarters.

The 5000 households are chosen at random so that the sample represents the US population. Apart from attrition, each household remains in the Survey for 5
quarters. Thus, in each quarter, roughly one-fifth of the households drop out and are replaced by a new group. The fact that the CEX is a rotating panel puts some constraints on the kind of approximating models one can entertain. For example, it makes it difficult to investigate models in which long lags of consumption affect current period utility. Another limitation concerns estimation of household-specific parameters. Parameters indexed by \( i \) are unidentified in the cross section (there is a distinct parameter for each cross-sectional unit), and must be estimated from time-series variation. The fact that households participate in the Survey for only 5 quarters means that there is little information of this kind.\(^{10}\)

For my purposes, the Survey’s chief advantage is that it collects expenditure data on a wide variety of goods and services. In contrast, much of the micro literature on consumption (e.g., Hall and Mishkin, 1982; Zeldes, 1989; Altug and Miller, 1990; Cochrane, 1991; Mankiw and Zeldes, 1991; Jacobs, 1999) uses data from the PSID, which reports expenditures only for food. Data on food consumption would be appropriate if preferences were separable between food and other consumption goods, but this condition appears to be problematic. For example, Attanasio and Weber use CEX data to test the separability condition and find strong evidence against it. They also point out that because food is a necessity, it is not representative of consumption as a whole and is poorly suited for investigating issues related to intertemporal substitution or asset pricing. The availability of expenditure data on a variety of goods makes the CEX an attractive alternative to the PSID.

In constructing a summary measure of consumption expenditures, I sought to mimic the national income and product accounts definition of non-durables and services, making adjustments where required by theory or data constraints. Thus, consumption is measured by the sum of expenditures on food, alcohol, tobacco, clothing, household operations, utilities, and non-durable components of transportation expenses. In contrast to the NIPA, expenditures on education and health care were excluded from services because they seem more like durable or investment goods. In addition, pension contributions were excluded because they are a form of savings. Finally, because of problems with top-coding in the CEX, the rental-equivalence measure of housing consumption was excluded. The resulting measure of consumption is similar to that used by Attanasio and Weber.

3.2. Sample selection criteria

The paper analyses two subsets of the CEX, which are summarized in Table 1. The base sample consists of all households that satisfy three conditions. First, they must reside in urban areas. The BLS did not sample rural households in the early years of the Survey, and I discarded them from the later years to maintain consistency. Second, there must be enough information to compute household consumption expenditures. Thus, the correlation between \( R_{xt} \) and \( g_t \) cannot be consistently estimated.
growth. This requires matching households in consecutive quarters, and those for which either \( c_t \) or \( c_{t-1} \) is missing were discarded. Third, households must file complete income reports. There are legitimate concerns about measurement error in the data, especially about whether households are diligent in keeping records and responding to questionnaires. To screen out some of the less diligent, I discarded households who filed incomplete income reports. Income data are not used in the analysis, but I suspect there is a correlation between diligence in reporting income and consumption, and this filter may reduce measurement error in the latter.

The second sample is a subset of the first and seeks to identify households who participate in financial markets. The theory summarized in the previous section applies to households that hold stocks and bonds, and it is well known that many households do not. The CEX reports some (limited) information about asset holdings, and this was used to construct a subsample of households who hold financial assets. Members of the base sample were included in this sample if they satisfied at least one of three additional conditions. First, they were included if they reported holding stocks or bonds. Ideally, we want households that they hold both stocks and bonds, but the Survey does not provide enough information to identify separately stock and bondholders. We can identify only the union of the two sets. Alternatively, households were included if they reported receiving dividend or interest income. Many households report receiving the flow of asset income but not holding the stock of assets, and I interpret this as prima facie evidence of unreported assets. Households were also included if they reported contributions to pension funds, which presumably hold stocks and bonds on their behalf. On average, roughly 40 percent of the households in the base sample qualified for this subsample. In contrast, Mankiw and Zeldes (1991) reported that 27.6 percent of US households hold stocks, and Haliassos and Bertaut (1995) reported an upper bound of 36.8 percent. Thus, the selection criteria may be too inclusive.

Both samples run from 1980.Q2 through 1994.Q4, with two missing quarters. The missing quarters resulted from problems in matching households across interviews. Households were matched using the Survey’s internal identification number, which is supposed to be unique and to remain unchanged across interviews, but in two quarters the matching procedure broke down. One problem occurred with the 1982.Q1 survey, when a very large fraction of matches with 1981.Q4 appeared to be invalid. To cross-validate ID matches, I also compared certain demographic

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Criteria for sample selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Label</td>
<td>Description</td>
</tr>
<tr>
<td>Base sample</td>
<td>All urban households reporting consumption expenditures in consecutive quarters and filing complete income reports</td>
</tr>
<tr>
<td>Stock and bondholders</td>
<td>The subset of households also reporting ownership of stocks or bonds, or receipt of dividend or interest income, or contributions to pension funds</td>
</tr>
</tbody>
</table>
variables in consecutive quarters. For example, household members should have the same race and gender in consecutive quarters, their age should be the same or increase by 1 year, and so on. In most quarters, nearly all the ID matches were confirmed, but in 1982.Q1 almost all were rejected. The reason is unclear, but it may reflect a systematic error in assigning, coding, or extracting household ID numbers (non mea culpa). In any case, because there were few seemingly valid matches, I chose to treat this quarter as missing. Similarly, between the 1985.Q4 and 1986.Q1 surveys, the BLS made some procedural changes and assigned a new set of ID numbers to existing panel members. Because the new and old ID numbers could not be matched, this quarter was also treated as missing.

Whether the missing quarters are problematic for estimating the sample moments in Eq. (2.10) depends on why they are missing. Essentially, we are interested in the vector time-series, \( \{R_{xt}, \mu_2, \mu_3\} \). Excess returns are observed throughout the sample, but the cross-sectional factors have missing elements. The missing data would be problematic (non-ignorable) if the probability of observing \( \mu_2 \) and \( \mu_3 \) were related to \( R_{xt} \) or to their own realizations, but this does not appear to be the case. Instead, it is more likely that the missing values are simply due to internal changes in survey procedures or to errors in assigning or recording household ID numbers. Because it is unlikely that the probability of observing \( \mu_2 \) and \( \mu_3 \) depends on their own realizations, the data can be regarded as “missing at random”, in the terminology of Little and Rubin (1987). Similarly, because it is unlikely that the probability of observing \( \mu_2 \) and \( \mu_3 \) depends on the realization of \( R_{xt} \), the data can be regarded as “observed at random”. Since both conditions are satisfied, the data are “missing completely at random”, and the missing observations are ignorable. In this case, consistent estimates of the moments of interest can be obtained from available-case methods (e.g., see Little and Rubin, 1987, Chapter 3). The missing quarters are therefore only a minor annoyance.

4. Empirical results

4.1. A benchmark first-order model

To establish a benchmark, I begin by reporting results for a version of the model that abstracts from idiosyncratic variation. If we were to ignore the higher-order factors in Eq. (18), the equity premium would be

\[
ER_{xt} = \frac{-\text{cov}[R_{xt}, \exp(-z\mu_1)]}{E[\exp(-z\mu_1)]}.
\]

Roughly speaking, this yields a version of an aggregate consumption-based model.

---

11 Prior researchers have not reported encountering this problem. However, a BLS employee told me that the early surveys were re-processed a few years ago, and he suspects the ID numbers were scrambled when this was done.
A time series for $\mu_{1t}$ was constructed by taking the cross-sectional average of $g_{it}$ period-by-period. Excess returns were measured as the difference between returns on the value-weighted NYSE portfolio and 3-month Treasury bills. The implicit consumption deflator for non-durables and services was used to convert nominal units into real values. The moments $E[\exp(-\alpha \mu_{1t})]$ and $\text{cov}[R_{xt}, \exp(-\alpha \mu_{1t})]$ were estimated by GMM, averaging over available observations, and heteroskedasticity and autocorrelation consistent (HAC) covariance matrices were computed to account for sampling error in the estimates.

Fig. 1 reports the results. The horizontal line illustrates the sample equity premium, and the lines fanning out from the origin represent model-generated values. The center line among the latter shows point estimates as a function of $\alpha$, and the other two lines mark the upper and lower bounds of a centered 90 percent confidence band. Confidence intervals were computed by applying the delta method to Eq. (22) and using the HAC covariance matrices. Results are reported for $\alpha$ ranging from 1 to 15. Estimates based on total household consumption are shown on the left-hand column, and those for consumption per household member are shown on the right.

The figure confirms the well-known conclusion that an aggregate version of the model does not account for the equity premium. Model-generated risk premia are positive but small at low degrees of risk aversion. As $\alpha$ rises, the model becomes somewhat more plausible, as model-generated values rise to around 1.5–2 percent. In addition, the upper bounds of the confidence bands approach 4 percent as $\alpha$
approaches 15. Thus, the burden of explaining the equity premium falls on higher-order factors.

4.2. A second-order model

Next, I consider a model in which the equity premium depends on both the cross-sectional mean and variance, but in which the cross-sectional skewness is suppressed. A time series for $\mu_{2t}$ was constructed by taking the cross-sectional variance on a period-by-period basis, $\mu_{2t} = (1/N) \sum_i (g_{it} - \mu_{1t})^2$. One set of realizations is plotted in Fig. 2, along with realizations for excess returns. The figure uses data on consumption per household members for stock and bondholders, but the other samples are similar. Table 2 summarizes some of its key properties.

One motivation for studying models with incomplete markets is that idiosyncratic variation in consumption may help resolve the volatility puzzle associated with aggregate consumption-based models. This puzzle follows from the results of Hansen and Jagannathan (1991), who derived a lower bound on the variance of stochastic discount factors and demonstrated that there is too little volatility in

![Time Series Plot](image)

![Scatter Plot](image)

Fig. 2. Realizations of the cross-sectional variance.
models based on aggregate consumption. Because household consumption is much more variable, models based on uninsured idiosyncratic risk can resolve this puzzle.

Indeed, there is an enormous amount of volatility in household consumption growth. Table 2 reports that the average cross-sectional variance is around 14 or 15 percent, which means that the quarterly standard deviation is on the order of 35–40 percent! The variance is smaller for stock and bondholders than for all urban households, as the former are presumably better able to smooth idiosyncratic shocks by trading securities. But the difference between the two samples is minor relative to the level of both. This amount of volatility seems too big to be true, and it suggests that there is a great deal of measurement error in the data. I return to this issue below, and attempt to determine whether the results are robust to certain kinds of contamination.

Resolving the volatility puzzle is a step in the right direction, but it is not sufficient for explaining the equity premium. Cochrane and Hansen (1992) and others emphasize that in addition to the volatility puzzle there is also a correlation puzzle. That is, in standard aggregate consumption-based models discount factors are not only too smooth but also too weakly correlated with stock returns. In order to resolve the equity premium puzzle, we need a risk factor that covaries with excess returns, and $\mu_2t$ does not seem promising in this regard.

Table 2 shows that the covariance between excess returns and the idiosyncratic factor is close to zero. Point estimates are on the order of tenths of a basis point, with robust standard errors on the order of 0.8–1.7 basis points. The estimates are expressed as quarterly rates, but multiplying by 4 yields covariances on the order of only 1–3 basis points, with standard errors as large as 7 basis points. At most, it appears that this covariance could be on the order of 10–20 basis points at annual rates. That being the case, $\mu_2t$ does not appear to be a promising candidate for explaining the equity premium.

Fig. 3 confirms this impression. It plugs $\mu_1t$ and $\mu_2t$ into Eq. (18)\(^{12}\) and plots model-generated equity premia as a function of $\alpha$. The horizontal line again marks the sample equity premium. As before, the upper and lower lines fanning out from the origin mark the boundaries of a 90 percent confidence interval, and the center line marks the point estimate. The results are essentially the same as those for the

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**Table 2**

Properties of the second-order factor

<table>
<thead>
<tr>
<th></th>
<th>Total Consumption</th>
<th>Per capita Consumption</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{E}(\mu_2t)$</td>
<td>0.147</td>
<td>0.155</td>
<td></td>
</tr>
<tr>
<td>$\text{cov}(\mu_2t,R_{xt})$</td>
<td>$-0.6$</td>
<td>$-0.6$</td>
<td></td>
</tr>
<tr>
<td>Base sample</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td>$\text{cov}(\mu_2t)$</td>
<td>$0.136$</td>
<td>$0.143$</td>
<td>$0.3$</td>
</tr>
<tr>
<td>Stock and bondholders</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.8)</td>
</tr>
</tbody>
</table>

*Note:* This table reports the mean value of $\mu_2t$ along with its covariance with $R_{xt}$. HAC standard errors shown in parentheses. The units are basis points at quarterly rates.

\(^{12}\)For now, $\mu_3t$ is still suppressed.
benchmark model. Estimated risk premia are positive but small at low degrees of risk aversion, and they increase with $\alpha$. For $\alpha = 15$, the point estimates are around 2 percent, and model upper bounds are in the vicinity of 4–5 percent. The main difference from the benchmark model is that confidence bands fan out a bit more quickly, but high degrees of risk aversion are still needed to generate an economically significant risk premium.

4.3. A third-order model

The next model includes all three cross-sectional moments. Fig. 4 plots a set of realizations of $\mu_{3t}$, based on data on consumption per household members for stock and bondholders, and Table 3 summarizes its properties. On average, the cross sections are a bit skew to the left, as required, but the degree of skewness is only weakly correlated with excess returns. The covariance has the correct sign, but it is small in magnitude. Thus, the individual contribution of $\mu_{3t}$ is small.

Fig. 5 plots the equity premium implied by Eq. (18); this measures the joint contribution of $\mu_{1t}$, $\mu_{2t}$, and $\mu_{3t}$. For stock and bondholders, the point estimates rise to around 1–2 percent for $\alpha = 10$, and then level off. For all households, the point estimates continue to increase with $\alpha$ and reach 4.25 or 4.5 percent for $\alpha = 15$. The estimates are less precise than in the previous models, and model upper bounds increase more rapidly with $\alpha$. For stock and bondholders, model upper bounds approach 4.5–5.75 percent for $\alpha = 15$, and for all households they cross the observed

![Graphs of total household consumption and consumption per household member](image-url)
premium for $\alpha$ around 11 or 12. Therefore, for high degrees of risk aversion, the observed equity premium lies within the upper tail of the model distribution. But for more plausible degrees of risk aversion, say $\alpha \leq 5$, the implied equity premia are bounded above by 2.5 percent.

Table 3
Properties of the third-order factor

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Consumption</th>
<th>Per capita</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E(\mu_{3t})$</td>
<td>$\text{cov}(\mu_{3t}, R_{xt})$</td>
<td>$E(\mu_{3t})$</td>
<td>$\text{cov}(\mu_{3t}, R_{xt})$</td>
</tr>
<tr>
<td>Base sample</td>
<td>$-16.1$</td>
<td>0.4</td>
<td>$-22.8$</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>(5.6)</td>
<td>(0.6)</td>
<td>(6.1)</td>
<td>(0.5)</td>
</tr>
<tr>
<td>Stock and bondholders</td>
<td>$-17.0$</td>
<td>0.1</td>
<td>$-25.8$</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>(7.1)</td>
<td>(0.6)</td>
<td>(7.7)</td>
<td>(0.6)</td>
</tr>
</tbody>
</table>

*Note: This table reports the mean value of $\mu_{3t}$ along with its covariance with $R_{xt}$. HAC standard errors shown in parentheses. The units are basis points at quarterly rates.
4.4. Sensitivity analysis

Finally, I performed two calculations to follow up on questions about the baseline results. One question concerned whether the results are sensitive to gross outliers in the data, which occur in every quarter. To investigate the role of outliers, I computed cross-sectional consumption factors using “Huberized” data, and I found that the results were essentially the same as those reported above. Outliers do not account for the weak correlation between cross-sectional consumption factors and excess returns.

Another question concerned the length of the holding period. The foregoing results are based on a quarterly holding period, but Daniel and Marshall (1997, 1998) suggest that consumption-based asset pricing models may provide a better approximation over longer horizons. Accordingly, I computed results for a 3 quarter holding period, which is the longest for which household consumption growth can be calculated using CEX data. Again, I found that the results were very similar.

5. Measurement error

As noted above, measured consumption growth has a cross-sectional standard error on the order of 35–40 percent per quarter, which is too big to be true. This

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section considers whether the results are robust to measurement error. The first subsection computes a bias adjustment motivated by a particular model of measurement error and a worst-case calibration of the measurement error variance. The second re-estimates cross-sectional factors using an instrumental variable strategy. For the most part, the results are robust, especially for calibrations with low degrees of risk aversion.

5.1. A particular form of measurement error

One generic concern about survey data concerns the diligence (or lack thereof) with which respondents keep records and answer questionnaires. Despite the precautions taken in choosing subsamples and bounding outliers, there are still indications of substantial measurement error. To the extent that measurement error inflates estimates of the cross-sectional variance, it may bias model-generated equity premia toward zero. In this subsection I consider the extent to which the results are robust to this source of bias.

In what follows, I assume that measured consumption growth, \( g_{it} \), equals true consumption growth, \( \gamma_{it} \), plus an error, \( \eta_{it} \):

\[
g_{it} = \gamma_{it} + \eta_{it}. \tag{23}
\]

I also assume that \( \eta_{it} \) has mean zero and that it is distributed independently and symmetrically across households. The independence assumption presumes that the Survey is well designed but that households are negligent in responding. Flaws in survey design would introduce a common element in \( \eta_{it} \) and invalidate this assumption. But if the primary source of measurement error is negligence, errors by one household are likely to be unrelated to errors by others, and in large cross sections the mean error will be zero. Finally, I also assume that \( \eta_{it} \) is independent of the state of the economy. Although the Survey may be noisy, it seems unlikely that the degree of noisiness depends on economic fundamentals.

According to these assumptions, the first- and third-order factors are well-measured in large cross sections,

\[
\hat{\mu}_{1t} = \mu_{1t},
\hat{\mu}_{3t} = \mu_{3t} \tag{24}
\]

and measurement error contaminates the cross-sectional variance,

\[
\hat{\mu}_{2t} = \mu_{2t} + \sigma_{\eta t}^2. \tag{25}
\]

Here \( \sigma_{\eta t}^2 \) represents the measurement error variance at time \( t \). Substituting these expressions into (18) shows how measurement error affects model-generated equity premia:

\[
\mathbb{E}R_{xt} = \frac{-\text{cov}[R_{xt}, \exp(-\alpha \mu_{1t})](1 + \alpha^2/2(\mu_{2t} + \sigma_{\eta t}^2) - (\alpha^3/6)\mu_{3t})]}{\mathbb{E}[\exp(-\alpha \mu_{1t})(1 + \alpha^2/2(\mu_{2t} + \sigma_{\eta t}^2) - (\alpha^3/6)\mu_{3t})]} \tag{26}
\]

The measurement error variance appears in both the numerator and the denominator. In the numerator, it affects the equity premium only to the extent
that it is correlated with excess returns. Focusing on this term, to reverse the results reported above, the Survey would have to be more accurate in periods when excess returns are low. While households may vary in the degree of care they exercise in responding to the Survey, it seems unlikely that the degree of diligence varies with the state of the economy. The choice of how much care to exercise may depend on the opportunity cost of time, but measures of opportunity cost such as real wages are themselves weakly correlated with the stock market. If measurement error is independent of the state of the economy, as assumed above, then \( \sigma^2 \) drops out of the numerator and (26) simplifies to

\[
E_{xt} = -\frac{\text{cov}(R_{xt}, \exp(-\alpha \mu_{1t})(1 + (\alpha^2/2)\mu_{2t} - (\alpha^3/6)\mu_{3t}))}{E[\exp(-\alpha \mu_{1t})(1 + \alpha^2/2(\mu_{2t} + \sigma^2_{\mu t}) - (\alpha^3/6)\mu_{3t})]}.
\]

(27)

According to this expression, we need only consider whether there is enough contamination in the denominator to reverse our conclusions.

If we assume that measurement error inflates the average variance by a factor \( k \),

\[
E\mu_{zt} = k E\mu_{zt},
\]

(28)

then (27) can be written in terms of observables as

\[
E_{xt} = -\frac{\text{cov}(R_{xt}, \exp(-\alpha \mu_{1t})(1 + (\alpha^2/2)\hat{\mu}_{2t} - (\alpha^3/6)\hat{\mu}_{3t}))}{E[\exp(-\alpha \mu_{1t})(1 + \alpha^2/2(\mu_{2t} + \sigma^2_{\mu t}) - (\alpha^3/6)\hat{\mu}_{3t})]}.
\]

(29)

To calibrate \( k \),\(^{14}\) suppose that the degree of measurement error is independent of age and that the log of household consumption is a random walk. Then, the increase in the cross-sectional dispersion of consumption in levels over the life cycle provides an unbiased estimate of \( \mu_{2t} \). According to estimates by Deaton and Paxson (1994), the within-cohort, cross-sectional variance for consumption increases by 69 basis points per year, implying that \( E(\mu_{2t}) \approx 0.0017 \) per quarter. In contrast, the measured cross-sectional variance for consumption growth is roughly 0.14 per quarter, which implies that measurement error accounts for more than 98 percent of the measured variation! Accordingly, I set \( k = 0.14/0.0017 \) and use (29) to calculate a bias-corrected estimate of the equity premium. The results are shown in Fig. 6, which reports estimates for \( \alpha \) ranging from 1 to 10.

Measurement error is not important in calibrations with low risk aversion. For example, for stock and bondholders the point estimates remain below 1.6 percent for \( \alpha \leq 5 \), and the upper bounds are 3.7 percent or less for \( \alpha \leq 4.5 \). To develop some intuition for the small \( \alpha \) case, notice that a high degree of measurement error undermines the model’s ability to satisfy the volatility bound when consumers are not too risk averse. The promise of models like this is based in part on the fact that household consumption growth is more volatile than aggregate consumption growth. But if most of the measured volatility is noise and household consumption

\(^{14}\)I am grateful to the referee for suggesting this calibration.
growth is actually only a bit more volatile than in the aggregate, then the model will not deliver on this promise. Fullfilling the volatility bound is necessary for satisfying the Euler equation, and if the model fails in the former dimension it must also fail in the latter.

On the other hand, measurement error is important in calibrations involving higher values of $\alpha$, and it becomes increasingly problematic as $\alpha$ increases. As shown in the figure, model-generated equity premia and their standard errors increase rapidly with $\alpha$. For stock and bondholders, the point estimates cross the observed premium for $\alpha = 9.5$. More importantly, the confidence bands fan out very quickly, and the calculations become uninformative for $\alpha > 5$. For example, for stock and bondholders a centered 90 percent confidence interval ranges from $-2.3$ to $7$ percent for $\alpha = 6$, and it encompasses the interval $-8.7$ to 24 percent for $\alpha = 10$.

Thus, the results are inconclusive $\alpha \geq 6$. The combination of idiosyncratic risk and a moderate degree of risk aversion could account for the equity premium, provided that most of the measured variation is due to noise. Fig. 6 assumes that it is, but it is hard to say whether the data are actually this noisy.

Indeed, estimates reported in the next section suggest that this calibration is better interpreted as an upper bound on the degree of measurement error. To jump ahead, the next section uses an instrumental variable to “clean” the data before constructing cross-sectional factors. The variance of the fitted values from the data-cleaning regression provides a lower bound on the variance of actual consumption growth, and this lower bound is about 5 percent greater than the value inferred from Deaton...
and Paxson’s estimate.\textsuperscript{15} Thus, Fig. 6 should be interpreted as a worst-case scenario, illustrating the effects of the maximum potential degree of contamination. In the worst case, the results are robust for $z \leq 5$, but they are inconclusive for higher values.

If the inconclusive range were restricted to values of $z$ that are grossly implausible a priori, we could stop here. But since the inconclusive range includes some plausible values of $z$, I also consider another complementary approach to dealing with measurement error.

5.2. Job loss as an instrumental variable

Accordingly, I consider an alternative strategy that uses an instrumental variable to clean the consumption data before estimating cross-sectional moments. In this context, an instrument should be correlated with true consumption growth but uncorrelated with the measurement error. Thus, I seek a variable that is associated with uninsured movements in consumption, but which is relatively well-measured and unlikely to be reported with error.

The results of Cochrane (1991) and Mace (1991) suggest a candidate. Both papers report that job loss is uninsurable and correlated with idiosyncratic variation in consumption. But because changes in employment status are memorable (indeed, sometimes traumatic), they are likely to be reported more accurately than consumption. Hence, changes in job status are likely to be uncorrelated with measurement error in consumption growth.

In what follows, I continue to assume that the measurement error is additive in logs, has mean zero, and is independent across households. According to these assumptions, average consumption growth is well measured, and the error contaminates the idiosyncratic component:

$$\frac{g_{it}}{C_{0 m}} = \left( \frac{g_{it}}{C_{0 m}} - \mu_{1t} \right) + \eta_{it}.$$  \hfill (30)

To clean the data, I project the measured idiosyncratic component, $g_{it} - \mu_{1t}$, onto variables measuring changes in employment status, and then construct cross-sectional factors from the fitted values.

Following Mace (1991), I construct a set of dummy variables to measure changes in job status. The Survey collects information on employment status in the first and last interviews in which a household participates. There are eight possible outcomes, depending on the behavior of the reference person, spouse, and others in the household: (1) the reference person is employed but the spouse and others are not; (2) the reference person and spouse are both employed; (3) all three are employed; (4) the reference person and others are employed but the spouse is not; (5) only the spouse is employed; (6) the spouse and others are employed; (7) only others are

\textsuperscript{15} Of course, this interpretation rests on the assumption that the instrument is valid. If it were sufficiently highly correlated with measurement error in consumption growth, the variance of the fitted value could exceed the true variance of consumption growth. My reasons for assuming that the instrument is uncorrelated with the noise in consumption are given below.
employed; and (8) no one is employed. By combining this information across interviews, one can construct a set of $8^2$ dummy variables summarizing all possible pairs of job status outcomes.

Because the employment status variable is collected only in the first and last interviews, the variable measuring changes in job status span an interval of 3 quarters. Hence, the appropriate measure of consumption growth is $\log(\frac{c_{it}}{c_{it-3}})$, and the appropriate holding period for stocks and bonds is 3 quarters. The IV estimates are most closely related to the longer horizon results mentioned above.

The data-cleaning regressions take the following form:

$$\log(\frac{c_{it}}{c_{it-3}}) = \theta_0 + \sum_{j=1}^{64} \theta_j E_{ijt} + u_{it}, \quad (31)$$

where $E_{ijt}$ is the $j$th job status dummy for household $i$ in period $t$. The results of these regressions are similar to those reported by Mace. The coefficients are highly jointly significant and for the most part have intuitively plausible signs. The employment dummies account for 2–3 percent of the variation in measured consumption growth,\textsuperscript{16} and the implied volatility of fitted values is more plausible. For example, in the raw data $\log(\frac{c_{it}}{c_{it-3}})$ has a cross-sectional standard deviation of roughly 50 percent at an annual rate, but the fitted values from this regression have a cross-sectional standard deviation of about 8.5 percent at an annual rate.\textsuperscript{17}

Fig. 7 reports results for a three-factor model in which the fitted values from these regressions are used to compute cross-sectional moments. The results are similar to those reported in Section 4. The idiosyncratic factors still covary weakly with excess returns, and model equity premia are 1 percent or less for $\alpha < 10$. As $x$ approaches 15, the point estimates increase to 2 or 2.5 percent, depending on the sample and measure of consumption, and the upper confidence bounds approach 5 percent.

6. Conclusion

This paper uses household consumption data to investigate whether uninsured idiosyncratic risk accounts for the equity premium. The analysis is relevant to models like the one in Constantinides and Duffie in which households can smooth consumption over time but not across all states of nature. Following Mankiw, the paper develops an equilibrium factor model in which risk premia are related to the covariance between an asset’s return and moments of the cross-sectional distribution for consumption growth. Data from the Consumer Expenditure Survey are used to

\textsuperscript{16}The low $R^2$ statistics do not necessarily imply that these are poor instruments, for they represent the percentage of the variance of the error-ridden consumption measure, not of true consumption growth.

\textsuperscript{17}The Deaton–Paxson calibration implies a cross-sectional standard deviation of 8.25 percent, which is less than that of the fitted values. Since the variance of actual consumption growth must be at least as large as the variance of the fitted values of the regression, it follows that the Deaton–Paxson calibration understates the variance of actual consumption growth and overstates the measurement error variance. This is why I interpret the calibration as a worst-case scenario.
construct a time series of cross-sectional moments, and risk factors are computed by estimating their covariance with excess stock returns.

In one respect, the results confirm a common intuition. Household consumption is far more volatile than aggregate consumption, and accounting for this idiosyncratic variation helps resolve Hansen and Jagannathan’s volatility puzzle. But cross-sectional consumption factors are weakly correlated with excess returns, and at low degrees of risk aversion they generate equity premia of 2 percent or less. Furthermore, for preferences involving low degrees of risk aversion, the results are robust to measurement error.

The analysis abstracts from transactions costs, borrowing and short-sales constraints, and other asset market frictions. Within the class of frictionless pricing models, the results suggest that idiosyncratic risk is not a promising candidate for explaining the equity premium. They leave open the possibility that the puzzle can be resolved by combining market frictions with uninsured idiosyncratic shocks.

References


