INTEREST RATES AND CURRENCY PRICES
IN A TWO-COUNTRY WORLD

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This paper is a theoretical study of the determination of prices, interest rates and currency exchange rates, set in an infinitely-lived two-country world which is subject both to stochastic endowment shocks and to monetary instability. Formulas are obtained for pricing all equity claims, nominally-denominated bonds, and currencies, and these formulas are related to earlier, closely related results in the theories of money, finance international trade.

1. Introduction

This paper is a theoretical study of the determination of prices, interest rates and currency exchange rates, set in an infinitely-lived two-country world which is subject both to stochastic endowment shocks and to monetary instability. The objectives of the study, or more exactly, the limits to the study's objectives, are in large measure dictated by the nature of the model's simplifying assumptions. In this introduction, then, I will first describe the common features of the models themselves, and then consider the range of substantive questions on which these models seem likely to shed some light.

In its real aspects, the model is a variation on that developed in Lucas (1978). Traders of both countries are identical, with preferences defined over the infinite stream of consumption goods. Goods are non-storable, arriving as unproduced endowments, following a Markov process. Agents are risk averse, so they will be interested in pooling these endowment risks, and since they have identical preferences, an equilibrium in which all agents hold the same portfolio will, if ever attained, be indefinitely maintained. This perfectly

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1See also Breeden (1979), Bröck (1979), Cox et al. (1978), Danthine (1977) and LeRoy (1973). Much of this literature can be traced back to Merton (1973), to which the reader with deeper genealogical interests is referred.
pooled equilibrium is the one studied, in various forms, below. Since equilibrium quantities consumed are, in this exchange system, dictated by nature, the analysis of the real system involves simply reading the Arrow–Debreu securities prices off the appropriate marginal rates of substitution. This is carried out in section 2.

In section 3, a single ‘world’ currency is introduced, with its use motivated by a ‘finance constraint’ of the form proposed by Clower (1967) and Tsiang (1956), to the effect that goods must be purchased with currency accumulated in advance of the period in which trading takes place. With a constant supply of money, or currency, the real aspects of equilibrium replicate exactly those of the barter equilibrium of section 2. When the money supply is stochastic, the formulas for securities prices require modification.

Section 4 introduces national currencies, together with a free market or ‘flexible exchange rate’ system under which currencies may be traded, along with other securities, prior to shopping for goods. In section 5, the consequences of imposing a specific form of exchange rate fixing are examined. The normative conclusion reached from comparing these two regimes is a reproduction of the equivalence result reached earlier, and for basically identical reasons, by Helpman (1979). Concluding comments are contained in sections 6 and 7.

The aspirations of this study are difficult to assess, for it is in some respects highly ambitious and, in others, very modest. The framework here proposed provides one way of integrating monetary theory, domestic and international, with the powerful apparatus of modern financial economics. It is capable of replicating all of the classical results of monetary theory as well as the main formulas for securities pricing that the theory of finance produces, and of suggesting modifications to the latter theory suited to an unstable monetary environment. There is little doubt that the main task of monetary economics now is to catch up with our colleagues in finance, though the question of how this may best be done must be regarded as considerably more open.

On the side of modesty, it must be conceded that when this integration is carried out as is done here, many, perhaps most, of the central substantive questions of monetary economics are left unanswered. These failings will appear below more nakedly than is customary in the monetary literature, so much so that they may well appear to be failings of the particular approach taken here as opposed to those of this literature in general. I do not believe this to be the case.

I take the term ‘finance constraint’ from Kohn (1980), who traces the history of what I had been calling the ‘Clower constraint’ back to important earlier contributions by Robertson (1940) and Tsiang (1956), as well as forward to Tsiang’s (1980) recent paper. Kohn’s paper, which does not in any way detract from Clower’s (1967) contribution also deals decisively with some common criticisms of this point of departure in monetary theory.
2. A barter model

Though the main concern of this paper is with alternative monetary arrangements, it is convenient to begin with an analysis of a barter equilibrium. The demography, technology and preferences of this barter economy will remain unchanged in the monetary variations discussed later.

Consider a world economy with two countries. These countries have identical constant populations; all variables will then be expressed in per (own country) capita terms. Each citizen of country 0 is endowed each period with \( \xi \) units of a freely transportable, non-storable consumption good, \( x \). Each citizen of country 1 is endowed with \( \eta \) units of a second good, \( y \). These endowments \( \xi \) and \( \eta \) are stochastic, following a Markov process with transitions given by

\[
\Pr \{ \xi_{t+1} \leq \xi', \eta_{t+1} \leq \eta' \mid \xi_t = \xi, \eta_t = \eta \} = F(\xi', \eta', \xi, \eta).
\]

Assume that the process \( \{\xi_t, \eta_t\} \) has a unique stationary distribution \( \Phi(\xi, \eta) \). The realizations \( \xi, \eta \) are taken to be known at the beginning of the period, prior to any trading, but no information (other than full knowledge of \( F \)) is available earlier.

Each agent in country 1 wishes to maximize

\[
E \left\{ \sum_{i=0}^{\infty} \beta^i U(x_{it}, y_{it}) \right\}, \quad 0 < \beta < 1,
\]

where \( x_{it} \) is consumption in country 1 in period \( t \) of the good \( x \), and \( y_{it} \) is consumption of the good \( y \). The function \( U \) and the discount factor \( \beta \) are common to both countries. \( U \) is assumed to be bounded, continuously differentiable, increasing in both arguments, and strictly concave. The remainder of the paper will be concerned with resource allocation in this abstract world under alternative market arrangements.

The arrangement considered in this section is one of complete markets in the sense of Arrow (1964) and Debreu (1959), under which agents trade in both goods, spot and in advance, contingent on all possible realizations of the shock process \( \{\xi_t, \eta_t\} \). In setting out the notation for such an equilibrium, I will exploit the simplicity of the present set-up to the full.

The preferences of agents have been assumed independent of their nationalities, so that agents differ, if at all, only in their endowments. Moreover, agents are risk-averse so that in the face of stochastically varying endowments, one would expect them to use available securities markets to pool these risks. In this context, pooling must come down to an exchange of claims on 'home' endowment for claims on 'foreign' endowment in return. Perfect pooling, in this sense would involve agents of each country owning half the claims to 'home' endowment and half of the foreign endowment. The
equilibrium constructed below is one in which agents begin perfectly pooled in this sense and remained so pooled under all realized paths of the disturbances. Under these circumstances, the world economy becomes virtually identical to that studied in Lucas (1978), with a single representative consumer consuming half of the endowments of both goods, or \( \left( \frac{x}{2}, \frac{y}{2} \right) \) each period, and holding the 'market portfolio' of such securities as are traded. Our analytical task will be to price these securities.

Let \( s = (\xi, \eta) \) be the current state of the system. Take the price of all goods, current and future, to be functions of the current state \( s \), with the understanding that prices are assumed stationary in the sense that the same set of prices is established at \( s \) independent of the calendar time at which \( s \) may be realized. Then knowledge of the equilibrium price functions together with knowledge of the transition function \( F(s', s) = F(\xi', \eta', \xi, \eta) \) amounts to knowledge of the probability distribution of all future prices, or rational expectations. In what follows, agents are assumed to have such knowledge.

In view of the simplicity of the model under study, it is evident that although all Arrow-Debreu contingent claim securities can be priced, only a very limited set of securities is needed to represent the 'market portfolio' that traders will hold in equilibrium. I will proceed under the following, wholly arbitrary, conventions as to which goods and securities will be traded, indicating at various points below how other securities may easily be priced as well.

For a system in any current state \( s \), let the current spot price of good \( x \) be unity, so that all other prices will be in terms of current \( x \)-units. Let \( p_x(s) \) be the spot price of good \( y \), in \( x \)-units, if the system is in state \( s \). Let \( a_x(s) \) be the current \( x \)-unit price of a claim to the entire future (from tomorrow on) stream \( \{ \xi_t \} \) of the endowment of good \( x \), and \( q_x(s) \) the current price of a claim to the future stream \( \{ \eta_t \} \).

With these conventions set, consider an individual trader entering a period endowed with \( \theta \) units of wealth, in the form of claims to current and future goods, valued in current \( x \)-units. His objects of choice are current consumptions \( (x, y) \), at spot prices \( (1, p_y(s)) \), equity shares \( \theta_x \) in future endowments \( \{ \xi_t \} \) at the price per share \( q_x(s) \), and shares \( \theta_y \) in future \( \{ \eta_t \} \), priced at \( q_y(s) \). His budget constraint is thus

\[
x + p_x(s)y + q_x(s)\theta_x + q_y(s)\theta_y \leq \theta.
\]

For a given portfolio choice \( (\theta_x, \theta_y) \), his wealth in \( x \)-units as of the beginning of the next period will, if next period's state is \( s' \), be given by

\[3\text{This restriction of the analysis to a particular stationary equilibrium obviously must leave open questions involving the stability of equilibrium, or of whether a system beginning with agents imperfectly pooled would tend over time to approach the perfectly pooled equilibrium studied below. For reasons given in Lucas and Stokey (1982) and Nairay (1981), time-additive preferences of the form (2.1) probably imply a negative answer to this stability question.}\]
\[ \theta' = \theta_x[\xi' + q_x(s')] + \theta_y[p_y(s')\eta' + q_y(s')]. \]  

(2.3)

With this investment in notation, one can write out a functional equation for the value \( v(\theta, s) \) of the objective (2.1) for a consumer (of either nationality) who finds himself in state \( s \) with wealth \( \theta \) and proceeds optimally. It is

\[
v(\theta, s) = \max_{x, y, \theta, s'} \{ U(x, y) + \beta \int v(\theta', s') f(s', s) ds' \},
\]

(2.4)

subject to the constraint (2.2), where \( \theta' \) is given by (2.3) and where \( f \) is the transition density for the transition function \( F. \)

The first order conditions for this problem are (2.2), with equality, and

\[
U_x(x, y) = \lambda,
\]

(2.5)

\[
U_y(x, y) = \lambda p_y(s),
\]

(2.6)

\[
\beta \int v(\theta', s') [\xi' + q_x(s')] f(s', s) ds' = \lambda q_x(s),
\]

(2.7)

\[
\beta \int v(\theta', s') [p_y(s')\eta' + q_y(s')] f(s', s) ds' = \lambda q_y(s).
\]

(2.8)

Moreover, we know that the multiplier \( \lambda \) is the derivative of the maximized objective function \( v(\theta, s) \) with respect to the right-hand side of (2.2), or that

\[
v_\theta(\theta, s) = \lambda.
\]

(2.9)

In a perfectly-pooled equilibrium, we know that each trader consumes his share of both endowments, so that \((x, y) = (\zeta, \eta)\). Hence from (2.5) and (2.6), the equilibrium spot prices of \( y \) in terms of \( x \) is

\[
p_y(s) = U_y(\zeta, \eta)/U_x(\zeta, \eta) = U_y(s)/U_x(s),
\]

(2.10)

where the second equality defines a shorthand that will be used frequently below.

Also in equilibrium, each trader begins and ends a period with the identical portfolio of equity claims \( \theta_x = \theta_y = \frac{1}{2} \). Then from (2.3), (2.5), (2.7) and (2.9), shares in the \( \{\xi_t\} \) process are priced by

\[
q_x(s) = \beta [U_x(s)]^{-1} \int U_x(s')[\xi' + q_x(s')] f(s', s) ds'.
\]

(2.11)

Symmetrically (almost) from (2.5), (2.5), (2.8) and (2.9) shares in the \( \{\eta_t\} \) process are priced by

*For a rigorous treatment of an equation essentially identical to (2.4), see Lucas (1978). I am proceeding here at a much less formal level.
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\[ q_x(s) = \beta[U_x(s)]^{-1} \int U_x(s')[p_y(s') \eta' - q_y(s')] f(s', s) \, ds'. \]  

(2.12)

These formulas may be compared to their counterpart (6) in Lucas (1978). Either may be solved 'forward' to give the current price in terms of future dividends only. Thus from (2.11)

\[ q_x(s) = [U_x(s)]^{-1} \sum_{t=1}^{\infty} \beta^t E[\xi_t U_x(s_t) | s_0 = s]. \]  

(2.13)

Eq. (2.2) may similarly be solved 'or \( q_x(s) \). If \( U_x(s) \) were constant, (2.11) and (2.13) would be entirely familiar theoretical relationships between equity prices and dividends.

In addition to pricing equities, this theory can price any one-period Arrow-Debreu security. Thus let \( A \) be any \( s' \)-set to which \( F(s', s) \) assigns probability, let \( \chi_A(s') \) be the characteristic function of this set (\( \chi_A(s') = 1 \) if \( s' \in A \) and 0 otherwise) and let \( q^{(1)}(A, s) \) be the price today, if today's state is \( s \), of a claim to one unit of good \( x \) tomorrow if \( s' \in A \) and 0 otherwise. If it is possible to purchase \( z \) units of such a security (or sell \( -z \) units) the consumer's problem (2.4) becomes

\[ v(\theta, s) = \max_{x, y, \theta_y, \theta_z} \{ u(x, y) + \beta \int v(\theta', s') f(s', s) \, ds' \}, \]  

(2.14)

subject to, in place of (2.2),

\[ x + p_y(y) y + q_x(s) \theta_x + q_y(s) \theta_y + q^{(1)}(A, s) z \leq \theta \]  

(2.15)

and with tomorrow's wealth \( \theta' \) given by

\[ \theta' = \theta_x[\xi' + q_x(s')] + \theta_y[p_y(s') \eta' + q_y(s')] + z \chi_A(s'), \]  

(2.16)

in place of (2.3). The first-order condition for \( z \) for this problem is

\[ \beta \int v(\theta', s') \chi_A(s') f(s', s) \, ds' = \lambda q^{(1)}(A, s). \]  

(2.17)

Now the equilibrium level of \( z \) must be zero, since all \( x \)-units are already claimed by equity holders, so all other equilibrium prices are as determined above. Hence applying the facts \( \lambda = v_{\theta}(\theta, s) = U_x(s) \) to (2.17) gives

\[ q^{(1)}(A, s) = \beta[U_x(s)]^{-1} \int A_U(x(s')) f(s', s) \, ds'. \]  

(2.18)

It will be convenient below to have a notation for the 'density function' \( q(s', s) \) corresponding to the function \( q^{(1)}(A, s) \). Let
\[ q(s', s) = \beta [u_x(s)]^{-1} u_x(s') f(s', s), \text{ so that} \]

\[ q^{(1)}(A, s) = \int_A q(s') \, ds'. \tag{2.20} \]

Loosely, \( q(s', s) \) prices a claim to one unit of \( x \) contingent on next period's state being \( s' \), today's state being \( s \).

Having priced one-period securities in (2.18), the recursive character of the model can be used to price \( n \)-period securities via the Markovian formula

\[ q^{(n)}(A, s) = \int q^{(n-1)}(A, u) q^{(1)}(du, s), \quad n = 2, 3, \ldots, \tag{2.21} \]

or, in terms of the density \( q(s', s) \).

\[ q^{(n)}(A, s) = \int q^{(n-1)}(A, u) q(u, s) \, du, \quad n = 2, 3, \ldots. \tag{2.22} \]

Here \( q^{(n)}(A, s) \) is the price, if today's state is \( s \), of a unit of good \( x \) \( n \) periods hence contingent on the system's being in a state in \( A \) at that date.

In addition to pricing all claims to returns made risky by nature, the theory can price arbitrary, man-made lotteries. Thus let \( g(u, s', s) \) be a probability density for \( u \), conditioned on \( (s', s) \), and let it be possible to purchase or sell at the price \( r(s) \) per unit, \( z \) units of a claim to \( u \) units of \( x \) delivered tomorrow, where \( u \) is drawn from \( g(u, s', s) \). Then by reasoning identical to that leading to the formula (2.18) one arrives at the lottery ticket price formula:

\[ r(s) = \beta [U_x(s)]^{-1} \int U_x(s') u g(u, s', s) f(s', s) \, du \, ds'. \tag{2.23} \]

Notice that if \( u \) and \( s' \) are independent the integral on the right-hand side of (2.23) factors and, recalling (2.19) one obtains

\[ r(s) = \int q(s', s) f(s', s) \, ds' \cdot \int u g(u, s) \, du. \tag{2.24} \]

That is, the price of a lottery ticket is the price of one unit of future \( x \), with certainty, times the mean return (in units of \( x \)) per lottery ticket. Where is the risk premium associated with the variability of \( u \)? It is absent, as it should be, since in a competitive market no one is in a position to impose risk on anyone else, and no premium need be charged for risks not borne.

### 3. Monetary models

The preceding section provides a complete theory of equilibrium goods and securities pricing for a two-good, barter exchange economy. The
remainder of the paper considers a variety of alternative monetary arrangements for this same world economy. In all models studied, the use of 'money' or 'currency' will be motivated by a constraint imposed on all traders to the effect that goods can be purchased only with currency accumulated in advance. The idea, as sketched in Lucas (1980), is that under certain circumstances currency can serve as an inexpensive record-keeping device for decentralized transactions, enabling a decentralized system to imitate closely a centralized Arrow–Debreu system. I will not elaborate on these features of the technology that make 'decentralized' exchange economical, relative to 'centralized'.

The timing of trading is taken to be the following. At the beginning of a period, traders from both countries meet in a centralized marketplace, bringing securities and currency holdings previously accumulated, and engage in perfectly competitive securities trading. Before the trading opens, the current period's real state, \( s = (\varepsilon, \eta) \), is known to all, as are any current monetary shocks. At the conclusion of securities trading, agents disperse to trade in goods and currencies. I find it helpful to think of each trader as a two-person household, in which one partner harvests the endowment and sells it for currency to various strangers while the other uses the household's currency holdings to purchase goods from other strangers, with no possibility of intra-day communication between them, but this little story plays no formal role in the analysis. At the end of a period, agents consume their goods and add cash receipts from endowment sales to their securities holdings.

Given this timing of trading, and given the presence of any securities earning a positive nominal return in some currency, it is evident that agents will hold non-interest-bearing units of that currency in exactly the amount needed to cover their perfectly predictable current-period goods purchases. This extremely sharp distinction between 'transactions' and 'store of value' motives for holding various assets is, for some purposes, much overdrawn, but for other purposes it is extremely convenient, as it collapses current period 'goods demand' and 'currency demand' into a single decision problem.

In the economy under study, let \( M_t \) nominal dollars per capita (of each country, or \( 2M_t \) in total) be in circulation, so that there is a single 'world' money, and the world economy behaves, as in section 2, as a single two-good system. Prior to any trading in period \( t \), let each trader's money holdings be augmented by a lump-sum dollar transfer of \( w_t M_{t-1} \), so that the money supply evolves according to

\[
M_{t+1} = (1 + w_{t+1})M_t. \tag{3.1}
\]

See Howitt (1974) and Lucas (1980) for scenarios which try to make this reference to a decentralized exchange of money and goods more concrete and hence better motivated for present purposes.
That is, $M_i$ denotes the post-transfer, pre-trading per capita supply of money for period $t$. Let $\{w_t\}$ follow a Markov process, possibly related to the real process $\{s_t\}$, with the transition function

$$H(w', w, s', s) = \Pr\{w_{t+1} \leq w' \mid w_t = w, s_{t+1} = s', s_t = s\}$$

and a corresponding transition density $h(w', w, s', s)$. Think of $w_t$ as being known, along with $s_t$ prior to any period $t$ trading.

Now let $p_x(s, M)$ be the dollar price of a unit of good $x$, when the real state of the economy is $s$ and when post-transfer dollar balances are $M$, and let $p_x(s)$ be the relative price of $y$ in terms of $x$-units. Since all currency is, by hypothesis, spent on current goods, we have

$$2M = p_x(s, M)[\xi + p_y(s)\eta]$$

so that nominal prices follow:

$$p_x(s, M) = \frac{2M}{\xi + p_y(s)\eta}.$$  \hspace{1cm} (3.2)

This is the unit-velocity version of the quantity theory of money to which the Clower constraint leads in the absence of a 'precautionary motive' for money holding.

To determine the behavior of equilibrium goods and securities prices, i will seek an equilibrium, analogous to that constructed in section 2, in which agents from both countries begin in a situation of equal wealth and maintain this situation over time. Let there be two securities traded, in addition to currency: a perfectly divisible claim to all of the dollar receipts from the current and future sale of the process $\xi$, priced (in $x$-units) at $q_x(s, w)$, and a claim to the $\eta$ process, priced at $q_y(s, w)$.

Now consider a resident of either country, beginning a period with post-transfer claims of $x$-unit value $\theta$. Let the world be in state $(s, w, M)$ and denote the agent’s optimum value function by $v(s, w, M, \theta)$. His initial decision, as he engages in securities trading, is to divide $\theta$ among a portfolio $(\theta_x, \theta_y)$ of equity claims, at the prices $q_x(s, w)$ and $q_y(s, w)$, and dollars of currency $m$ at the price $[p_x(s, M)]^{-1}$ given in (3.2). In this choice, he faces the constraint

$$\frac{m}{p_x(s, M)} + q_x(s, w)\theta_x + q_y(s, w)\theta_y \leq \theta.$$  \hspace{1cm} (3.3)

After completing securities trading, he uses currency to finance goods purchases $(x, y)$ at the $x$-unit prices $(1, p_y(s))$. Thus his finance constraint is
\[ x + p_x(y) y \leq \frac{m}{p_x(s, M)}. \]  

(3.4)

A given set of choices \( m, \theta_x, \theta_y, x \) and \( y \) will dictate a beginning-of-next-period asset position \( \theta' \) as follows. His sources of funds in dollars are unspent currency carried over from the current period, \( m - p_x(s, M)(x + p_x(s)y) \), dividends and the new market value of his \( \{ \xi \} \) holdings, \( \theta_x[p(s, M) \xi + p_x(s', M') q_x(s', w')] \), dividends and the market value of his \( \{ \eta \} \) holdings, \( \theta_y[p_x(s, M)p_y(s) \eta + p_x(s', M') q_y(s', w')] \), and his next-period money transfer \( w' M \). Since \( \theta' \) is measured in \( x \)-units, each of these terms must be deflated by \( p_x(s', M') \). Then

\[
\begin{align*}
\theta' &= \frac{1}{p_x(s', M')} \left[ m - p_x(s, M)(x + p_x(s)y) \right] \\
&+ \frac{p_x(s, M)}{p_x(s', M')} \left[ \xi \theta_x + p_x(y) \eta \theta_y \right] + q_x(s', w') \theta_x + q_y(s', w') \theta_y \\
&+ \frac{w'M}{p_x(s', M')}.
\end{align*}
\]

(3.5)

The monetary analogue to (2.4) is then

\[
\nu(s, w, M, \theta) = \max_{x, y, m, \theta_x, \theta_y} \left\{ U(x, y) + \beta \int \nu(s', w', M', \theta') dF dH \right\},
\]

subject to (3.3) and (3.4), where \( \theta' \) is given by (3.5) and where \( dF \) and \( dH \) abbreviate \( f(s', s) ds' \) and \( h(s', s, w', w) dw' \), respectively.

Now \( m \) can be eliminated between (3.3) and (3.4) to give

\[
x + p_x(s)y + q_x(s, w) \theta_x + q_y(s, w) \theta_y \leq \theta.
\]

(3.7)

If the finance constraint (3.4) is binding in all states, the first term on the right-hand side of (3.5) will be zero. Replacing \( p_x(s) \) with the values given at \( (s, M) \) and \( (s', M') = (s', M(1 + w')) \) by (3.2), (3.5) can be replaced by

\[
\begin{align*}
\theta' &= \frac{\xi' + p_x(s') \eta'}{(1 + w')(\xi' + p_x(s') \eta')} \left[ \xi \theta_x + p_x(s) \eta \theta_y \right] + q_x(s', w') \theta_x \\
&+ q_y(s', w') \theta_y + \frac{1}{1 + w'} (\xi' + p_x(s') \eta').
\end{align*}
\]

(3.8)

With these simplifications, it is clear that \( \nu(s, w, M, \ell) \) does not depend on \( M \), and (3.6) can be replaced by
\[ v(s, w, \theta) = \max_{x, y, \theta_x, \theta_y} \{ U(x, y) + \beta \int v(s', w', \theta') dF dH \} \quad (3.9) \]

subject to (3.7) and with \( \theta' \) given by (3.8).

The first-order conditions for this problem are

\[ U_x(x, y) = \lambda, \quad (3.10) \]
\[ U_y(x, y) = \lambda p_x(s), \quad (3.11) \]
\[ \beta \int v(s', w', \theta') \left[ q_x(s', w') + \frac{\xi' + p_x(s')\eta'}{1 + w'} \frac{\xi}{\xi + p_x(s)\eta} \right] dF dH = \lambda q_x(s, w), \quad (3.12) \]
\[ \beta \int v(s', w', \theta') \left[ q_y(s', w') + \frac{\xi' + p_x(s')\eta'}{1 + w'} \frac{p_y(s)\eta}{\xi + p_x(s)\eta} \right] dF dH = \lambda q_y(s, w). \quad (3.13) \]

In addition

\[ v(s, w, \theta) = \lambda \quad (3.14) \]

holds. These are analogues to (2.5)–(2.9).

In the equilibrium here conjectured, quantities of current goods are \((x, y) = (\frac{1}{2} \xi', \frac{1}{2} \eta')\) and a trader beginning a period with the equity holdings \((s, w) = (\frac{1}{2}, \frac{1}{2})\) will choose to end with \((\theta_x, \theta_y) = (\frac{1}{2}, \frac{1}{2})\). At these consumption levels, (3.10) and (3.11) are satisfied with the same relative price \(p_x(s)\) given in (2.10), and \(\lambda = U_x(\frac{1}{2} \xi', \frac{1}{2} \eta') = U_x(s)\). Then (3.12) and (3.13) become

\[ U_x(s)q_x(s, w) = \beta \int U_x(s') \left[ q_x(s', w') + \frac{\xi' + p_x(s')\eta'}{1 + w'} \frac{\xi}{\xi + p_x(s)\eta} \right] dF dH, \quad (3.15) \]
\[ U_y(s)q_y(s, w) = \beta \int U_x(s') \left[ q_y(s', w') + \frac{\xi' + p_x(s')\eta'}{1 + w'} \frac{p_y(s)\eta}{\xi + p_x(s)\eta} \right] dF dH. \quad (3.16) \]

Evidently, the portfolio \((\theta_x, \theta_y) = (\frac{1}{2}, \frac{1}{2})\) is feasible for an agent beginning a period with a \(\theta\)-value equal to one-half the world’s money supply and one-half the outstanding equity shares. [See (3.3).] Evaluating the right-hand side of (3.8) at \((\theta_x, \theta_y) = (\frac{1}{2}, \frac{1}{2})\) gives

\[ \theta' = \frac{1}{2} \left[ q_x(s', w') + q_y(s', w') + \xi' + p_x(s')\eta' \right] \]
so that this portfolio choice maintains the perfectly pooled equilibrium. Hence (3.15) and (3.16) are, as conjectured, equilibrium equity prices and (2.11) continues to describe equilibrium goods prices.

It is necessary also to verify that equilibrium nominal interest rates are strictly positive under all states, since this equilibrium has been obtained under the provisional hypothesis that the finance constraint is always binding. To do so, it is necessary to price dollar-denominated one-period bonds, which can be done as follows. A claim to one dollar next period is a claim to \([p_x(s',M')]^{-1}\) units of \(x\) next period, where \(M' = M(1+w')\) is next period's post-transfer money supply. From (3.2), then, a claim to a dollar one period hence is a claim to \([2M(1+w')]^{-1}[\xi' + p_y(s')\eta']\) units of \(x\), one period hence. Using the 'density' \(q(s',s)\) defined in (2.19), the equilibrium price today, in \(x\)-units, of the claim is

\[
\frac{\beta}{2M} \left[U_x(s)\right]^{-1} \int U_x(s')[\xi' + p_x(s')\eta'](1+w')^{-1} f(s',s') \cdot h(w',w,s,s') \, ds' \, dw'.
\]

Its price in dollars is then \(p_x(s,M)\) times this quantity, or applying (3.2) again

\[
b(s,w) = \frac{\beta}{U_x(s)\xi' + U_y(s)\eta'} \cdot \frac{U_x(s')\xi' + U_y(s')\eta'}{1+w'} \int f(s',s) \, ds' \, dw',
\]

where \(b(s,w)\) is the dollar price today of a claim on one dollar tomorrow.

Eq. (3.17) is a version of the familiar decomposition of the nominal interest rate \((b^{-1} - 1)\) into a 'real rate of interest' and an 'expected inflation premium', but in a context in which these terms have a definite meaning and in which agents' attitudes toward risk are taken fully into account. The term 'real rate' is inherently ambiguous in a multi-good economy, but the factor

\[
\beta \int \frac{U_x(s')\xi' + U_y(s')\eta'}{U_x(s)\xi' + U_y(s)\eta'} f(s',s) \, ds'
\]

is a decent enough index number of the 'own rates' of interest on goods \(x\) and \(y\), and describes how nominal interest rates would behave under a regime of perfectly stable money, or \(w_t = 0\) with probability one, for all \(t\). If money is not perfectly stable, the integrand of the term (3.4), will in equilibrium be divided by \(1+w'\), integrated with respect to the distribution \(H(w',w,s,s')\) of the next monetary injection \(w'\); and the resulting function of next period's real state \(s'\) will be integrated with respect to \(s'\). This is the way rational risk-averse agents will assign an 'inflation premium' onto the nominal interest rate in situations where current conditions, real and monetary, convey information on future money growth.
Now, as already observed, (3.2) will hold in equilibrium in all states only if nominal interest rates are positive in all states. Hence the restriction

$$b(s, w) < 1 \quad \text{for all } (s, w)$$

must be added in what follows. Eq. (3.17) displays the requirements imposed by (3.19): A high subjective discount rate (low $\beta$), low $s$ variability, and high average inflation all work to make (3.19) more likely to hold.

It is illuminating to compare the equity price formulas (3.15) and (3.16) to the equity prices $q_A(s)$ and $q_s(s)$ given in (2.11) and (2.12). In the barter economy of section 2, the price $q(s) = q_A(s) + q_s(s)$ of a claim to the entire world's output sequence satisfies, adding (2.11) and (2.12)

$$q(s) = \beta[U_A(s)]^{-1} \int U_A(s')[q(s') + \xi' + p_A(s')\eta'] dF.$$

(3.20)

In the monetary economy, the price $q(s, w) = q_A(s, w) + q_s(s, w)$ obtained by adding (3.15) and (3.16) satisfies

$$q(s, w) = \beta[U_A(s)]^{-1} \int U_A(s')[q(s', w') + \frac{\xi' + p_A(s')\eta'}{1 + w'}] dF \, dH.$$

(3.21)

[Both (3.20) and (3.21) may be solved forward to obtain analogues to (2.13).]

The formulas (3.20) and (3.21) differ by the factor $(1 + w')^{-1}$ that deflates the real 'dividend' in (3.21). The point is that in a monetary economy an equity claim is a claim to dollar receipts, and this claim may be diluted (or enhanced) by monetary transfers. Agents in a monetary economy are free to exchange all of the 'real' securities available to them in section 2 [so that, for example, $q(s)$ as given by (3.20) continues to price total world output correctly in the monetary economy], but it is no longer possible for all private portfolios together to claim all real output. The 'inflation tax' must be paid by some one.

Notice also that, depending on the joint distribution $H(w', w, s', s)$, monetary transfers may well have a differential effect on equity prices. The integrands on the right-hand side of (3.15) and (3.16) permit arbitrary correlations between monetary transfers $w'$ and real shocks $\xi'$ and $\eta'$. In the present context of an exchange system with identical agents, nothing can affect consumption patterns and welfare, but relative prices are clearly not invariant to the nature of monetary-fiscal policy. Not only is money not 'neutral' but there is a variety of possibilities for non-neutral effects.

Finally, notice that under a perfectly stable monetary policy, with the transfer shock $w'$ identically zero, a one-period nominal bond is the exact equivalent to an equity claim on next period's output. From (3.17), the $x$-unit
price of a claim to all of tomorrow's money is, under the policy \( w' = 0 \),

\[
2M \cdot [p_x(s, M)]^{-1} \int \frac{U_x(s') \xi' + U_y(s') \eta'}{U_x(s) \xi + U_y(s) \eta} \, dF
\]

which, using (3.2) and (2.10), equals

\[
\beta [U_x(s)]^{-1} \int U_x(s') [\xi' + p_y(s') \eta'] \, dF.
\]

This expression is identical to the 'dividend' term in the equity price formula (3.21), when \( w' = 0 \).

In this model, nothing is gained by economizing on the number of securities traded, but it is of some interest, I think, that with stable monetary policy, a single dollar-denominated bond is the equivalent of a fully diversified equity claim to 'world output' one period hence. As soon as money becomes variable this simplicity is lost and additional securities are needed. It may be the case that in situations in which costs are associated with multiplying the number of distinct securities held, this loss of simplicity is one of the welfare costs of monetary instability.

4. A national currency, flexible exchange rate model

In this section, the timing and monetary conventions of section 3 will be retained but instead of a single world currency, two national currencies will circulate. These currencies will be exchanged freely at a centralized securities market, along with any other securities people wish to trade, prior to trading in goods. As in section 3, it will be assumed that nominal interest rates for bonds denominated in either currency are positive in all states, so that the finance constraints for both currencies are always binding.

Let there by \( M_t \) 'dollars' in circulation after any transfers have occurred in period \( t \), and \( N_t \) 'pounds'. These currency supplies are assumed to evolve according to

\[
M_{t+1} = (1 + w_{0,t+1}) M_t, \quad (4.1)
\]

\[
N_{t+1} = (1 + w_{1,t+1}) N_t, \quad (4.2)
\]

Karakken and Wallace (1978) study equilibrium with multiple currencies, but in a setting in which traders are free to use any currency in any transaction. (provided it is acceptable to both parties in the exchange). In the present paper, the question of which sellers will accept which currency is settled at the outset, by convention [see (4.3) and (4.4)]. This starting point obviously precludes making progress on some of the fundamental questions posed in Karaken and Wallace (1978).
where the transitions for the process \( \{ w_t \} = \{ w_c, w_i \} \) are given by

\[
K(w', w, s', s) = \Pr \{ w_{0t}, w_{1t} + 1 \leq w_0', w_{1t} + 1 \leq w_1' \mid w_{0t} = w_0, w_{1t} = w_1, s_{t+1} = s', s_t = s \}.
\]

Each citizen of country 0 receives a lump-sum dollar transfer of \( w_0 M_{t-1} \) at the beginning of \( t \); each citizen of 1 receives the pound transfer \( w_1 N_{t-1} \).

With the finance constraint binding, equilibrium nominal goods prices are simply

\[
p_x(s, M) = M/\xi, \tag{4.3}
\]

\[
p_y(s, N) = N/\eta. \tag{4.4}
\]

analogous to (3.2). Letting \( p_x(s) \) denote, as before, the price of \( y \) in \( x \)-units, the equilibrium exchange rate ($$/£$$) is given by the purchasing-power-parity (i.e., arbitrage) formula

\[
e(s, M, N) = p_x(s, M)p_y(s)[p_x(s, N)]^{-1} = \frac{M}{N} \frac{\eta}{\xi} p_y(s). \tag{4.5}
\]

Notice that this formula for the exchange rate depends on the relative currency supplies in exactly the way one would expect on quantity-theoretic grounds. It will also vary with real endowments, in a manner that depends on the derivatives of

\[
\frac{\eta}{\xi} p_y(s) = \frac{\eta}{\xi} \frac{U_x(\frac{1}{\xi}, \frac{1}{\eta})}{U_x(\frac{1}{\xi}, \frac{1}{\eta})}.
\]

To see what is involved, consider the case where \( U \) is homothetic, so that the marginal rate of substitution is a positive, negatively-sloped function \( g(r) \), say, of the endowment ratio \( r = \eta/\xi \) only. Then the dollar price of pounds will increase with an increase in British output relative to the U.S. if

\[
\frac{d}{dr} g(r) g(r)[1 + r (1 + g'(r)/g(r))] > 0.
\]

The reverse sign would occur in the case where relative prices are so sensitive to relative quantity changes that the terms of trade 'turn against' a high output country; the case Bhagwati (1958) and Johnson (1955) called 'immiserizing growth'.

This discussion of the relationship of exchange rate behavior to the curvature of indifference curves has an 'elasticities approach' flavor to it. Yet
the formula (4.5) is also consistent with the 'monetary approach' to exchange rate determination, being based on relative money supplies and demands. The reason these two approaches are so compatible in the present context is that the extreme 'transactions demand' emphasis implicit in the use of the finance constraint makes the 'stock' demand for money and the 'flow' demand for goods equivalent.\footnote{See Stockman (1980) for a closely related, earlier discussion.}

As in section 3, securities pricing will be studied under the provisional hypothesis that agents of both countries hold identical portfolios. Having obtained prices under this hypothesis, it will then be verified that this is in fact equilibrium behavior. As always, there is a great deal of latitude as to which limited set of specific securities is assumed to be traded in equilibrium. I will select a set that facilitates comparison with the analysis of sections 2 and 3.

Let \( q_x(s, w) \) be the price, in \( x \)-units, of claim to all of the dollar receipts of the \( \zeta_t \) process and let \( q_y(s, w) \) be the \( x \)-unit price of the pound receipts of the \( \eta_t \) process. Agents hold these two securities in a portfolio \((\theta_x, \theta_y)\). In addition, since monetary transfers accrue (by assumption) to nationals of each country, agents will want to pool this monetary form of endowment risk. Let \( r_x(s, w) \) be the price, in \( x \)-units, of an equity claim to all future periods' dollar transfers, \( w_0M \), and let \( r_y(s, w) \) be the \( x \)-unit price of all future periods' pound transfers \( w_1N \). Let \((\psi_x, \psi_y)\) denote an agent's holding of these two instruments. Then the portfolio constraint for an agent beginning a period with \( x \)-unit holdings of amount \( \theta \) is, analogous to (3.3),

\[
\frac{m}{p_x(s, M)} + e(s, M, N)n \frac{p_y(s, M)}{p_x(s, M)} + r_x(s, w)\psi_x + r_y(s, w)\psi_y + q_x(s, w)\theta_x + q_y(s, w)\theta_y \leq \theta. \tag{4.6}
\]

His finance constraints, analogous to (3.4), are

\[
p_x(s, M)x \leq m, \tag{4.7}
\]

\[
p_y(s, M)y \leq n. \tag{4.8}
\]

Consolidating these constraints and using (4.5) gives the analogous to (3.7):

\[
x + p_x(s)y + r_x(s, w)\psi_x + r_y(s, w)\psi_y + q_x(s, w)\theta_x + q_y(s, w)\theta_y \leq \theta. \tag{4.9}
\]

For a citizen of country 0, the beginning-of-next-period wealth (in \( x \)-units)
For a citizen of country 1, the last term on the right-hand side of (4.10) is 
\[ \frac{1}{p_x(s', M')} [m - p_x(s, M)x + e(s', M', N')(n - p_y(s, M)y)] \]

\[ + \frac{p_x(s, M)}{p_x(s', M')} \xi \theta_x + \frac{e(s', M', N')p_x(s, N)}{p_x(s', M')} \eta \theta_y \]

\[ + q_x(s', w') \theta_x + q_y(s', w') \theta_y \]

\[ + \frac{w_0' M}{p_0(s', M')} \psi_x + \frac{e(s', M', N')w_1' N}{p_0(s', M')} \psi_y \]

\[ + r_x(s', w') \psi_x + r_y(s', w') \psi_y \]

\[ + \frac{w_0' M}{p_x(s', M')} \]

(4.10)

For a citizen of country 1, the last term on the right-hand side of (4.10) is 
\[ [p_x(s', M')]^{-1} e(s', M', N')w_1' N \] and (4.10) is otherwise the same for him as for the country 0 citizen.

With the constraints (4.7) and (4.8) binding, the first term on the right-hand side of (4.10) is zero. The remaining terms can be simplified using the nominal price formulas (4.3)-(4.5), so that (4.10) reduces to the analogue of (3.8):

\[ \theta' = \frac{\xi'}{1 + w_0'} \theta_x + \frac{\eta'}{1 + w_1'} p_x(s') \theta_y + q_x(s', w') \theta_x \]

\[ + q_y(s', w') \theta_y + \frac{w_0'}{1 + w_0'} \xi'(1 + \psi_x) + \frac{w_1'}{1 + w_1'} p_y(s') \eta' \psi_y \]

\[ + r_x(s', w') \psi_x + r_y(s', w') \psi_y \]

(4.11)

(This is for country 0. The modification for country 1 is obvious.) The problem facing the agent is then given by

\[ v(s, w, \theta) = \max_{x, y, \alpha, \beta, \nu, \theta'} \{ U(x, y) + \beta \int v(s', w', \theta') \, dF \, dK \}, \]

subject to (4.9), with \( \theta' \) given by (4.11).

The development of the first-order conditions for this problem is sufficiently close to the preceding section that it need not be repeated. In a
symmetric equilibrium, the agent must buy \((x,y) = (\xi, \eta), (\theta_x, \theta_y) = (\frac{1}{2}, \frac{1}{2})\), and \((\psi_x, \psi_y) = (-\frac{1}{2}, \frac{1}{2})\). A country 1 agent holds \((\psi_x, \psi_y) = (\frac{1}{2}, -\frac{1}{2})\) and otherwise behaves identically. In such an equilibrium equity prices are given by the analogues to (3.15)–(3.16):

\[
U_x(s)q_x(s, w) = \beta \int U_x(s') \left[ q_x(s', w') + \frac{\xi'}{1 + w_0} \right] dF dK, \tag{4.13}
\]

\[
U_x(s)q_y(s, w) = \beta \int U_x(s') \left[ q_y(s', w') + \frac{\eta'}{1 + w_1} p_y(s') \right] dF dK. \tag{4.14}
\]

The prices of the claims to future monetary transfers are similarly given by

\[
U_x(s)r_x(s, w) = \beta \int U_x(s') \left[ r_x(s', w') + \frac{w_0'}{1 + w_0} \xi' \right] dF dK, \tag{4.15}
\]

\[
U_x(s)r_y(s, w) = \beta \int U_x(s') \left[ r_y(s', w') + \frac{w_1'}{1 + w_0} p_y(s') \eta' \right] dF dK. \tag{4.16}
\]

As in section 3, it is necessary to determine the conditions under which nominal interest rates will be strictly positive. A claim to one dollar one period hence is a claim to \(M^{-1}(1+w_0)^{-1}\xi'\) x-units and hence has a current x-unit price of

\[
\beta [U_x(s)]^{-1} M^{-1} \int U_x(s') (1+w_0)^{-1} \xi' dF dK.
\]

Its dollar price is therefore

\[
b_x(s, w) = \beta \int \frac{U_x(s') \xi'}{U_x(s) \xi' 1 + w_0} dF dK. \tag{4.17}
\]

Similarly, a claim to a pound one period hence has the current pound value:

\[
b_y(s, w) = \beta \int \frac{U_y(s') \eta'}{U_y(s) \eta' 1 + w_1} dF dK. \tag{4.18}
\]

The discussion following eq. (3.17) is applicable to (4.17)–(4.18) as well.

5. A national currency, fixed exchange rate model

In this section, the timing, monetary conventions, and market structure of section 4 will be maintained without change. The objective of the analysis
will be to find a symmetric, perfectly pooled equilibrium in which the exchange rate is maintained at a constant level through central bank intervention in the currency market.

Not infrequently, fixed exchange rate regimes are discussed as though they were equivalent to a single currency regime such as that analyzed in section 3. Thus if there are $M$ and $N$ in circulation, and if the exchange rate is fixed at $e$, then one could call $M + \varepsilon N$ the ‘world money supply’ and let this magnitude play the role of $M$ in section 3. This is where the analysis of this section is headed, too, but in order to gain some insight into the conditions under which this simplifying device is legitimate, it is best to begin at a prior level. Accordingly, the existence of differentiated national currencies in the sense of section 4, and a currency-and-securities market operating under the same rules, are both assumed here. Hence, if the exchange rate is to be fixed, someone or some agency has to do something to make it fixed. I will assign this role to a single, central authority, holding reserves of both currencies, trading in spot currency markets so as to maintain the exchange rate $e$ at some constant value $\varepsilon$.

To analyze such a regime under rational expectations, it is necessary either to assume that the behavior of this central authority, in combination with the behavior of monetary policy and real shocks in the two countries, is consistent with the permanent maintenance of the rate $\varepsilon$, or to incorporate into the analysis the possibility of devaluations and the consequent speculative activity this possibility would necessarily involve. I will take the former, much simpler, course.

Let the authority begin (and also end) a given period with total reserves of dollar value $D$, possibly after receiving new currency transfers from one or both countries. Let its holdings after all securities trading is completed be $R$ and $S$ so that at the conclusion of trading

$$D - R + \varepsilon S$$

must hold. Under the hypothesis, provisionally maintained here, that nominal interest rates are uniformly positive, eqs. (4.3) and (4.4) will continue to hold, but with $M$ and $N$ replaced by the quantities $M - R$ and $N - S$ of these currencies remaining in private circulation. Then the formula (4.5) for the equilibrium exchange rate becomes

$$\varepsilon = \frac{M - R \eta}{N - S \zeta} p_j(s).$$

This model of an exchange rate fixing institution is taken from Heipman (1979), where national central banks are also considered.
Given $\bar{c}$, given the value of $s = (\xi, \eta)$ selected by nature, and given the two national money supplies $M$ and $N$, (5.1) and (5.2) are two equations in the end-of-period reserve levels $R$ and $S$. Viability of the fixed rate regime, then, requires that $R > 0$ and $S > 0$ for all possible states $(s, M, N)$. It is readily seen that these two inequalities are equivalent to

$$D > N\bar{e} - M\frac{\eta}{\xi}p_y(s), \quad \text{and}$$

$$D > M - N\bar{e}\left|\frac{\eta}{\xi}p_x(s)\right|. \quad (5.3)$$

To interpret these conditions, suppose that the positive random variable $(\eta/\xi)p_y(s)$ ranges in value from zero to infinity. Then for (5.3) and (5.4) to hold for all states of nature $s$ the stabilizing authority must hold reserves of dollar value $D$ exceeding both the dollar value of pounds outstanding $N\bar{e}$ [inequality (5.3)] and all outstanding dollars $M$ [inequality (5.4)]. Tighter bounds on the range of $(\eta/\xi)p_y(s)$ would permit smaller reserves. With constant money supplies $M$ and $N$ (or with $w_{0t} = w_{1t} = 0$ for all $t$) it is clear that a sufficiently large reserve level $D$ can always be selected.

With $M_t$ and $N_t$ drifting over time, even if the drifts $w_{0t}$ and $w_{1t}$ are perfectly correlated, it is clear that no constant reserve level $D$ can maintain (5.3) and (5.4) forever. Surely this cannot be surprising. It is equally clear that by augmenting reserves appropriately from time to time the inequalities (5.3) and (5.4) can be indefinitely maintained. In this rather weak and obvious sense, then, the maintenance of fixed exchange rate requires coordination in the monetary policies of the two countries and of the stabilizing authority. At the same time, there may remain a good deal of latitude for independent monetary policies on a period-to-period basis. Indeed, over a sample period in which no devaluations occur, the inequalities (5.3) and (5.4) should probably be viewed as placing no econometrically useful restrictions on the joint distribution of the processes $w_{0t}, w_{1t}$.

With (5.3) and (5.4) maintained, then, the rest of the analysis is precisely that of the single-money world economy studied in section 3. Now $M_t - R_t + \bar{e}(N_t - S_t)$, or 'world money' plays the role of $M_t$ in section 3. The Markov processes governing the motion of world money would have to be derived from the behavior of the two monetary policies and the stabilizing authority. and might not be first-order. Modifying the analysis of section 3 to incorporate higher-order processes on the monetary shock is not a difficult exercise. Of course, the requirement (3.19) that nominal interest rates be positive is presupposed in this adaptation, too.

In summary, then, it is possible to devise a pegged exchange rate regime under which the Pareto-optimal resource allocation obtained under a flexible
rate system is replicated exactly, provided only that the authority responsible for maintaining the fixed rate is armed with sufficient reserves. This conclusion does not, of course, rest on the notion that price fixing is innocuous in any general sense, but rather on the function served by the particular prices that appear in this model. In the barter allocation of section 2, a full list of Arrow–Debreu contingent claim securities is available. In the monetary model of section 3 money is introduced in addition to these contingent claim securities, motivated by the idea that current goods purchases are carried out in a decentralized, anonymous fashion. With stable money, this monetary modification does not disturb the relative price configuration of section 2.

In section 4, a second money was introduced and trade in the two currencies was permitted. Again, with stable money supplies, relative prices and quantities are not altered. This redundant security does no harm. It also does no good, however, and thus when it is effectively removed, as in the present section, the efficiency properties of the real resource allocation are left undisturbed. The price-fixing involved does not (or need not) alter the relative price of any pair of goods, as it does in the classic case for flexible rates constructed by analogy to ordinary commodity price pegging. Neither does it introduce any new options, as it does in Mundell’s (1973) defense of a ‘common currency’.

One frequently sees exchange rate regimes compared in terms of where it is that certain shocks get ‘absorbed’. In the present model, with perfectly flexible prices in all markets, ‘shock absorption’ is easy and the issue of which prices respond to which shocks is of no welfare consequence. However, the two regimes do differ radically in their implications for the volatility of domestic nominal prices, and a comparison may be suggestive in thinking about extensions of the model to cover situations in which nominal price instability is associated with real pain.

Consider only regimes with perfectly stable money supplies, $M$ and $N$, so that the only shocks are to $\xi$ and $\eta$. In the flexible rate regime, nominal prices in country 0 are given in (4.3). Here $p_x(s, M)$ responds to changes in home endowment with an elasticity of minus one, and to changes in the foreign endowment not at all. In the fixed rate regime, $p_x(s, M)$ is given by $(M-R)/\xi$, where reserves $R$ also fluctuate stochastically. Solving for $M-R$ from (5.1) and (5.2), one obtains

$$M-R = \left[1 + \frac{\eta}{\xi} p_x(s)\right]^{-1}[M + \bar{\epsilon}N - D],$$

so that the domestic price level is just

$$p_x(s, M) = \left[\frac{\xi + \eta p_x(s)}{\xi} \right]^{-1}[M + \bar{\epsilon}N - D].$$
or world money in private circulation divided by world output, valued in \( x \)-units. Now if world output fluctuates less than output in each individual country, domestic price levels have less 'shock absorbing' to do under fixed than flexible rates. This observation is very much in the spirit of Mundell's argument in Mundell (1973).

To what extent these results, and those of Helpman (1979) and Helpman and Razin's (1981) earlier work should be taken to bear on the controversy over which set of international monetary institutions are to be preferred in practice is difficult to determine. I suspect that the central issue in this debate is whether one takes a nationalist or an internationalist point of view toward relations among countries. If so, economic analysis cannot be expected to resolve the question directly, but it may contribute indirectly to its resolution by making it more difficult for contestants to defend essentially political conclusions on the basis of what seem to be 'purely' economic arguments.

6. Possible relaxations

Of the many ways in which the models in this paper differ from reality, four seem to me likely to be the most crucial in applications: the assumed absence of production, the implication that the velocity of circulation is fixed, independent of interest rates and income, the implication that all agents hold identical portfolios, and the absence of business cycle effects. The purpose of this section is to discuss briefly the likely causes and/or consequences of these presumed deficiencies in the model.

Production can be introduced into the barter model of section 2, so long as the one consumer device is retained. Using the connection between competitive equilibria and Pareto-optima, one can obtain the optimum (and equilibrium) quantities produced and consumed, and insert these quantities into the marginal-rate-of-substitution formulas used in section 2 to price securities. See Brock (1979).

In the monetary economics of sections 3–5, matters are not so simple. As in Grandmont and Younes (1973), the Clower constraint sets up a 'wedge' between the private and social returns to capital and labor. Factors of production utilized today produce goods consumed today, but since factors are paid at the end of the period, the private trade-off involves exchanging effort today for consumption tomorrow. With a positive discount rate, this difference matters. These observations are valid even under a perfectly stable monetary policy; with stochastic variability in the latter, still more complications are involved. These are not difficulties of formulating a coherent definition of an equilibrium with production, but they are barriers to applying the solution methods used in Brock (1979) or Lucas (1978) and hence challenges to future research.
The unit-velocity prediction (really, assumption) of Clower-based monetary models is a great convenience theoretically, as we have seen earlier in this paper, but a serious liability in any empirical application one can imagine. It arises because of the way information is assumed to flow in the model: first, people learn exactly how much they will buy in the current "period" and at what price. second, they execute these purchases using currency balances finely tuned for this purpose. By reversing this sequence by, for example, making people commit themselves to money holdings prior to learning the current value of the shocks $\zeta$ and $\eta$, or by introducing non-insurable, personal shocks as in Lucas (1980), one can introduce a precautionary motive to money demand that leads to a richer and more conventional treatment of velocity. These modifications lead to complications of their own, however, and I thought it best to abstract from them in this first pass at a set of problems which is complicated enough in its own right.

Of course, even if modified to incorporate a precautionary motive, any Clower-based model assigns a heavy burden to the idea of a "period", and one is definitely not supposed to let the length of a period tend to zero and hope that the predictions of such a model will be unchanged. This observation is sometimes raised as a criticism of models of this class. If such criticism were accompanied by examples of serious monetary theory which does not have this property, it would have considerably more force.²

The fact that, in equilibrium, all traders in the world hold the identical market portfolio is a simplification that is absolutely crucial to the mode of analysis used above. It is also grossly at variance with what we know about the spatial distribution of portfolios: Americans hold a disproportionately high fraction of claims to American earnings in their portfolios, Japanese a high fraction of Japanese assets, and so on. For that matter, neighborhood savings-and-loan banks attract local savings, mostly, and invest it in local assets, mostly, even within a single city in a single country.

Why is this? Much of conventional trade theory explains this simply by forgetting the existence of international capital markets, in certain selective ways.¹⁰ A real answer must have something to do with the local nature of the information people have, but it is difficult to think of models that even make a beginning on understanding this issue. It is encouraging that the theory of finance has obtained theories of securities price behavior that do very well empirically based on this common portfolio assumption, even though their predictions on portfolio composition are as badly off as those of this paper.

Finally, these models contain nothing that I would call a "business cycle".

²The finance constraint idea can be adapted to continuous time models [see Frenkel and Helpman (1980)], in which case the relevant "period" becomes a fixed lag between the date of sale and the date of receipt of payment.

¹⁰An exception is Weiss's (1980) analysis.
There is real variability, due to endowment fluctuations, and monetary variability, due to unstable fiscal policy, but the only connection between these two kinds of shocks arises because policies may react to endowment movements. There is no sense in which real movements are induced by monetary instability. There is no doubt that the absence of such effects must limit the ability of models of this general class to fit time series, though the seriousness of this limitation for relatively smooth episodes such as the post-World War II period is not well-established.

7. Conclusion

This paper has been devoted to the development of a simple prototype model capturing certain real and monetary aspects of the theory of international trade. Its results consist mainly of the re-derivation within a unified framework of a number of familiar formulas (or close analogues thereto) from the theories of finance, money and trade. Perhaps the best way to sum up, then, is simply to provide a compact index of these formulas.

The formula for equity pricing in an 'efficient market' in a real system is given in (2.11) [or (2.13)] in a form that reflects agents' aversion to risk; (2.23) adapts this formula to any arbitrary, related security. Modifications of these formulas suited to an erratic, monetary environment are given in (3.15)–(3.16) (for the one-money cases) and (4.13)–(4.14) (for the two-money case).

The 'equation of exchange' for determining domestic prices appears as (3.2) and (4.3)–(4.4). A version of the Fisherian formula for expressing the nominal interest rate in terms of its real and nominal determinants is given in (3.17) and again in (4.17)–(4.18). The purchasing-power-parity law of exchange rate determination is given in (4.5).

I found it striking that all of these formulas — really, every main result in classical monetary theory and the theory of finance — fall out so easily, once an investment in notation is made. This seems to me an encouraging feature of models based on the finance constraint. It remains to be seen, however, whether models of this type can be pushed into genuinely new substantive territory.

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