A Habit-Based Explanation of the Exchange Rate Risk Premium

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ABSTRACT

This paper presents a model that reproduces the uncovered interest rate parity puzzle, based on a time-varying business-cycle related risk premium. Agents have preferences with external habits. During bad times in the home market, when the domestic habit is close to domestic consumption level, the exchange rate becomes more sensitive to domestic than to foreign aggregate consumption growth shocks. As a result, the foreign currency depreciates in case of a negative consumption growth shock at home. Hence, investing in foreign currency is risky in bad times, when domestic interest rates are low relative to foreign interest rates.

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According to the standard uncovered interest rate parity (UIP) condition, the expected change in exchange rate should be equal to the interest rate differential between foreign and domestic risk-free bonds. A simple regression of exchange rate changes on interest rate differentials should produce a regression coefficient of 1. Instead, empirical work following Hansen and Hodrick (1980) and Fama (1984) consistently reveals a regression coefficient that is smaller than 1 and very often negative. The international economics literature refers to these negative UIP slope coefficients as the UIP puzzle or forward premium anomaly.

Negative slope coefficients mean that currencies with higher than average interest rates actually tend to appreciate. Investors in foreign one-period discount bonds thus earn the interest rate spread plus the bonus from the appreciation of the currency during the holding period. As a result, the failure of the UIP condition implies positive predictable excess returns when investing in high interest rate currencies and negative excess returns for investing in low interest rate currencies. There are two possible explanations for predictable excess returns: time-varying risk premia and/or expectational errors. In this paper, I assume that expectations are rational and I develop a risk premium explanation of the forward premium puzzle. In data generated by a calibrated version of my model, I replicate the negative UIP slope coefficient.

To do so, I adapt the workhorse model of modern finance. In my model, the stand-in investor has external habit preferences over consumption. This is in the spirit of Campbell and Cochrane (1999), but with one key difference: in bad times, when consumption is close to the habit level and investors are more risk-averse, risk-free rates are low. Goods can be shipped between countries, but investors incur international trade costs.

Using this model, I obtain three novel results. First, in my model, UIP fails, as it does in the data. This failure is the result of a time-varying, counter-cyclical risk premium. When the domestic stand-in investor is more risk-averse than his foreign counterpart, the domestic investor expects a positive risk premium for investing in foreign currency. Why? Complete markets imply that the foreign currency appreciates when the foreign inter-temporal marginal rate of substitution (or stochastic discount factor) is higher than the domestic one. In equilibrium, domestic consumption growth shocks swamp the effect of foreign consumption shocks on the exchange rate simply because the home investor is more risk averse. As a result, when the domestic economy receives a negative consumption
growth shock, the foreign currency depreciates, leaving the home investor exposed to more domestic consumption risk. Vice versa, when the domestic economy receives a good shock, the foreign currency appreciates. Investing in foreign currency is clearly very risky when the home investor’s consumption is close to his habit; the investor therefore wants to be compensated by a large risk premium. But when the domestic investor is less risk averse, foreign consumption shocks dominate the exchange rate, which is no longer risky. Under such conditions, the exchange rate provides a consumption hedge, and the domestic investor expects a negative excess return.

As a result, the domestic investor expects to receive a positive foreign currency excess return in bad times when he is more risk-averse than his foreign counterpart. In my model, times of high risk-aversion correspond to low interest rates at home. Thus domestic investors expect positive currency excess returns when domestic interest rates are low and foreign interest rates are high. Hence, the expected domestic currency excess returns increase sharply with (foreign minus domestic) interest rate differentials, and this produces a negative UIP coefficient in frictionless asset markets.

Second, the model replicates the key empirical finding of Lustig and Verdelhan (2007): portfolios of high interest rates currencies offer high excess returns because they tend to depreciate in bad times for a US investor. This is also the case in the model: the foreign currency tends to depreciate more for a given low consumption growth when the foreign interest rate is higher. This can be tested on simulated data by regressing changes in real exchange rates on consumption growth and consumption growth interacted with the interest rate differential between two countries. In the model, the coefficient of this interaction term is clearly positive and significant. Additionally, the model provides a clear rationale for building such portfolios of currency excess returns. Ranking countries on interest rates is equivalent to sorting them on the level of the state variable (the surplus-consumption ratio), which changes with risk-aversion. By taking averages of excess returns inside each portfolio when idiosyncratic (foreign consumption growth) shocks cancel out, one focuses on expected excess returns, the object of interest when looking at the problem from a financial perspective.

Third, the model offers a clear link between currency and equity risk premia, through interest rates, and thus suggests new empirical exercises. In the model, low domestic interest rates (i.e lower than abroad) imply high currency excess returns, and low domestic
interest rates (i.e lower than usual) imply high Sharpe ratios. To test this implication, I turn to currency portfolios of developed countries sorted on interest rates. I compute the average stock market returns for the last fifty years (denominated in local currencies) for each portfolio. I find that countries that offer high currency excess returns to the US investor offer low equity Sharpe ratios to local investors. The rationale is the same as before: when foreign interest rates are high, the foreign currency offers high excess returns, but the foreign stock market does not. This result is consistent with previous findings.

The link between interest rates and currency returns comes from any test that rejects the UIP condition, and the literature abounds. For the US, the link between nominal interest rates and equity excess returns is known since Fama and Schwert (1977), and Campbell and Yogo (2006) show that it is robust. For France, Germany, Japan and Switzerland over the 1990-2003 period, Hau and Rey (2004) find that a negative shock to the foreign stock market return (relative to the US) leads to an appreciation of the foreign currency.

I start by studying an endowment economy, where each country consumes its endowment and trade is impossible. The endowment process is calibrated to match the equilibrium consumption process in the data. In this model, I derive closed form expressions for the currency excess return and the UIP slope coefficient. I calibrate the model using the first and second moments of consumption growth and real interest rates, and the maximal Sharpe ratio. The model reproduces the forward premium puzzle, delivering a negative UIP coefficient. The mean, standard deviation and autocorrelation of the consumption growth rate, real interest rate, price-dividend ratio, return on the market and long-term real yield are in line with their empirical counterpart. Yet, the simulation highlights two weaknesses of this simple model: the simulated real exchange rate is too volatile and too correlated with consumption growth.

In addition, I estimate the model by minimizing pricing errors from Euler conditions. As there is only one source of shocks in each country, pricing kernels can be theoretically recovered using either consumption data or interest rates. I use two sets of currency excess returns as test assets. I first consider the investment opportunities of an American investor in 8 other OECD countries. I then focus on the 8 portfolios of currency excess returns built in Lustig and Verdelhan (2007). By taking into account many investment opportunities in currencies, these portfolios create a large cross-section of excess returns, without imposing the estimation of a large variance-covariance matrix. Following Hansen,
Heaton, and Yaron (1996), a continuously-updating general method of moments (GMM) estimator is used. Estimates based either on consumption data or on interest rate data lead to reasonable parameters when pricing the currency excess returns of an American investor. Furthermore, the hypothesis that the pricing errors are zero cannot be rejected at conventional confidence levels.

Finally, I show how to derive the post-trade consumption allocations starting from endowment processes in both countries, and allowing agents to trade. I consider proportional and quadratic trade costs and derive optimal exports. I simulate the model for a range of trading costs found in the international economics literature, and I focus on its two previously highlighted weaknesses. Although the model still implies volatile stochastic discount factors that match other asset prices, it now reproduces the variance of the changes in the real exchange rate. We know since Mehra and Prescott (1985) and Hansen and Jagannathan (1991) that stochastic discount factors must have a large variance in order to price stock excess returns. In complete markets, the change in the real exchange rate is theoretically equal to the ratio of foreign and domestic stochastic discount factors. Thus, as Brandt, Cochrane, and Santa-Clara (2006) show, volatile stochastic discount factors may imply very volatile exchange rates if countries do not share some risk, increasing the correlation between discount factors. In the present model, endowment shocks are independent and identically distributed, uncorrelated across countries, but countries share some risks because trade costs are finite. As a result, the variance of the theoretical exchange rate remains low. The model however cannot at this stage fully account for the Backus and Smith (1993)’s puzzle. The correlation between differences in consumption growth and changes in the real exchange rate is no longer equal to one as with power utility, but it is higher than in the data, because only one source of shocks drives all variables. Yet, Burstein, Eichenbaum, and Rebelo (2005) estimate that at least 50% of the variation in real exchange rates is due to changes in the relative prices of non tradable goods across countries. Taking this into account, I show that the introduction of non tradable goods in the model, by acting as a source of measurement errors on the real exchange rate, lowers significantly its correlation with relative consumption growth rates.

I turn now to the existing literature, first focusing on habit preferences and then on solutions to the UIP puzzle. Some examples of habit preferences are Sundaresan (1989),

This paper adds to a large body of empirical and theoretical work on the UIP condition. On the empirical side, most papers test the UIP condition on nominal variables. Two recent studies, however, relate the puzzle to real variables. Hollifield and Yaron (2003) decompose the currency risk premium into conditional inflation risk, real risk, and the interaction between inflation and real risk. They find evidence that real factors, not nominal ones, drive virtually all of the predictable variation in currency risk premia. Lustig and Verdelhan (2007) find that real aggregate consumption growth risk is priced on currency markets. As a result, this model focuses on real risk, abstracting from money and inflation.

On the theory side, numerous studies have attempted to explain the UIP puzzle under rational expectations, but few models reproduce the negative UIP slope coefficient. Table (I) presents a synthesis of the assumptions and results of these attempts. I report here the four most successful studies. Frachot (1996) shows that a financial two-country Cox, Ingersoll, and Ross (1985) framework can account for the UIP puzzle but it does not provide an economic interpretation of the currency risk premium. Alvarez, Atkeson, and Kehoe (2005) use endogenously segmented markets to qualitatively generate the forward premium anomaly. In their model, higher money growth leads to higher inflation, thus inducing more agents to enter the asset market because the cost of non-participation is higher, and decreasing risk premia. Bacchetta and van Wincoop (2006) develop a model where investors face costs of collecting and processing information. Because of these costs, many investors optimally choose to assess available information and revise their portfolios infrequently. Thus, rational inattention produces a negative UIP coefficient.
along the lines suggested by Froot and Thaler (1990) and Lyons (2001): if investors are slow to respond to news of higher domestic interest rates, there will be a continued reallocation of portfolios towards domestic bonds and a appreciation of the currency subsequent to the shock. Finally, Bansal and Shaliastovich (2007), following Colacito and Croce (2005) and extending Bansal and Yaron (2004)'s model to a two-country setting, rely on a perfect cross-country correlation among shocks to the long run components of consumption growth rates to reproduce the UIP puzzle.

The rest of the paper is organized as follows: section I outlines the two-country one-good model, abstracting from international trade in order to obtain closed-form expressions on currency risk premia. Section II reports simulation results on stock, bond and currency returns. Section III presents estimation exercises using either consumption data or interest rates to compute stochastic discount factors. Section IV shows how to compute optimal international trade and reports simulation results with proportional and quadratic trade costs. Section V concludes.

I. Model

In the model, there are two countries (with same initial wealth) and one good. In this section, I consider post-trade allocations. First, I describe preferences and define the real exchange rate and the currency risk premium. Then, I derive a closed form expression for the currency excess return and provide an interpretation to the UIP puzzle.

A. Habit-based preferences

In each country, a representative agent is characterized by external habit preferences similar to Campbell and Cochrane (1999) but with time-varying risk-free rates. The agent maximizes:

$$E \sum_{t=0}^{\infty} \beta^t (C_t - H_t)^{1-\gamma} - 1 \over 1 - \gamma,$$

where $\gamma$ denotes the risk-aversion coefficient, $H_t$ the external habit level and $C_t$ consumption. The external habit level can be interpreted as a subsistence level or as a social externality. In each country, the habit level is related to consumption through the follow-
Table I: Summary of the literature

The table presents a survey of the results obtained on the UIP puzzle (empirically, the UIP slope coefficient $\alpha$ is often negative) and the volatility puzzle ($\sigma^2_{\Delta e} > \sigma^2_p > \sigma^2_{t-1}$, in the data, where $\sigma^2_{\Delta e}$ is the variance of the change in exchange rates, $\sigma^2_p$ is the variance of the currency risk premium and $\sigma^2_{t-1}$ is the variance of the interest rate differential).

<table>
<thead>
<tr>
<th>Papers</th>
<th>Features</th>
<th>UIP puzzle</th>
<th>Volatility puzzle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lucas (1982)</td>
<td>Two-country, cash-in-advance</td>
<td>$\alpha \simeq 1$</td>
<td>$\sigma^2_{\Delta e} &gt; \sigma^2_{t-1}, \sigma^2_p$</td>
</tr>
<tr>
<td>Bekaert (1996)</td>
<td>Lucas (1982) + Habit persistence</td>
<td>$\alpha &lt; 1/2$</td>
<td>$\sigma^2_{\Delta e} &gt; \sigma^2_{t-1}, \sigma^2_p$</td>
</tr>
<tr>
<td>Moore and Roche (2002)</td>
<td>Lucas (1982) + Habit persistence + Limited participation</td>
<td>$0 &lt; \alpha &lt; 1$</td>
<td>$\sigma^2_{\Delta e} &gt; \sigma^2_p &gt; \sigma^2_{t-1}$</td>
</tr>
<tr>
<td>Alvarez et al. (2005)</td>
<td>Lucas (1982) + Endogenously segmented markets</td>
<td>$\alpha &lt; 0$ for $\pi &lt; \overline{\pi}$</td>
<td>$\sigma^2_{\Delta e} &gt; \sigma^2_{t-1}$</td>
</tr>
<tr>
<td>Sarkissian (2003)</td>
<td>Heterogeneity</td>
<td>$0 &lt; \alpha &lt; 1$</td>
<td>$\sigma^2_{\Delta e} &gt; \sigma^2_p &gt; \sigma^2_{t-1}$</td>
</tr>
<tr>
<td>Lyons (2001)</td>
<td>Arbitrage zone + Limited adaptation</td>
<td>$-1 &lt; \alpha &lt; 3$</td>
<td>n.a</td>
</tr>
<tr>
<td>Bacchetta and van Wincoop (2005)</td>
<td>Information costs</td>
<td>$\alpha &lt; 0$</td>
<td>$\sigma^2_{\Delta e} &gt; \sigma^2_p &gt; \sigma^2_{t-1}$</td>
</tr>
<tr>
<td>Gourinchas and Tornell (2004)</td>
<td>Limited rationality</td>
<td>$\alpha &lt; 0$</td>
<td>$\sigma^2_{\Delta e} &gt; \sigma^2_p &gt; \sigma^2_{t-1}$</td>
</tr>
<tr>
<td>Obstfeld and Rogoff (1995)</td>
<td>Monopolists + Sticky prices + UIP</td>
<td>$\alpha = 1$</td>
<td>$\sigma^2_{\Delta e} = \sigma^2_{t-1}, \sigma^2_p = 0$</td>
</tr>
<tr>
<td>Chari et al. (2002)</td>
<td>Monopolists + Sticky prices</td>
<td>n.a</td>
<td>n.a</td>
</tr>
<tr>
<td>Bansal and Shaliastovich (2006)</td>
<td>Epstein and Zin (1989)</td>
<td>$\alpha &lt; 0$</td>
<td>$\sigma^2_{\Delta e} &gt; \sigma^2_p &gt; \sigma^2_{t-1}$</td>
</tr>
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</table>
ing autoregressive process of the surplus consumption ratio $s_t \equiv (C_t - H_t)/C_t$:

$$s_{t+1} = (1 - \phi)\bar{s} + \phi s_t + \lambda(s_t)(\Delta c_{t+1} - g).$$  \hspace{1cm} (2)

Lowercase letters correspond to logs, and $g$ is the average consumption growth rate. The sensitivity function $\lambda(s_t)$ describes how habits are formed from past aggregate consumption. I assume that in both countries idiosyncratic shocks to consumption growth are i.i.d log-normally distributed:

$$\Delta c_{t+1} = g + u_{t+1}, \text{ where } u_{t+1} \sim \text{i.i.d. } N(0, \sigma^2).$$

Moreover, to keep the model simple and tractable, I assume that the two endowment shocks $u_{t+1}$ and $u^*_t$ are independent across countries.\(^1\) I refer to ‘bad times’ as times of low surplus consumption ratio (when the consumption level is close to the habit level), and use ‘negative shocks’ to refer to negative consumption growth shocks $u$.

**External habits**  In each country, the habit level depends only on domestic, not foreign, consumption, and on aggregate, not individual, consumption. Thus, the inter-temporal marginal rate of substitution is given by:

$$M_{t+1} = \beta \frac{U_c(C_{t+1}, X_{t+1})}{U_c(C_t, X_t)} = \beta \left( \frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma} = \beta e^{-\gamma[g + (\phi - 1)(s_t - \bar{s}) + (1 + \lambda(s_t)) (\Delta c_{t+1} - g)]}. \hspace{1cm} (3)$$

The dynamics of the surplus consumption ratio in Campbell and Cochrane (1999)’s model are described by:

$$\lambda(s_t) = \frac{1}{\bar{S}} \sqrt{1 - 2(s_t - \bar{s})} - 1, \text{ when } s \leq s_{\text{max}}, 0 \text{ elsewhere},$$

where $\bar{S}$ and $s_{\text{max}}$ are respectively the steady-state and upper bound of the surplus-consumption ratio. $\bar{S}$ measures the average gap, in percentage points, between consump-

\(^1\)These assumptions can be relaxed in the simulation. Baxter and Crucini (1995) find that productivity shocks in the US and Europe exhibit a low positive correlation of 0.22. Taking this correlation into account decreases the volatility of real exchange rates - see equation (12) - but it does not modify substantially the results obtained on the forward premium.
tion and habit levels. Assuming that \( \tilde{S} = \sigma \sqrt{\frac{\gamma}{1 - \phi}} \) and \( s_{\text{max}} = \tilde{s} + (1 - \tilde{S}^2) / 2 \) leads to linear time-varying risk-free rates:

\[
r_t = \tilde{r} - B (s_t - \tilde{s}),
\]

where \( \tilde{r} = -\ln(\beta) + \gamma g - \frac{\gamma^2 \sigma^2}{2\tilde{s}} \) and \( B = \gamma (1 - \phi) - \frac{\gamma^2 \sigma^2}{2\tilde{s}} \). Interest rates are constant when \( B = 0 \). To study the UIP puzzle, this is obviously not an interesting case. When \( B < 0 \), interest rates are low in bad times and high in good times.

When the interest rate is allowed to fluctuate, the model resembles the affine framework proposed by Cox, Ingersoll, and Ross (1985), which Frachot (1996) has shown reproduces the forward premium.\(^2\) Campbell and Cochrane (1999)'s model does not correspond to a narrow definition of affine representations of the yield curve, because the market price of risk is not linear in the state variable \( s \) (the surplus-consumption ratio). The model belongs however to the generalized class of affine factor models because its market price of risk can be written as a linear function of the sensitivity function \( \lambda(s) \).

\[ \text{B. Real exchange rates and currency risk premium} \]

I now turn to the trading arrangements in securities markets and their implications for real exchange rates and currency risk premia.

**Real exchange rates** There are no arbitrage opportunities and financial markets are complete.\(^3\) In each country, at each date, a representative investor has access to a domestic bond that pays off one unit of domestic consumption next period in all states of the world and to a foreign bond that pays off one unit of foreign consumption next period in all states of the world. The Euler equation for a foreign investor buying a foreign bond with

\( \text{Frachot (1996) shows that a two-country version of Cox, Ingersoll, and Ross (1985) produces, for certain parameter values, a negative U.I.P slope coefficient. This framework, however, offers no obvious economic explanation for the foreign currency risk premia. The U.I.P slope coefficient is equal to} \) \( (1 - e^{-\lambda}) / (1 - \frac{A^d(1)}{A^d(1) + \alpha}) \) where \( \lambda \), \( \alpha \) and \( A^* \) are diffusion parameters, and \( A^d \) satisfies a unidimensional Riccati differential equation.

\( \text{Assuming the “law of one price on the asset markets” implies the existence of a stochastic discount factor} \ M_{t+1}. \text{ Assuming the “absence of arbitrage” is stronger: it implies the existence of a positive} \ M_{t+1}, \text{ see Cochrane (2001). I use the latter assumption because it also implies the uniqueness of} \ M_{t+1} \text{ in complete markets. Note that the form of the utility function in this paper guarantees that} \ M_{t+1} > 0. \)
return $R_{t+1}^*$ is: \( E_t(M_{t+1}^* R_{t+1}^*) = 1 \). The Euler equation for a domestic investor buying the same foreign bond is: \( E_t(M_{t+1} R_{t+1}^* Q_{t+1} Q_t) = 1 \), where \( Q \) is the real exchange rate expressed in domestic goods per foreign good. Because the stochastic discount factor is unique in complete markets, the change in the real exchange rate is defined as the ratio of the two stochastic discount factors at home and abroad:

\[
\frac{Q_{t+1}}{Q_t} = \frac{M_{t+1}^*}{M_{t+1}}. 
\]

The real exchange rate is here the rate at which the two countries do not want to trade further.

**Exchange rate risk premium** The exchange rate risk premium is the excess return of a domestic investor who borrows funds at home, buys foreign currency, lends on the foreign market for one period and finally reconverts his earnings to the original currency. Thus, in logs, the foreign currency excess return \( r_{t+1}^e \) is equal to:

\[
r_{t+1}^e = \Delta q_{t+1} + r_t^* - r_t, 
\]

where \( r_t \) and \( r_t^* \) are respectively the domestic and foreign risk-free real interest rates. The domestic investor gains \( r_t^* \), but he has to pay \( r_t \), and he loses if the dollar appreciates in real terms - \( q \) decreases - when his assets are abroad.

Backus, Foresi, and Telmer (2001) show that the expected foreign currency excess return is equal to one half of the difference between conditional variances of the two pricing kernels. Assuming log-normal stochastic discount factors leads to:

\[
\begin{align*}
  r_t &= -\log E_t M_{t+1} = -E_t m_{t+1} - \frac{1}{2} Var_t(m_{t+1}), \\
  r_t^* &= -\log E_t M_{t+1}^* = -E_t m_{t+1}^* - \frac{1}{2} Var_t(m_{t+1}^*). 
\end{align*}
\]

The expected change in the exchange rate is then:

\[
E_t(\Delta q_{t+1}) = E_t(m_{t+1}^*) - E_t(m_{t+1}) = -r_t^* + r_t - \frac{1}{2} Var_t(m_{t+1}^*) + \frac{1}{2} Var_t(m_{t+1}).
\]
Thus, the expected foreign currency excess return is equal to:
\[ E_t(r_{t+1}^e) = \frac{1}{2} Var_t(m_{t+1}) - \frac{1}{2} Var_t(m_{t+1}^*) . \] (7)

Equation (7) shows that, in order to obtain predictable currency excess returns, one needs conditional heteroskedasticity of discount factors, which implies here time-variation in Sharpe ratios. Standard homoskedastic models are generally incapable of delivering time-varying risk premia. Thus, with iid consumption growth shocks, power utility cannot deliver a solution to the UIP puzzle, but habit preferences may do so. Such preferences, defined using the difference between consumption and habit, lead to a local risk-aversion coefficient proportional to \( C/(C - H) \), which is the inverse of the surplus-consumption ratio. As a result, one needs time-variation in the surplus-consumption ratio and thus the habit level to produce time-variation in risk-aversion and predictable currency returns.

C. A solution to the UIP puzzle

In this paper, I assume that consumption growth shocks are iid and uncorrelated across countries. Under these assumptions, I can derive a closed-form expression for the currency excess return that highlights the rationale and mechanism of the model.

To simplify the notation, I assume that the preferences of domestic and foreign investors are characterized by the same underlying structural parameters: same risk-aversion coefficient \( \gamma = \gamma^* \), same persistence and steady-state value of the surplus-consumption ratio \( \phi = \phi^* \) and \( S = S^* \), same mean and volatility of consumption growth \( g = g^* \) and \( \sigma = \sigma^* \). With these preferences, the variance of the log stochastic discount factor is equal to:
\[ Var_t(m_{t+1}) = \frac{\gamma^2 \sigma^2}{S^2} [1 - 2(s_t - \bar{s})] , \] (8)

Another way to obtain the same result is to start from the definitions of the log currency risk premia in Lustig and Verdelhan (2007) for the domestic and foreign investors.

In any model where the SDF is lognormal, then the maximal Sharpe ratio is approximately equal to the standard deviation of the log SDF. The maximal Sharpe ratio is \( SR_t = E_t(M_{t+1})/\sigma_t(M_{t+1}) \), where \( E_t(M_{t+1}) = exp[E_t(m_{t+1}) + 1/2 \sigma_t(m_{t+1})] \). As a result, \( SR_t \approx \sigma_t(m_{t+1}) \).

Consider \( U(t) = (C_t - H_t)^{1-\gamma}/(1 - \gamma) \) and define the local risk-aversion coefficient as \( \gamma_t = -C_tU_{cc}(t)/U_c(t) \). Then, \( \gamma_t = \gamma C_t/(C_t - H_t) = \gamma/S_t \).
and equation (7) leads to the following expected currency excess return:

\[ E_t(\Delta q_{t+1}) = E_t(\Delta q_{t+1}) + r^*_t - r_t = \frac{\gamma^2 \sigma^2}{S^2}(s^*_t - s_t). \] (9)

In this case, the real interest rate differential is simply:

\[ r_t - r^*_t = -B(s_t - s^*_t). \] (10)

As a result, the expected change in exchange rate is equal to:

\[ E_t(\Delta q_{t+1}) = [1 + \frac{1}{B} \frac{\gamma^2 \sigma^2}{S^2}] [r_t - r^*_t] = \gamma(\frac{1}{B} - \phi) [r_t - r^*_t]. \] (11)

An interpretation of the UIP puzzle  This model reproduces the UIP puzzle. In this framework, the UIP slope coefficient no longer needs to be equal to unity, even if consumption shocks are simply iid. Since the risk premium depends on the interest rate gap, the coefficient \( \alpha \) in a UIP regression can be below 1 and, when \( B < 0 \), even negative.

What is the intuition for this result? First, exchange rates covary with consumption growth shocks and command time-varying consumption risk premia. In this model, the local curvature of the utility function is equal to \( \gamma/S_t \), thus a low surplus consumption ratio (when consumption is close to the habit level) makes the agent more risk-averse. Using equations (3) and (5), the change in the real exchange rate is:

\[ \Delta q_{t+1} = k_t + \gamma[1 + \lambda(s_t)](\Delta c_{t+1} - g) - \gamma[1 + \lambda(s^*_t)](\Delta c^*_{t+1} - g), \]

where \( k_t \) summarizes all variables known at date \( t \). In bad times, when the domestic investor is more risk averse than his foreign counterpart, \( s_t < s^*_t \) and \( 1 + \lambda(s_t) > 1 + \lambda(s^*_t) \). In this case, domestic consumption shocks dominate the effect of foreign consumption shocks on the exchange rate. As a result, when the domestic economy receives a negative consumption growth shock in bad times, the foreign currency depreciates. Thus, the foreign currency depreciates in response to a negative consumption growth shock and exposes the home investor to more domestic consumption growth risk. Vice-versa, when the domestic economy receives a positive consumption growth shock, the foreign currency
appreciates. The foreign currency is a risky investment when the domestic investor is more risk averse than his foreign counterpart, and the investor wants to be compensated through a positive excess return. As a result, the domestic investor gets a positive excess return if he is more risk averse than his foreign counterpart. However, when the domestic investor is less risk averse than the foreign investor, the exchange rate actually becomes dominated by foreign consumption shocks and, in equilibrium, the foreign investor receives a positive excess return. The interpretation of the risk premium is perfectly symmetric, thus taking into account that a positive excess return for the domestic investor means a negative one for the foreign investor. The currency risk premium is time-varying because risk-aversion is time-varying too.

Second, times of high risk aversion correspond to low interest rates. In bad times, when consumption is close to the subsistence level, the surplus consumption ratio $s_t$ is low, the domestic agent is very risk-averse and domestic interest rates are low. As we have seen, a domestic investor expects to receive a positive foreign currency excess return in times when he is more risk-averse than his foreign counterpart. Thus the domestic investor expects positive foreign currency excess returns when domestic interest rates are low and foreign interest rates are high. This translates to a UIP coefficient less than 1. It is negative because in times of high risk-aversion, a small consumption shock has a large impact on the change in marginal utility, and the stochastic discount factor has a considerable conditional variance $\text{Var}_t(m_{t+1})$. As a consequence, when interest rates are low, the conditional variance of the stochastic discount factor and the excess return are high, and domestic currency excess returns increase sharply with interest rate differentials.

We can reinterpret this result using Backus, Foresi, and Telmer (2001) conditions to reproduce the UIP puzzle: a negative correlation between the difference in conditional means and the half difference in conditional variances of the two pricing kernels, and a greater volatility of the latter. The difference in conditional means of the pricing kernels is equal to $\gamma(1 - \phi)(s_t - s^*_t)$. The currency risk premium, which is the half difference in conditional variances of the two pricing kernels, is given in equation (9). The two are clearly negatively correlated. The risk premium has a larger variance than the difference in conditional means if $\gamma^2\sigma^2/\tilde{S}^2$ is above $\gamma(1 - \phi)$, which is the case for pro-cyclical interest rates ($B < 0$). As a result, the UIP coefficient is negative. Note however that this model satisfies the conditions of Proposition 2, page 16 of Backus, Foresi, and Telmer (2001).
As a consequence, it can reproduce the UIP puzzle but only at the price of potentially negative real interest rates, which is clearly the case when $B < 0$. In a model with storable goods, negative real risk-free rates are an undesirable feature. Empirically though, I find that the frequency of negative real rates simulated by the model is in line with the one measured in the data.

**Long run values of exchange rates**  *In the very long run, the risk premium disappears if the two countries have the same intrinsic characteristics.* If the two countries are similar, then the average real risk-free rate is the same in both countries. Taking unconditional expectations of equation (11) shows that the change in the real exchange rate and the risk premium are on average equal to zero. In the long run, two similar countries satisfy the purchasing power parity condition.

**Currency and equity risk premia**  If two countries have different structural parameters however, steady-state currency excess returns and changes in the real exchange rate are not zero. The steady-state currency excess return is related to the half-difference in maximal steady-state Sharpe ratios (squared):

$$r^e = \frac{1}{2} \gamma^2 \sigma^2 - \frac{1}{2} \gamma^* \sigma^* - \frac{1}{2} \frac{S^2}{S^*} \approx \frac{1}{2} \frac{SR^2}{S^*} - \frac{1}{2} \frac{SR^*}{S^2}.$$  

Countries with high compensation for risk, where the effective coefficient of risk-aversion is high, offer low currency excess returns at the steady-state. The link between currency excess returns and Sharpe ratios would hold with any lognormal SDF; it is already apparent in equation (7). As a motivational exercise, let us look at a cross-section of currency and equity excess returns. Panel A in Table II reports average stock market Sharpe ratios and average currency excess returns over the post-Bretton Woods period for the following countries: Australia, Canada, France, Germany, Italy, Japan, Switzerland and United Kingdom. The country with the highest equity Sharpe ratio (Switzerland) is by far the one offering the lowest currency excess return to the US investor. Its Sharpe ratio is bigger than the US one and the corresponding excess return is negative. Note however two important caveats: as indicated by the standard errors, country by country average currency excess returns are not significantly different from zero (which would be the case
in a symmetric model); equity may not be mean-variance efficient, and thus stock market Sharpe ratios may not correspond to maximal Sharpe ratios.

The model goes beyond an unconditional correlation between currency and risk premia though; it singles out interest rates as the relevant conditioning variables. In the model, low domestic interest rates (i.e., lower than abroad) imply high currency excess returns, and low domestic interest rates (i.e., lower than usual) imply high Sharpe ratios. To highlight the link between currency and equity premia, conditional on interest rates, I turn to currency portfolios of countries sorted on interest rates and compute the average stock market returns (denominated in local currencies) inside each portfolio. Panel B in Table II reports the average currency and stock market excess returns of the 8 currency portfolios proposed in Lustig and Verdelhan (2007). Each period, countries are ranked on the basis of their interest rate at the end of the previous period. The first portfolio contains low interest rate currencies and the last portfolio contains high interest rate currencies. Using this ranking, I allocate stock market excess returns (expressed in foreign currencies) into the same 8 portfolios and compute mean excess returns for each portfolio. As a result, I obtain currency and stock market excess returns conditioned on the level of the foreign interest rate.7

As expected from the UIP literature and shown in Lustig and Verdelhan (2007), low interest rate countries offer low currency excess returns, while high interest rate countries offer high currency excess returns. The novelty of this table is that countries offering high currency excess returns for the US investor offer low equity Sharpe ratio to the local investor. The spread between currency excess returns in the first and last portfolios is highly significant (with a mean value of $-4.65$ percent and a standard error of 0.95). Likewise, the Sharpe ratio obtained on the spread between stock market excess returns in the first and last portfolios is significant (with a mean value of 0.36 percent and a standard error of 0.15). The rationale is the same as before: when foreign interest rates are high, the foreign currency offers high excess returns, but the foreign stock market does not. As a result, there seems to be a clear link between currency and equity risk premia, and the interest rate appears to be the relevant risk indicator. Table II is clearly not a full test of

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7I consider only developed countries. The list of countries used and details about the portfolios' construction are available in Lustig and Verdelhan (2007). I approximate expected excess returns with realized ones.
Table II
Currency and Equity Risk

Panel A presents average currency excess returns $E(r^e)$ for an American investor and Sharpe ratio $SR$ for foreign investors on foreign stock market excess returns (denominated in foreign currencies). The countries considered are: Australia, Canada, France, Germany, Italy, Japan, Switzerland and United Kingdom. Data are quarterly, from Global Financial Data. The period is 1971:I-2004:IV. Panel B presents average currency excess returns $E(r^e)$ for an American investor and Sharpe ratio $SR$ for foreign investors on foreign stock market excess returns for 8 portfolios of developed countries sorted on interest rates. Standard errors are reported between brackets. They are obtained by bootstrapping estimations 10,000 times (i.e drawing with replacement under the assumption that excess returns are i.i.d). Data are quarterly, from Global Financial Data. The period is 1953:I-2002:IV. In both panels, currency excess returns are computed using Treasury bill yields and exchange rates, with the United States as the domestic country. Sharpe ratio are computed using ex-post stock market returns, expressed in foreign currencies, and foreign equivalent of Treasury bill yields. All moments are annualized.

**Panel A: 8 Foreign Countries**

<table>
<thead>
<tr>
<th></th>
<th>AU</th>
<th>CN</th>
<th>FR</th>
<th>GR</th>
<th>IT</th>
<th>JP</th>
<th>SW</th>
<th>UK</th>
<th>US</th>
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</thead>
<tbody>
<tr>
<td>$E(r^e)$</td>
<td>1.42</td>
<td>0.74</td>
<td>2.01</td>
<td>1.54</td>
<td>1.76</td>
<td>1.25</td>
<td>−0.64</td>
<td>1.80</td>
<td>−−</td>
</tr>
<tr>
<td></td>
<td>[2.01]</td>
<td>[0.89]</td>
<td>[2.15]</td>
<td>[2.21]</td>
<td>[2.13]</td>
<td>[2.68]</td>
<td>[2.94]</td>
<td>[1.90]</td>
<td></td>
</tr>
<tr>
<td>$SR$</td>
<td>0.17</td>
<td>0.11</td>
<td>0.12</td>
<td>0.08</td>
<td>−0.02</td>
<td>0.16</td>
<td>0.34</td>
<td>0.21</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>[0.20]</td>
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<td>[0.19]</td>
<td>[0.18]</td>
<td>[0.19]</td>
<td>[0.18]</td>
<td>[0.23]</td>
<td>[0.20]</td>
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**Panel B: 8 Portfolios**

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<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(r^e)$</td>
<td>−1.59</td>
<td>0.78</td>
<td>0.63</td>
<td>0.91</td>
<td>0.63</td>
<td>2.02</td>
<td>1.60</td>
<td>2.69</td>
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<tr>
<td></td>
<td>[1.52]</td>
<td>[1.29]</td>
<td>[1.49]</td>
<td>[1.31]</td>
<td>[1.44]</td>
<td>[1.17]</td>
<td>[1.21]</td>
<td>[1.19]</td>
</tr>
<tr>
<td>$SR$</td>
<td>0.58</td>
<td>0.52</td>
<td>0.29</td>
<td>0.38</td>
<td>0.51</td>
<td>0.39</td>
<td>0.28</td>
<td>0.24</td>
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<tr>
<td></td>
<td>[0.15]</td>
<td>[0.16]</td>
<td>[0.13]</td>
<td>[0.14]</td>
<td>[0.14]</td>
<td>[0.13]</td>
<td>[0.16]</td>
<td>[0.14]</td>
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II. Simulation

In this section, I first describe the calibration and then the simulation results of a model where endowment shocks correspond to post-trade allocations. Finally, as a reality check, I compute time-series of the stochastic discount factor, the surplus consumption ratio and
the local curvature using actual US consumption data.

A. Method

I assume that two countries, for example the United States and United Kingdom, can be characterized by the same set of parameters \((g, \sigma, \beta, \gamma, \phi)\) and that endowment shocks are not correlated across countries.

**Calibration** I fix \(\gamma\) to 2, which is a common value in the real business cycle literature and the value chosen by Campbell and Cochrane (1999) and Wachter (2006) in their simulations. To determine the remaining five independent parameters of the model, I target five simple statistics: the mean \(g\) and standard deviation \(\sigma\) of real per capita consumption growth, the mean \(\bar{r}\) and standard deviation \(\sigma_r\) of the real interest rate and the steady-state Sharpe ratio \(SR\). I calibrate the first three moments directly from the data and obtain closed-form expressions for the last two moments.\(^8\) These five statistics are measured over the 1947:2-2004:4 period for the US economy. Per capita consumption data of non durables and services are from the BEA. US interest rates, inflation and stock market excess returns are from CRSP (WRDS). The real interest rate is the return on a 90-day Treasury bill minus the expected inflation. I compute expected inflation with a one-lag two-dimensional VAR using inflation and interest rates. The Sharpe ratio is the ratio of the unconditional mean of quarterly stock excess returns on their unconditional standard deviation. Table III summarizes the parameters used in this paper. They are close to the ones proposed by Campbell and Cochrane (1999) and Wachter (2006). The habit process is very persistent (\(\phi = 0.994\)), and consumption is on average 7 percent above the habit level, with a maximum gap of 12 percent (respectively 6% and 9% in Campbell and Cochrane (1999)).

**Simulation method** From 100,000 endowment shocks and the parameters above, I build surplus consumption ratios, stochastic discount factors, interest rates in both coun-

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\(^8\)An exact closed-form expression for the standard deviation of the interest rate is difficult to obtain, but I choose the parameters using a simple approximation: supposing that \(\lambda(s_t)\) remains equal to its steady-state value \((\lambda(\bar{s}) = (1 - \bar{S})/\bar{S})\), the variance of the interest rate is close to \((\sigma/\bar{S})^2B^2/(1 - \phi^2)\), where \(S\) is defined in terms of \(\sigma, \gamma, \phi\) and \(B\).
Table III
Calibration Parameters

The table presents the parameters of the model and their corresponding values in Campbell and Cochrane (1999) and Wachter (2006). Data are quarterly. The reference period is here 1947:2-2004:4 (1947-1995 in Campbell and Cochrane (1999), 1952:2-2004:3 in Wachter (2006)). Per capita consumption of non durables and services is from the BEA web site. Interest rates and inflation data are from CRSP(WRDS). Expected inflation is computed using a one-lag two-dimensional VAR using inflation and interest rates. The real interest rate is the return on a 90-day Treasury bill minus the expected inflation. The UIP coefficient is computed using the US-UK exchange rate. UK interest rates, inflation rates and exchange rates are from Global Financial Data.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td><strong>Calibrated parameters</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$g$ (%)</td>
<td>0.53</td>
<td>0.47</td>
<td>0.55</td>
</tr>
<tr>
<td>$\sigma$ (%)</td>
<td>0.51</td>
<td>0.75</td>
<td>0.43</td>
</tr>
<tr>
<td>$\pi$ (%)</td>
<td>0.34</td>
<td>0.23</td>
<td>0.66</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.99</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td>$B$</td>
<td>−0.01</td>
<td>−</td>
<td>0.01</td>
</tr>
<tr>
<td><strong>Implied parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.00</td>
<td>0.97</td>
<td>0.98</td>
</tr>
<tr>
<td>$\overline{S}$</td>
<td>0.07</td>
<td>0.06</td>
<td>0.04</td>
</tr>
<tr>
<td>$S_{\text{max}}$</td>
<td>0.12</td>
<td>0.09</td>
<td>0.06</td>
</tr>
</tbody>
</table>
tries and the implied exchange rate. To compute moments on the price-dividend ratio (using the price of a consumption claim), stock market returns and real yields, I use the numerical algorithm developed by Wachter (2005). Appendix A details the procedure.

B. Results

Table IV first reports mean, standard deviation and autocorrelation of consumption growth, real interest rate, real exchange rate change, log price dividend ratio, market return (measured as a return on a consumption claim) and real five-year yield. It also presents the correlation between changes in exchange rates and relative consumption growth, between the equity excess return and either the log price dividend ratio or the risk-free rate. I first review the UIP and equity premium puzzles, comparing this model with the recent empirical findings of Lustig and Verdelhan (2007), and then turn to the implied real yields and the link between consumption growth and exchange rates.

**UIP and equity premium puzzles** The calibration targets the first two moments of consumption growth and real interest rates and the equity Sharpe ratio, and the simulation successfully reproduces their empirical counterparts. Let us now focus on moments not used in the calibration. First and foremost, the model delivers a UIP slope coefficient $\alpha$ that is negative ($-1.3$) and in line with its empirical value. Second, the model implies a sizable equity premium of 5.4%, and a reasonable volatility of the market return (12.6%). The mean log price dividend ratio and its standard deviation are in line with the data. As a result, the model can reproduce both the equity premium puzzle and the UIP puzzle. Third, the model reproduces the stark contrast between the very low persistence of the exchange rates changes and the high persistence of interest rate differentials. The model slightly underestimates the persistence of consumption growth and overestimates the persistence of the risk-free rate, but it gives a close fit for exchange rates, market returns, price-dividend ratios and real yields. The empirical low serial correlation of real risk-free rates seems however an artefact due to the assumption on expected inflation. In the US, nominal interest rates are highly autocorrelated at both annual and quarterly frequencies, real interest rates are highly autocorrelated at annual frequencies, but quarterly real interest rates here are not because quarterly expected inflation (computed here using a
### Table IV
Simulation Results: Post-Trade Consumption

The table first presents the mean, standard deviation and autocorrelation of consumption growth $\Delta c$, risk-free interest rates $r^f$, changes in real exchange rates $\Delta q$, log price-dividend ratio $pd$, stock market risk premium $r_m$ and real five-year yield $y^5$. All moments are annualized. The table then reports the correlation between the stock market excess return and the log dividend price ratio $\rho_{t+1}^{r_m - r_f}$, the correlation between stock market excess return and the risk-free rate $\rho_{t+1}^{r_m - r_f}$, the correlation between the consumption growth differential and changes in real exchange rates $\rho_{t+1}^{\Delta c^\ast - \Delta c}$ and the UIP slope coefficient $\alpha$. Standard errors are reported in brackets. In both panels, the last three columns correspond to actual data for the US and the US-UK exchange rate over the 1947:II-2004:IV period (1997:IV-2006:IV for real yields from US inflation-indexed bonds).

<table>
<thead>
<tr>
<th></th>
<th>Simulation Results</th>
<th>Actual Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean (%) Std (%)</td>
<td>Autoc. Mean (%) Std (%) Autoc.</td>
</tr>
<tr>
<td>$\Delta c$</td>
<td>2.11 1.03 0.00</td>
<td>2.10 1.35 0.17</td>
</tr>
<tr>
<td>$r^f$</td>
<td>1.64 1.85 0.98</td>
<td>1.40 1.99 0.76</td>
</tr>
<tr>
<td>$\Delta q$</td>
<td>7.65 40.65 0.01</td>
<td>0.38 10.29 0.06</td>
</tr>
<tr>
<td>$pd$</td>
<td>354.73 57.68 0.99</td>
<td>479.12 42.12 0.95</td>
</tr>
<tr>
<td>$r_m$</td>
<td>5.43 12.63 0.01</td>
<td>8.63 16.70 -0.04</td>
</tr>
<tr>
<td>$y^5$</td>
<td>1.19 2.05 0.97</td>
<td>2.64 1.15 0.97</td>
</tr>
</tbody>
</table>

|                  | Coef. s.e. | Coef. s.e. |
| $\rho_{t+1}^{\Delta q, \Delta c^\ast - \Delta c}$ | 0.82 [0.01] | −0.04 [0.13] |
| $\rho_{t+1}^{r_m - r_f}$ | 0.09 [0.02] | −0.14 [0.07] |
| $\rho_{t+1}^{r_m - r_f}$ | −0.08 [0.02] | −0.03 [0.07] |
| $\alpha$         | −1.26 [0.51] | −1.29 [0.64] |
bivariate VAR) is volatile. However, real yields computed from inflation-indexed bonds are highly autocorrelated. To sum up, this framework can reproduce at the same time the first two moments of consumption growth and risk free rates and the UIP and equity premium puzzles. This is the main achievement of the model.

Moreover, the model links the predictable components of currency and equity excess returns. In the model, bad times correspond to low risk-free rates and high currency risk premia. Empirically, this result is supported by a vast literature on the UIP condition: a UIP slope coefficient below unity implies that currency excess returns are higher when domestic interest rates are lower. In the model, bad times correspond to high risk-aversion and a high Sharpe ratio; when interest rates are low, equity excess returns are high. We know since Fama and Schwert (1977) that the same relationship holds in the data as well. More recently, using efficient tests of stock market predictability, Campbell and Yogo (2006) reject the null of no predictability for the risk-free rate at monthly and quarterly frequencies over the 1952-2002 period. The model implies that high price dividend ratio are positively correlated with high excess return; the data used in this paper suggest the opposite, but Campbell and Yogo (2006) cannot reject the null of no predictability for the price-dividend ratio.

Link to previous work  The model and its simulation highlight the results in Lustig and Verdelhan (2007). First, the simulation replicates their key empirical finding. Second, the model provides a rationale for building portfolios of currencies to study exchange rate risk.

Lustig and Verdelhan (2007) find that high interest rate currencies provide high excess returns because these currencies tend to depreciate in bad times for the American investor. The model replicates this finding. A simple way to see this is to consider the following regression of changes in exchange rates on domestic consumption growth and domestic consumption growth interacted with the interest rate differential expressed in percentage points:

$$\Delta q_{t+1} = \beta_0 + \beta_1 \Delta c_{t+1} + \beta_2 \Delta c_{t+1} (r^*_t - r_t) + \epsilon_{t+1}. $$

A positive coefficient $\beta_2$ indicates that the foreign currency tends to appreciate ($q$ increases) more in good times for a domestic investor when the foreign interest rate is high.
Likewise, the foreign currency tends to depreciate more in bad times when the foreign interest rate is high. As a result, when the foreign interest rate is higher, it means more consumption growth risk for the domestic investor. In the model, $\beta_2$ is positive and significant ($\beta_2 = 16$ with a standard error of 0.6).

Currency portfolios offer empirically three advantages: by conditioning on the interest rate, they create a large average spread in excess returns between low and high interest rate portfolios, which is more than twice the average spread for any two given countries; they keep the number of covariances that must be estimated low; they allow to continuously expand the number of countries studied as financial markets open up to international investors, thus including data from the largest possible set of countries. The model offers a simple theoretical motivation for building these currency portfolios. Using first-order Taylor approximation, the ex-post currency excess return for country $i$ is:

$$r_{t+1}^{ex-post,i} \simeq -\gamma \left[ (\phi - 1)(s_i - s_t) + \frac{1}{S}(u_{t+1}^i - u_t) \right] - B(s_i - s_t),$$

$$\simeq E_t(r_{t+1}^{i}) - \frac{\gamma}{S}(u_{t+1}^i - u_{t+1}).$$

In the model, time series averages of currency excess returns for any given currency would be zero in a long sample. But, by ranking countries on foreign interest rates, currency portfolios sort countries on the level of the state variable $s^i$, which changes with risk-aversion. By taking averages of excess returns inside each portfolio, provided that the idiosyncratic risks $u_{t+1}^i$ cancel out, one focuses on non-zero expected excess returns, which is the object of interest from a financial perspective.

**Real yields** I now turn to the model’s implications for the real term structure. Pro-cyclical real risk-free rates imply a slightly downward sloping average yield curve. The simulated real yield on a 5-year note is 0.4 percentage point lower than the 3-month interest rate. Table VIII in the appendix reports results on the US and UK real yield curves, along with simulated results. Empirical evidence on the average slope of the yield curve are inconclusive. Using inflation-indexed bonds from 1983 to 1995, Evans (1998)

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documents that real term premia in the United Kingdom are significantly negative (−2%). I extend his results using the Bank of England zero-coupon real yields and a Nelson and Siegel (1987) interpolation to obtain yields for the same maturities as in the model. I find a flat yield curve from 1995 to 2006. J. Huston McCulloch has constructed interpolated real yield curves from US TIPS data starting in 1997. US data show an upward-sloping real yield curve. With short samples and, potentially, liquidity issues, the empirical slope of the real yield curve obtained from inflation-indexed bonds remains an open question. Decomposing nominal yields into real and inflation-related components, Ang, Bekaert, and Wei (2007) find that the unconditional real rate curve is fairly flat around 1.3%, which is close to the value of the 5-year real rate in the model. Cochrane and Piazzesi (2006) note that the real yield curve should be downward sloping when inflation is stable. In this case, interest rate variation comes from variation in real rates, and long-term bonds are safer investments for long-term investors because rolling over short term bonds runs the reinvestment risk that short term rates will change. Note that the model also implies real bond risk premia, thus rejecting the expectation hypothesis on real yields. These bond risk premia are negative on average, and time-varying. The classic Campbell and Shiller (1991) test of the expectation hypothesis leads to negative and significant slope coefficients (−0.01 at the five year horizon). Inflation indexed bonds in the US produce a similar negative coefficient, but the sample is too short to be conclusive.

Exchange rates and consumption This paper proposes a simple, fully developed model that replicates the UIP puzzle, links currency to equity and bond risk premia. However the model, because of its simplicity, has two major shortcomings: it implies a strong and positive correlation between changes in exchange rate and consumption growth and a high volatility of real exchanges rates.


\[ y_{t+1}^{n-1} - y_t^n = \beta_0 + \frac{\beta_1}{n-1} (y_t^n - y_t^1) + \varepsilon_{t+1}. \]

Under the expectation hypothesis, the slope coefficient \( \beta_1 \) is equal to one.
confirm their findings. Backus and Smith (1993) note that in complete markets and with power utility, the change in the real exchange rate is equal to the relative consumption growth in two countries times the risk-aversion coefficient ($\Delta q_{t+1} = -\gamma[\Delta c^*_{t+1} - \Delta c_{t+1}]$), thus implying a perfect correlation between the consumption growth and real exchange rate variations. In the model presented here, the presence of habits leads to a lower correlation than with power utility, but the model still implies too high a correlation between real exchange rates and consumption growth rates because a single source of shocks drives all variables.

Moreover, simulated real exchange rates vary here three times more than in the data. This result can be related to the definition of the exchange rate in complete markets in equation (5), which implies that the variance of the real exchange rate changes is equal to:

$$\sigma^2(\Delta q) = \sigma^2(m) + \sigma^2(m^*) - 2\rho(m, m^*)\sigma(m)\sigma(m^*).$$  \hspace{1cm} (12)

To fit the equity premium, we know since Mehra and Prescott (1985) and Hansen and Jagannathan (1991) that the variance of the stochastic discount factor has to be high. Taking into account the low correlation among consumption shocks across countries, and thus the low correlation of stochastic discount factors with CRRA preferences, Brandt, Cochrane, and Santa-Clara (2006) show that the actual exchange rate is much smoother than the theoretical one implied by asset pricing models. The same tension is present here, because, when countries do not trade, the standard deviation of the change in exchange rate is proportional to the Sharpe ratio.\textsuperscript{11} Thus, one cannot obtain a high Sharpe ratio and a low exchange rate volatility at the same time.

The model clearly needs to be refined. In the last section, I introduce international trade and nontradables and study their implications regarding the volatility of real exchange rates and their correlation with consumption growth.

C. Actual data

Figure 4 shows the time-series of the surplus consumption ratio, stochastic discount factor and local risk curvature for an American investor. The figure is based on the same set of

\textsuperscript{11}The variance of real exchange rate appreciation is here at the steady-state: $\langle Var_t(\Delta q_{t+1}) \rangle_{\text{Steady-state}} = 2(\gamma \sigma/\bar{S})^2 = 2SR^2$.  

25
parameters presented in the first column of Table III, but the simulation uses only actual US consumption growth data for the 1947:II–2004:IV period.

Feeding the model with actual consumption growth shocks provides a simple reality check. It appears that the surplus consumption ratio varies between 4% and 12%. Thus, the local curvature, computed as $\gamma/S_t$, fluctuates between 15 and 60 and is much higher than the risk-aversion coefficient. The resulting stochastic discount factor is volatile till the mid-50s and then fluctuates around unity. The implied real interest rates are sometimes negative, reaching a minimum value of $-0.4\%$, but negative values do not happen more often than in US ex-ante real interest rates (computed as indicated in section II - A). This reality check also shows that habits are well defined. Ljungqvist and Uhlig (2003) argue that, in some cases, habit levels in Campbell and Cochrane (1999)’s model may decrease following a sharp increase in consumption. By construction of the model, an infinitesimal rise in consumption always increases habit levels. With actual data, the case described by Ljungqvist and Uhlig (2003) never happens.

### III. Estimation

The calibration exercise has shown that for parameter values close to the ones used in this literature, the model can reproduce the first two moments of consumption growth and interest rates and features of the equity and currency markets. In this section, I present results of a direct estimation of the model on foreign currency excess returns. I look for the structural parameters of stochastic discount factors (risk-aversion $\gamma$, persistence $\phi$, average surplus consumption ratio $\bar{S}$) that minimize the pricing errors of the Euler equation. I check that the model is not rejected by the data, and that its parameters imply pro-cyclical real interest rates.

#### A. Method

The model can be estimated without linear approximation by computing the sample equivalent of the Euler equation:

$$E_t[M_{t+1}R_{t+1}^{e,i}] = 0,$$
where $R_{t+1}^i = (1 + r_t^i)Q_{t+1}^i/Q_t^i - (1 + r_t)$ represents the currency excess return of investing in country $i$ and $Q^i$ and $r^i$ are respectively the real exchange rate and the real interest rate of country $i$. Theoretically, the model has only one kind of shock that drives both consumption and interest rate processes. Thus I estimate the stochastic discount factor $M_{t+1}$ using either Treasury Bills or consumption data. In each case, I conduct two different experiments:

- first, the model is estimated using moments implied by the pricing behavior of an American investing in 8 other OECD countries (Australia, Canada, France, Germany, Italy, Japan, Switzerland, United Kingdom). Estimations are run over the 1971 : I - 2004 : IV period, for which interest rates and exchange rates are available for all countries considered. The model predicts that average currency excess returns should be zero between similar countries. Thus, the estimation is run on conditional moments, using a constant and the domestic lagged interest rate as instruments. As a result, this setup gives 16 moments that allow for the estimation of the three parameters ($\gamma$, $\phi$ and $\tilde{S}$).

- second, the model is estimated using the 8 portfolios of currency excess returns proposed in Lustig and Verdelhan (2007). Estimations are run over the post-Bretton Woods 1971 : I - 2002 : IV period, for which these portfolios are available.

Estimations rely on the continuously-updating estimator studied by Hansen, Heaton, and Yaron (1996). The estimator is implemented over a grid of potential parameters and seeks to minimize a weighted average of pricing errors. Standard errors are computed using GMM asymptotic theory following Hansen (1982) for the three structural parameters and by delta-method for the implied UIP coefficients. Possible parameters’ ranges are deduced from the empirical literature on foreign exchange risk premia (the persistence coefficient $\phi$ should be above 0.8 and below unity) and habit-based models (the steady-state surplus-consumption ratio $\tilde{S}$ should be positive and below 10%). The coefficient $\gamma$, which is related to risk-aversion, is assumed to be positive and below 20.

### B. Results

Table V presents the estimated values of the model’s three structural parameters, the minimized criterion $J$ and the corresponding $p$-value $p = 1 - \chi^2(J, N - 3)$ testing the null hypothesis that pricing errors are zeros. The table also reports the implied UIP
slope coefficient $\alpha_{\text{implied}} = \gamma (1 - \phi) / B$ that the structural parameters would deliver in a two-country symmetric model with post-trade consumption data. Panel A reports results obtained using only consumption growth to compute stochastic discount factors. Panel B reports results obtained using only interest rates to compute stochastic discount factors.

The three structural parameters are estimated within their proposed ranges, and no corner solution is reached, except in one case where $\phi$ reaches its upper bound. $P$-values range from 49% to 82%. $\gamma$ coefficients vary between 3 and 11 depending on the set of excess returns and pricing kernels considered. The persistence parameter $\phi$ is estimated around 0.99 with relatively high standard errors. Average surplus consumption ratios range values between 3% and 7% which translate in habits ranging from 93% to 97% of consumption. In simulations assuming post-trade consumption shocks, these parameters would deliver small negative UIP coefficients $\alpha$.

The estimated values of the model’s three structural parameters seem reasonable and line with the literature on domestic excess returns. Chen and Ludvigson (2004) estimate habit-based models without imposing the functional form of habit preferences. They conclude that in order to match moment conditions corresponding to Fama-French portfolios, habits should be equal to a large fraction of current consumption (97% on average). Using a simulation-based method, Tallarini and Zhang (2005) estimate Campbell and Cochrane (1999)'s model on US domestic assets (assuming a constant real risk-free interest rate). They find that the persistence coefficient $\phi$ is above 0.9 and the risk-aversion coefficient equal to 6.3.

All estimations imply negative values for $B$, i.e pro-cyclical interest rates, which is consistent with recent results found in the real interest rate literature. Challenging previous findings from Stock and Watson (1999), Dostey, Lantz, and Scholl (2003) conclude that the ex-ante real rate is contemporaneously positively correlated with GDP and with lagged cyclical output. Likewise, Ang, Bekaert, and Wei (2007) find that the US real rate is pro-cyclical.

**IV. International trade**

In this section, I show how to compute optimal international trade starting from endowment processes and reports simulation results obtained with proportional and quadratic
Table V
Estimation Results

The table presents the estimated values of the model’s three structural parameters (risk-aversion $\gamma$, persistence $\phi$, average surplus consumption ratio $\overline{S}$ in percentage) and the implied UIP slope coefficient $\alpha_{implied} = \gamma(1 - \phi)/B$. It also reports the number of excess returns $N$, the minimized criterion $J$ and the corresponding $p$-value $p = 1 - \chi^2(J, N - 3)$ testing the null hypothesis that the pricing errors are zeros. Panel A reports results obtained using only consumption growth to compute stochastic discount factors. Panel B reports results obtained using only interest rates to compute stochastic discount factors. In columns 2 and 4, the estimation uses the currency excess returns of an American investor in 8 other OECD countries (Australia, Canada, France, Germany, Italy, Japan, Switzerland, United Kingdom). Using a constant and the US interest rate as instruments, the estimation is run on 16 moment conditions. In columns 3 and 5, the estimation uses the 8 portfolios of currency excess returns proposed in Lustig and Verdelhan (2007). These portfolios are built by sorting currencies on interest rates. Data are quarterly. The sample is 1971:II-2004:IV for individual currencies and 1971:II-2002:IV for currency portfolios. Standard errors are reported between brackets.

<table>
<thead>
<tr>
<th></th>
<th>Panel A: Using consumption</th>
<th>Panel B: Using interest rates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8 Countries</td>
<td>8 Portfolios</td>
</tr>
<tr>
<td>$N$</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>$J$</td>
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<tr>
<td>$p$</td>
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<td>$\gamma$</td>
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</tr>
<tr>
<td></td>
<td>[3.09]</td>
<td>[1.15]</td>
</tr>
<tr>
<td>$\phi$</td>
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<td>0.99</td>
</tr>
<tr>
<td></td>
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<td>[0.14]</td>
</tr>
<tr>
<td>$\overline{S}$</td>
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<td>2.50</td>
</tr>
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<td></td>
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<td>[0.06]</td>
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<td>$\alpha_{implied}$</td>
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<td>$-0.02$</td>
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<td></td>
<td>[0.02]</td>
<td>[0.02]</td>
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</table>
trade costs. Finally, I investigate the role of non tradable goods.

A. Optimal exports

I assume that international shipping costs have two components. The first one is the usual iceberg-like trade cost. When a unit of the good is shipped, only a fraction $1 - \tau$ arrives to the foreign shore. The second component is a quadratic cost, which captures the capacity constraints of international trade and ensures that the total cost of trade increases with the volume of international trade. Thus, this quadratic cost is assumed to be proportional (with coefficient $\delta$) to the ratio of exports to endowments as in Backus, Kehoe, and Kydland (1992).

Let $X_t$ denote the amount of the good exported from a domestic to a foreign country at time $t$. A superscript $*$ refers to the same variable for the foreign country. To find the optimal amount of exports $X_t \geq 0$ and $X_t^* \geq 0$, I consider the planning problem:

$$
\text{Max } \sum_{t=0}^{\infty} \beta^t \left( \frac{(C_t - H_t)^{1-\gamma} - 1}{1-\gamma} + \frac{(C_t^* - H_t^*)^{1-\gamma} - 1}{1-\gamma} \right),
$$

subject to:

$$
C_t = Y_t - X_t + X_t^*(1 - \tau - \frac{\delta X_t^*}{2Y_t^*}) \quad \text{and} \quad C_t^* = Y_t^* - X_t^* + X_t(1 - \tau - \frac{\delta X_t}{2Y_t}),
$$

where $Y_t$ denotes the domestic endowment. If one country exports, the other does not as there is only one good in the model. Let us assume first that the domestic country exports ($X_t \geq 0$, $X_t^* = 0$). The first order condition with respect to $X_t$ is then:

$$
-[Y_t - X_t - H_t]^{-\gamma} + [1 - \tau - \frac{\delta X_t}{2Y_t}][Y_t^* + X_t(1 - \tau - \frac{\delta X_t}{2Y_t})] - H_t^* = 0.
$$

The optimal amount of exports is the solution to equation (15) provided that it is positive and satisfies the following conditions: exports are below endowments, consumptions are above habit levels in both countries, and a positive fraction of the export makes it to the shore. A closed form solution can be found for log utility ($\gamma = 1$) or when there is no quadratic cost. Appendix A studies the different solutions with and without quadratic cost.
costs.

The case of foreign country exports is obviously symmetric. If the foreign country exports \(X_t = 0, X_t^* \geq 0\), the first order condition with respect to \(X_t^*\) is then:

\[
-\left[Y_t^* - X_t^* - H_t^*\right]^{-\gamma} + [1 - \tau - \delta X_t^*][Y_t + X_t^*(1 - \tau - \delta X_t^* Y_t)] - H_t^\gamma = 0. \tag{16}
\]

If there are no positive solutions to both export problems, then countries consume their endowments. There is a no-trade zone in which the marginal utility gain of shipping a good is more than offset by the trade cost. Figure 1 summarizes the different cases.

When there is trade, one first-order condition \((15)\) or \((16)\) of the social planner’s problem is satisfied, and the countries share risk. When there is no trade, the real exchange rate is determined on the asset market as the ratio of the two marginal utilities of consumption. To summarize, the real exchange rate \(Q_t\) can take the following values:

- If the domestic country exports, \(Q_t = \frac{1}{1 - \tau - \delta X_t^* / Y_t^*}\);
- If the foreign country exports, \(Q_t = 1 - \tau - \delta X_t^* / Y_t\);
- If there is no trade, \(Q_{t+1} = (\frac{Y_t^{*} - X_t^{*}}{Y_t - X_t})^{-\gamma}\).

Introducing quadratic costs has an interesting implication for real exchange rates. Without quadratic costs, real exchange rates fluctuate between two constant boundaries when there is no trade and remain on a boundary when one country exports, as shown by Dumas (1992). With quadratic costs, real exchange rates are never constant even when countries export. The setting presented here relates to a large literature in international economics. Proportional (iceberg-like) shipping costs were first proposed by Samuelson (1954), and then used by Dumas (1992), Sercu, Uppal, and Hulle (1995), Sercu and Uppal (2003) and Obstfeld and Rogoff (2000) to study real exchange rates. Yet, none of these papers tackle the forward premium puzzle, and Hollifield and Uppal (1997) show that proportional trade costs are not enough to reproduce the UIP puzzle when agents are characterized by constant relative risk-aversion (CRRA). They find that the implied UIP slope coefficient is never negative, not even for extreme levels of constant risk-aversion or trade costs.
B. Simulation results with trade

To simulate the model with trade, I need to calibrate additionally the proportional and quadratic trade costs. Anderson and van Wincoop (2004) provide an extensive survey of the trade cost literature and conclude that total international trade costs, which include transportation costs and border-related trade barriers, represent an ad-valorem tax of about 74%. Obstfeld and Rogoff (2000) assume a conservative trade cost of 25%. I simulate the model with a proportional trade cost $\tau$ equal to 0 (no trade cost), 25%, 50% and 75%, and a quadratic trade cost $\delta = 0$ or $\delta = 0.2$ as in Backus, Kehoe, and Kydland (1992). The latter value ensures that trade costs increase with trade, but reasonably so: when a country imports the equivalent of 20% of its endowment, trade cost increase by 2 percentage points. All simulations use the same set of parameters reported in Table III, but with a higher volatility of endowment shocks. This value is chosen to match the standard deviation of US net income, which is defined as the sum of consumption in non durables and services and net exports. Net income is more volatile than consumption growth: its standard deviation is 0.66% per quarter versus 0.51% for consumption growth.

When countries can trade, they share risk and their consumption growth is less volatile than their endowment shocks. This in turn decreases the standard deviation of real interest rates and real exchange rates. The UIP coefficient remains negative and in the 95% confidence interval of its empirical counterpart. Thus this model reproduces the forward premium puzzle for reasonable levels of international trade costs. I detail below the impact of proportional and quadratic trade costs on trade and the exchange rate distribution.

Proportional trade costs Let us first consider the case of proportional trade costs. Figure 2 reports the time-series of the real exchange rate, the surplus-consumption ratios and the exports/endowments ratios for both countries during the first 10,000 periods

---

12Border-related trade barriers represent a 44% cost. This estimate is a combination of direct observation and inferred costs. Transportation costs represent 21%.
13The same logic as in the previous section applies here. When the domestic investor is more risk-averse, the foreign currency is dominated by domestic consumption growth shocks, and it is a risky investment for the domestic investor. Moreover, when countries share risk, consumption growth shocks are positively correlated. In this case, when the domestic investor is less risk-averse than the foreign investor, the foreign currency can even provide a consumption hedge.
Table VI  
Simulation Results: Trade

The table presents the standard deviation (\(\sigma\)) of real per capita consumption growth, the mean (\(\bar{r}\)) and standard deviation (\(\sigma_r\)) of the real interest rate and the standard deviation (\(\sigma_{\Delta q}\)) and autocorrelation (\(\rho_{\Delta q_t, \Delta c_{t-1}}\)) of the real exchange rate. \(\Delta q_t\) denotes the correlation between the consumption growth differential and changes in exchange rate. \(T\) denotes the mean openness ratio. \(\alpha\) denotes the UIP slope coefficient and \(s.e\) the associated standard error. The parameter \(\tau\) determines the size of the proportional trade cost while \(\delta\) determines the importance of the quadratic trade cost. The last column corresponds to actual data for the US and the US-UK exchange rate over the 1947:II-2004:IV period. Data are quarterly. All moments are annualized.

<table>
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<tr>
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<th>Simulation Results</th>
<th>Data</th>
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<tbody>
<tr>
<td></td>
<td>(\tau = 0)</td>
<td>(\tau = 0.25)</td>
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<tr>
<td>(\sigma) (%)</td>
<td>1.02</td>
<td>0.99</td>
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<tr>
<td>(\bar{r}) (%)</td>
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<td>1.30</td>
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<td>(\sigma_r) (%)</td>
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<td>(\sigma_{\Delta q}) (%)</td>
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<td>(\rho_{\Delta q_t, \Delta c_{t-1}})</td>
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</tr>
<tr>
<td>(T) (%)</td>
<td>18.76</td>
<td>13.14</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>[0.00]</td>
<td>[0.29]</td>
</tr>
<tr>
<td>(s.e)</td>
<td>[0.42]</td>
<td>[0.38]</td>
</tr>
</tbody>
</table>

Panel B: Proportional and quadratic cost \(\delta = 0.2\)

<table>
<thead>
<tr>
<th></th>
<th>(\tau = 0)</th>
<th>(\tau = 0.25)</th>
<th>(\tau = 0.5)</th>
<th>(\tau = 0.75)</th>
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<tbody>
<tr>
<td>(\sigma) (%)</td>
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<td>1.01</td>
<td>1.05</td>
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<tr>
<td>(\bar{r}) (%)</td>
<td>1.34</td>
<td>1.44</td>
<td>1.36</td>
<td>1.42</td>
</tr>
<tr>
<td>(\sigma_r) (%)</td>
<td>1.64</td>
<td>1.50</td>
<td>1.61</td>
<td>1.80</td>
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<tr>
<td>(\sigma_{\Delta q}) (%)</td>
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<td>10.99</td>
<td>10.94</td>
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<tr>
<td>(\rho_{\Delta q_t, \Delta c_{t-1}})</td>
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<td>0.74</td>
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</tr>
<tr>
<td>(T) (%)</td>
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<td>12.84</td>
<td>8.07</td>
<td>7.36</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>-0.84</td>
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<td>-1.29</td>
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<td>(s.e)</td>
<td>[0.42]</td>
<td>[0.38]</td>
<td>[0.10]</td>
<td>[0.14]</td>
</tr>
</tbody>
</table>

33
of a simulation. Countries trade when their endowments imply differences in marginal utility of consumption that are not offset by trade costs. When countries trade, the real exchange rate is constant, equal to $1/(1 - \tau)$ or $1 - \tau$ depending on whether the domestic or foreign country exports. When there is no trade, the real exchange rate fluctuates between these bounds. Thus, with a low trade cost, the exchange rate mostly bounces back and forth between two boundaries and spend most of its time on the boundaries. This has implications for both trade openness and exchange rate volatility.

First, the model implies high openness ratios, computed as the average of imports and exports divided by net income. For trade costs equal to 50%, the average openness ratio is equal to 15%, which is above the empirical counterpart (8% on average over the 1947: I – 2004: IV period for the US).\(^{14}\)

Second, trade costs lower the volatility of simulated real exchange rates; it is for example divided by more than 5 times at trade costs of 50%. As Seru and Uppal (2003) noted, the lower the trade cost, the smaller the no-trade zone and the lower the exchange rate variance. When countries trade, SDFs become positively correlated. At the limit, when there is no trade cost, countries share risk perfectly and the real exchange rate is constant (see Table VI column 2, panel A).

**Quadratic trade costs** Introducing proportional trade costs lowers the real exchange rate volatility, but it implies that the real exchange rate is often constant, which is counterfactual. Adding quadratic trade costs leads to more reasonable patterns as shown in figure 3 and in panel B of Table VI. Even when countries trade, the real exchange rate is no longer constant and it can exceed the previous two fixed boundaries. The increasing marginal trade cost works against large import volumes, even when endowments imply large differences in marginal utility of consumption. Thus trade openness is reduced to 8% for trade costs equal to $\tau = 0.50\%$ and $\delta = 0.2$. For the same parameters, the volatility of the simulated exchange rate roughly matches its empirical counterpart. The model cannot reproduce with the same set of parameters both the pre- and post-Bretton Woods exchange rate volatilities because we know since Baxter and Stockman (1989) that

\[\text{Note that this estimate takes into account all international trade with the US and not only bilateral US-UK trade. One would expect the openness ratio to be smaller and more volatile for one particular bilateral trade than for the sum of all exports and imports.}\]
real consumption growth shocks have similar volatilities in both sub-periods. Explaining differences in exchange rate regimes is however beyond the scope of this paper.

C. Non tradable goods

I have considered so far only one good and assumed that it is tradable. Yet, Burstein, Eichenbaum, and Rebelo (2005) and Burstein, Eichenbaum, and Rebelo (2006) estimate that at least 50% of the variation in real exchange rates is due to changes in the relative prices of non tradable goods across countries. I now introduce non tradable goods in the model.

Let us assume that preferences are defined over total consumption \( C \), which combines consumption over tradable goods \( C_T \) and non-tradable goods \( C_N \) through a CES function:

\[
C_t = \left[ \psi_t^\theta \left( C_T^\theta \right)^\frac{\theta-1}{\theta} + \left( 1 - \psi_t \right)^\theta \left( C_N^\theta \right)^\frac{\theta-1}{\theta} \right]^\frac{1}{\theta}.
\]

where \( \psi_t \) denotes a preference shock and \( \theta \) the elasticity of substitution between tradable and non tradable goods. The domestic household receives an endowment each period in tradable and non-tradable goods. Assume that the domestic and foreign household can also trade a bond denominated in units of aggregate consumption. The Euler equations of the domestic and foreign investors lead to the definition of the real exchange rate as in equation (5), but the marginal utilities of consumption are now defined with respect to tradable goods. Let us define \( \xi_t \) as:

\[
\xi_t = \frac{\partial C_t}{\partial C_t^\theta} = \left( \frac{\psi_t C_t}{C_t^\theta} \right)^\frac{1}{\theta}.
\]

The real exchange rate is the ratio of two marginal utilities of consumption times the ratio of domestic and foreign wedges \( \xi_t \) and \( \xi_t^* \):

\[
Q_t = \frac{U_{C,t}^*}{U_{C^T,t}} = \frac{U_{C,t}^*}{U_{C,t}} \frac{\xi_t^*}{\xi_t}. \tag{17}
\]

\( U_{C,t}^* / U_{C,t} \) corresponds to the ratio of marginal utilities of aggregate consumption; this is the object studied so far in this paper. The second ratio, \( \xi_t^* / \xi_t \) is new; it is time-
varying if the relative endowments of tradable and non-tradable goods or the relative preference parameters change. We can map the previous simulations into this framework by reinterpreting consumption growth shocks as shocks to aggregate consumption, and not simply shocks to consumption of tradable goods. In this case, the law of motion of the state variable and real interest rates are not modified, but the real exchange rate should now be computed according to equation (17). I interpret the ratio of the domestic and foreign wedges $\xi_t$ and $\xi^*_t$ as a measurement error affecting changes in real exchange rates. I simulate the model with the same parameters as before (with $\tau = 0.25$ for trade costs). Table VII shows that small measurement errors greatly reduce the correlation between real exchange rates and relative consumption growth. The simulated correlation is then within two standard deviations of the actual point estimate. Compared to the case of a single tradable good, the volatility of real exchange rates nearly doubles, and the model thus attributes only 50% of the real exchange rate variations to tradables.

This is however not a complete solution to the Backus and Smith (1993) puzzle, which would for example require a careful calibration of the wedges $\xi_t$ and $\xi^*_t$ and departing from the complete market assumption, which is beyond the scope of this paper. I only wish here to highlight two directions for future research, that may prove fruitful. First, Lustig and Verdelhan (2007) show that the correlation between consumption growth and exchange rates depends on interest rates differentials. Because the correlation switches sign when the interest rate differential fluctuates, a simple unconditional measure might not show an existing link between exchange rates and consumption growth. Second, as pointed out by Brandt, Cochrane, and Santa-Clara (2006), understanding exchange rates in incomplete markets seem crucial. Chari, Kehoe, and McGrattan (2002) show that relaxing the complete markets assumption is not enough to solve the puzzle, but Benigno and Thoenissen (2006) claim that a model with incomplete markets and non-traded intermediate goods goes a long way towards its solution.

V. Conclusion

The failure of the UIP condition implies positive predictable excess returns when investing in high interest rate currencies and negative excess returns when investing in low interest rate currencies. I show in this paper that one of the workhorse models in consumption-
based asset pricing, designed to match some salient features of stock returns, is also consistent with the stylized facts of foreign currency returns.

The model has two main characteristics: time-varying risk aversion and trade costs. In this model, the domestic investor expects positive excess returns in times when he is more risk-averse than his foreign counterpart. The same reasoning applies naturally to the foreign investor. Times of high risk-aversion correspond to low interest rates. Thus, the domestic investor expects a positive risk premium when interest rates are lower at home than abroad.

Model simulations reproduce the usual negative covariance between exchange rate variations and interest rate differentials, while simultaneously delivering a sizable equity Sharpe ratio. Proportional and quadratic trade costs deliver real exchange rates that are neither stale nor too volatile, even as consumption processes among countries are uncorrelated. The model’s estimation gives reasonable parameters, thus rationalizing the exchange rate risk premium.

Many models in macroeconomics and international finance do not produce time-varying risk premia and thus follow the UIP condition to define the link between exchange rates and interest rates. The results reported in this paper offer an interesting alternative that could be incorporated in larger models with production, investment and savings decisions. Lettau and Uhlig (2000) show that Campbell and Cochrane (1999) preferences deliver overly smooth consumption in a real business cycle framework. Agents are very risk-averse locally, meaning that the inter-temporal elasticity of substitution is very low. This leads to a desire to use labor to radically smooth consumption. However, this difficulty might be overcome by introducing pre-determined labor, a time-to-plan assumption and/or adjustment costs and two separate sectors.
Table VII
Simulation Results: Impact of Measurement Errors

The table first reports the mean and standard deviation of consumption growth $\Delta c$ and changes in real exchange rates $\Delta q$. All moments are annualized. The table then reports the correlation between the consumption growth differential and changes in real exchange rates $\rho_{\Delta q_t, \Delta c^*_t - \Delta c_t}$ and the UIP slope coefficient $\alpha$. Standard errors are reported in brackets. The last two columns correspond to series simulated with measurement errors. In both cases, the proportional trade cost is equal to 0.25.

<table>
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<tr>
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<th>Model without measurement errors</th>
<th>Model with measurement errors</th>
</tr>
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<td>Mean(%) Std(%)</td>
<td>Mean(%) Std(%)</td>
</tr>
<tr>
<td>$\Delta c$</td>
<td>2.11 0.99</td>
<td>2.11 0.99</td>
</tr>
<tr>
<td>$\Delta q$</td>
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<td>0.12 12.67</td>
</tr>
<tr>
<td>$\rho_{\Delta q_t, \Delta c^*_t - \Delta c_t}$</td>
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<td>0.27 [0.01]</td>
</tr>
<tr>
<td>$\alpha$</td>
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<td>-3.01 [0.41]</td>
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</table>

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References


A. Simulation Method

I first draw 110,000 i.i.d endowment shocks and delete the first 10,000. From the 100,000 endowment shocks and the parameters of the model, I build the endowment process. To compute the price-dividend ratio and bond prices as a function of the surplus consumption ratio, I use the numerical algorithm developed by Wachter (2005). I choose a grid of 100 points in which $S$ ranges from $5e^{-4}$ to $S_{\text{max}}$. I refer the reader to Wachter (2005) for details, and I focus here on the specific difficulties of this two-country economy. Solving the social planner program presents two challenges that I briefly describe below.

A. Habit and Consumption

Trade at date $t+1$ in equations (15) and (16) depend on the habit level at date $t+1$. The habit level cannot be computed using the exact law of motion described in equation (2) because it requires the value of consumption at date $t+1$, which in turn depends on trade at date $t+1$. But Campbell and Cochrane (1999) chose the sensitivity function $\lambda(s_t)$ so that the habit level at date $t+1$ does not actually depend on consumption level on the same date. This can be shown using a first order Taylor approximation of the law of motion of the habit level $h_{t+1}$ when $s_t$ is close to its steady-state value $\bar{s}$ and the consumption growth $\Delta c_{t+1}$ is close to its average $g$ (see footnote 1 page 6 of Campbell and Cochrane (1995)):

$$h_{t+1} = \phi h_t + [(1 - \phi)h + g] + (1 - \phi)c_t.$$  \hspace{1cm} (18)

Equation (18) gives a first guess for the habit level at date $t+1$, thus allowing the computation of trade and consumption at date $t+1$. This new estimate of consumption is used to compute the habit level using the exact law of motion and the process is iterated until convergence.

B. Optimal Trade

No quadratic cost When there is no quadratic cost, the domestic country exports when $(Y^*_t - H^*_t)(1 - \tau)^{-\frac{1}{2}} < (Y_t - H_t)$. If this condition is verified, the optimal amount
of exports is derived from the first-order condition (15):

$$X_t = \frac{Y_t - H_t - (1 - \tau)^{-\frac{1}{\gamma}}(Y_t^* - H_t^*)}{1 + (1 - \tau)^{1 - \frac{1}{\gamma}}}$$

Similarly, the foreign country exports when $(Y_t^* - H_t^*) > (1 - \tau)^{-\frac{1}{\gamma}}(Y_t - H_t)$. If this condition is verified, the optimal amount of exports is equal to:

$$X_t^* = \frac{Y_t^* - H_t^* - (1 - \tau)^{-\frac{1}{\gamma}}(Y_t - H_t)}{1 + (1 - \tau)^{1 - \frac{1}{\gamma}}}.$$ 

As a result, there is no trade when $(1 - \tau)^{\frac{1}{\gamma}} \leq (Y_t^* - H_t^*)/(Y_t - H_t) \leq (1 - \tau)^{-\frac{1}{\gamma}}$.

**Quadratic costs** In the presence of quadratic costs, there is no closed form solution for the optimal amount of exports (except for log-utility).

To find the optimal amount of exports, let us define and minimize the following function $f$ derived from the first-order condition (15):

$$f(X_t) = -[Y_t - X_t - H_t]^{-\gamma} + [1 - \tau - \delta \frac{X_t}{Y_t^*}] [Y_t^* + X_t (1 - \tau - \delta \frac{X_t}{2 Y_t^*}) - H_t^*]^{-\gamma}.$$ 

The solution $X_t$ to $f(X_t) = 0$ has to satisfy three conditions. First, a country cannot export more than its endowment; thus $X_t$ is in the interval $0 \leq X_t \leq Y_t$. Second, habit preferences prevent consumption from falling below the habit level in both countries; thus $X_t \leq Y_t - H_t$ and $Y_t^* + X_t (1 - \tau - \delta \frac{X_t}{2 Y_t^*}) - H_t^* \geq 0$. The latter condition imposes that $X_t \in [x_{1,t}, x_{2,t}]$ where $x_{1,t} = Y_t^* (1 - \tau - \sqrt{\Delta_t})/\delta$ and $x_{2,t} = Y_t^* (1 - \tau + \sqrt{\Delta_t})/\delta$ when $\Delta_t = (1 - \tau)^2 + 2 \delta (Y_t^* - H_t^*)/Y_t^* > 0$. Third, the foreign country imports $X_t$ only if a positive fraction of the good makes it to its shore, thus $0 \leq X_t \leq 2 Y_t^* (1 - \tau)/\delta$. To satisfy the three conditions $X_t$ has to be in the interval $[0, \min(Y_t - H_t, 2 Y_t^* (1 - \tau)/\delta)] \cap [x_{1,t}, x_{2,t}]$.

Note that when the endowment level is above the habit $(Y_t^* - H_t^* > 0)$, then $\Delta_t > 0$, $x_{1,t} < 0$ and $x_{2,t} > 2 Y_t^* (1 - \tau)/\delta$. Thus, the solution of the maximization problem is in the interval $[0, \min(Y_t - H_t, 2 Y_t^* (1 - \tau)/\delta)]$. In this case, over this simple interval, a solution
exists if and only if:
\[
\frac{Y_t^* - X_t^*}{Y_t - X_t} < (1 - \tau)^{\frac{1}{\gamma}}.
\] (19)

Note that \(f\) is decreasing:
\[
f'(X_t) = -\gamma[Y_t - X_t - H_t]^{-\gamma - 1} - \frac{\delta}{Y_t^*}[Y_t^* + X_t(1 - \tau - \frac{\delta}{2 Y_t^*}) - H_t^*]^{-\gamma} \]
\[
- \gamma[1 - \tau - \frac{\delta}{Y_t^*}]^2[Y_t^* + X_t(1 - \tau - \frac{\delta}{2 Y_t^*}) - H_t^*]^{-\gamma - 1}.
\]
Thus, there exists an optimal amount of exports if \(f(0) > 0\) and \(f(\min[Y_t - H_t, 2Y_t^*(1 - \tau)/\delta]) < 0\). The first boundary condition \(f(0) > 0\) is equivalent to condition (19). This boundary condition also defines cases when the domestic country exports under no quadratic costs.

Let us check that the second boundary condition \(f(\min[Y_t - H_t, 2Y_t^*(1 - \tau)/\delta]) < 0\) is always satisfied. When \(Y_t - H_t \geq 2Y_t^*(1 - \tau)/\delta\), the boundary condition \(f(2Y_t^*(1 - \tau)/\delta) < 0\) is always satisfied:
\[
f(2Y_t^*(1 - \tau)/\delta) = -[Y_t - 2Y_t^*(1 - \tau)/\delta - H_t]^{-\gamma} - [1 - \tau][Y_t^* - H_t^*]^{-\gamma} < 0.
\]
When \(Y_t - H_t \leq 2Y_t^*(1 - \tau)/\delta\), there also exists a solution to \(f(X_t) = 0\) because \(f_{X_t \to Y_t - H_t}(X_t) \to -\infty.\)

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Table VIII
Real Yield Curve

The table reports average, standard deviation and autocorrelation of real yields in actual data and in the model. Panel A reports evidence obtained on inflation-indexed bonds in the UK and the US for different maturities. Data for the UK come from Evans (1998) and the Bank of England’s website. Missing data points are obtained using a Nelson and Siegel (1987) interpolation. Data for the US come from J. Huston McCullogh’s website. Panel B reports equivalent results obtained with the model.

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<th>5 years</th>
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B. Figures

Figure 1. The figure presents the optimal exports’ problem with proportional and quadratic trade costs. The horizontal axis correspond to domestic consumption net of domestic habit, $C - H$. The vertical axis correspond to foreign consumption net of foreign habit, $C^\star - H^\star$. Assume that the two countries are characterized by point $A$ where endowments (net of habit levels) are given. If there are only proportional costs, the foreign country exports $X^\star_1$ units. For each unit that the foreign country exports, the domestic country receives $(1 - \tau)$. Thus, the slope between $A$ and $B$ is $-1/(1 - \tau)$. At point $B$, the real exchange rate is equal to $(1 - \tau)$. If there are proportional and quadratic costs, the foreign country exports $X^\star_2$ units. The quadratic trade cost incurred is equal to $\delta X^\star_2$. At point $C$, the real exchange rate is equal to $(1 - \tau - \delta X^\star_2)$. 

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Figure 2. Snapshot of a simulation with proportional trade costs (first 10,000 periods). The first panel presents the real exchange rate. The second panel presents the surplus consumption ratios in the two countries. The last two panels present the exports/endowments ratios ($X/Y$ and $X^{*}/Y^{*}$) at home and abroad. The trade cost parameters are $\tau = 25\%$ and $\delta = 0$. 
Figure 3. Snapshot of a simulation with proportional and quadratic trade costs (first 10,000 periods). The first panel presents the real exchange rate. The second panel presents the surplus consumption ratios in the two countries. The last two panels present the exports/endowments ratios \((X/Y)\) and \((X^*/Y^*)\) at home and abroad. The trade cost parameters are \(\tau = 25\%\) and \(\delta = 0.2\).
Figure 4. Reality check. Stochastic discount factor (SDF), surplus consumption ratio (SP) and local curvature for an American investor computed with actual US consumption data only over the 1947 : II – 2004 : IV period using the parameters presented in the first column of Table III with $\tau = 1$ and $\delta = 0$. 