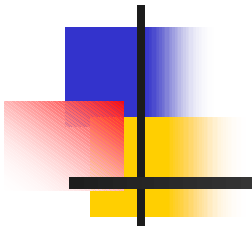


Asset Pricing Models

The CAPM

The Market Model

The Arbitrage Pricing Model

Asset Pricing Model 1

The Capital Asset Pricing Model

ASSUMPTIONS

A1: risk averse investors maximize expected utility

A2: one period horizon where expected returns and standard deviations fully describe the distribution of returns over the investor's horizon

A3: assets are infinitely divisible

A4: risk free asset exists

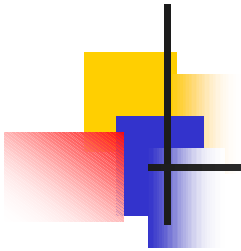
A5: no taxes nor transaction costs

A6: borrowing and lending at the risk free rate for all market participants

A7: investors are price takers: they do not have information or abilities better than the best in the market

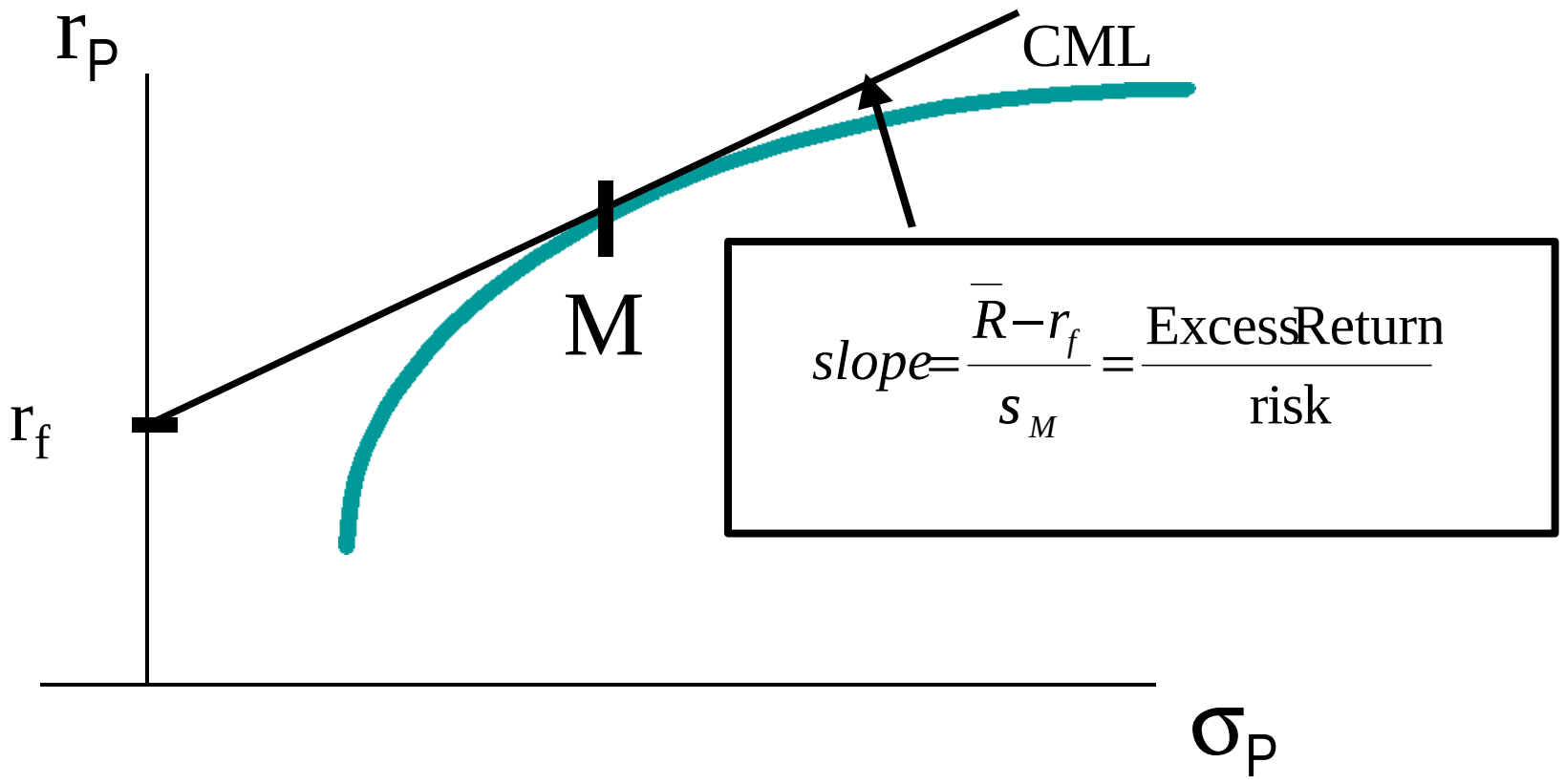
A8: homogeneous expectations

THE CAPITAL MARKET LINE (CML)



- If the market is efficient, investors will choose optimal portfolios based on minimizing risk and maximizing return
- With risk-free borrowing and lending the set of choices is represented by a line that is tangent to the curved efficient frontier.
- the new efficient frontier that results from risk free lending and borrowing
- both risk and return increase in a linear fashion along the CML

THE CAPITAL MARKET LINE



THE CAPITAL MARKET LINE: Separation Theorem

- the division between the investment decision and the financing decision
- to be somewhere on the CML, the investor initially
  - decides to invest
  - based on risk preferences makes a separate financing decision **to borrow or to lend**

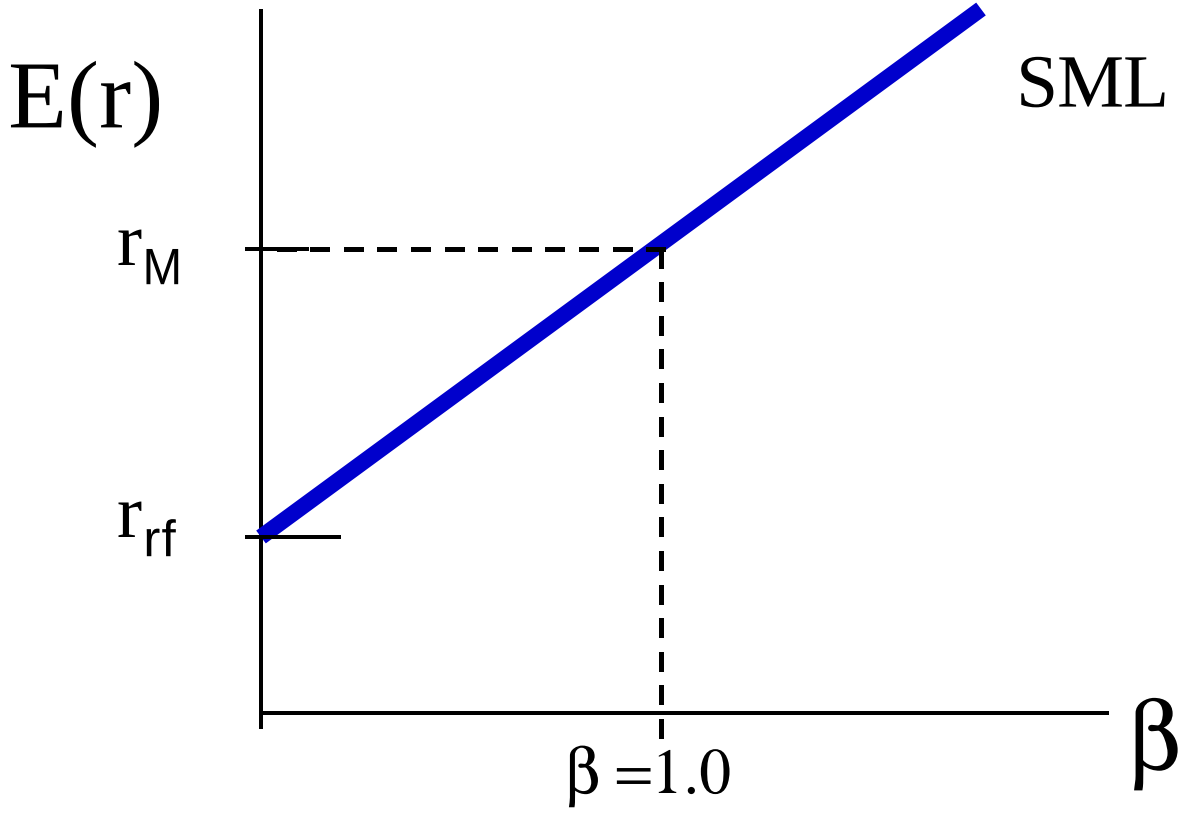
THE MARKET PORTFOLIO

- DEFINITION: the portfolio of all risky assets which are traded in the market
- ATTRIBUTES
  - Value weighted
  - Completely diversified—no unsystematic risk
- CAPM THEORY: the Market portfolio is the tangency portfolio (M) in the CML

THE SECURITY MARKET LINE (SML)

- DEFINITION: the security market line expresses the linear relationship between the return investors require of a risky asset and its risk
- WHERE DOES THE SML COME FROM? the security market line is the mathematical result of finding the optimal portfolio weights in the capital market line and then manipulating the first order conditions of a lagrange function.
- WHAT IS ITS IMPORTANCE? the security market line shows the *only* measure of risk that investors care about is the *beta* of the asset (stock, portfolio, or any other traded asset).

THE SECURITY MARKET LINE (SML) Graphically



THE SECURITY MARKET LINE (SML) Equations

$$E(R_i) = r_{rf} + \left[\frac{\overline{R}_M - r_f}{s_m^2} \right] s_{i,m}$$

or

$$E(R_i) = r_f + (\overline{R}_M - r_f) b_{i,M}$$

where

$$b_{i,M} = \frac{s_{i,M}}{s_M^2} = \frac{\text{covariance}\ (R_i, R_M)}{\text{variance}(R_M)}$$

THE SECURITY MARKET LINE (SML) FACTS

- The BETA of the value-weighted market is 1.0
- The BETA of a portfolio is the weighted average of the betas of its component securities

$$b_{P,M} = \sum_{i=1}^{N} X_i b_{i,M}$$

Asset Pricing Model 2

The Market Model

Assumptions of the Market Model

- Assumptions: return on a risky asset is related to the return on a market index in a regression model

$$r_{it} = a_i + b_i R_{Mt} + e_{it}$$

Regression assumptions

assumption 1: $E(\varepsilon_{it})=0$, means the average effect of omitted variables is zero

assumption 2: $COV(\varepsilon_{it-i}, \varepsilon_{it})=0$, means the omitted variables from date *t-i* do not cause errors on date *t*

assumption 3: $COV(\varepsilon_{it}, R_{Mt})=0$, means that the omitted variables do not affect the market index

Differences between the MARKET MODEL and the CAPM

Theoretical Motivation is different:the market model is not an equilibrium model like the CAPM:

- It does not make any assumptions about how investors optimize their portfolio
- It simply makes the assumption about the statistical relationship with the market.

Practical Implementation is different (slightly) the market model uses an assumed market index (you choose).The CAPM uses only one, the value-weighted market portfolio

Systematic and Unsystematic Risk in the Market Model

If the market model is true, then the variance of stock *i,* its total risk, can be partitioned into

Total Risk = Systematic Risk + Unsystematic Risk

$$s_i^2 = b_i^2 s_M^2 + s_{ei}^2$$

The only source of systematic risk is the market.

Why partition risk?

- Every asset pricing model specifies that investors are rewarded for bearing "systematic risk" not "unsystematic risk"
- Systematic risk in the market model is $b_i s_M^2$
- Unsystematic risk (sometimes called "Idiosyncratic risk") in the market model is s_{ei}^2
  - not related to beta
  - risky assets with larger amounts of will s_{ei}^2 not have larger E(r)

Asset Pricing Model 3

The Arbitrage Pricing Model

Assumptions

- The APT is an ***equilibrium*** factor model of security returns the driving force is Arbitrage
- **Assumption 1:** A pure arbitrage portfolio should earn a very small return (it does *not* have to be zero)
  - **Definition:** Pure Arbitrage Portfolio:

  - Zero investment (Long & short)
  - Zero sensitivity to the factors driving the market
  - In a well-functioning market competition should make profits on pure arbitrage portfolios very small.

Assumption 2: FACTOR MODEL

$$r_{it} = a_i + b_{1i}F_{1t} + b_{2i}F_{2t} + \cdots + b_{Ki}F_{Kt} + e_{it}$$

where r is the return on security i

b_{Li} is the coefficient of the factor*L* for asset *i*

F_{Lt} is the value of the factor*L* at time *t*

ε_{it} is the error term for asset *i* at time *t*

The market model is simply a one-factor model. A factor model is a multiple regression model and has the same statistical assumptions as the market model.

In general there can be a “K” factor model.

Determination of security prices

If the assumptions are true, it can be shown (using linear algebra) that all assets will have a linear relationship with the "b" coefficients for a given time period

$$E(r_i) = r_f + l_1 b_{1i} + l_2 b_{2i} + \cdots + l_k b_{Ki}$$

Where:

$\mathbf{r_i}$ is the return on the security *i*

l_L = the return investors require because of factor *L*'s risk.

$\mathbf{b_{Li}}$ = the coefficient for factor L (Its exactly like beta).

$l_L \mathbf{b_{Li}}$ = the return that investors require for bearing the risk imposed by the factor.

Systematic and Non-Systematic Risk in the Arbitrage Pricing model

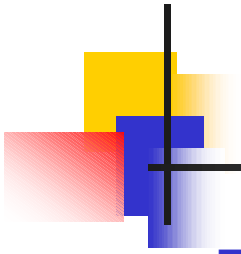
In all cases Total risk = [systematic risk] + unsystematic risk.

If there is a one factor model then the variance of stock *i*, is partitioned as in the market model:

$$s_i^2 = [b_i^2 s_M^2] + s_{ei}^2$$

If there are K factors then the regression assumption of the factor model results in the partition:

$$s_i^2 = [b_{1i}^2 s_1^2 + b_{2i}^2 s_2^2 + \cdots + b_{Ki}^2 s_K^2] + s_{ei}^2$$



Summary

- The CAPM is an *equilibrium* model derived from assumptions about investors
- The Market Model is a purely statistical model mainly used to estimate beta and distinguish market risk from non-market risk
- The Arbitrage Pricing Model is an *equilibrium* model that looks like the market model but has more factors.