

Return

Definitions of Returns

Holding Period Returns:
$$HPR = \frac{E+I-B}{B} = \frac{E+I}{B} - 1$$

- where: I=income, E= ending value, B= beginning value

Arithmetic returns:
$$A = \frac{HPR_1 + HPR_2 + \dots + HPR_N}{N}$$

- where $HPR_1 = HPR$ for period 1, $HPR_2 = HPR$ for period 2, etc

Geometric Returns:

$$GEO = \prod_{i=1}^N (1 + HPR_i) - 1 = \left(\left(\frac{E_1 + I_1}{B_0} \right) \left(\frac{E_2 + I_2}{E_1 + I_1} \right) \dots \left(\frac{E_N + I_N}{E_{N-1} + I_{N-1}} \right) \right)^{\frac{1}{N}}$$

$$GEO = \left(\frac{E_N + I_N}{B_0} \right)^{\frac{1}{N}} \text{ if all Income is reinvested}$$

- for continuously compounded returns: $GEO = (1/n)\ln[E/B]$
- E/B is called a “wealth relative” and represents the growth of \$1 over the N periods

Dollar weighted returns (= internal rate of return)

$$B = \frac{\pm I_1}{(1+r)} + \frac{\pm I_2}{(1+r)^2} + \dots + \frac{\pm I_{N-1}}{(1+r)^{N-1}} + \frac{E}{(1+r)^N}$$

Comparison of The Returns

Dollar Weighted vs Time Weighted vs Arithmetic

example:

<i>Year</i>	<i>Per dollar Returns</i>	<i>End of Year Contributions</i>	<i>Account Value</i>
1994		\$2,000	\$2,000
1995	-50%	\$1,000	\$2,000
1996	100%	- 0 -	\$4,000

Dollar Weighted (=Internal Rate of Return): $r=$

Time-Weighted (= Geometric return for multiple rates): $GEO=$

Arithmetic: $A=$

Which return to use?

Geometric Return

- represents the accurate picture of the *past*. It tells you how much money was made per dollar invested.
- is the best estimate of a future, terminal wealth ratio
 - Suppose we randomly draw five returns from a sample of 1,000 returns and calculate the terminal value of \$1 invested at each of the five returns. We repeat this experiment 200 times. The geometric average of the 1,000 returns will be the most likely terminal value.
- AIMR requires that the geometric return be used in its “Performance Presentation Standards”

Arithmetic Return

- represents the best guess of next year’s (or any year’s) holding period return.
- Formally, the arithmetic return is the unbiased, minimum variance estimator of the mean of the population of holding period returns.

Internal Rate of Return

- The internal rate of return is appropriate when you want to determine the total effect of the choice of the investment and the timing of the choice.

The Case of the Dreyfus Aggressive Growth Fund.

How Dollar-Weighted Returns Can Be Useful

- The Dreyfus Corporation, a mutual fund group formed in 1947, started the Dreyfus Aggressive Growth (DAG) open-end mutual fund on September 29, 1995. (To “start a fund” means to register it with the SEC and to offer its shares to the public)
- The prospectus of the DAG stated that initial public offerings purchased by the Dreyfus group would be allocated equitably among Dreyfus funds. However, the Dreyfus Corporation favored the DAG with IPOs purchased by the fund group during the first half of 1996. DAG was favored over other funds in the fund family and over the three other funds managed by the same portfolio manager.
- The total returns and Assets for 1996 are below:

<i>Date</i>	<i>Monthly Return</i>	<i>Assets</i>
Dec-95		\$ 4.43
Jan-96	12.4%	\$ 6.05
Feb-96	14.1%	\$ 10.81
Mar-96	10.0%	\$ 24.42
Apr-96	7.7%	\$ 57.71
May-96	10.4%	\$ 154.40
Jun-96	-10.0%	\$ 128.94
Jul-96	-13.6%	\$ 106.45
Aug-96	6.6%	\$ 121.50
Sep-96	0.1%	\$ 126.16
Oct-96	-13.6%	\$ 99.97
Nov-96	-3.2%	\$ 91.71
Dec-96	3.6%	\$ 92.21

Computing the Dollar Weighted Return

- Suppose the fund was owned by one person who put in \$4.43 at the end of January and took out the net cashflow of the fund at the end of each month, liquidating it at the end of December.
- Suppose cash inflows are negative and cash outflows are positive.
- To distinguish cash inflows from the increase in value of the fund use the following equation:

$$\text{End of month } t \text{ cash flow} = (\text{Assets at end of month } t) - (\text{Assets at end of month } t-1) * (1 + \text{HPR}_t)$$

- This assumes that the cash flow came at the end of the month. If the cash flow came at the beginning of the month write

$$\text{Beginning of month } t \text{ cash flow} = (\text{Assets at end of month } t) / (1 + \text{HPR}_t) - (\text{Assets at end of month } t-1)$$

- What is the IRR of the investor in the fund?
- In reality this will be the return of the IRR of all investors in the fund.
- Note it will not be the return of the person who invested money in December 1995 and took out his money in December 1996 because that assumes no cash outflows during the year.

What Happened to Dreyfus

according to: United States Securities and Exchange Commission administrative proceedings file no. 3-10201

- On May 10, 2000 the United States Securities and Exchange Commission and the Dreyfus Corporation agreed that the Dreyfus would consent to the orders of the Commission without admitting or denying the Commission's findings.
- The Commission ordered that:
 - Dreyfus and the portfolio manager, Micheal Schonberg, be censured.
 - Dreyfus cease and desist from committing any future violation of the Investment Company Act of 1940.
 - Dreyfus pay \$1,000,000 fine.
 - Schonberg is suspended for nine months from the investment business
 - Dreyfus had to retain a consultant to conduct a comprehensive review of policies and procedures for (a) IPO allocations (b) performance advertising (c) code of ethics (d) Dreyfus's supervision of (a)-(c). The consultant will make recommendations submit a report to the SEC within 90 days.
 - The Dreyfus Board of Directors adopt the recommendations within 90 days of the consultant's report.