

Review of Mortgage Mathematics

Objective: The goal of this assignment is to review the mathematics underlying common types of mortgages including fixed rate (FRM) and adjustable rate mortgages (ARM), graduated payment mortgages (GPM), and reverse annuity mortgages (RAM). This is a necessary prelude to the structuring of derivative mortgage securities using pools of different types of mortgages. These derivative mortgage securities include but are not limited to principal only (PO) and interest only (IO) stripped mortgaged backed securities (MBS), collateralized mortgage obligations (CMOs), and planned amortization classes (PACs) among others.

Assignment: This is an *individual assignment*. Please turn in a hard copy of your computer output. Please download the spreadsheet that accompanies this case. The spreadsheet contains several templates to help you in the modeling process.

1. **Blended APR on Fixed Rate Mortgages (20 Points):** Assume that a borrower takes out two mortgage loans to finance the purchase of his property. Following are the terms of each loan:

	<u>1st Mortgage</u>	<u>2nd Mortgage</u>
Interest Rate	5.0%	6.5%
Term (in Years)	30	5
Payments per Year	12	12
Loan Amount	\$359,000	\$41,000
Points	.0325	.0025
Other Loan Fees	\$995	\$99



- Prepare an amortization schedule for each mortgage
- Calculate the official annual percentage rate (APR) using either the RATE function or the IRR function in Excel for each mortgage. The APR is the effective borrowing cost.
- Calculate what the blended APR (borrowing cost) on the combined first and second mortgage is to the borrower if he pays off the loan at the end of year 3.

2. **Adjustable Rate Mortgage (ARM) (20 Points):** Assume that a lender offers the borrower an adjustable rate mortgage based on the following terms at the beginning of January 1999:

Loan Amount	\$359,000	Margin on ARM	.025
Initial Interest Rate (1 st year)	.0373	Interest Rate Caps	
ARM Index ¹	1 year T-Bill	Annually	.02
Adjustment Period ²	Annually	Lifetime	.05
Amortization Period (Years)	30 years	Negative Amortization	Yes/125% of Orig. Loan
Points on Loan	.0238		

¹Yield on United States Treasury Securities adjusted to a constant maturity of 1 year

²We assume here that the new interest rate on the ARM mortgage is set in January of each year. The new rate remains constant for one year and then is reset in January of the subsequent year.

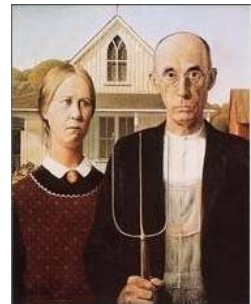
Complete the amortization table in the ARM template. The yield on Treasury Securities adjusted to a constant maturity of 1 year is located in the "Treasury constant maturity 1yr" worksheet. Is there any period of time when negative amortization occurs? If so, when does this occur? Prepare a graph of the ARM loan balance over time. In addition to this, prepare another graph showing the principal and interest on this ARM over time.

3. Reverse Annuity Mortgage (RAM) (30 points)³: Introduced in the late 1980s, reverse annuity mortgages allow older consumers the opportunity to convert the equity in their homes to cash while retaining home ownership. In contrast to a "regular" mortgage, where the borrower makes monthly payments to the lender, a reverse mortgage, the borrower receives money from the lender. In general, borrowers do not have to repay the RAM for as long as they live in their home. In return, the lender holds some — if not most or all — of a borrower's home's equity. The proceeds of the loan are tax-free, there are no minimum income requirements, and for most reverse mortgages, the money can be used for any purpose.

There are several types of reverse mortgages:

- The federally insured Home Equity Conversion Mortgage (HECM), administered by the Department of Housing and Urban Development (HUD)
- Single-purpose reverse mortgages, usually offered by state or local government agencies for a specific reason
- Proprietary reverse mortgages, offered by banks, mortgage companies, and other private lenders and backed by the companies that develop them.

To qualify for a reverse mortgage, you must be at least 62 and have paid off all or most of your home mortgage. Income is generally not a factor, and no medical tests or medical histories are required. If you seek an HECM, you also must undergo free mortgage counseling from an independent government-approved "housing agency." Financial institutions offering proprietary reverse mortgages may require similar counseling or homeowner education. The amount you can borrow depends on your age, the equity in your home, the value of your home, and the interest rate. If it's an HECM, federal law limits the maximum amount that can be paid out. You can be paid in a lump sum, in monthly advances, through a line of credit, or a combination of all three. Depending on the plan, reverse mortgages generally allow homeowners to retain title to their homes until they permanently move, sell their home, die, or reach the end of a pre-selected loan term. Generally, a move is considered permanent when the homeowner has not lived in the home for 12 *consecutive* months. So, for example, a person could live in a nursing home or other medical facility for up to 12 months before the reverse mortgage would be due.



However, be aware that reverse mortgages tend to be more costly than traditional loans because they are rising-debt loans. The interest is added to the principal loan balance each month. So, the total amount of interest owed increases significantly with time as the interest compounds. Reverse mortgages use up all or some of the equity in a home. That leaves fewer

³The description of a reverse annuity mortgage is taken almost verbatim from <http://www.ftc.gov/bcp/conline/pubs/homes/rms.htm>

assets for the homeowner and his or her heirs. Lenders generally charge origination fees and closing costs; some charge servicing fees. How much is up to the lender. Interest on reverse mortgages is not deductible on income tax returns until the loan is paid off in part or whole. Because homeowners retain title to their home, they remain responsible for taxes, insurance, fuel, maintenance, and other housing expenses.

Suppose that your grandparents are considering a RAM to supplement their income from social security. They have already paid off their mortgage. An appraiser has determined that the value of their house is \$650,000. A lender is willing to lend them a RAM based on a 50% loan to value ratio (LTV) for 10 years⁴ at 5.25% per annum. Using the “3a. Reverse Mortgage (RAM)” template, generate an amortization table. What is the cumulative principal amount that the borrower receives? Why is this less than the loan amount of the RAM? What is the unpaid RAM balance at the end of year 5 (month 60)? Prepare a chart showing the unpaid loan balance of the RAM over time.

Notes to Calculations:

- Ending RAM balance at the end of month 120 (10 years) is equal to the amount borrowed. The amount borrowed is known as the Contract Loan Amount

$$\text{Contract Loan Amount} = \text{Value of House} * \text{LTV}$$

- For a RAM, the beginning balance in month 1 is equal to \$0.
- To obtain the payment to the borrower by the lender use the PMT function e.g.

$$\text{PMT}(\text{rate}, \text{nper}, \text{pv}, \text{fv}, \text{type})$$

Where rate = monthly interest rate; annual rate ÷ 12
 nper = total number of monthly payments for the loan; term in years * 12
 pv = principal of the loan; beginning balance in each period
 fv = -1 * Contract Loan Amount

- $\text{Current Loan Amount}_T = \text{Beginning Loan Amount}_T + \text{Payment to Borrower}_T$.
- $\text{Accrued Interest} = 0$ if month = term in years * 12 = last month of loan
 $= \text{Current Loan Amount}_T * \text{monthly interest rate}$
- $\text{Ending Balance}_T = \text{Current Loan Amount}_T + \text{Accrued Interest}_T$.



⁴The term of the loan is set such that it is less than the life expectancy of the RAM borrower.

4. **Graduated Payment Mortgage (GPM)** (30 points): A borrower is applying for a \$359,000 GPM loan for at 5.25% interest for 30 years. The graduation rate is 7.5% per year with payments remaining constant from year 6 through year 30. Debt payments are due at the end of each month.



- a. What is the initial payment? To calculate the initial payment, you can use the Solver function in Excel. To access this function, pull down **Tools** menu and click on **Solver**. The Target Cell is the Total PV of all of the debt service payments (initial loan amount). You will be iterating on the initial loan payment (put in a guess).
- b. Given your solution in part a, generate an amortization table for the GPM using the “4b. GPM AmortTable” template in your workbook. In what month does the GPM mortgage begin to amortize (no negative amortization)?
- c. Prepare a graph of the outstanding loan balance of this GPM over time.