

Understanding Mortgage Prepayments

1. **Call Valuation** (60 points): CS First Boston (CSFB) has recently hired you as part of their Structured Finance group. The first assignment that your boss gives to you is to value the embedded call option in mortgages. Suppose that the current spot interest rate is 5.5%. Assume that CSFB uses a 1.055 factor for interest rate increases and a .975 factor for interest rate decreases and that the probability for up (down) is 50% e.g. $\pi_u = \pi_d = .5$ and periods are 1 year in length. If

Contract Interest Rate: 5.5% Refinancing Costs (Points on New Loan): 0 points
Principal: \$10,000 Payments Per Annum: 1 payment per year
Term of Loan: 4 years

- Generate the five period (Years 0-4) Interest Rate Tree.
- Value the Non-Callable Mortgage Bond. Please show your intermediate mortgage values in a five period "lattice".
- Value the Prepayment (Call) Option on the Callable Mortgage. Please show your intermediate option values in a four period "lattice". Also report what the exercise price is in each of the 5 periods ($t=0,1,2,3,4$)
- Value the Callable Mortgage. Please show your intermediate values in a five period "lattice".

2. **Cash Flows based on PSA Speed** (40 points): Calculate the cash flows for the first 30 months on the following mortgage pass-through:

Original Mortgage Balance	\$500,000 (in 000s)	Term	360 months
Interest Rate	5.3%	PSA	100%
Servicing Fee ¹	0.5%		

- What would the cash flows for the first 30 months be if the PSA is 100%?
- What would the cash flows for the first 30 months be if the PSA increased to 125%?
- What would the cash flows for the first 30 months be if the PSA decreased to 80%?
- Graph your cash flows under the various PSAs (100%, 125%, 80%) on the same chart. The y-axis is the cash flows and the x-axis is the months.
- What would the cash flows for the first 30 months be if you used the following PSA vector:

$$\begin{bmatrix} \text{Month : 1 - 15} & 125\% \text{ PSA} \\ \text{Month : 16 - 30} & 100\% \text{ PSA} \end{bmatrix}$$

¹The servicing fee includes GNMA timing insurance