

Problem Set 3-Financial Re-Engineering and Resulting Yields
Strips, CMOs, and Option-Adjusted Spreads

1. Suppose that the underlying mortgage pass-through which is used to create a stripped MBS (mortgage-backed security) has the following features:

Original Mortgage Balance	\$100,000 (in 000s)
Interest Rate	9.5%
Servicing Fee	0.5%
Pass-Through Rate	9.0%
Term	360 months

a. If the PSA is 100%, what are the cash flows to the holder of a PO (Principal-Only) Strip and the holder of an IO (Interest-Only) Strip respectively for the first 30 months?

b. If the current price of the PO strip is \$40,000 what is the monthly IRR and bond equivalent yield if the PSA is 100%. What if the PSA is 125%? What if the PSA is 150%? Please report your answers in a table e.g.

PSA	100%	125%	150%
IRR	_____	_____	_____
BEY	_____	_____	_____

c. If the current price of the IO strip is \$60,000 what is the monthly IRR and bond equivalent yield if the PSA is 100%. What if the PSA is 125%? What if the PSA is 150%? Please report your answers in a table e.g.

PSA	100%	125%	150%
IRR	_____	_____	_____
BEY	_____	_____	_____

2. Suppose that the underlying mortgage pass-through which is used to create a collateralized mortgage-backed security (CMO) has the following features:

Original Mortgage Balance	\$100,000 (in 000s)
Interest Rate	9.0%
Servicing Fee	0.5%
Pass-Through Rate	8.5%
Term	360 months

We wish to create a CMO with 4 tranches (bond classes) - Class A, Class B, Class C and Class Z having the following attributes:

Class	Par Value	Coupon Rate	Stated Maturity
A	\$22,800	7.50%	5-9 years
B	\$25,300	7.75%	9-14 years
C	\$42,700	8.25%	12-17 years
Z	\$9,200	9.00%	28-30 years

Assumptions:

- The coupon rate of interest is paid currently on tranches A, B, and C but not on tranche Z until principal on the other tranches is repaid. All current amortization of principal and prepayments from the entire mortgage pool is allocated first to tranche A.
- Payments into the pool occur *monthly*.
- No consideration of any reinvestment of interim cash flows.
- Rate of Prepayment is 115% PSA

Questions:

- What are the cash flows to each class for the full 360 months?
- Graph the flows of CMO Principal to Classes A, B, C, and Z on the same graph over the full term 360 months (you may have to use the “skipping” feature in your spreadsheet to plot the graph)
- What is the yield to maturity (IRR) for each tranche assuming the following initial bond prices

Class	Bond Price (32nd)
A	98-05
B	97-23
C	93-02
D	71-27

- The attached handout by Jeff Berenbaum, Fall 1991, “Negative Convexity”, *Secondary Mortgage Markets* 8(3): 32-33, allows you to set up a spreadsheet to show how negative convexity can arise as a result of rapid prepayments.
 - Set up the Negative Convexity spreadsheet using the article and replicate the graph given on page 32 for negative convexity.
 - Make a duplicate copy of your spreadsheet and “as an experiment, replace the look-up table with one that assumes borrowers react event more strongly to changes in interest rates e.g.,

double the prepayment speeds for positive spreads and cut in the prepayment speeds for negative spreads”. Show that the bottom curve in Figure 1 on page 32 becomes even more negatively convex.

- c. What is the expected price of the mortgage pool assuming that yields will vary between 8%-12% with an equal likelihood/probability of occurring e.g. the probability that the yield will be 8% is equal to the probability that the yield will be at 11%.

Hint: “Averaging the results for many different yield assumptions would better approximate the true value of a mortgage compared to using one interest rate.”

Negative Convexity

A spreadsheet illustrates that ignoring prepayment options leads to overvaluing mortgages.

by Jeffrey S. Berenbaum

Prepayment options are exercised by borrowers. As a result, mortgage prepayments speed up and slow down exactly when investors would prefer the reverse. For example, when interest rates fall, investors would like their mortgage-backed securities with relatively high coupons to remain outstanding, yet borrowers prepay more rapidly.

This Lotus 1-2-3 spreadsheet can be used to show that adverse prepayments result in "negative convexity," that is, mortgage prices fall by more when interest rates rise than they increase when interest rates fall.

The spreadsheet values a pool of mortgages with the same interest rates, remaining terms and ages by calculating their cash flows for 360 months and discounting them at the yield on new mortgages. A user-entered, look-up table assigns a prepayment rate to the mortgage pool based on the spread between the mortgage interest rate and the current market rate. This percentage of the loans prepays fully; the calculations assume no curtailments.

The prepayment rates in the look-up table are expressed using the Public Securities Association (PSA) standard. Thus, the calculation of prepayments in the spreadsheet depends on the mortgage age as well as the interest rate.

At 100 percent PSA, a pool of mortgages prepays at 0.2 percent (annually) in the first month; this rate increases by 0.2 percent each month for 30 months until it levels off at 6 percent prepayments annually.

In the hypothetical look-up table shown, mortgages at the current rate (spread = 0) prepay at 180 percent PSA, that is 1.8 times the prepayment rates for 100 percent PSA. Mortgages below the current rate (spread < 0) prepay more slowly than 180 percent PSA. The prepayment rate slows

down less as the spread gets more negative, reflecting housing turnover and other factors that cause prepayments even for loans with attractive rates.

Mortgages above the current rate (spread > 0) prepay more rapidly than 180 percent PSA. Initially the increase is relatively small; then the spread reaches levels where many borrowers find it worthwhile to refinance.

Figure 1 shows the results of using the spreadsheet to value a pool of one-year-old, 30-year, 10 percent mortgages assuming that current interest rates vary from 7 to 13 percent. The two curves are drawn using different prepayment assumptions.

The bottom curve is based on the look-up table. It exhibits negative convexity. Because of the shape of the curve, the value of the 10 percent mortgage at any point on the curve will be higher than if its value is calculated by taking an average of two possible outcomes, one with higher and one with lower rates. For example, the value of the 10 percent mortgages at 10 percent is par (100), while the average of

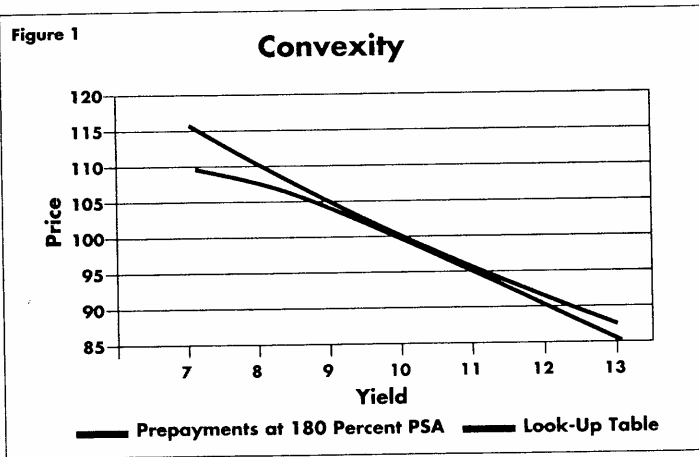
the values at 9 and 11 percent (104.49 and 95.31) is 99.9.

The top curve in Figure 1 was drawn by changing all the PSA speeds in the look-up table to 180 percent PSA, regardless of current market rates. In other words, the investor does not face the adverse choice of the borrower. In this case the curve is positively convex.

As an experiment, this look-up table could be replaced with one that assumes borrowers react even more strongly to changes in interest rates. For example, prepayment speeds could be doubled for positive spreads and be cut in half for negative spreads. The bottom curve in Figure 1 would become even more negatively convex.

Note that the look-up table only has values for spreads between -3 and 3. For lower spreads the value will not be calculated; for higher spreads the calculation uses the PSA for spreads equal to 3. However, the table can be expanded.

Should this spreadsheet be used to value mortgages? Because of negative convexity,

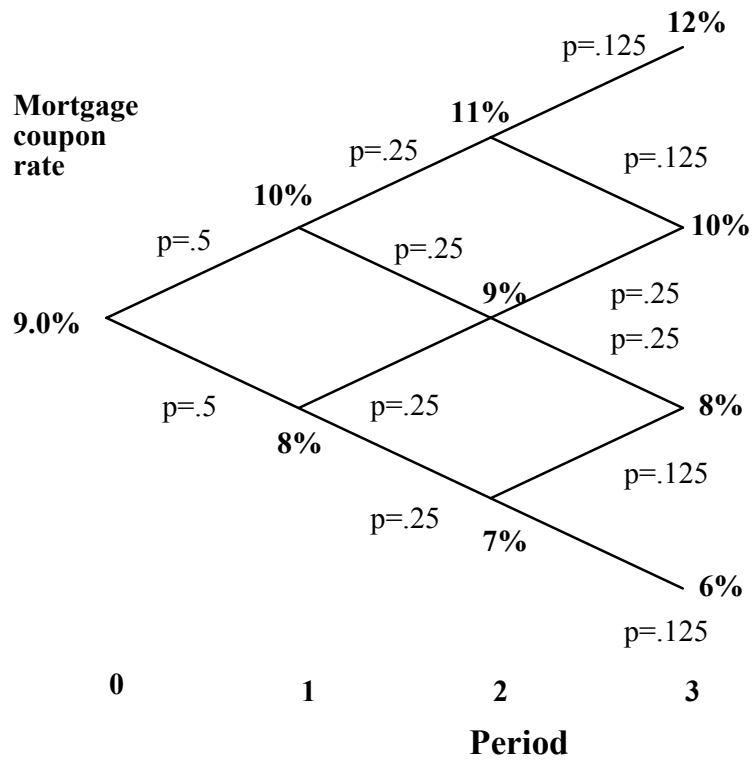


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4. Calculate the option-adjusted spread (OAS) and the yield to maturity on a mortgage pass-through given the following assumptions:

- i) Borrowers only prepay due to refinancing mortgages at a lower rate
- ii) The current zero coupon yield curve for T bonds is flat and the discount rate on T bonds = 8%
- iii) The mortgage coupon rate is 11% on an outstanding mortgage pool with an outstanding principal balance of \$1,000,000. The mortgages have a 3-year maturity and pay principal and interest only once at the end of each year.
- iv) Mortgage loans are fully amortized and there is no servicing fee
- v) The current mortgage rate (y) is 9%. Interest rate movements over time change maximum of 1% up or down each year. The time path of interest rates follow a binomial process.
- vi) Because of prepayment penalties and other refinancing costs, mortgagees don't begin to prepay until mortgage rates, in any year, fall 3% or more below the mortgage coupon rate for the pool (11% in our example)
- vii) With prepayments present, cash flows in any year can be either the promised debt service, the promised debt service + repayment of outstanding principal, or cash flow = 0 if all mortgages have been prepaid or paid off in the previous year

Also, assume that the time path of mortgage interest rates over 3 years with the associated probabilities (p) is as given below



Hints in Calculating OAS:

Recall that

$$P = \frac{E(CF_1)}{(1 + r_1 + OAS)} + \frac{E(CF_2)}{(1 + r_2 + OAS)^2} + \frac{E(CF_3)}{(1 + r_3 + OAS)^3}$$

where P = Price of Mortgage Pass-through

r_1 = Discount rate on 1-year, zero-coupon Treasury bond

r_2 = Discount rate on 2-year, zero-coupon Treasury bond

r_3 = Discount rate on 3-year, zero-coupon Treasury bond

OAS = Option adjusted spread on mortgage pass-through

therefore since you are solving for the unknown variable, you can use the SOLVER algorithm in EXCEL. (The new version of Lotus for Windows has a similar feature)

Note: This is the final problem set. The **problems are due on April 23.**