Professor Crocker H. Liu   Revised: November 5, 2001
Equity Securitization of Real Estate

Problem Set 5
Real Estate Portfolio Analysis

Objective: The objective of this assignment is to introduce students to some of the issues surrounding adding real estate to a portfolio of stocks, bonds, and other assets. Typically, real estate is regarded as an alternative asset class used primarily for diversification purposes.

Assignment: Download the real estate data from my website (rees_MPT2002.xls) and use the downloaded spreadsheet to answer the following questions. Please highlight your answers in yellow and turn in a hard copy of your results. ***This is an individual assignment***.

1. Allocation to Real Estate: How Much? Practitioners have cited several studies in making their investment allocations to real estate. These studies include the work of Hoag (1980), Fogler (1984), and Firstenberg, Ross, and Zisler (1988) among others.[1] The data from these studies are located in the worksheets labeled Hoag, Fogler, and FRZ () respectively. The major difference among the datasets involves how real estate returns are measured. Actual transaction prices of industrial real estate are used in the Hoag study. In the Fogler study, assumptions are made about the attributes of the return on real estate. More specifically, real estate is presumed to have the same risk as stock ($\sigma_{\text{Real Estate}} = \sigma_{\text{Stock}}$), with the return on real estate = return on stock - 2% (200 basis points) due to the illiquid nature of real estate. In addition to this, Fogler assumes that real estate is not correlated with either stocks or bonds ($\rho_{\text{Ppty,Stock}} = 0$, $\rho_{\text{Ppty,Bonds}} = 0$). In contrast to these studies, Firstenberg, et al (FRZ) look alternatively at the return on equity REITs, returns imputed by "capping out" the income from NCREIF, and returns obtained by un-smoothing the NCREIF appraisal based returns using a fourth order autoregressive process (AR(4)).

Using the means and covariances of returns in each of the appropriate worksheets, calculate the return on the portfolio and the weight for stocks, bonds, and real estate for the levels of portfolio risk given. Assume that there is no risk free rate in the economy and that no short sales are allowed. Compare the results from these different datasets. What is the minimum and maximum allocation for real estate? Please discuss. You can use the Solver subroutine in Excel to do the portfolio allocation. An example of how to use the Solver subroutine is provided in the appendix to this handout. If the Solver sub-rountine is not visible in the Tools submenu, click on Add-Ins… and choose Solver.

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[1]See Firstenberg, Paul, Stephen Ross, and Randall Zisler, 1988. "Real Estate: The whole story", Journal of Portfolio Management, pp. 22-34; H. Russell Fogler, Winter 1984. "20% in Real Estate: Can Theory Justify It?", Journal of Portfolio Management, pp. 6-13; and James Hoag, 1980. "Towards Indices of Real Estate Value and Return. Journal of Finance 35(2): 569-580.

2. Revisiting Attribution Analysis of a Real Estate Mutual Fund. In problem set #3, you were asked to perform an attribution analysis of Cohen and Steers Realty Shares (ticker symbol: CSRSX). The issue that we wish to explore in this problem set is whether we could have done "better" e.g., higher returns for the same level of risk as CSRSX by using mean-variance optimization. To obtain the initial set of weights, use data from January 1994 through December 1998 from the worksheet labeled "CSRSX". In performing the mean-variance optimization, set the standard deviation of your portfolio equal to the standard deviation of the returns on CSRSX over the January 1994 through December 1998. Also, set the mean return equal to the average return over the January 1994 through December 1998 period. To calculate the next set of weights, use data from February 1994 through January 1999 e.g., drop one month at the beginning and add one month at the end of the data series. In performing the new mean-variance optimization, set the standard deviation of your portfolio equal to the standard deviation of the returns on CSRSX over the February 1994 through January 1999. Also, set the mean return equal to equal to the average return over the February 1994 through January 1999 period. Repeat this process. In doing your mean-variance optimization, assume that there is no risk free asset in the economy and that short sales are not allowed (all assets are held either in zero or positive amounts). To calculate the covariances, use the covariance option in the Data Analysis subroutine located in the Tools submenu.[2] You are trying to maximize the return and have the standard deviation of the portfolio as a constraint. Given the set of weights at time T (prior month), calculate what the return on the portfolio would be in the next period (month). An example of this is given in the worksheet labeled “CSRSX MPT Weights”. Be sure to keep changing the standard deviation (constraint). Compare the results of this analysis with the attribution results that you obtained in an earlier homework. Are the portfolio weights from using a mean-variance algorithm comparable the actual CSRSX weights as revealed by the attribution analysis over time? Please discuss the differences. Plot the return from the CSRSX versus the return using the mean-variance algorithm for CSRSX over time. Would it pay to employ a mean-variance framework? If transaction costs are .005 or .5%, would it make sense to use a mean-variance framework?

[2]If the Data Analysis subroutine is not available, pull down the Tools submenu, select Add-Ins …, click on Analysis Toolpak, and then click on OK. You should now see the Data Analysis subroutine under the Tools submenu.

3. The Asian Crisis and Real Estate Allocation: A commonly reported driver of the Asian crisis, which occurred around July 1997, was the exposure of major Asian banks to real estate. Allegedly, as real estate markets plummeted, banks suffered enormous losses due to their exposures to real estate developers, these problems then spread to the rest of the financial sector[3]. An interesting observation comes from Paul Krugman's recent book, *The Return of Depression Economics*:

> "*How did a few bad real estate loans and a botched devaluation in Thailand - a small, faraway country of which most people knew little - sent dominoes toppling from Indonesia to South Korea*?"

We wish to examine the structural relationship between equity and real estate markets[4] in eight major Asian countries: China, Hong Kong, Indonesia, Malaysia, The Philippines, South Korea, Taiwan and Thailand. Using information on average returns (means), their riskiness (variances) in each country, and their co-movements with equity and real estate in other countries located in the worksheets labeled "PreBreak(Means & Cov)" and "PostBreak(Means & Cov)", calculate the mean-variance frontiers associated with the pre-crisis and post-crisis period and plot the two frontiers on the same graph. Please discuss your findings. Did the efficient frontier of stocks and real estate remain stationary during the pre- and post-crisis periods? Did investing in real estate securities in each country add any added value/benefits over and above investing in the common stock of each country? Prior to the crisis, in which countries should we have invested in their real estate securities? After the crisis, were there any countries in which it made sense (from an MPT perspective) to invest in their real estate securities?

Please turn in a hard copy of your work.

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[3] For an excellent overview of the role of real estate in the Asian crisis focusing on Thailand, see Betrand Renaud, 1999, "Real Estate and the Asian Crisis: Lessons of the Thailand Crisis," Capital Markets Development Department, World Bank.; see also his earlier analysis of global real estate cycles: Betrand Renaud, 1997, "The 1985 to 1994 Global Real Estate Cycle: An Overview," *Journal of Real Estate Literature* 5: 13-44.

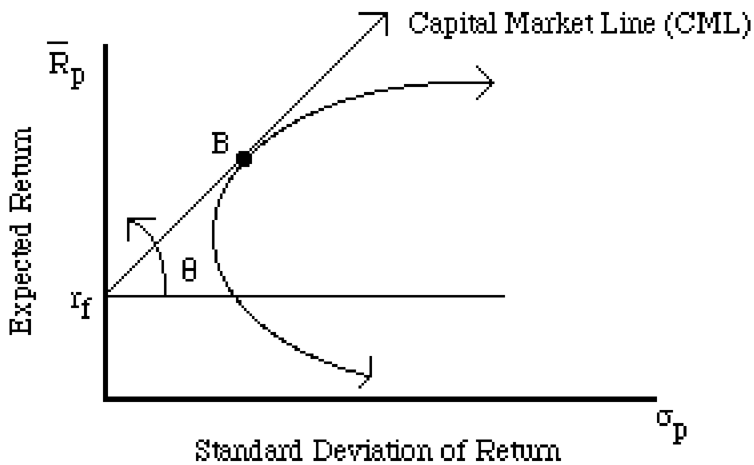
[4] Note that because of a lack of reliable data, we do not use data on direct real estate investment; the results developed here are for real estate securities. Another argument for using real estate securities is that ownership of the underlying real estate was prohibited in some countries prior to the crisis.

Appendix A: Implementing MPT Analysis in Excel

Efficient Frontier: set of portfolios that has the maximum return for every given level of risk, or the minimum risk for every level of return.

Scenario ❶*: *Short Sales Allowed, Riskless Borrowing and Lending Exist

Efficient Frontier Assuming Short Sales and a Riskless Asset



riskless lending and borrowing rate implies that a single portfolio of risky assets (portfolio B = best choice) is preferred to all other portfolios.

- **Best choice B** portfolio plots on the ray connecting the riskless asset (r_f) and a risky portfolio (B) that lies furthest in a counterclockwise direction

  Note: This is the ray with the *largest* slope.

  Recall that the slope of the line connecting r_f and a risky portfolio is

$$\text{Slope} = \frac{\bar{R}_p - r_f}{\sigma_p} = \left[\frac{\text{Expected Return on Portfolio - Riskfree Rate}}{\text{Standard Deviation of Portfolio Return}}\right] \qquad (1)$$

$$= \text{Excess Return on Portfolio Per Unit of Total Risk}$$

Intuition: Our objective is to find that risky portfolio B with the largest excess return per unit of risk. In searching for that risky portfolio, the sum of the proportions invested in the assets equals 100%.

Decision Problem: Mathematically, our objective function and constraints are as follows

Maximize $\theta = \dfrac{\bar{R}_p - r_f}{\sigma_p}$ — maximize the excess return per unit of risk

subject to: $\sum_{i=1}^{n} X_i = 1$ — sum of % invested in each stock equals 100%

where
n = number of stocks (assets)
X_i = % or proportion invested in the ith stock
$\bar{R}_p = X_1\bar{R}_1 + X_2\bar{R}_2 + ... + X_n\bar{R}_n$ (mean return on a portfolio)
$\sigma_p = [X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + ... + X_n^2\sigma_n^2 + 2X_1X_2\sigma_{12} + 2X_1X_3\sigma_{13} + ... + 2X_{n-1}X_n\sigma_{n-1,n}]^{.5}$

Example 1[5]: Calculating Optimal Weights when Short Sales are Allowed and Riskless Borrowing and Lending Exists

Consider the following 3 securities

			Covariance Matrix		
Security	Mean Return	Variance	Stock1	Stock2	Stock3
Colonel Motors	14%	36	36	9	18
Separated Edison	8%	9	9	9	18
Unique Oil	20%	225	18	18	225

If the riskless lending and borrowing rate is 5%, what is the optimal proportion to invest in each security?

To solve this problem, we will use the Solver subroutine in Microsoft's Excel spreadsheet software. Solver is an optimization/resource allocation tool. To use Solver, there are three types of cells

target cell: cell that you want to minimize (e.g. risk), maximize (e.g. return), or set to a certain value.

changing cells: cells that you want to be adjusted until a solution is found. Excel allows you to specify up to 200 changing cells.

[5]From E. Elton and M. Gruber, Modern Portfolio Theory and Investment Analysis, Wiley, New York, NY, 2nd Edition, 1984

constraint cells: cells that must fall within certain limits or satisfy target values. Excel allows you to specify a maximum of 500 constraints, 2 for each changing cell plus 100 additional constraints for a total of no more than 1000 cells in a problem.

Instructions: Open the Excel Spreadsheet and using the information given above, perform the following operations (an example of what your spreadsheet should resemble follows subsequently)

Step 1:Type the names of the stocks in column A. (We have used A4 thru A6)

Step 2: Enter the raw mean returns of the stocks in column B (e.g. B4 thru B6)

Step 3: Enter the excess mean returns of the stocks in column C (e.g. C4 thru C6). Excess returns are the mean return minus the riskfree rate.

Stock	Mean Return		Rf	XRtn
Colonel Motors	14%	-	5%	= 9%.
Separated Edison	8%	-	5%	= 3%
Unique Oil	20%	-	5%	= 15%

Step 4: Enter the covariance matrix in column D thru column F (e.g. D4 thru F6)

Step 5: Type the initial weights directly above the covariance matrix. (e.g. D2,E2,F2 in our example) Note: since we have 3 stocks, we initially let each stock have an equal weight in our portfolio e.g. weight = 1/n = 1/3 where n is the number of stocks.

$X_1 = X_{\text{Colonel Motors}} = 1/3 = .33333....$
$X_2 = X_{\text{Separated Edison}} = 1/3 = .33333....$
$X_3 = X_{\text{Unique Oil}} = 1/3 = .33333....$

Use cell referencing to copy the initial weights directly above the covariance matrix and put them in column G (G4 thru G6) in our example. In other words, let G4 = D2, G5 = E2, and G6 = F2.

Intuition: This will prevent double counting when we set our ***changing cells*** to solve for the optimal portfolio weights.

Step 6: Sum the initial weights in columns D2, E2, and F2 and put the result in F9 e.g. F9 = sum(D2:F2)

Intuition: Recall that the sum of the % invested in each stock must equal 100% so $X_1 + X_2 + X_3 = X_{\text{Colonel Motors}} + X_{\text{Separated Edison}} + X_{\text{Unique Oil}} = 1$. This is our ***constraint cell***.

Step 7: Calculate the excess mean return to the portfolio and put the result in C9 e.g. C9 = mmult(D2:F2,C4:C6)

$R_p = X_1 * XRtn_{\text{Colonel Motors}} + X_2 * XRtn_{\text{Separated Edison}} + X_3 * XRtn_{\text{Unique Oil}}$

Step 8: Calculate the variance of the portfolio and put the result in G9 e.g. G9 = mmult(mmult(D2:F2,D4:F6),G4:G6)

Intuition: we use the Excel command for matrix multiplication to calculate the variance of the portfolio which is equal to

$$\sigma^2 = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} .33 & .33 & .33 \end{bmatrix} \begin{bmatrix} 36 & 9 & 18 \\ 9 & 9 & 18 \\ 18 & 18 & 225 \end{bmatrix} \begin{bmatrix} .33 \\ .33 \\ .33 \end{bmatrix}$$

Step 9: Calculate theta which represents the excess mean portfolio return per unit of risk and put the result in G12 e.g. G12 = C9/sqrt(G9)

Intuition: Recall that our objective function is to maximize theta e.g.

$$\text{Maximize } \theta = \frac{\bar{R}_p - r_f}{\sigma_p}$$

	A	B	C	D	E	F	G
1	**Efficient Stock Portfolio**						
2				0.333333	0.33333	0.333333	Weight
3		Return (Ri)	Excess Rtn		Covariance		
4	Colonel Motors	14	9	36	9	18	= D2
5	Separated Edison	8	3	9	9	18	= E2
6	Unique Oil	20	15	18	18	225	= F2
7							
8			Port XRtn			Total Wts	Port Var
9			=mmult(D2:F2,C4:C6)			=sum(D2:F2)	=mmult(mmult(D2:F2,D4:F6),G4:G6)
10							
11							Theta
12							=C9/sqrt(G9)

Your spreadsheet should be identical to the following:

	A	B	C	D	E	F	G
1	**Efficient Stock Portfolio**						
2				0.333333	0.33333	0.333333	Weight
3		Return (Ri)	Excess Rtn		Covariance		
4	Colonel Motors	14	9	36	9	18	0.33333333
5	Separated Edison	8	3	9	9	18	0.33333333
6	Unique Oil	20	15	18	18	225	0.33333333
7							
8			Portfolio XRtn			Total Wts	Portfolio Var
9			9			1	40
10							
11							Theta
12							1.4230249

Step 10: Pull Down the **Tools** submenu and chose the ***Solver*** subroutine. Fill in the boxes as follows:

Set Target Cell: $G$12 The target cell is the cell you're maximizing/minimizing

Equal to: ⊙ Max

By Changing Cells: $D$2:$F$2

Subject to the Constraints: $F$9 = 1 Note: you need to click on the **Add** button to add this constraint. The cell reference is $F$9, pull down the arrow and choose =, then type 1 in the constraint: box. Click on the **OK** button.

Step 11: Click on the Solve button. Your optimal weights should be as follows

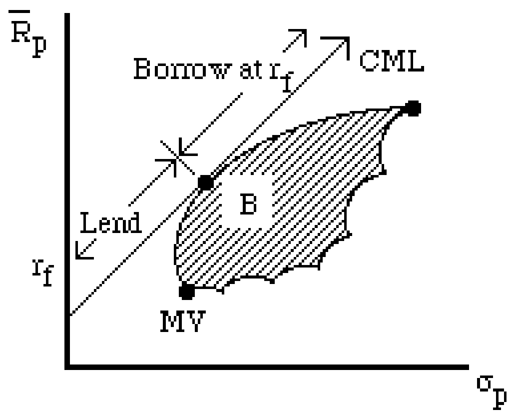
$X_1 = X_{\text{Colonel Motors}} = .778$
$X_2 = X_{\text{Separated Edison}} = .056$
$X_3 = X_{\text{Unique Oil}} = .167$

So 77.8% should be invested in Colonel Motors, 5.6% in Separated Edison, and the remaining 16.7% in Unique Oil.

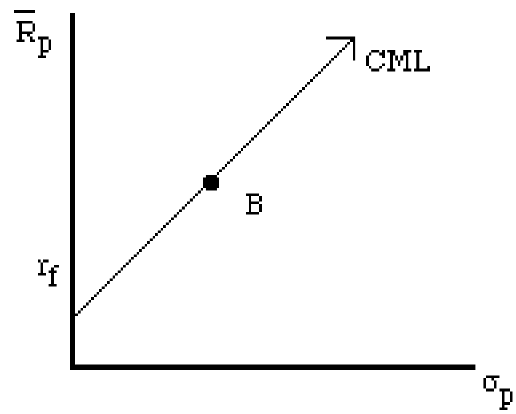
Scenario ❷: Short Sales Aren't Allowed with Riskless Borrowing/Lending at r_f

If we change the preceding scenario so that no short sales are allowed but unlimited riskless borrowing and lending exists at the riskfree rate, then our opportunity set and efficient set will change because we have once again changed our assumptions. Following is a graphical depiction of the opportunity set and efficient set assuming no short sales are allowed with unlimited riskless borrowing and lending at r_f.

OPPORTUNITY SET
r_f Exists, No Short Sales



EFFICIENT SET
r_f Exists, No Short Sales



Observe: As in the case where short sales are allowed, one portfolio is optimal or "best". However, investors cannot hold risky securities in negative amounts because of the no short sale restriction.

Decision Problem: The investor's decision problem is similar to Scenario ❶ above except that a new constraint is added to the equation

$$\text{Maximize } \theta = \frac{\bar{R}_p - r_f}{\sigma_p}$$

subject to: $\sum_{i=1}^{n} X_i = 1$ sum of % invested in each stock equals 100%

$X_i, X_j \geq 0$ for all i, j Each risky security held in nonnegative amounts (new constraint)

Example: Continuing with our preceding example involving three stocks, Colonel Motors, Separated Edison, and Unique Oil, we would make the following modification in step 2 as follows

Pull Down the **Tools** submenu and chose the ***Solver*** subroutine. Fill in the boxes as follows:

Set Target Cell: $G$12 — The target cell is the cell you're maximizing/minimizing

Equal to: ⊙ Max

By Changing Cells: $D$2:$F$2

Subject to the Constraints: $F$9 = 1
(new constraint) $D$2 >= 0
(new constraint) $E$2 >= 0
(new constraint) $F$2 >= 0

Note: you need to click on the **Add** button to add this constraint. The cell reference is $F$9, pull down the arrow and choose =, then type 1 in the constraint: box. Click on the **OK** button.

Observation: In our example, since all three stocks have positive weights, it is as if the no short sale constraint is the same as the short sale constraint. This might not necessarily be the case.