1. **Bond yields.** Our mission is to explore the relation between the price of a bond and its yield. Suppose we have a 5-year bond with annual coupons of $c$ and a principle of 100. Thus an owner gets a cash flow of $c$ after years one to four and $100 + c$ after year five. If the bond sells for price $q$, it’s common to express it as the discounted cash flow

$$q = \frac{c}{1+y} + \frac{c}{(1+y)^2} + \frac{c}{(1+y)^3} + \frac{c}{(1+y)^4} + \frac{(c+100)}{(1+y)^5}.$$  

The discount rate $y$ is referred to as the yield or **yield to maturity**. Equally common is to use $y$ as a way to report the price, since knowing $y$ is enough to compute the price (plug it into the equation).

(a) Plot the price $q$ against a grid of points $y$ between 0.00 and 0.10. How does the price vary as we change the yield?

(b) Suppose the price is $q = 102$. Use your graph to estimate the yield $y$.

(c) Write a bisection program to find the yield $y$ associated with price $q = 102$. *Comment: See the Matlab guide to anonymous functions and the root-finding code posted on the course outline for examples.*

(d) How does your answer change if the price is $q = 99$. Why?

(e) Optional (for aficionados only). If we define $d = 1/(1+y)$, we see that the bond price is a polynomial in $d$:

$$-q + cd + cd^2 + cd^3 + cd^4 + (c + 100)d^5 = 0.$$  

Since it’s a polynomial of degree 5, it has five roots. What happened to the other ones when we computed the yield earlier?

2. **Finding calls from puts.** If we know put prices at given strikes $k$, we can compute the call prices at the same strikes from the put-call parity relation. And vice versa.

Here’s an example. Consider a one-year put option on a stock currently selling for 100. The option with strike price $k = 95$ has a price of 2. The one-period bond price is $q_1^t = 0.99$. What is the price of a call option at the same strike?

3. **Black-Scholes-Merton volatility.** Our mission here is to examine the role of the mysterious volatility parameter $\sigma$ in the BSM formula. The calculations refer to put options on the S&P 500 exchange-traded fund, ticker symbol SPY. You can look up prices at *Yahoo Finance* for various strikes and maturities. Or use a Bloomberg machine.

We’ll use these inputs: The current price of the underlying is $s_t = 208$. The price of a one-year bond is $q_1^t = 1.00$. 

(a) Consider a put option with strike price $k = 180$. If volatility $\sigma = 0.10$, what is the price of the option? If $\sigma = 0.20$?

(b) Plot the option price against volatility for $\sigma$ between 0.01 and 0.30. What does it look like? Can you say why?

(c) Suppose the price of the option is 6. What value of $\sigma$ does that correspond to?