Abstract

In a rational-expectation model of international portfolio and consumption decisions, international home bias in equities depends on the correlation between non-diversifiable labor income risk and the cross-country equity returns, when agents have log utility in consumption. We show that there is weak empirical evidence for this channel. Moreover standard preferences fail to account for other empirical evidence on international asset prices.

We propose an alternative environment with model uncertainty populated by the sophisticated agents of the robust-control theory of Hansen and Sargent (2005). Maintaining the assumption of unitary intertemporal elasticity of substitution, we show that home bias in equity can also depend on the correlation between equity returns and the real exchange rate and its weight depends on a measure of the distrust that the agent has with respect to the objective probability distribution. This hedging component, which mainly refers to long-run risk in real exchange rate, is more relevant from an empirical point of view. The proposed model is successful along other dimensions, where instead the standard rational-expectation model fails.
1 Introduction

Home bias in equities is one of the most persistent observations in international finance. Investors hold a large share of their wealth in domestic securities more than what would be dictated by the share of these securities in the world market.\(^1\) Per se, this is not a puzzle. A growing body of the literature has proposed portfolio models that can account for a partially diversified portfolio. The current explanations range from the existence of information frictions to trade costs in goods and asset markets, home bias in consumption, sticky prices, terms of trade movements.\(^2\)

Many progresses have been made, in particular in developing general equilibrium analyses of portfolio choices, but there are still several shortcomings in current models that require further investigation.

First, a common assumption in the current literature is that of complete financial markets. This is a convenient device to obtain a closed-form solution, but an unrealistic assumption to describe the current stage of financial integration, as argued among others by Obstfeld (2006).

Second, the structure of the preferences used is based on the current expected-utility paradigm with an isoelastic-utility flow. This assumption, as it is well known, has counterfactual implications. On one side, together with the complete-market assumption, these preferences create a strong link between the consumption differentials across countries and the real exchange rate. In particular, consumption should fall in one country relative to the other while the real exchange rate appreciates. Instead, in the data, the cross-country differential in consumption growth and the real exchange rate are only weakly correlated. This is called the Backus-Smith anomaly.\(^3\) On the other side, this structure of preferences is unable to match other asset price moments as the high and volatile returns on equities and the shape and volatility of the yield curve. Models that pretend to explain portfolio choices cannot fail to account for movements in asset prices, because both are the faces of the same coin.

Last but not least, most of the current models derive the portfolio shares as a function of primitive parameters, like the risk-aversion coefficient, the share of traded goods, etc... This is clearly a welcome feature of general equilibrium models, but has the drawback of hiding the hedging relationships based on observable variables that are at the root of the portfolio decisions. When the determinants of the portfolio shares are written in terms of covariances between the asset returns and the source of risk, in most of the cases, the models fail to solve the portfolio home-bias puzzle.\(^4\)

This paper first contributes to the literature by proposing a simple dynamic general equilibrium model of consumption and portfolio choices under incomplete financial markets. In particular asset trading includes equities and bonds in two currency markets. The determinants of the portfolio decisions are written in terms of observable variables like the covariances between the relevant excess returns and the risk to hedge. Under rational expectations and log utility, the model implies that the cross-country movement in non-diversifiable labor income is the only risk that needs to be hedged in international financial markets.

This result confirms the analyses of Baxter and Jermann (1997), Bottazzi et al. (1998), Coeurdacier and Gourinchas (2008) and Heatcote and Perri (2004), but comes with two qualifi-

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\(^1\)See Bertaut and Grever (2004).


\(^3\)Backus and Smith (1993).

\(^4\)Coeurdacier and Gourinchas (2008) and van Wincoop and Warnock (2006) discuss a similar issue.
ocations. First, in a dynamic model, the relevant covariances and variances, although constants, are all conditional on previous-period information. This is not an innocuous result when the model is tested against the data. Second, there are three excess returns which are relevant for the portfolio allocation. The excess return of foreign equity with respect to domestic equity and its covariance with the non-diversifiable income risk determines the split between domestic and foreign equity. The excess return of domestic equity versus domestic bonds and its respective covariance determines the portfolio composition between equities and bonds. Finally the excess return of foreign versus domestic bonds determines the portfolio composition between the two bonds. All these covariances are also conditional on the remaining excess returns. There is a bias in holding one of the available assets when this asset pays well when needed, in particular when agents face an unfavorable shock to non-diversifiable income.

Using data on US and other G7 countries, we do not find important quantitative evidence that supports this channel. The model generally implies full portfolio diversification and moreover a foreign-equity bias when the asset menu includes only equities.

Real exchange rate risk is another hedging reason that results from portfolio models, as in the analyses of Adler and Dumas (1983), Coeurdacier and Gourinchas (2008) and van Wincoop and Warnock (2007). This channel would be relevant in our model were we relaxing the assumption of log utility. However, there are two reasons why we do not explore this direction: on one side, the estimates of the intertemporal elasticity of substitution point to numbers very close to the unitary value, as discussed by Vissing-Jørgensen and Attanasio (2003); on the other side, the risk aversion coefficient should be increased to make this component relevant from a quantitative point of view with the cost of rising the volatility of the risk-free rate and falling in the risk-free rate puzzle.

In this paper we show that we can still recover a similar hedging component under the assumption of an unitary elasticity of substitution while avoiding the risk-free rate puzzle. To obtain this result, we use preferences in which there is a distinction between the parameter that captures how agents would like to shift consumption intertemporally and the parameter that measures how agents would like to shift consumption intratemporally across different states of nature – a measure of risk aversion. To this purpose, we analyze an environment in which agents face model uncertainty in the sense that they surround the true probability distribution with nearby distributions statistically difficult to detect in finite samples. In particular, we use the sophisticated agents of the robust-control literature, developed by Hansen and Sargent (2005), which makes their decisions taking into account the worst-case scenario. Under log-utility these preferences are observationally equivalent to the non-expected recursive preferences of Epstein and Zin (1989) and Kreps and Porteus (1978).

This framework is desirable for at least two reasons. On the one hand, these preferences have been shown to be successful in matching some properties of financial data, like the equity premium puzzle and the slope of the yield curve. On the other hand, model uncertainty acts like a preference shock in standard preferences and generates a multiplicative perturbation in the stochastic discount factor, relaxing the link between cross-country consumption and the real exchange rate even when markets are complete.

The reason for why we can recover the hedging component with respect to real exchange rate risk is that the fear of model misspecification translates into bad news on the cross-country risk.

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6Barillas et al. (2007) among others make this parallel.
7See Barillas et al. (2006) and Piazzesi and Schneider (2006).
8Pavlova and Rigobon (2007) studies the role of preference shocks for the determination of asset prices and exchange rate.
consumption profiles which ultimately depend on the real exchange rate movements. News on current and future real exchange rate appreciation are bad news with respect to the expected consumption profile. Investors would like to invest more in securities that provide a good hedge against this risk.

In our empirical analysis, we show that this channel is more relevant from a quantitative point of view than the hedging component due to non-diversifiable income risk and is able to explain a good proportion of the asset home bias and other empirical facts. This result seems surprising given that van Wincoop and Warnock (2006) show that the real exchange rate does not covary much with the excess return on equities, once conditioning on bond returns. But, in our model, what matters is the covariance between the present discounted value of the surprises in the real-exchange-rate growth and the excess return on equities. Hedging long-run risk in the real exchange rate is more relevant and equities are a good hedge with respect to that risk once we appropriately weigh for risk aversion.

Finally, we show that our model is also able to account for other stylized empirical facts like those documented by Tille (2005) – that the US is a net creditor in equity instruments and a net debtor in bond instruments – and Lane and Shambaugh (2008) – that advanced countries tend to have a positive overall exposure to foreign-currency denominated instruments and a negative exposure to foreign-currency debt instruments.

This work is in discrete time, but it is related to continuous-time literature on portfolio choices under ambiguity. Maenhout (2004, 2006) develops a modification of the continuous-time robust-control literature to study portfolio and consumption choices in a partial-equilibrium dynamic model. To get a closed-form solution he adopts a transformation of the objective function of the decision makers that changes the penalization of entropy from a constant Lagrange multiplier into a function of the value function. This modification deeply changes the nature of the approach proposed by Hansen and Sargent (2005) in a way that it is not comparable with the one proposed here.\cite{9} Uppal and Wang (2003) and Epstein and Miao (2005) are related papers that instead use ambiguity aversion based on recursive multiple priors. In particular, Epstein and Miao (2005) develop a two-country continuous-time dynamic general equilibrium model. In contrast to this paper, they focus on a complete-market allocation. Most importantly, their conclusion for asset home bias depends on imposing the assumption that agents have more ambiguity in the foreign asset’s return. Van Nieuwerburgh and Veldkamp (2007) model an economy with imperfect information in which agents can learn and acquire better information on domestic and foreign stocks. However, to get home bias they have to assume that each home investor has prior information about home asset’s payoff which is slightly more precise than the prior information foreigners have. Instead, in this paper, model uncertainty creates a departure from full portfolio diversification that can go in either directions, to justify more or less home bias without necessarily assuming that home agents have more ambiguity or less information with respect to foreign asset’s return. The results of this paper depends instead on the sign of data covariances.

This work is structured as follows. Section 2 discusses the structure of model uncertainty. In Section 3 we present the model. We contrast the equilibrium portfolio allocation, implied by the standard framework with rational expectations in Section 4 with those those implied by model uncertainty, in Section 5. Section 6 presents the empirical analysis and evaluate the empirical relevance of the model.

\footnote{See the discussion in Pathak (2002).}
2 Model Uncertainty

We characterize model uncertainty as an environment in which agents are not sure about the true probability distribution and might instead act using a nearby distorted ‘subjective’ probability distribution. Consider a generic state of nature $s_t$ at time $t$ and define $s^t$ as the history $s^t \equiv [s_t, s_{t-1}, \ldots, s_0]$. Let $\pi(s^t)$ be the ‘objective’ probability measure on histories $s^t$. Decisionmakers are instead endowed with a different probability measure, a ‘subjective’ one, denoted with $\tilde{\pi}(s^t)$ which is absolutely continuous with respect to the ‘objective’ measure. Absolute continuity is obtained by using the Radon-Nykodym derivative.\footnote{This way of constructing subjective probability measures is borrowed from Hansen and Sargent (2005, 2007).} First, we assume that the two probability measures agree on which events have zero probability. Moreover there exists a non-negative martingale $G(s^t)$ with the property

$$E(G_t) \equiv \sum_{s^t} G(s^t)\pi(s^t) = 1 \quad (1)$$

such that

$$\hat{E}(X_t) \equiv \sum_{s^t} \tilde{\pi}(s^t)X(s^t) = \sum_{s^t} G(s^t)\pi(s^t)X(s^t) \equiv E(G_t X_t) \quad (2)$$

for a generic random variable $X(s^t)$ where we have defined $E(\cdot)$ and $\hat{E}(\cdot)$ the expectation operators under the ‘objective’ and ‘subjective’ probability measures, respectively. Since $G_t$ is a martingale, it is also the case that

$$E(G_{t+1}X_t) = E(G_t X_t). \quad (3)$$

In particular $G(s^t)$ is a probability measure, which is equivalent to the ratio $\tilde{\pi}(s^t)/\pi(s^t)$, that allows a change of measure from the ‘objective’ to the ‘subjective’ measure. Moreover, since $G_t$ is a martingale, we can define its increment $g(s_{t+1}|s^t)$ as $g(s_{t+1}|s^t) \equiv G(s^{t+1})/G(s^t)$ with the property that $E_t g_{t+1} = 1$. We can further write

$$\hat{E}_t(X_{t+1}) = E_t(g_{t+1}X_{t+1}) \quad (4)$$

for a generic random variable $X_{t+1}$, where $E_t(\cdot)$ and $\hat{E}_t(\cdot)$ denote the conditional-expectation operators under the two measures. Note also that $g(s_{t+1}|s^t) = \tilde{\pi}(s_{t+1}|s^t)/\pi(s_{t+1}|s^t)$. High values of $g(s_{t+1}|s^t)$ imply a higher probability that the decisionmakers give to state $s_{t+1}$ conditional on $s^t$.

As in Hansen and Sargent (2005), it is convenient to define a measure of the distance between the ‘objective’ and ‘subjective’ probabilities given by entropy

$$E_t(g_{t+1} \ln g_{t+1})$$

which approximately measures the variance of the distortions in the beliefs, with the property of being equal to zero when there are indeed no distortions, i.e. $g(s_{t+1}|s^t) = 1$ for each $s_{t+1}$. In particular in what follows, it is useful to exploit the discounted version of entropy discussed in Hansen and Sargent (2005) and given by

$$E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} G_t E_t(g_{t+1} \ln g_{t+1}) \right\}, \quad (5)$$

where $0 < \beta < 1$. A high value of entropy can be interpreted as a very large distance between the ‘subjective’ and the ‘objective’ beliefs. On the contrary a low value of entropy implies beliefs that are not too distorted or far from the ‘objective’ ones.
3 Model

We consider a model with two countries, that we denote domestic and foreign, and two representative agents, each belonging to a country. Representative agents supply a fixed amount of labor.\footnote{The model can be also modified to include an elastic labor supply, without changing our results.} In each country, there is a continuum of firms that produce a continuum of goods in a market characterized by monopolistic competition. All goods are traded. Households enjoy consumption of both domestic and foreign goods and can trade in a set of financial assets. There are four assets traded in the international markets: two risk-free nominal bonds denominated in each of the currency and shares in two equities that represent participation in the dividends of domestic and foreign firms, respectively.

The representative agent in the domestic economy maximizes utility given by

$$\hat{E}_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \log c_t \right\}$$

(6)

where $\beta$, with $0 < \beta < 1$, is the intertemporal discount factor and $\hat{E}_{t_0}(\cdot)$ is the time-$t_0$ expectation operator taken with respect to the distorted probability measure. As discussed in the previous section, this subjective, distorted, probability measure is absolutely continuous with respect to the objective one and satisfies the property (4). Therefore, the expected utility can be written also in terms of the objective distribution as

$$\hat{E}_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \log c_t \right\} = E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} G_t \log c_t \right\}$$

where we have normalized $G_{t_0} = 1$. The representative agent in the other country has similar preferences but a possibly different subjective probability measure and so a different expectation operator $\hat{E}_{t_0}^*(\cdot)$. The utility flow is logarithmic in the consumption index $c$ which is defined by

$$c \equiv \left[ n^{\frac{1}{\vartheta}} (c_H)^{\frac{\vartheta-1}{\sigma_t}} + (1-n)^{\frac{1}{\vartheta}} (c_F)^{\frac{\vartheta-1}{\sigma_t}} \right]^{\frac{1}{\vartheta-1}},$$

where $c_H$ and $c_F$ are consumption sub-indexes of the continuum of differentiated goods produced respectively in country $H$ and $F$

$$c_H \equiv \left[ \int_0^1 c(h)^{\frac{\sigma_t-1}{\sigma_t}} dh \right]^{\frac{1}{\sigma_t-1}}, \quad c_F \equiv \left[ \int_0^1 c(f)^{\frac{\sigma_t-1}{\sigma_t}} df \right]^{\frac{1}{\sigma_t-1}}.$$

In particular, $c(h)$ and $c(f)$ denote the consumption of the individual goods $h$ and $f$ produced in the domestic and foreign country, respectively. In particular $n$, with $0 < n < 1$, is the weight given to the consumption of domestic goods and $\vartheta$, with $\vartheta > 0$, is the intratemporal elasticity of substitution between the bundles $c_H$ and $c_F$. Instead $\sigma_t$ is the time-varying elasticity of substitution across the continuum of measure one of goods produced in each country, with $\sigma_t > 1$. The appropriate consumption-based price indices expressed in units of the domestic currency are defined by

$$P \equiv \left[ n(P_H)^{1-\vartheta} + (1-n) (P_F)^{1-\vartheta} \right]^{\frac{1}{1-\vartheta}},$$

(7)
with

\[ P_H \equiv \left[ \int_0^1 p(h)^{1-\sigma_t} dh \right]^{\frac{1}{1-\sigma_t}}, \quad P_F \equiv \left[ \int_0^1 p(f)^{1-\sigma_t} df \right]^{\frac{1}{1-\sigma_t}}. \]

A similar structure of preferences holds for the foreign agent with the appropriate asterisks. In particular the weight \( n^* \) in the consumption index might not be equal to \( n \) capturing the presence of consumption home bias, when \( n > n^* \), and implying variation over time in the real exchange rate.

In each country, there is a continuum of firms of measure one producing the goods in a monopolistic-competitive market. A domestic firm of type \( h \) has a CRS production technology

\[ y_t(h) = Z_t^\phi l_t^{1-\phi} \]

where \( Z_t \) is a natural resource available in the country and \( l_t \) denotes labor which is paid at the wage rate \( W_t \); \( \phi \) is a parameter with \( 0 < \phi \leq 1 \). When \( \phi = 1 \), the model boils down to an endowment economy.

Prices are set without frictions and the law-of-one price holds. Equilibrium implies that prices are equalized across all firms within a country and set as a time-varying markup \( \mu_t \equiv \sigma_t/[(\sigma_t - 1)(1 - \phi)] > 1 \) over the marginal costs

\[ P_{H,t} = \mu_t \frac{W_t l_t}{y_{H,t}}, \]

which implies that the wage payments are inversely related to the mark-up

\[ W_t l_t = \frac{P_{H,t} y_{H,t}}{\mu_t}. \]

Firms make profits and distribute them in the form of dividends. The aggregate dividends in the domestic economy are given by

\[ D_{H,t} = P_{H,t} y_{H,t} - W_t l_t = \frac{(\mu_t - 1)}{\mu_t} P_{H,t} y_{H,t}, \]

which shows instead that dividends are positively correlated with the mark-up. The model would then be able to generate a negative correlation between dividends and non-diversifiable labor income. Anticipating the discussion of the next section, the possibility that non-diversifiable labor income correlates negatively with the equity return gives rise to an hedging motif for holding domestic equity and might rationalize the existence of equity home bias. This channel has been emphasized and debated in the recent literature and, for comparisons, we allow for this possibility on theoretical grounds.\(^{12}\) When \( \phi = 1 \) we are in a pure endowment economy with all income diversifiable. In this case \( \mu_t \) goes to infinity.

The market of foreign goods works in a similar way with the appropriate modifications. There are two equity markets one for each country, with shares that are traded internationally. The stock-market prices, in the respective currency, are \( V_{H,t} \) and \( V_{F,t}^* \) for the domestic and foreign country, respectively. Households can also trade in two risk-free nominal bonds, denominated in units of the two currencies, with one period maturity. The flow-budget constraint of the domestic agent is

\[ B_{H,t} + S_{t} B_{F,t} + x_{H,t} V_{H,t} + x_{F,t} S_{t} V_{F,t}^* \leq R_{H,t} B_{H,t-1} + S_{t} R_{F,t} B_{F,t-1}^* \]

\[ + x_{H,t-1}(V_{H,t} + D_{H,t}) + x_{F,t-1} S_{t}(V_{F,t}^* + D_{F,t}) + W_t l_t - P_t c_t \quad (8) \]

\(^{12}\)Mark-up shocks can fall in the category of redistributive shocks, discussed by Coeurdacier and Gourinchas (2008) and Coeurdacier et al. (2007).
where $B_{H,t}$ and $B_{F,t}$ are the amounts of nominal bonds, in units of the two currencies, held at time $t$, where $R_{H,t}$ and $R_{F,t}^{*}$ are the risk-free returns from period $t-1$ to period $t$, in the respective currencies; $x_{H,t}$ and $x_{F,t}$ are the shares held by the domestic agent of the domestic and foreign equity, respectively. Finally $S_{t}$ is the nominal exchange rate, the price of foreign currency in terms of domestic currency. The flow budget constraint (8) can be written in a more compact form as

$$A_{t} = R_{p,t}A_{t-1} + W_{t}l_{t} - P_{t}c_{t}$$

(9)

where we have defined

$$A_{t} \equiv B_{H,t} + S_{t}B_{F,t} + x_{H,t}V_{H,t} + x_{F,t}S_{t}V_{F,t}$$

and

$$R_{p,t} = \alpha_{H,t-1}R_{H,t} + \alpha_{F,t-1}R_{F,t}^{*}S_{t-1} + \alpha_{H,t-1}^{e}R_{H,t}^{e} + \alpha_{F,t-1}^{e}R_{F,t}^{e}S_{t-1},$$

in which $\alpha_{H,t}$, $\alpha_{F,t}$, $\alpha_{H,t}^{e}$, $\alpha_{F,t}^{e}$ represent the shares of wealth that the domestic agent invests in the domestic bond, foreign bond, domestic equity and foreign equity, respectively, with the property

$$\alpha_{H,t} + \alpha_{F,t} + \alpha_{H,t}^{e} + \alpha_{F,t}^{e} = 1.$$

(10)

Moreover $R_{H,t}^{e}$ and $R_{F,t}^{e}$ are the returns in the two stock markets in their respective currencies. We can also express the flow budget constraint in real terms – in units of the domestic consumption index – writing

$$a_{t} = r_{p,t}a_{t-1} + \xi_{t} - c_{t}$$

(11)

where

$$r_{p,t} = \alpha_{H,t-1}r_{H,t} + \alpha_{F,t-1}r_{F,t}^{*}q_{t-1} + \alpha_{H,t-1}^{e}r_{H,t}^{e} + \alpha_{F,t-1}^{e}r_{F,t}^{e}q_{t-1}$$

in which lower-case variables denote the real counterpart of the upper-case variable; $\xi_{t} \equiv W_{t}l_{t}/P_{t}$ while $q_{t}$ is the real exchange rate defined as $q_{t} \equiv S_{t}P_{t}^{*}/P_{t}$.

The domestic agent’s optimization problem is to maximize (6) under the flow budget constraint (11) and appropriate no-Ponzi game conditions by choosing consumption and the portfolio allocation.

### 3.1 Optimality conditions

The optimality condition with respect to consumption implies an orthogonality condition, in expectation, between the real stochastic discount factor and the real portfolio return

$$\hat{E}_{t}(m_{t+1}r_{p,t+1}) = 1,$$

(12)

where $m_{t+1}$ is the real stochastic discount factor defined as

$$m_{t+1} \equiv \beta \frac{c_{t}}{c_{t+1}}.$$

(13)

A similar condition applies to the foreign economy

$$\hat{E}_{t}^{*}(m_{t+1}^{*}r_{p,t+1}^{*}) = 1,$$

(14)

\[13\] See the appendix for details on the derivations and definitions.
where the foreign stochastic discount factor is defined as

\[ m_{t+1}^* = \beta \frac{c_t^*}{c_{t+1}}. \]  

(15)

The optimality conditions with respect to the portfolio allocation imply a set of four restrictions for each agent, one for each asset, given by

\[ \hat{E}_t (m_{t+1} r_{H,t+1}) = 1, \]
\[ \hat{E}_t (m_{t+1} r_{F,t+1} \frac{q_{t+1}}{q_t}) = 1, \]
\[ \hat{E}_t (m_{t+1} r_{H,t+1} \frac{q_{t+1}}{q_t}) = 1, \]
\[ \hat{E}_t (m_{t+1} r_{F,t+1} \frac{q_{t+1}}{q_t}) = 1. \]

(16)

(17)

(18)

(19)

Equilibrium in the goods market requires the production of each good to be equal to world consumption

\[ y_{H,t} = c_{H,t} + c_{H,t}^*, \]
\[ y_{F,t} = c_{F,t} + c_{F,t}^*. \]

The labor markets are in equilibrium at the exogenously supplied quantities of labor

\[ l_t = \bar{l}_t, \]
\[ l_t^* = \bar{l}_t^*. \]

Bonds are in zero-net supply worldwide

\[ B_{H,t} + B_{H,t}^* = 0, \]

and

\[ B_{F,t} + B_{F,t}^* = 0. \]

Equity shares sum to one

\[ x_{H,t} + x_{H,t}^* = 1, \]
\[ x_{F,t} + x_{F,t}^* = 1. \]

Given the path of the stochastic disturbances \{\bar{l}_t, \bar{l}_t^*, Z_t, Z_t^*, \mu_t, \mu_t^*\} an equilibrium is an allocation of quantities \{c_t, c_{H,t}, c_{F,t}, c_t^*, c_{H,t}^*, c_{F,t}^*, c_{H,t}^*, c_{F,t}^*, \alpha_{H,t}, \alpha_{H,t}^*, \alpha_{F,t}, \alpha_{F,t}^*, \alpha_{H,t}^*, \alpha_{H,t}^*, \alpha_{F,t}^*, \alpha_{F,t}^*, \alpha^*, \alpha_t^*\} and prices \{r_{H,t}, r_{F,t}, r_{H,t}^*, r_{F,t}^*, q_t, P_{H,t}/P_{F,t}, w_t, w_t^*\} such that each agent’s consumption, portfolio shares and wealth are optimal given prices; and goods, labor, asset markets are in equilibrium.

Although we have written a general equilibrium model, in the next section, we show that we do not really need to solve all the model to understand the determinants of the portfolio choices. Instead, we can determine the portfolio shares \{\alpha_{H,t}, \alpha_{H,t}, \alpha_{H,t}^*, \alpha_{H,t}^*, \alpha_{H,t}^*, \alpha_{H,t}^*, \alpha_{F,t}, \alpha_{F,t}^*, \alpha_{F,t}^*, \alpha_{F,t}^*\} by taking as given the path of prices \{r_{H,t}, r_{F,t}, r_{H,t}^*, r_{F,t}^*, q_t\} together with the processes of non-diversifiable labor incomes \{\xi_t, \xi_t^*\}. This is a convenient result because forces our portfolio implications to be compatible with observable variables, which represent a harder test for the model. Since the stochastic structure of the model is rich enough, it should be eventually possible
to build processes for the shocks or in any case to enrich the stochastic structure in a way to match the observed prices.

Recent papers in the literature on international portfolio choices assume a general equilibrium structure and explain portfolio choices in terms of primitive parameters or shocks. However, most of these models would be less successful if the portfolio choices were analyzed under data restrictions on prices and returns.\textsuperscript{14}

4 A simple case: no model uncertainty

We start with the simple case in which there is no model uncertainty, meaning that ‘subjective’ and ‘objective’ probability measures coincide. For a generic random variable $X_{t+1}$, it follows that $E_tX_{t+1} = E_t^*X_{t+1} = E_tX_{t+1}$. Accordingly, we can write each set of orthogonality conditions (16)–(19), by taking the difference between the two in each set, as

\begin{align*}
E_t \left[ \left( m_{t+1} - m_{t+1}^* \frac{q_t}{q_{t+1}} \right) \frac{q_{t+1}}{q_t} \right] &= 0, \quad (20) \\
E_t \left[ \left( m_{t+1} - m_{t+1}^* \frac{q_t}{q_{t+1}} \right) \frac{q_{t+1}}{q_t} \right] &= 0, \quad (21) \\
E_t \left[ \left( m_{t+1} - m_{t+1}^* \frac{q_t}{q_{t+1}} \right) r_{r,t+1}^e \right] &= 0, \quad (22) \\
E_t \left[ \left( m_{t+1} - m_{t+1}^* \frac{q_t}{q_{t+1}} \right) r_{F,t+1}^e \frac{q_{t+1}}{q_t} \right] &= 0. \quad (23)
\end{align*}

The above four conditions now require the cross-country difference in the real stochastic discount factors, evaluated in the units of the domestic discount factor, to be orthogonal to the assets returns.

First, we solve for the portfolio allocation under the assumption that all income is diversifiable. This is the case of pure endowment economies in which production comes only from natural resources, i.e. $\phi = 1$ and $\xi_t = \xi_t^* = 0$. Under this restriction and log utility, consumption in each country is proportional to the financial wealth

\begin{align*}
c_t &= \frac{1}{\beta} a_t, \quad c_t^* = \frac{1}{\beta} a_t^*, \quad (24)
\end{align*}

where financial wealth evolves according to the following law of motion

\begin{align*}
a_t &= \beta r_{p,t} a_{t-1}, \quad a_t^* = \beta r_{p,t}^* a_{t-1}^*, \quad (25)
\end{align*}

The portfolio allocation can be simply characterized by guessing that in equilibrium

\begin{align*}
m_{t+1} = m_{t+1}^* \frac{q_t}{q_{t+1}}, \quad (26)
\end{align*}

through which (20)–(23) are automatically satisfied. Equation (26) holding means that risk is completely shared across the two agents. We can further write (26), by using (13), (15), (24) and (25), as

\begin{align*}
r_{p,t+1} &= r_{p,t+1}^* \frac{q_{t+1}}{q_t}.
\end{align*}

\textsuperscript{14}See van Wincoop and Warnock (2006), Coeurdacier and Gourinchas (2008) for a related argument and for models that are instead evaluated under data restrictions.
Our guess is verified when $\alpha_{H,t} = \alpha_{F,t} = \alpha_{H,t}^* = \alpha_{F,t}^* = 0$, $\alpha_{H,t}^c = \alpha_{H,t}^{*c} = 1/2$, $\alpha_{F,t}^c = \alpha_{F,t}^{*c} = 1/2$. which is indeed a feasible solution. In equilibrium, households do not hold any wealth in the bond markets and instead hold all the wealth in the equity market in the same proportion. In this case there is full sharing of risk and full international portfolio diversification and the model fails to account for the home bias in assets observed in the data. Moreover, this striking conclusion holds irrespectively of the degree of home bias and the elasticity of substitution between home and foreign goods.\footnote{See Coeurdacier and Gourinchas (2008) and Heatcote and Perri (2004) for a similar result obtained under the assumption that markets are indeed locally complete. Our model nests also the one-good model when $\vartheta$ goes to infinity.}

We now allow for non-diversifiable labor income. This small variation complicates the model solution in such a way that we are no longer able to get it in a non-linear closed form.\footnote{Van Wincoop and Warnock (2006) obtained a closed-form solution, but in a partial-equilibrium two-period model. Coeurdacier and Gourinchas (2008), Coeurdacier et al. (2007), Heatcote and Perri (2004), Kollman (2006) obtain closed-form solutions by assuming that markets are locally complete.}

We can still derive many insights by using the approximation methods developed by Devereux and Sutherland (2006) and Tille and van Wincoop (2006). First, we solve for the paths of consumption and wealth, given returns and the steady-state portfolio shares, then we use this result to solve for the steady-state portfolio shares as a function of prices, returns and non-diversifiable labor income.

In what follows, a variable with an ‘upper-bar’ denotes the steady state and a ‘hat’ denotes the log-deviation with respect to the steady state. A first-order approximation of the Euler conditions (12) and (14) implies

\begin{align}
E_t \Delta \hat{c}_{t+1} &= E_t \hat{r}_{p,t+1}, \\
E_t \Delta \hat{c}^*_{t+1} &= E_t \hat{r}^*_{p,t+1}.
\end{align}

In particular, the portfolio returns can be written in a first-order approximation as

\begin{align}
\hat{r}_{p,t+1} &= \hat{r}_{H,t+1} + \hat{\alpha}^c \text{e}r_{t+1}, \\
\hat{r}^*_{p,t+1} &= \hat{r}_{H,t+1}^* + \hat{\alpha}^{*c} \text{e}r_{t+1} - \Delta \hat{q}_{t+1},
\end{align}

where we have defined

\[ \hat{\alpha} \equiv \begin{bmatrix} \hat{\alpha}_F \\ \hat{\alpha}_H^c + \hat{\alpha}_F^c \end{bmatrix}, \quad \hat{\alpha}^* \equiv \begin{bmatrix} \hat{\alpha}_F^* \\ \hat{\alpha}_H^{*c} + \hat{\alpha}_F^{*c} \end{bmatrix}, \]

and the vector of excess returns as

\[ \text{e}r_{t} \equiv \begin{bmatrix} \hat{r}_{F,t} + \Delta \hat{q}_{t} - \hat{r}_{H,t} \\ \hat{r}_{H,t}^* - \hat{r}_{H,t} \\ \hat{r}_{F,t}^* + \Delta \hat{q}_{t} - \hat{r}_{H,t}^* \end{bmatrix}. \]

In a first-order approximation, the no-arbitrage conditions imply that excess returns have zero means, $E_t \text{e}r_{t+1} = 0$. It follows, using equations (27) and (28), that the expected cross-country differential in the growth of consumption depends on the expected depreciation in the real exchange rate, as it is standard with incomplete markets,

\[ E_t \Delta \hat{c}^R_{t+1} = E_t \Delta \hat{q}_{t+1}, \]

where an upper-script $R$ denotes the difference between the domestic and foreign variables.
A first-order approximation of the flow budget constraint (11) together with the budget constraint of the foreign agent implies

\[ \beta \hat{a}_t^R = \hat{a}_{t-1}^R + \hat{x}' \text{exr}_t + \Delta \hat{q}_t + \beta s_\xi \hat{\xi}_t^R - \beta s_c \hat{c}_t^R \]  

(30)

where \( s_\xi \) is the steady-state ratio between non-traded income and financial wealth, given by

\[ s_\xi \equiv \bar{\xi}/\bar{a}, \]

which we have assumed to be the same across countries; \( s_c \) is the steady-state ratio between consumption and financial wealth and is such that \( s_c = (1 - \beta)/\beta + s_\xi \). Moreover we have defined the vector

\[ \bar{\lambda} \equiv \begin{bmatrix} 2(\bar{\alpha}_F^H + \bar{\alpha}_F^F) - 2 \\ 2\bar{\alpha}_F^c - \bar{1} \end{bmatrix}. \]

The set of difference equations (29) and (30) can be solved to obtain relative consumption and wealth (\( \hat{c}_t^R, \hat{a}_t^R \)) as a function of the states (\( \hat{a}_{t-1}^R, \hat{q}_{t-1} \)) and the processes of excess returns, relative non-diversifiable income and the real exchange rate \( \{\text{exr}_t, \hat{\xi}_t^R, \hat{q}_t\} \). In particular, we obtain

\[ \hat{c}_t^R - \hat{q}_t = \frac{(1 - \beta)}{\beta s_c} (\hat{a}_{t-1}^R - \hat{q}_{t-1}) + \frac{(1 - \beta)}{\beta s_c} \hat{x}' \text{exr}_t + \frac{(1 - \beta) s_\xi}{s_c} \text{Et} \sum_{T=t}^{\infty} \beta^{T-t} (\hat{\xi}_T^R - \hat{q}_T), \]

(31)

\[ (\hat{a}_t^R - \hat{q}_t) = (\hat{a}_{t-1}^R - \hat{q}_{t-1}) + \bar{\lambda}' \text{exr}_t + s_\xi (\hat{\xi}_t^R - \hat{q}_t) - (1 - \beta) s_\xi \text{Et} \sum_{T=t}^{\infty} \beta^{T-t} (\hat{\xi}_T^R - \hat{q}_T). \]

(32)

We can now determine the portfolio shares by using a second-order approximation of the moment conditions (20)–(23). In particular we just need three restrictions to determine the vector \( \bar{\lambda} \). We can write

\[ \text{Et} \left[ (\Delta \hat{c}_{t+1}^R - \Delta \hat{q}_{t+1}) (\hat{r}_{F,t+1} + \Delta \hat{q}_{t+1} - \hat{r}_{H,t+1}) \right] = 0, \]

\[ \text{Et} \left[ (\Delta \hat{c}_{t+1}^R - \Delta \hat{q}_{t+1}) (\hat{r}_{F,t+1} - \hat{r}_{H,t+1}) \right] = 0, \]

\[ \text{Et} \left[ (\Delta \hat{c}_{t+1}^R - \Delta \hat{q}_{t+1}) (\hat{r}_{F,t+1} + \Delta \hat{q}_{t+1} - \hat{r}_{H,t+1}) \right] = 0. \]

By using equations (31)–(32) in the above conditions, we can obtain the steady-state vector of portfolio shares as

\[ \bar{\lambda} = -s_\xi \frac{\beta}{1 - \beta} \Sigma_t^{-1} \text{Et}(\text{exr}_t \cdot \varepsilon_{t,t+1}) \]

(33)

where we have defined \( \varepsilon_{t,t+1} \) as the news at time \( t + 1 \) in the growth path of the cross-country non-diversifiable labor incomes (in units of the same currency), given by

\[ \varepsilon_{t,t+1} = \sum_{j=0}^{\infty} \beta^j \left[ \text{Et} \left( \Delta \hat{\xi}_{t+1,j}^R - \Delta \hat{q}_{t+1,j} \right) - \text{Et} \left( \Delta \hat{\xi}_{t+1,j}^R - \Delta \hat{q}_{t+1,j} \right) \right], \]

(34)

and \( \Sigma_t \) is the time–t conditional variance-covariance matrix of the vector of excess returns \( \text{exr}_{t+1} \).

Equation (33) determines the portfolio allocation in the steady state. When \( s_\xi = 0 \), we confirm the previous result of the simple model in which all income risk is tradeable: indeed \( \bar{\lambda} = 0 \) and accordingly \( \bar{\alpha}_k^c = \bar{\alpha}_k^F = 1/2 \) and \( \bar{\alpha}_k^c = \bar{\alpha}_k^F = 0 \), for \( k = H, F \). When there is non-diversifiable income, there is a departure that depends on the covariances between labor-income risk and the excess returns.

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The set of conditions in (33) can be written in a more simple form as

$$\tilde{\alpha}_F = -\frac{s_\xi}{2} - \frac{1}{1 - \beta} \frac{\beta}{ \text{var}_t(\hat{r}_{F,t+1} + \hat{\Delta}q_{t+1} - \hat{r}_{H,t+1})} \text{cov}_t(\varepsilon_{l,t+1}, \hat{r}_{F,t+1}, \hat{r}_{F,t+1} | \hat{\Delta}q_{t+1} - \hat{r}_{H,t+1}) | \hat{\Delta}q_{t+1} - \hat{r}_{H,t+1}, \hat{r}_{F,t+1}, \hat{r}_{F,t+1} | \hat{\Delta}q_{t+1} - \hat{r}_{H,t+1})},$$  \hspace{1cm} (35)$$

$$\tilde{\alpha}_H + \tilde{\alpha}_F = 1 - \frac{s_\xi}{2} \frac{1}{1 - \beta} \frac{\beta}{ \text{var}_t(\hat{r}_{H,t+1} + \hat{\Delta}q_{t+1} - \hat{r}_{H,t+1})} \text{cov}_t(\varepsilon_{l,t+1}, \hat{r}_{H,t+1}, \hat{r}_{H,t+1} | \hat{\Delta}q_{t+1} - \hat{r}_{H,t+1}) | \hat{\Delta}q_{t+1} - \hat{r}_{H,t+1}, \hat{r}_{H,t+1}, \hat{r}_{H,t+1} | \hat{\Delta}q_{t+1} - \hat{r}_{H,t+1})},$$  \hspace{1cm} (36)$$

$$\tilde{\alpha}_F = \frac{s_\xi}{2} \frac{1}{1 - \beta} \frac{\beta}{ \text{var}_t(\hat{r}_{F,t+1} + \hat{\Delta}q_{t+1} - \hat{r}_{F,t+1})} \text{cov}_t(\varepsilon_{l,t+1}, \hat{r}_{F,t+1}, \hat{r}_{F,t+1} | \hat{\Delta}q_{t+1} - \hat{r}_{F,t+1}) | \hat{\Delta}q_{t+1} - \hat{r}_{F,t+1}, \hat{r}_{F,t+1}, \hat{r}_{F,t+1} | \hat{\Delta}q_{t+1} - \hat{r}_{F,t+1})},$$  \hspace{1cm} (37)$$

with variances and covariances which are conditional on selected excess returns, where \( \text{exr}^{ib}, \text{exr}^{ie} \) and \( \text{exr}^{de} \) denote the excess returns on international bonds, international equity and domestic equity, respectively, defined as

$$\text{exr}^{ib}_{t+1} = \hat{r}_{F,t+1} + \hat{\Delta}q_{t+1} - \hat{r}_{H,t+1},$$  \hspace{1cm} (38)$$

$$\text{exr}^{ie}_{t+1} = \hat{r}_{F,t+1} + \hat{\Delta}q_{t+1} - \hat{r}_{H,t+1},$$  \hspace{1cm} (39)$$

$$\text{exr}^{de}_{t+1} = \hat{r}_{F,t+1} - \hat{r}_{H,t+1}. $$  \hspace{1cm} (40)$$

Using (35) to (37) together with (10), we are able to determine the split of wealth across the different assets. We can start our discussion with equation (37) which determines whether there will be home bias in equity holdings. This happens when, conditional on the other excess returns, the excess return on international equity covaries positively with the surprises in the cross-country differential in the growth of non-diversifiable labor income. Suppose that the domestic agent receives a bad shock regarding the domestic non-diversifiable labor income, then it will hold a larger share of wealth in domestic equity when, under these circumstances, the return on the domestic equity is higher relative to foreign equity. The portfolio decision depends on whether domestic equity is a good hedge or not with respect to labor-income risk, relative to foreign equity.

Equation (36) instead determines the proportion invested in the overall equity market relative to the bond market. When \( s_\xi = 0 \), the agent would like to invest all its wealth in equities, as we previously discussed. Instead when \( s_\xi \neq 0 \) and \( \varepsilon_{l,t+1} \) covaries positively with the excess return of domestic equity over domestic bonds, then the home agent will also take an overall positive position in the bond markets. It will hold a positive share of wealth in bonds whenever in the face of a bad shock to labor income, the return on the domestic bond relative to equity pays better, i.e. when bonds are, in relative terms, a good hedge with respect to labor-income shocks.

Finally, equation (35) describes the position taken in the foreign bond market and as a consequence in the domestic bond market, given the overall position described in (36). When the covariance between \( \varepsilon_{l,t+1} \) and the excess return of the foreign bond with respect to the domestic bond is positive, the foreign bond does not pay well when needed. In this case the domestic agent would like to take a short position in the foreign bond market. Note that this does not necessarily imply a long position in the domestic bond market. Indeed, the overall position depends on equation (36), as previously discussed.

Although simpler versions of (35) and (37) have been treated in the literature, to our knowledge this is the first complete treatment in a dynamic general equilibrium model with incomplete markets.
Simple cases are nested in the above framework. When there is only trading in equities, the relevant condition for determining the portfolio allocation boils down to

\[ \hat{\alpha}_F^e = \frac{1}{2} - \frac{s}{2} \frac{\beta}{1 - \beta} \frac{\text{cov}(\bar{\varepsilon}_{l,t+1}, \bar{r}_{F,t+1} + \Delta \bar{q}_{t+1} - \bar{r}_{H,t+1})}{\text{var}(\bar{r}_{F,t+1}) + \Delta \bar{q}_{t+1} - \bar{r}_{H,t+1}}. \] (41)

The domestic agent holds a smaller share of its wealth in the foreign equity market when the excess return of the foreign stock with respect to the domestic one covaries positively with the surprises in the domestic-versus-foreign non-diversifiable labor incomes. Note that now these covariances are no longer conditional on the other excess returns, but they are only conditional on time \( t \). There is home bias in equity holdings when home equity is a good hedge with respect to non-diversifiable income risk.

A popular argument for international diversification being worse is the neoclassical model of Baxter and Jermann (1997) in which labor income and dividends are correlated. In this case, the above covariance would be negative implying even larger holdings of foreign assets. Heatcote and Gourinchas (2008) instead show a case in which the correlation can become positive when there is capital accumulation, or decumulation, and home bias in consumption preferences. Coeurdacier and Gourinchas (2008) discuss several theoretical cases that can rationalize a positive covariance and then imply home-bias in equity.\(^{17}\)

Our theoretical model shows that the covariance can be positive or negative depending on the strength of the mark-up shocks. Conditional on a positive mark-up shocks, profits and dividends increase whereas labor income decreases which might imply a negative correlation between labor-income risk and the return on domestic equity. In this case, the domestic agent would hold more of its own asset to hedge with respect to labor-income risk. However, at the end, whichever channel is relevant is a question of empirical evaluation of the covariances involved in (41).

Coeurdacier and Gourinchas (2008) consider a model in which agents can also trade in bonds, but in which shocks have a certain property of symmetry such that each country bond position is balanced to zero, so that a long position in one bond corresponds necessarily to a short position in the other. In our model, this is nested by requiring that \( \hat{\alpha}_H + \hat{\alpha}_F = 0 \). It follows that the relevant conditions for determining the portfolio allocations are

\[ \hat{\alpha}_F = -\frac{s}{2} \frac{\beta}{1 - \beta} \frac{\text{cov}(\bar{\varepsilon}_{l,t+1}, \bar{r}_{F,t+1} + \Delta \bar{q}_{t+1} - \bar{r}_{H,t+1})}{\text{var}(\bar{r}_{F,t+1}) + \Delta \bar{q}_{t+1} - \bar{r}_{H,t+1}} \] (42)

\[ \hat{\alpha}_F^e = \frac{1}{2} - \frac{s}{2} \frac{\beta}{1 - \beta} \frac{\text{cov}(\bar{\varepsilon}_{l,t+1}, \bar{r}_{F,t+1} + \Delta \bar{q}_{t+1} - \bar{r}_{H,t+1})}{\text{var}(\bar{r}_{F,t+1}) + \Delta \bar{q}_{t+1} - \bar{r}_{H,t+1}}. \] (43)

The above two conditions are similar to the ones discussed in Coeurdacier and Gourinchas (2008) under their log-utility case. However, there are two important differences: 1) variances and covariances are conditional on the previous-period information while in their model they are unconditional;\(^{18}\) 2) \( \bar{\varepsilon}_{l,t+1} \) captures the surprises in the present discounted value of cross-country labor income and not just its current value.\(^{19}\)

We can easily generalize the model to recover another hedging motif – the one with respect to real-exchange-rate risk – if we were to assume non-log utility, as in the analysis of Adler and Dumas (1983), Coeurdacier and Gourinchas (2008) and van Wincoop and Warnock (2006). There are two important reasons for why we do not follow this strategy: on one side estimates

\(^{17}\)See also Coeurdacier et al. (2007) and Engel and Matsumoto (2006).

\(^{18}\)Conditional and unconditional moments in general coincides with white-noise processes.

\(^{19}\)In the next section, we discuss how our empirical counterpart differs from Coeurdacier and Gourinchas (2008).
of the intertemporal elasticity of substitution are not far from the unitary value, as discussed in Vissing-Jørgensen and Attanasio (2003); on the other side, if we were to increase the risk-aversion coefficient to enhance the importance of hedging the real-exchange-rate risk we would lower the intertemporal elasticity of substitution, and amplify in a counterfactual way the volatility of the risk-free rate. We follow a different strategy.

5 Portfolio Choices under Model Uncertainty

When agents face model uncertainty, they might use distorted probability distributions to form “subjective” conditional expectations. The latter are linked to the “objective” conditional expectations through the martingale increment \( g_t \), as shown in (4). In particular we can write conditions (20)–(23) as

\[
E_t \left[ \left( m_{t+1}g_{t+1} - m_t^*g_t^* \frac{q_t}{q_{t+1}} \right) \right] = 0,
\]

\[
E_t \left[ \left( m_{t+1}g_{t+1} - m_t^*g_t^* \frac{q_t}{q_t} \right) \frac{q_{t+1}}{q_t} \right] = 0,
\]

\[
E_t \left[ \left( m_{t+1}g_{t+1} - m_t^*g_t^* \frac{q_t}{q_{t+1}} \right) r_{e,t+1}^H \right] = 0,
\]

\[
E_t \left[ \left( m_{t+1}g_{t+1} - m_t^*g_t^* \frac{q_t}{q_{t+1}} \right) r_{e,t+1}^F \right] = 0.
\]

This set of equations implies the three restrictions needed to determine the portfolio allocation. In a second-order approximation they read as

\[
E_t \left[ (\Delta \hat{c}^R_{t+1} - \Delta \hat{q}_{t+1} - \hat{g}^R_{t+1})(r_{F,t+1}^e + \Delta \hat{q}_{t+1} - \hat{r}_{e,H,t+1}) \right] = 0,
\]

\[
E_t \left[ (\Delta \hat{c}^R_{t+1} - \Delta \hat{q}_{t+1} - \hat{g}^R_{t+1})(\hat{r}_{e,H,t+1} - \hat{r}_{e,H,t+1}) \right] = 0,
\]

\[
E_t \left[ (\Delta \hat{c}^R_{t+1} - \Delta \hat{q}_{t+1} - \hat{g}^R_{t+1})(\hat{r}_{e,F,t+1}^e + \Delta \hat{q}_{t+1} - \hat{r}_{e,F,t+1}^e) \right] = 0.
\]

The optimal portfolio allocation is going to be affected by the factor \( g^R_{t+1} \) which measures the cross-country difference between the subjective and objective probability distributions. So far, we have put only a minimal structure on \( g_{t+1} \) and \( g^*_t \). We now enrich our set of assumptions by endogeneizing the way beliefs are distorted. Specifically, we consider the sophisticated agents of the robust-control theory of Hansen and Sargent (2005, 2007). Decisionmakers fear model misspecification and surround the objective probability distribution with nearby distributions which are absolutely continuous as in the framework discussed in Section 1. In particular they make their choices with the fear of the worst possible distribution. This distribution is chosen by an ‘evil’ agent to minimize the utility of the decisionmaker under an entropy constraint of the form similar to (5), that measures the distance between the distorted and the objective beliefs. In a more formal way, \( \{g_t\} \) is chosen to minimize

\[
E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} G_t \ln C_t \right\}
\]

under the constraint

\[
E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} G_t \beta E_t (g_{t+1} \ln g_{t+1}) \right\} \leq k,
\]

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and the restrictions given by the martingale assumption on $G_t$,

$$G_{t+1} = g_{t+1} G_t,$$

(47)

$$E_t g_{t+1} = 1,$$

(48)

where $k$ measures an upper-bound limit on the entropy constraint, meaning that the distance between the distorted and the objective beliefs is bounded.

Hansen and Sargent (2005) propose an alternative formulation of this problem in which the entropy constraint is added to the utility of the agent to form a modified objective function

$$E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} G_t \ln c_t \right\} + \theta E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} G_t \beta E_t (g_{t+1} \ln g_{t+1}) \right\},$$

(49)

where $\theta > 0$. The problem of the ‘evil’ agent becomes that of choosing the path $\{g_t\}$ to minimize (49) under the constraints (47) and (48). Higher values of $\theta$ implies less fear of model misspecification since the ‘evil’ agent is penalized more by raising entropy when it minimizes the utility of the decisionmaker. When $\theta$ goes to infinity, the optimal choice of the ‘evil’ agent is to set $g_{t+1} = 1$ at all times, so that there is no model uncertainty. The problem of the decisionmaker is instead that of maximizing (49) taking into account the minimizing action of the evil agent. As discussed in the literature, among others by Barillas et al (2008), it can be shown that the solution of this max-min optimization problem implies a transformation of the original utility function (49) into a non-expected recursive utility function of the form

$$v_t = c_t^{1-\beta} \left( \left[ E_t (v_{t+1})^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \right)^{\beta},$$

(50)

This risk-adjusted utility function coincides with that of the preferences described in Kreps and Porteus (1978) and Epstein and Zin (1989) when $\gamma$, the risk-aversion coefficient of the Kreps-Porteus-Epstein-Zin preferences, is related to $\theta$ through the following equation

$$\theta = \frac{1}{(1-\beta)(\gamma - 1)},$$

(51)

The two frameworks coincide in terms of their implications for the equilibrium allocation, but the assumptions under the two models are different and in particular the parameters $\theta$ and $\gamma$ have different interpretations.\textsuperscript{20} As discussed in Barillas et al. (2008), $\gamma$ represents the risk-aversion coefficient whereas $\theta$ is a measure of the doubts that the decisionmaker has with respect to the model probability distribution. While $\gamma$ can indeed be calibrated as a parameter capturing the risk-aversion coefficient, $\theta$ can be related to the detection error probability: the more the doubts, the lower the $\theta$, the lower the detection error probability.

At this point, the reader might wonder why we need to assume near rationality instead of just assuming Kreps-Porteus-Epstein-Zin preferences. Beside the appeal of describing agents which in the real world are not so sure about the structure of the model, the framework with model uncertainty is also observationally equivalent to a model in which there are preference shocks, which Pavlova and Rigobon (2007) have shown to be successful in explaining several features of asset prices in open economies. The Hansen-Sargent sophisticated agent seeking robust decisions, indeed, is just one of the possible classes of agents acting in a framework characterized by model uncertainty and distorted beliefs, as defined in Section 2. In our future research agenda we

\textsuperscript{20}The two models are observationally equivalent only with log utility.
plan to investigate whether this more general environment can help to describe other puzzles in international finance.

We can now solve for $g_{t+1}$ by noting that the real stochastic discount factor associated with the preferences (50) is

$$m_{t+1} = \beta \frac{c_t}{c_{t+1}} \left( V_t^{1-\gamma} \right)$$

so that $g_{t+1}$ is given by

$$g_{t+1} = \left( \frac{v_t^{1-\gamma}}{E_t(V_{t+1}^{1-\gamma})} \right).$$

Notice that in (50) we can scale continuation values by consumption to get

$$\frac{v_t}{c_t} = \left[ E_t \left( \frac{v_{t+1} c_{t+1}}{c_t} \right)^{1-\gamma} \right]^{\frac{\beta}{\gamma}},$$

showing that $g_{t+1}$ can be related to current and future consumption path. Indeed, in a first-order approximation, which suffices to evaluate (44)–(46), we can write

$$\hat{g}_{t+1} = - (\gamma - 1) \sum_{j=0}^{\infty} \beta^j \left[ E_t \Delta \hat{c}_{t+1+j} - E_t \Delta \hat{c}_{t+1+j} \right],$$

where $\hat{g}_{t+1}$ increases when the agent receives bad news with respect to the consumption-growth profile.\(^{21}\) Recall that $g(s_{t+1}|s^t)$ is equivalent to the ratio between the subjective and objective probabilities, $\hat{\pi}(s_{t+1}|s^t)/\pi(s_{t+1}|s^t)$. Higher values of $g(s_{t+1}|s^t)$ implies that the agent is putting more probability on those states of nature where there are bad news on the consumption-growth profile. When $g(s_{t+1}|s^t)$ increases, the stochastic discount factor increases, measuring the appetite for receiving additional wealth. In this case, the agent would like to hold assets that pay well, when indeed there are bad news on the consumption-growth profile.

The above derivations apply also to the foreign agent. Thereby, in the symmetric case of equal fear of misspecification ($\gamma = \gamma^*$) we can write that the relative difference across $g$ depends negatively on the surprises in the consumption-growth differential across countries:

$$\hat{g}_{t+1}^R = - (\gamma - 1) \sum_{j=0}^{\infty} \beta^j \left[ E_t \Delta \hat{c}_{t+1+j}^R - E_t \Delta \hat{c}_{t+1+j}^R \right].$$

One important thing to notice is that the first-order approximation of the model equilibrium conditions is not affected by the assumption that agents have distorted beliefs. Indeed $g_{t+1}$ and $g_{t+1}^*$ enter the Euler equations, but, up to first-order, their expected values are zero, since they are martingales. However, $g_{t+1}$ and $g_{t+1}^*$ enter indirectly the first-order approximation because they affect the coefficients of the approximation which depend on the steady-state portfolio allocation. As we have shown, indeed, the steady-state shares depend on the ratio of second-order moments.

It follows from the above that (29) and (30) still hold and can be used to write (52) as

$$\hat{g}_{t+1} = - (\gamma - 1) \frac{1 - \beta}{\beta s_c} \lambda_{\text{exr} t+1} - (\gamma - 1) \varepsilon_{q, t+1} - (\gamma - 1) \frac{s_c}{s_c} \varepsilon_{l, t+1},$$

\(^{21}\)Hansen et al. (2008) show how to derive $g_{t+1}$ in a closed-form solution including risk-premia terms, which, however, are not important in our approximation for computing the steady-state portfolio shares.
where we have defined the time-$t$ surprises in the real exchange rate growth as
\[ \varepsilon_{q,t+1} = \sum_{j=0}^{\infty} \beta^j [E_{t+1} \Delta \hat{q}_{t+1+j} - E_t \Delta \hat{q}_{t+1+j}] . \] (53)

Therefore, the left-hand side of the orthogonality conditions (44)–(46) can be written as
\[ (\Delta \hat{c}^R_{t+1} - \Delta \hat{q}_{t+1} - \hat{g}^R_{t+1}) = \gamma(1 - \beta) \bar{\lambda} \text{exr}_{t+1} + (\gamma - 1) \varepsilon_{q,t+1} + \gamma \frac{s_c}{s_c} \varepsilon_{l,t+1}, \]
from which it follows that (44)–(46) imply
\[ \bar{\lambda} = -s_c \frac{\beta}{1 - \beta} \frac{\Sigma_t^{-1} E_t (\text{exr}_{t+1} \cdot \varepsilon_{l,t+1}) - s_c (\gamma - 1) \frac{\beta}{1 - \beta} \Sigma_t^{-1} E_t (\text{exr}_{t+1} \cdot \varepsilon_{q,t+1})}{s_c} . \] (54)

Equation (54) determines the steady-state portfolio shares under fear of model misspecification. In particular, there is an additional term with respect to equation (33) which depends on the covariances between the excess returns and the surprises in the real exchange rate. When beliefs are not distorted (i.e. as $\theta \to \infty$) $\gamma$ is equal to 1 and then (54) coincides with (33).

In contrast with the rational-expectation case, therefore, model uncertainty and fear of misspecification imply a departure from full diversification, even when all income risk is tradeable, i.e. $s_c = 0$. In particular the second term on the right-hand-side of (54) captures the hedging motif with respect to the real exchange rate risk. The importance of this term depends on the parameter $\gamma$: the higher $\gamma$, the more important this component.

In particular, equation (54) shows that, on top of the first component already discussed in Section 4, there should be a bias with respect to domestic equity when, conditional on the other returns, domestic equity pays well relative to foreign equity when there are news about an appreciation of the real exchange rate. Bonds should be held when they pay better than equities when needed, and a higher proportion of foreign bonds when their return is negatively correlated with the surprises in the real exchange rate, conditioning on the other shocks.

The intuition is the following. Under model uncertainty agents might have a distorted probability distribution. In particular, when they make robust choices, this distortion comes from the fear of the worst-case scenario. The decisionmaker tends to give higher probability to the states of nature in which he/she gets bad news with respect to consumption growth. Bad news of domestic consumption growth relative to foreign consumption growth are captured by movements in the real exchange rate, in particular by an unexpected appreciation of the real exchange rate ($q$ falls).

There is an important parallel and a qualification to add at this point. The additional component in equation (54), which captures the hedge against real exchange rate risk, would be also present in a model with non-distorted beliefs and non-unitary risk aversion/or intertemporal elasticity of substitution. Condition (54) would be also valid in that model. However, there is an important difference. In a rational-expectation model, risk aversion is the reciprocal of the intertemporal elasticity of substitution so by rising risk aversion, to make the second component larger, the intertemporal elasticity of substitution is lowered, as we have already discussed. By lowering the intertemporal elasticity of substitution the volatility of the risk-free rate increases in a counterfactual way. With our preference specifications, the intertemporal elasticity of substitution is tied to one, which is also a value close to the empirical estimates, and the parameter $\gamma$ can increase giving more weight to the second component without affecting the volatility of
the risk-free rate. Moreover, what matters is not only the current real exchange rate risk but also the future values. As \( \beta \) gets close to one, only the long-risk remains relevant. Indeed in this case, \( \varepsilon_{q,t+1} \) becomes proportional to the revisions in the expectations regarding the long-run real exchange rate

\[
\varepsilon_{q,t+1} \cong E_{t+1}q_{\infty} - E_{t}q_{\infty}.
\]

We can get further insights by looking at the simple case in which only equities are traded. It can be shown that

\[
\alpha_F = \frac{1}{2} - 1 \frac{\beta}{2} \frac{1 - \beta}{\gamma} \frac{\text{cov}_t(q_{t+1}, \hat{r}_{F,t+1}^e + \hat{r}_{H,t+1}^e - \hat{r}_{H,t+1}^e)}{\text{var}_t(\hat{r}_{F,t+1}^e + \hat{r}_{H,t+1}^e)} - \frac{1}{2} \frac{\gamma - 1}{\gamma} \frac{\beta}{1 - \beta} \frac{\text{cov}_t(q_{t+1}, \hat{r}_{F,t+1}^e + \hat{r}_{H,t+1}^e - \hat{r}_{H,t+1}^e)}{\text{var}_t(\hat{r}_{F,t+1}^e + \hat{r}_{H,t+1}^e - \hat{r}_{H,t+1}^e)}. \tag{55}
\]

In this case, on top of equation (41), agents would like to hold more domestic assets if their return is high when the real exchange rate is expected to appreciate. It requires that \( \varepsilon_{q,t+1} \) covaries positively with the excess returns of foreign-versus-domestic equity, \( \hat{r}_{F,t+1}^e + \hat{r}_{H,t+1}^e - \hat{r}_{H,t+1}^e \). As the fear of model misspecification increases, then, this additional hedging motif matters more for determining home bias in international portfolio choices. In the more general case in which also bonds are traded, the above conditions still hold, although now variances and covariances are conditional on the other excess returns. In particular we can write the third line of equation (54) as

\[
\alpha_F = \frac{1}{2} - 1 \frac{\beta}{2} \frac{1 - \beta}{\gamma} \frac{\text{cov}_t(q_{t+1}, \hat{r}_{F,t+1}^e + \hat{r}_{H,t+1}^e - \hat{r}_{H,t+1}^e | \text{ex}r_{ib}^{t+1}, \text{ex}r_{de}^{t+1})}{\text{var}_t(\hat{r}_{F,t+1}^e + \hat{r}_{H,t+1}^e | \text{ex}r_{ib}^{t+1}, \text{ex}r_{de}^{t+1})} - \frac{1}{2} \frac{\gamma - 1}{\gamma} \frac{\beta}{1 - \beta} \frac{\text{cov}_t(q_{t+1}, \hat{r}_{F,t+1}^e + \hat{r}_{H,t+1}^e - \hat{r}_{H,t+1}^e | \text{ex}r_{ib}^{t+1}, \text{ex}r_{de}^{t+1})}{\text{var}_t(\hat{r}_{F,t+1}^e + \hat{r}_{H,t+1}^e - \hat{r}_{H,t+1}^e | \text{ex}r_{ib}^{t+1}, \text{ex}r_{de}^{t+1})}. \tag{56}
\]

## 6 Empirical evidence

One of the appealing features of the theoretical model above is that it derives clear implications about the second moments of variables that are directly observable. These implications can be therefore empirically tested without too many assumptions on the empirical counterparts of our theoretical variables.

### 6.1 Data

To evaluate the implications of equations (33) and (54), we collect and use quarterly data for the G7 Countries, over the sample 1980q1-2007q4. We consider the US as the Home country and the aggregation of the rest of the G7 countries as the Foreign country.\(^{23}\)

\(\text{\textsuperscript{22}}\)See also Piazzesi and Schneider (2006) on how preferences of this kind are able to match moments on the US term structure.

\(\text{\textsuperscript{23}}\)In particular, we use data on aggregate nominal compensation of employees, from the OECD Quarterly National Accounts (**OCOS02B, where ** is the two-letter country code), the Consumer Price Indexes from the IFS database (**IHS0..F), nominal returns on short-term treasury bills from the IFS database (**IHS0..C), nominal National Price and Gross Return indexes on the domestic stock market, from MSCI Barra (MS****L), in local currency, and bilateral nominal exchange rate vis-à-vis the USD, constructed using the domestic stock-price indexes in USD, from the MSCI Barra (MS****S). Moving from the monthly National Price and Gross Return indexes from MSCI database, we construct series for the quarterly nominal returns on equity (\(R_{eq}^t\) following Campbell (1999).
To build a measure of relative labor income in units of US dollars, the variable \( \xi_i^R - \hat{q}_t \), we proceed as follows. We compute nominal labor income in US dollars for the Foreign country in period \( t \) as:

\[
S_t W_t^* \bar{l}_t = \sum_i \omega_{i,t} S_t W_t \bar{l}_{i,t},
\]

in which \( W_t \bar{l}_{i,t} \) is the nominal aggregate compensation for employees of country \( i \), \( S_t \) is the bilateral nominal exchange rate between country \( i \) and the dollar (US dollars for one unit of local currency), and \( \omega_{i,t} \) is the actual time-\( t \) GDP-weight of country \( i \) relative to the aggregation of the G6 countries:

\[
\omega_{i,t} = \frac{GDP_{i,t}}{\sum_i GDP_{i,t}}.
\]

We compute relative labor income in units of US dollars as the log difference between nominal aggregate compensation in the US and that of the rest of the world:

\[
\hat{\xi}_t = \log \left( \frac{W_t \bar{l}_t}{S_t W_t^* \bar{l}_t} \right) = \log \left( \frac{W_t \bar{l}_t}{S_t \bar{P}_t} \right) = \log \left( S_t \bar{l}_t \right).
\]

Accordingly, we define and compute the real exchange rate between the US and the G6 countries as:

\[
\hat{q}_t = \log \left( \frac{S_t \bar{P}_t}{P_t} \right) = \log \left( \frac{\sum_i \omega_{i,t} S_t \bar{P}_{i,t}}{P_t} \right),
\]

in which \( P_t \) is the CPI in local currency for country \( i \) and \( P_t \) is the CPI in the US.

Given nominal quarterly returns on the stock market, defined by \( R^e_{i,t} \) for each country \( i \) and \( R^e_{i,t} \) for the US, and nominal quarterly returns on bonds, defined by \( R^b_{i,t} \) for each country \( i \) and \( R^b_t \) for the US, we can obtain the real returns as \( r_{i,t} \equiv R_{i,t}P_{i,t-1}/P_{i,t} \) and \( r^b_t \equiv R^b_{i,t}P_{i,t-1}/P_{i,t} \) for each country \( i \) and for the US, respectively. We construct the three excess returns of interest as

\[
exr^e_t = \hat{r}^e_{F,t} + \Delta \hat{q}_t - \hat{r}^e_{H,t} = \log \left( \frac{\sum_i \omega_{i,t} \hat{r}^e_{i,t} \hat{q}_{i,t-1} \hat{q}_{i,t}}{\hat{r}^e_t} \right),
\]

\[
exr^b_t = \hat{r}^b_{F,t} + \Delta \hat{q}_t - \hat{r}^b_{H,t} = \log \left( \frac{\sum_i \omega_{i,t} \hat{r}^b_{i,t} \hat{q}_{i,t-1} \hat{q}_{i,t}}{\hat{r}^b_t} \right),
\]

\[
exr^{de}_t = \hat{r}^{de}_{F,t} - \hat{r}^e_{H,t} = \log \left( \frac{\hat{r}^{de}_t}{\hat{r}^e_t} \right).
\]

Table 1 reports some summary statistics for the variables of interest. We report the average level \( \mu(\cdot) \) and the standard deviation \( \sigma(\cdot) \), both annualized and in percentage points, the serial correlation coefficient \( \rho(\cdot) \) and the correlation with the growth rate in relative labor income \( \rho(\cdot, \Delta \hat{\xi}^R - \Delta \hat{q}) \) and with the real exchange rate \( \rho(\cdot, \hat{q}) \). These simple correlations already suggest that domestic equity seems a poor hedge against labor income risk, relative to foreign stocks, while both domestic equity and domestic bonds seem somewhat useful in providing the right co-movement to hedge against real exchange rate fluctuations. In the next sections we will refine and articulate these results.

\[24\text{To check for robustness, we repeated the analysis using average GDP-weights as an alternative aggregation methodology, as in Coeurdacier and Gourinchas (2008), and using both aggregate and per-capita levels for the quantity variables. None of our results is significantly affected.}\]
In order to evaluate the optimal portfolio allocation, we need to calibrate the steady-state ratio of consumption-to-financial wealth, $s_c$. To this end, we use the average financial wealth-to-disposable income ratio for the US as computed by Bertaut (2002), and the average consumption-to-disposable income ratio for the US, computed using data on personal consumption of non-durable goods and personal disposable income. The former, on a quarterly frequency, amounts to about 20, while the latter to around .3: by using these numbers we get a calibrated consumption-to-wealth ratio $s_c = 3/20 = .015$. We calibrate the quarterly time discount factor following Barillas et al (2006): $\beta = .995$. Accordingly, and using the value of $s_c$ obtained above, we can derive the steady-state value of the labor income-to-financial wealth ratio by using $s_\xi = s_c - (1 - \beta)/\beta$, from which it follows that $s_\xi = .01$.

### 6.2 The statistical model

We define the following vector

$$ Y_t \equiv \begin{bmatrix} \Delta \hat{\xi}_t - \Delta \hat{q}_t \\ \Delta \hat{q}_t \\ \hat{r}_{F,t} + \Delta \hat{q}_t - \hat{r}_{H,t} \\ \hat{r}_{H,t} - \hat{r}_{H,t} \\ \hat{r}_{e,F,t} + \Delta \hat{q}_t - \hat{r}_{e,F,t} \\ \hat{r}_{e,H,t} - \hat{r}_{e,H,t} \\ X_t \end{bmatrix}, $$

(57)

and estimate the following VAR(1) model

$$ y_t = A y_{t-1} + e_t, $$

(58)

in which $y_t \equiv Y_t - \bar{Y}$ is the demeaned data-vector and $e_t$ is distributed as a multivariate normal with zero mean and variance-covariance matrix $\Omega$.\textsuperscript{25} In the vector $Y$, we also include a series of additional controls, collected into the vector $X$, which might be useful in describing the dynamic path of the variables of interest. In practice, $X$ includes the growth rate in the consumption differential, the growth rate in relative GDP, the slope of the US term structure, the international excess return on ten-year government bonds and the growth rate in the US trade balance.\textsuperscript{26}

We define $\iota_z$ as the vector that selects the element $z$ from the vector $y$. In particular, the vector

\textsuperscript{25}The length of the VAR is chosen optimally using the Schwarz's Bayesian Criterion for each estimation, and turns out to be always 1.

\textsuperscript{26}Gourinchas and Rey (2007) show that the net export growth rate is a useful predictor for portfolio returns at long horizons, while the other variables are among the forecasting variables commonly used for predicting asset returns and labor income. See also Campbell (1996).
of excess returns, in deviation from its conditional mean, can be written as

\[
\text{exr}_{t+1} - E_t\text{exr}_{t+1} = \begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 & \ldots & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & \ldots & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & \ldots & 0
\end{bmatrix} (y_{t+1} - E_t y_{t+1}) = \begin{bmatrix}
\ell'_{qh} \\
\ell'_{de} \\
\ell'_{le}
\end{bmatrix} e_{t+1} = \ell'_{exr} e_{t+1}.
\]

We can then use the statistical model to evaluate \(\varepsilon_{l,t+1}\) and \(\varepsilon_{q,t+1}\), as

\[
\varepsilon_{l,t+1} = \sum_{j=0}^{\infty} \beta^j [E_{t+1}(\Delta^R \hat{\xi}_{t+1+j} - \Delta^R \hat{q}_{t+1+j}) - E_t(\Delta^R \xi_{t+1+j} - \Delta^R q_{t+1+j})] = \ell'_l (I - \beta A)^{-1} e_{t+1}
\]

\[
\varepsilon_{q,t+1} = \sum_{j=0}^{\infty} \beta^j (E_{t+1} \Delta^R \hat{q}_{t+1+j} - E_t \Delta^R q_{t+1+j}) = \ell'_q (I - \beta A)^{-1} e_{t+1}.
\]

Finally, by setting \(H \equiv (I - \beta A)^{-1}\), we can evaluate the relevant time-\(t\) conditional covariance vectors and matrices as implied by our estimated statistical model:

\[
E_t \{ (\text{exr}_{t+1} - E_t \text{exr}_{t+1}) \cdot \varepsilon_{l,t+1} \} = \ell'_l \Omega H \ell_t
\]

\[
E_t \{ (\text{exr}_{t+1} - E_t \text{exr}_{t+1}) \cdot \varepsilon_{q,t+1} \} = \ell'_q \Omega H \ell_q
\]

\[
\Sigma_t \equiv E_t \{ (\text{exr}_{t+1} - E_t \text{exr}_{t+1}) (\text{exr}_{t+1} - E_t \text{exr}_{t+1})' \} = \ell'_{exr} \Omega_{exr}. \]

Using the above estimated model, we can evaluate the theoretical implications of our framework, and relate the results to existing literature. At this point, it is important to underline that a common procedure in the literature is to rely on static models, and to evaluate covariances and variances using unconditional distributions (see van Wincoop and Warnock, 2006, and Coeurdacier and Gourinchas, 2008) or equivalently by running regressions of the form

\[
\varepsilon_{l,t+1} = \kappa_l + \psi'_l \text{exr}_{t+1} + u_{l,t+1}, \quad (59)
\]

\[
\varepsilon_{q,t+1} = \kappa_q + \psi'_q \text{exr}_{t+1} + u_{q,t+1}, \quad (60)
\]

for given parameters \(\kappa_k, \psi_k\), for \(k = l, q\) and well-behaved residuals \(u\), where indeed OLS regression coefficients imply

\[
\psi_k = \Sigma^{-1} E(\text{exr}_{t+1} \cdot \varepsilon_{k,t+1}). \quad (61)
\]

It is important to notice, however, that in the context of a general dynamic model this procedure is appropriate only as long as \(y_t\) is a multivariate white-noise process.

### 6.3 The case of no model uncertainty

In the absence of model uncertainty, with log utility, the only possible reason for home bias in equity is hedging against non-diversifiable labor income risk. In particular, this depends on the positive covariance between the present discounted value of domestic-versus-foreign labor income and the excess return of foreign-versus-domestic equity. This relation has been emphasized by several studies without reaching a clear consensus. Baxter and Jermann (1997) show that when equity is the only asset that can be traded internationally, the presence of non-diversifiable income risk actually implies a foreign-equity bias. On the other hand, Bottazzi et al. (1996) and more recently Coeurdacier and Gourinchas (2008) and Julliard (2003) bring evidence supporting the view that hedging against labor-income risk can explain some degree of home-bias in equity holdings. Heathcote and Perri (2004) and Coeurdacier and Gourinchas (2008), moreover, discuss some theoretical examples that can produce the required co-movements to explain home-bias.
We analyze this interaction in the context of our dynamic model, starting with a simple case in which the asset menu available for international trade includes only equities (henceforth Asset Menu I). In this case the relevant equilibrium condition, described by equation (41), involves a covariance-to-variance ratio which is conditional on time-\(t\) information, but unconditional on the residual asset space, the latter being empty.

To evaluate the relevant covariance and derive the portfolio allocation, we first estimate our statistical model.\(^{27}\) Using the output of the VAR we construct the surprises in the path of relative labor income across countries. In our model, we need to evaluate covariances and variances conditional on previous-period information, but for comparisons with Coeurdacier and Gourinchas (2008) we also compute the unconditional moments. The unconditional ratios are obtained through straightforward OLS projection of the surprises in relative labor income, obtained from the VAR, on the excess returns of the assets available to trade (in this case just the excess return of foreign-versus-domestic equity). Following equation (59), we obtain:

\[
\varepsilon_{l,t+1} = -0.473 \cdot (\hat{r}_{F,t+1} + \Delta \hat{q}_{t+1} - \hat{r}_{H,t+1}) + u_{l,t+1},
\]

where the standard error is reported in parenthesis. As it is clear from the equation above, it turns out that the relevant covariance-to-variance ratio is negative, statistically significant and economically large, and it is therefore unable to produce home-bias in equity, but rather implies a foreign-equity bias. This result on the one hand supports Baxter and Jermann (1997), and on the other hand weakens the argument of Heathcote and Perri (2007).

Notice, however, that this procedure is consistent with our theoretical model only as long as the statistical evidence suggests that the process \(y_t\) is in fact a multivariate white noise. While this representation is close to our findings, in some of our VAR specifications with no auxiliary variables, this is certainly not the case when we include additional regressors to help predicting the future path of labor income and excess returns. In this case, it is appropriate to compute the covariance-to-variance ratio conditional on time \(t\) following equation (41). To this end, we use the output of the estimated VAR model and compute the relevant ratios directly, using

\[
\text{cov}_t(\varepsilon_{l,t+1}, r_{F,t+1} + \Delta \hat{q}_{t+1} - \hat{r}_{H,t+1}) = \iota^t_i \Omega^H \iota^t_l
\]
\[
\text{var}_t(\hat{r}_{F,t+1} + \Delta \hat{q}_{t+1} - \hat{r}_{H,t+1}) = \iota^t_i \Omega \iota^t_{ie}.
\]

Also the computation of the conditional covariance-to-variance ratio supports the results of Baxter and Jermann (1997), that the portfolio diversification puzzle is even worse than expected. When the only asset that can be traded internationally is equity, the relevant covariance-to-variance ratio is negative and large, and the implied position on the international equity market is consistent with holding over 62% of financial wealth in foreign equity, as shown in Table 2.

The important observation of Coeurdacier and Gourinchas (2008) about this result is that, once also riskless bonds are traded, variances and covariances should be computed conditional on the other asset returns, as also shown by equations (42)–(43). Their claim is that, with the

\(^{27}\)For what concerns the statistical model, as a robustness check, we estimated three alternative specifications. The first specification is the minimal requirement to describe the model economy and include only data on relative labor income \((\Delta \hat{z}_R - \Delta \hat{q})\) and the excess returns on foreign equity. The second and third specifications augment the first one by introducing data on the residual excess returns. Moreover, for each of the specifications above, we also varied the informational content of the data-vector by adding the real exchange rate, in changes, and the auxiliary regressors included in \(X\). In the text we report results for the extensive specification only, corresponding to equation (57), since results are robust to the other alternatives. The full set of results is available upon request.
Table 2: Model with equities only – Rational Expectations

<table>
<thead>
<tr>
<th>Conditional Covariance-variance Ratios</th>
<th>(\text{cov}(\epsilon_{l,t+1}, \hat{r}<em>{F,t+1} + \Delta \hat{q}</em>{t+1} - \hat{r}_{H,t+1}))</th>
<th>-0.519</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{var}(\hat{r}<em>{F,t+1} + \Delta \hat{q}</em>{t+1} - \hat{r}_{H,t+1}))</td>
<td></td>
<td>0.026</td>
</tr>
</tbody>
</table>

Note: \(\bar{\alpha}_{eF}\) denotes the share of wealth invested in foreign equity

appropriate conditioning, the previous result would be overturned, and their empirical findings support this claim.

We repeat the analysis of Coeurdacier and Gourinchas (2008) within our dynamic framework. Accordingly, the asset menu, in this case, includes both equities and bonds, and the latter are balanced to an overall zero-position (for short, Asset Menu II).

Table 3 contrasts our findings with theirs.\(^{28}\) In the second column we report the findings of Coeurdacier and Gourinchas (2008) which show that, conditioning on the residual excess return, there is a positive covariance between the excess return on foreign-versus-domestic equity and non-diversifiable labor-income risk, whereas the unconditional covariance is instead negative. Thereby, they conclude that the results in Baxter and Jermann (1997) are driven by their particular asset structure, and do not hold when bonds are included. In the third column, we report the results of our estimation, which show instead a negative (although insignificant) conditional covariance.

Table 3: Unconditional covariance-variance ratios

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(r_{F,t+1} + \Delta \hat{q}<em>{t+1} - r</em>{H,t+1})</td>
<td>0.374</td>
<td>-0.017</td>
</tr>
<tr>
<td>(0.047)</td>
<td>(0.068)</td>
<td></td>
</tr>
<tr>
<td>(r_{F,t+1} + \Delta \hat{q}<em>{t+1} - r</em>{H,t+1})</td>
<td>-2.101</td>
<td>-1.030</td>
</tr>
<tr>
<td>(0.083)</td>
<td>(0.087)</td>
<td></td>
</tr>
</tbody>
</table>

Note: \(\text{standard errors in parentheses. \* Dependent variable is } \hat{r}_{w,t+1} - E_{t+1} \hat{r}_{w,t+1}. \** Dependent variable is } \epsilon_{l,t+1}.\)

The difference between the two results can be explained by the different approach to measure labor-income risk. Following Campbell (1996), Coeurdacier and Gourinchas (2008) use the unexpected component of the (home relative to foreign) return-to-labor, constructed according to

\[
\hat{r}_{t+1}^{w} - E_{t} \hat{r}_{t+1}^{w} = \sum_{j=0}^{\infty} \rho^{j}(E_{t+1} - E_{t})(\Delta \hat{q}_{t+1+j} - \Delta \hat{q}_{t+1+j})
\]

\[
- \sum_{j=1}^{\infty} \rho^{j}(E_{t+1} - E_{t})(\hat{r}_{F,t+1+j}^{e} - \hat{r}_{H,t+1+j}^{e} - \hat{r}_{F,t+1+j}^{e}) = (\mathbf{u}' - \rho \lambda_{e} \mathbf{A})(\mathbf{I} - \rho \mathbf{A})^{-1} \mathbf{e}_{t+1}, \quad (62)
\]

in which \(\rho \equiv 1 - s_{c}\) is a constant of linearization that depends on the average consumption-to-wealth ratio. Defined as such, the return-to-labor is likely, by construction, to be positively related with the excess return on foreign-versus-domestic equity. It is worth mentioning that this measure of labor-income risk is not directly implied by their theoretical model.

\(^{28}\)Note that we have defined the excess returns as foreign-versus-domestic returns, the opposite of Coeurdacier and Gourinchas (2008). Accordingly, for comparison, in Table 3 we report their results multiplied by -1.
Table 4: Model with equities and balanced bonds – Rational Expectations

<table>
<thead>
<tr>
<th>Conditional Covariance-variance Ratios</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\text{cov}(\tilde{\varepsilon}<em>{l,t+1}, \text{exr}</em>{ie,t+1})}{\text{var}(\text{exr}_{ie,t+1})} )</td>
<td>0.022</td>
</tr>
<tr>
<td>( \frac{\text{cov}(\tilde{\varepsilon}<em>{l,t+1}, \text{exr}</em>{ib,t+1})}{\text{var}(\text{exr}_{ib,t+1})} )</td>
<td>-1.185</td>
</tr>
</tbody>
</table>

Optimal Portfolio Allocation

| \( \bar{\alpha}_{e} \) | 0.495 |
| \( \bar{\alpha}_{F} \) | 0.287 |
| \( \bar{\alpha}_{e} + \bar{\alpha}_{F} \) | 0.782 |

Note: \( \bar{\alpha}_{e} \) denotes the share of wealth invested in foreign equity; \( \bar{\alpha}_{F} \) denotes the share of wealth invested in foreign bonds; \( \bar{\alpha}_{e} + \bar{\alpha}_{F} \) measures the overall share of wealth invested in foreign assets.

In our framework, instead, the model-based measure of non-diversifiable labor risk becomes the revision in the present-discounted value of cross-country labor income \( \tilde{\varepsilon}_{l,t+1} \), as shown by equation (34). Using this definition, we find that domestic equity is not a good hedge, even if we condition on bond returns. It is worth noticing that our measure of labor-income risk is instead similar to that of Baxter and Jermann (1997) which coincides with the first term on the right-hand-side of (62). Therefore, we reinforce Baxter and Jermann’s (1997) results even when we condition on other excess returns.

To derive the equilibrium portfolio allocations implied by our theoretical model, we compute the relevant covariance-to-variance ratios conditioning them also on the information set available at time-\( t \), and report the results in Table 4: the covariance between labor-income risk and the excess return on equities becomes of the right sign, but it is quantitatively negligible, and it does not imply a substantial degree of home-bias. On this respect, therefore, our results again contrast with Coeurdacier and Gourinchas (2008). However, we share the finding that agents should go long in foreign bonds and short in domestic bonds, with the counterfactual implication that almost 80% of domestic wealth is allocated to foreign assets.

We now turn to the more general specification of our model, by relaxing the assumption that the bond position is balanced (Asset Menu III). The relevant conditions are now (35), (36), (37). In this more general case, we can evaluate the ability of the model to replicate other stylized facts that are receiving increasing attention by the empirical literature. Tille (2005) reports a detailed breakdown of the composition of US foreign assets and liability, and documents that the US is a large net creditor in equity instruments, while a net debtor in bond instruments.

In our model we note that the steady-state net-foreign asset position (as a share of steady-state domestic wealth), defined by NFA, is given by

\[
NFA = \bar{\alpha}_{e}^{*} + \bar{\alpha}_{F} - \bar{\alpha}_{H}^{*} - \bar{\alpha}_{e}^{*} H.
\]

Moreover the net-foreign asset position in equities is given by \( NFE = \bar{\alpha}_{e}^{*} - \bar{\alpha}_{e}^{*} H \) and in bonds by \( NFB = \bar{\alpha}_{F} - \bar{\alpha}_{H}^{*} \). We write them as

\[
NFE = \bar{\alpha}_{e}^{*} + \bar{\alpha}_{e}^{*} - 1 \quad \text{(63)}
\]

\[
NFB = \bar{\alpha}_{F} + \bar{\alpha}_{H} \quad \text{(64)}
\]

\(^{29}\) Indeed, the only difference between (34) and Baxter-Jermann measure is the discount parameter: while they use \( \rho = 1 - s_{c} \), we use the time discount factor \( \beta \).

\(^{30}\) It can be shown that in a symmetric steady state in which \( \tilde{A} = \tilde{S} \tilde{A}^{*} \), \( \bar{\alpha}_{H}^{*} = 1 - \bar{\alpha}_{e}^{*} H \) and \( \bar{\alpha}_{H} = -\bar{\alpha}_{H} \).
Table 5: General model with equities and bonds – Rational Expectations

<table>
<thead>
<tr>
<th>Conditional Covariance-variance Ratios</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\text{cov}(\alpha_F, \text{exr}<em>{t+1}^H \mid \text{exr}</em>{t+1}^H, \text{exr}<em>{t+1}^H)}{\text{var}(\text{exr}</em>{t+1}^H \mid \text{exr}<em>{t+1}^H, \text{exr}</em>{t+1}^H)} )</td>
<td>0.049</td>
</tr>
<tr>
<td>( \frac{\text{cov}(\alpha_F, \text{exr}<em>{t+1}^H \mid \text{exr}</em>{t+1}^H, \text{exr}<em>{t+1}^H)}{\text{var}(\text{exr}</em>{t+1}^H \mid \text{exr}<em>{t+1}^H, \text{exr}</em>{t+1}^H)} )</td>
<td>-1.199</td>
</tr>
<tr>
<td>( \frac{\text{cov}(\alpha_F, \text{exr}<em>{t+1}^H \mid \text{exr}</em>{t+1}^H, \text{exr}<em>{t+1}^H)}{\text{var}(\text{exr}</em>{t+1}^H \mid \text{exr}<em>{t+1}^H, \text{exr}</em>{t+1}^H)} )</td>
<td>0.065</td>
</tr>
</tbody>
</table>

Optimal Portfolio Allocation

| \( \bar{\alpha}_F \) | 0.488 |
| \( \bar{\alpha}_H \) | 0.291 |
| \( \bar{\alpha}_F + \bar{\alpha}_H \) | 0.779 |
| \( \bar{\alpha}_H + \bar{\alpha}_F \) | 0.984 |

Note: \( \bar{\alpha}_F \) denotes the share of wealth invested in foreign equity; \( \bar{\alpha}_F \) denotes the share of wealth invested in foreign bonds; \( \bar{\alpha}_H + \bar{\alpha}_F \) measures the overall share of wealth invested in foreign assets; \( \bar{\alpha}_H + \bar{\alpha}_F \) measures the overall share of wealth invested in equity.

Accordingly, to replicate the empirical result of Tille (2005), our framework should imply that \( \bar{\alpha}_F + \bar{\alpha}_H > 1 \), and, equivalently, \( \bar{\alpha}_F + \bar{\alpha}_H < 0 \).

In a related paper, Lane and Shambaugh (2007, 2008) compute the foreign currency exposure (\( FX \)) of a large set of countries, defined as the difference between the share of foreign assets denominated in foreign currency and the share of foreign liabilities denominated in foreign currency, scaled by the size of the respective balance-sheet positions. In addition, they also decompose such index along several dimensions, and provide a measure of the net foreign currency exposure restricted to debt instruments (\( FX_{DEBT} \)). Their main finding is that there is a lot of heterogeneity across countries and time, with developed countries tending to have a positive exposure overall and a negative one when limited to debt instruments. In the notation of our model, a proxy of the overall exposure of the domestic country is given by \( FX = \bar{\alpha}_F + \bar{\alpha}_F \) while a proxy of the foreign-debt exposure is given by \( FX_{DEBT} = \bar{\alpha}_F \). Accordingly, the implications of our framework are consistent with the findings of Lane and Shambaugh (2008) if it turns out that \( \bar{\alpha}_F + \bar{\alpha}_F > 0 \) and \( \bar{\alpha}_F < 0 \).

To evaluate the implications of our general framework along these dimensions, we compute the time-\( t \) conditional covariance-to-variance ratios that are needed to derive the optimal portfolio allocation, and report the results in Table 5. Allowing for a non-zero position in the international bond market does not change the result of quasi-full international portfolio diversification. Indeed, empirical co-movements imply that domestic investors allocate more than three quarters of their wealth in foreign assets (\( \bar{\alpha}_F + \bar{\alpha}_F = 0.779 \)). Moreover, the co-movement of labor-income risk with the domestic equity premium implies evidence – although weak – of an overall long position in the international bond market (\( \bar{\alpha}_H + \bar{\alpha}_F < 1 \)), contrary to what documented by Tille (2005). Note that this result further exacerbates the inability of labor-income risk alone to support home-bias in equity: indeed, even though less than half of the steady-state wealth is allocated to foreign equities, the share allocated to domestic ones is also smaller than 50%, as can be seen by taking out \( \bar{\alpha}_F \) from the last line of Table 5. Finally, the long position on foreign bonds seems at odds with the empirical facts documented by Lane and Shambaugh (2008).

We will show in the next section that our model with distorted beliefs is more successful in
6.4 Real exchange rate risk and model uncertainty

In the above section we showed that there is no support for the view that domestic equity is a good hedge against non-diversifiable labor-income risk to explain the home-bias in US equity holdings. We now move to analyze the portfolio implications of model uncertainty, where we have shown that the fear of model misspecifications translates into real exchange rate risk that needs to be hedged, even with log-utility.

The role of hedging against real exchange rate fluctuations – as an explanation for the home-bias puzzle – which arises also in a rational-expectation model with non log-utility, has been recently questioned by van Wincoop and Warnock (2006) and Coeurdacier and Gourinchas (2008). Their main argument is based on the evidence that the covariance between real exchange rate changes and the excess return on foreign-versus-domestic equity becomes negligible once this covariance is taken conditional on other returns, like the excess return on riskless bonds.

The results of a simple OLS regression between real exchange rate changes and the vector of excess returns

$$\Delta \hat{q}_{t+1} = \kappa_q + \psi_{\text{exr}} t+1 + u_{q,t+1}, \quad (65)$$

are reported in Table 6. While the loading of the excess returns on foreign equity is significant and positive if equity is the only tradeable asset, once the vector of excess returns is augmented to include also the excess return on foreign-versus-domestic bonds, the covariance-to-variance ratio between the real exchange rate and the excess return on equity becomes negligible.

Table 6: Loadings of excess returns on real exchange rate depreciations

<table>
<thead>
<tr>
<th>loadings of:</th>
<th>equity only</th>
<th>equity and bonds</th>
<th>equity and bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{F,t+1} + \Delta q_{t+1} - r_{H,t+1}$</td>
<td>0.364 0.022</td>
<td>-0.022</td>
<td></td>
</tr>
<tr>
<td>(0.073) (0.068)</td>
<td>(0.071)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_{F,t+1} - r_{H,t+1}$</td>
<td>- 0.771</td>
<td>0.799</td>
<td></td>
</tr>
<tr>
<td>- (0.087)</td>
<td>(0.087)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_{H,t+1}$</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>(0.049)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: standard errors in parentheses. Dependent variable is $\Delta \hat{q}_{t+1}$.

In a rational-expectation model such small covariances (provided they are of the right sign) would require an unreasonably large degree of risk aversion to justify the hedging role of domestic equities, which would then open room for other puzzles, like the already mentioned risk-free rate puzzle.

Instead, our model with distorted beliefs provides two additional features which give a new role to real exchange rate risk: on the one hand, what matters is not only the current real-exchange-rate risk but also the revisions in the entire future expected path of the real exchange rate. What is relevant, therefore, is not so much the role of equity to hedge against short-run exchange rate risk, but rather its hedging properties against long-run fluctuations. On the other hand, for a given positive covariance between $\varepsilon_{q,t+1}$ and the excess return on equity, a stronger fear of model misspecifications translates into larger home biases, without requiring implausible coefficients of the intertemporal elasticity of substitution and therefore without falling in the risk-free rate puzzle. In what follows, we provide an empirical evaluation of these two additional features.
6.4.1 Short-run versus long-run risk

First, we study whether shifting from a short-run to a long-run perspective affects the hedging properties of equity with respect to real exchange rate risk. To this purpose, note that equation (53) can be written in terms of levels instead of growth rates:

\[ \varepsilon_{q,t+1} \equiv \sum_{j=0}^{\infty} \beta^j [E_{t+1} \Delta \hat{q}_{t+1+j} - E_t \Delta \hat{q}_{t+1+j}] = (1 - \beta) \sum_{j=0}^{\infty} \beta^j [E_{t+1} \hat{q}_{t+1+j} - E_t \hat{q}_{t+1+j}] . \] (66)

By looking at different terms in the summation above, we can investigate the co-movement between asset returns and surprises in the real exchange rate path, at different time-horizons. In particular, we can evaluate whether the hedging properties of equity and bonds change when the risk to be hedged is farther away in the future, as opposed to very soon.

Indeed, given our estimated model (58) and given the vector \( \iota_q \) which selects the depreciation rate from the vector \( y \), we can write the time-\( t+1 \) news about the real exchange rate \( k \)-periods ahead as

\[ (E_{t+1} - E_t) \hat{q}_{t+1+k} = \iota'_q \sum_{j=0}^{k} (E_{t+1} - E_t) y_{t+1+j} = \iota'_q \sum_{j=0}^{k} A^j e_{t+1} = \iota'_q (I - A)^{-1} (I - A^{k+1}) e_{t+1}, \]

and the news about the long-run component as

\[ \Delta E_{t+1} \hat{q}_\infty \equiv (E_{t+1} - E_t) \hat{q}_\infty = \iota'_q (I - A)^{-1} e_{t+1}, \]

in which \( \Delta E_{t+1} \) denotes the time-\( t+1 \) revisions in conditional expectations.

For each asset structure, we can use the above equation to evaluate the covariance-to-variance ratios with respect to all excess returns of interest, conditional on time-\( t \) information and on the residual asset space.

Indeed, for each time-horizon \( k = 0, 1, 2, \ldots \) we get:

\[ \Sigma_t^{-1} E_t ( \Delta E_{t+1} \hat{q}_{t+1+k} \cdot \text{exr}_{t+1}) = (\iota'_{\text{ext}} \Omega_{\text{ext}})^{-1} \iota'_q (I - A)^{-1} (I - A^{k+1}) \Omega_{\text{ext}}. \] (67)

Figure 1: The covariance-to-variance ratio between \( \Delta E_{t+1} \hat{q}_{t+1+k} \) and \( \text{exr}_{t+1} \), for increasing \( k \) (horizontal axis). Asset Menu I: equities only. Asset Menu II: equities and balanced bonds. Asset Menu III: general model with equities and bonds.

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Therefore, Figure 1 plots the covariance-to-variance ratios of the news in the real exchange rate path with the excess return on foreign-versus-domestic equity, against the time-horizon \( k \), under the three specifications: I) the model in which international trade is restricted to equities, II) the model in which there is trading in equities and bonds, but the latter are balanced; III) the general model with trading in equities and bonds. Figure 2 does the same for the excess return on foreign-versus-domestic bonds, for the two specifications including bonds (II and III). The first point (for \( k = 0 \)) in each plot corresponds to the covariance-to-variance ratio of a static model, in which only the short-run risk matters. Moving from the left to the right panel of Figure 1, the first point drops from .39 to virtually zero, implying that the hedging power of equity against real exchange rate risk fades away, when we condition on other excess returns and in particular on bonds. This is the core of the results in van Wincoop and Warnock (2006) and Coeurdacier and Gourinchas (2008).

However, we note that as we look at longer horizons the hedging properties of equity sharply increases, even when we condition on other excess returns. On the contrary, Figure 2 shows that the hedging properties of bonds marginally weaken. We view this evidence as suggesting that domestic equity can have a relatively more important role in hedging the real exchange rate risk at longer horizons so to explain the international home-bias puzzle.\(^{31}\)

### 6.4.2 The role of model uncertainty

Equation (54) shows that when agents have distorted beliefs and log-utility, the equilibrium international portfolio allocation is made of two components: a first one driven by the desire to insure against labor-income risk, which is common to the case of undistorted beliefs, and a second component instead directly driven by model uncertainty and related to hedging real-exchange-rate risk, whose weight depends on the extent to which agents fear model misspecifications (captured by \( \gamma \)).

Table 7 studies the implication when the only asset traded internationally is equity (Asset Menu I). The first two rows show that the conditional covariance between the surprises in the real exchange rate and the international excess return on equity is positive and economically

\(^{31}\)A recent literature documents the quantitatively substantial implications of long-run risk for asset valuation, in the context of non-expected utility frameworks. See, among others, Hansen et al. (2008), who also provide an interpretation related to model uncertainty.
Table 7: Model with equities only – Distorted Beliefs

<table>
<thead>
<tr>
<th>Covariance-variance Ratios</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\text{cov}(\epsilon_{l,t+1}, exr_{t+1})}{\text{var}(exr_{t+1})}$</td>
<td>-0.519</td>
<td>$\frac{\text{cov}(\epsilon_{q,t+1}, exr_{t+1})}{\text{var}(exr_{t+1})}$</td>
<td>0.490</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Optimal Portfolio Allocation**

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\bar{\alpha}_{F}$</th>
<th>$\bar{\alpha}_{F}^{e}$</th>
<th>$\bar{\alpha}_{F}$, second component</th>
<th>$\bar{\alpha}_{F}$, total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.626</td>
<td>0.626</td>
<td>0.626</td>
<td>0.626</td>
</tr>
<tr>
<td>2</td>
<td>0.500</td>
<td>0.318</td>
<td>0.209</td>
<td>0.173</td>
</tr>
<tr>
<td>5</td>
<td>0.500</td>
<td>0.318</td>
<td>0.209</td>
<td>0.173</td>
</tr>
<tr>
<td>10</td>
<td>0.500</td>
<td>0.318</td>
<td>0.209</td>
<td>0.173</td>
</tr>
<tr>
<td>20</td>
<td>0.500</td>
<td>0.318</td>
<td>0.209</td>
<td>0.173</td>
</tr>
</tbody>
</table>

Note: $\bar{\alpha}_{F}$ denotes the share of wealth invested in foreign equity; "first component" refers to hedging labor-income risk; "second component" refers to hedging real-exchange-rate risk.

large, while the covariance with labor-income risk is negative and even larger. As a consequence, the first component is qualitatively consistent with a foreign-equity bias, as we found under undistorted beliefs, while the second component is consistent with a home-bias. For $\gamma = 1$ the optimal distortion in agents beliefs is zero, and we get the rational-expectations result. When we consider model uncertainty, instead, the picture changes substantially: the more the doubts about the true model specification (the higher is $\gamma$), the more the second component becomes relevant and the more the portfolio allocation is biased toward domestic equity. Indeed, a moderate fear of misspecification is able to overturn the effect of hedging labor-income risk and produce a good degree of home-bias (over 70% of wealth allocated in domestic equity). By augmenting the asset structure to include also trading in riskless bonds (Asset Menu II), we get further qualifications to previous results, as shown in Table 8. First, as in Coeurdacier and Gourinchas (2008), the presence of bonds provides a valuable hedge against real exchange rate risk, and as a consequence the conditional covariance-to-variance ratio with equity becomes smaller, inducing a smaller hedging role for equities. However, unlike Coeurdacier and Gourinchas (2008), this role is not ruled out altogether because of the concern about both long-run risk and model uncertainty. First, a moderate concern about model misspecification is still going to induce a substantial home bias in equity holdings (over 60% allocated to domestic equity). Second, the hedging role of domestic bonds against the real exchange rate implies that, by increasing the doubts about the true model specification, the position on the foreign-bond market turns progressively from long to short.

Accordingly, domestic portfolio choices are strongly biased towards domestic assets (up to 85% of wealth invested in local assets), and the country has a negative foreign currency exposure in debt instruments, consistently with the stylized facts documented by Lane and Sham-baugh (2008).

Finally, we analyze the case in which we do not impose the zero-balanced position in the international bond market (Asset Menu III). Table 9 shows that the role of domestic equity as a hedge against real exchange rate risk is still twice as important as its role to hedge against labor-income risk, and is able to imply a substantial degree of home bias. In this respect, a

---

Piazzesi and Schneider (2006), in a non-expected utility framework, use $\gamma = 43$ to match the moments of the US term structure. See also Barillas et al. (2006) and Tallarini (2000) for using similar values to match the equity premium puzzle.
Table 8: Model with equities and balanced bonds – Distorted Beliefs

<table>
<thead>
<tr>
<th>Covariance-variance Ratios</th>
<th>0.022</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\text{cov}(\varepsilon_{l,t+1}, \text{exr}^{e}_{t+1}</td>
<td>\text{exr}^{b}<em>{t+1})}{\text{var}(\text{exr}^{e}</em>{t+1}</td>
</tr>
<tr>
<td>( \frac{\text{cov}(\varepsilon_{q,t+1}, \text{exr}^{e}_{t+1}</td>
<td>\text{exr}^{b}<em>{t+1})}{\text{var}(\text{exr}^{e}</em>{t+1}</td>
</tr>
<tr>
<td>( \frac{\text{cov}(\varepsilon_{l,t+1}, \text{exr}^{b}_{t+1}</td>
<td>\text{exr}^{e}<em>{t+1})}{\text{var}(\text{exr}^{b}</em>{t+1}</td>
</tr>
<tr>
<td>( \frac{\text{cov}(\varepsilon_{q,t+1}, \text{exr}^{b}_{t+1}</td>
<td>\text{exr}^{e}<em>{t+1})}{\text{var}(\text{exr}^{b}</em>{t+1}</td>
</tr>
</tbody>
</table>

### Optimal Portfolio Allocation

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( \alpha^{e}_{F} )</th>
<th>( \alpha^{F} )</th>
<th>( \alpha^{e}_{F} + \alpha^{F} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.495</td>
<td>0.287</td>
<td>0.782</td>
</tr>
<tr>
<td>2</td>
<td>0.495</td>
<td>0.287</td>
<td>0.782</td>
</tr>
<tr>
<td>5</td>
<td>0.495</td>
<td>0.287</td>
<td>0.782</td>
</tr>
<tr>
<td>10</td>
<td>0.495</td>
<td>0.287</td>
<td>0.782</td>
</tr>
<tr>
<td>20</td>
<td>0.495</td>
<td>0.287</td>
<td>0.782</td>
</tr>
</tbody>
</table>

**Note:** \( \alpha^{e}_{F} \) denotes the share of wealth invested in foreign equity; \( \alpha^{F} \) denotes the share of wealth invested in foreign bonds; \( \alpha^{e}_{F} + \alpha^{F} \) measures the overall share of wealth invested in foreign assets. “first component” refers to hedging labor-income risk; “second component” refers to hedging real-exchange-rate risk.

Further important implication of model uncertainty and the fear of misspecifications comes from the negative covariance between surprises in the real exchange rate path and the domestic equity premium. Indeed, a negative covariance implies that domestic equities pay relatively better than domestic bonds precisely when an unexpected appreciation of the real exchange rate occurs. In equilibrium, this implies that agents with distorted beliefs optimally take an overall short position in the international bond market in order to buy a higher share of domestic equity and hedge against the risk of a model misspecification. As \( \gamma \) increases, therefore, the overall share of wealth allocated in equity also increases, well above the unitary value. This result has important implications for the home-bias in equity, as shown by the last row of Table 9. The empirical co-movement of the domestic equity premium with real exchange rate risk indeed reinforces the role of domestic equity as a hedge against such risk, with respect to the previous specification, bringing the overall share of wealth optimally allocated to domestic equity to over 65 percentage points. Note that the result of an overall short position in the international bond market reconciles the model also with the stylized facts documented by Tille (2005), since now the model is able to produce a positive net position in equity instruments \((\alpha^{e}_{H} + \alpha^{F}_{e}) > 1\) coupled with a negative net position in debt instruments \((\alpha^{e}_{H} + \alpha^{F}_{F} < 0)\).

Overall, therefore, agents with distorted beliefs will tend to invest more in equity rather than in bonds, and within each class of assets they will tend to prefer domestic over foreign ones, because they provide a better hedge against both types of risks.

The role of model uncertainty in driving the equilibrium portfolio allocation is further explored in Figure 3, which plots the share of wealth invested in equity (domestic and overall) and foreign assets (bonds and overall) for increasing degrees of concern about model misspecifications.
Table 9: General model with equities and bonds – Distorted Beliefs

<table>
<thead>
<tr>
<th>Covariance-variance Ratios</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{cov}(\varepsilon_{t+1},\text{var}_{t+1}^F</td>
<td>\text{var}<em>{t+1}^F,\text{var}</em>{t+1}^e,\text{var}_{t+1}^b))</td>
<td>0.049</td>
<td>0.097</td>
<td>-1.999</td>
<td>0.771</td>
<td>0.065</td>
<td>-0.122</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Optimal Portfolio Allocation |  |  |  |  |  |  |  |
|-----------------------------|---|---|---|---|---|
| \(\alpha_P^\gamma\): first component | 0.488 | 0.488 | 0.488 | 0.488 | 0.488 |
| second component | 0.500 | 0.464 | 0.442 | 0.435 | 0.432 |
| total | 0.488 | 0.452 | 0.430 | 0.423 | 0.420 |
| \(\bar{\alpha}_P\): first component | 0.291 | 0.291 | 0.291 | 0.291 | 0.291 |
| second component | 0.000 | -0.286 | -0.458 | -0.515 | -0.544 |
| total | 0.291 | 0.004 | -0.168 | -0.225 | -0.253 |
| \(\alpha_P^\gamma + \bar{\alpha}_P\): total | 0.779 | 0.456 | 0.262 | 0.198 | 0.167 |
| \(\alpha_H^\gamma + \bar{\alpha}_P\): first component | 0.984 | 0.984 | 0.984 | 0.984 | 0.984 |
| second component | 1.000 | 1.045 | 1.073 | 1.082 | 1.086 |
| total | 0.984 | 1.030 | 1.057 | 1.066 | 1.071 |
| \(\alpha_H^\gamma\): total | 0.496 | 0.578 | 0.627 | 0.643 | 0.651 |

Note: \(\alpha_P^\gamma\) denotes the share of wealth invested in foreign equity; \(\bar{\alpha}_P\) denotes the share of wealth invested in foreign bonds; \(\alpha_F^\gamma + \bar{\alpha}_P\) measures the overall share of wealth invested in foreign assets; \(\alpha_H^\gamma + \bar{\alpha}_P\) measures the overall share of wealth invested in equity; \(\alpha_F^\gamma\) denotes the share of wealth invested in domestic equity. “first component” refers to hedging labor-income risk; “second component” refers to hedging real-exchange-rate risk.

Figure 3 synthetizes the main findings of this section. First, in the undistorted rational-expectations model, hedging against non-diversifiable labor-income risk is not sufficient to imply a disproportionate share of wealth optimally allocated to domestic equity, unlike in Coeurdacier and Gourinchas (2008), and instead implies a foreign-equity bias when international trade is restricted to equities only, as in Baxter and Jermann (1997). The implied foreign-currency exposure in debt instruments is, in this case, positive, as in Coeurdacier and Gourinchas (2008), and instead implies a foreign-equity bias when international trade is restricted to equity only. Moreover, the overall share allocated to equities, both domestic and foreign,
Figure 3: Optimal portfolio allocation: the effect of increasing degrees of near-rationality. Specification I: model with equities only. Specification II: model with equities and balanced bonds. Specification III: general model with equities and bonds.

increases above unity (top-right panel) implying an overall short position in the bond market. This reinforces the role of domestic equity as a good hedge against real exchange rate risk and makes the home country a net creditor in equity instruments and a net debtor in debt instruments, consistently with the evidence documented by Tille (2005). Finally, increasing the concerns about misspecifications yields also a short position in foreign bonds (bottom-left panel) coupled with a positive overall position in foreign assets (bottom-right panel), in line with the evidence of Lane and Shambaugh (2008).

7 Conclusions

The observation that international investors hold a disproportionate share of their wealth in domestic rather than foreign assets is one of the most persistent ones in international finance. This is named the international home-bias puzzle, that the literature has been dealing with for a couple of decades.

This paper develops a dynamic general equilibrium model of portfolio and consumption choices, with incomplete markets and distorted beliefs. Households might have a subjective probability distribution that is generally different from the unknown objective one (although the two distributions are close enough, in an absolute continuity sense, to be observationally equivalent in finite samples) and make robust optimal choices against the model uncertainty. This framework assigns a new role to real exchange rate risk for portfolio allocation even in a model with log-utility. Importantly, moreover, what matters is not only the short-run risk but also and foremost the long-run risk.

Within this framework we characterize optimal portfolio allocations in terms of covariances between measurable sources of risk to be hedged (non-diversifiable labor income risk and real exchange rate risk) and a vector of cross-country excess returns, and evaluate their empirical
relevance using financial and macro data on the G7 countries.

Our results suggest that, contrary to what claimed in recent related contributions, hedging against non-diversifiable labor-income risk is not sufficient to account for the lack of international portfolio diversification. Indeed, in a setting in which equity is the only available asset, correlations in financial data support a substantial foreign-equity bias, as in Baxter and Jermann (1997). Adding further assets does not help in identifying a clear role for this risk in explaining the home-bias puzzle. On the other hand, a moderate concern about model misspecification is able to generate an equilibrium home-bias in equity holdings, which is also quantitatively large in case equities are the only asset available for international trade, and allows to match other empirical facts.

The methodological contribution of the paper goes beyond the analysis of the home-bias puzzle. The class of preferences that we suggest, in fact, produces a perturbation of the equilibrium stochastic discount factor which decouples the measure of intertemporal elasticity of substitution with the risk-aversion coefficient, and can prove useful in addressing other failures of standard preference specifications along the asset-price dimension.\(^{33}\) It has been shown in closed-economy settings, that disentangling the elasticity of intertemporal substitution from the degree of risk aversion helps in accounting for the equity premium puzzle. Once we open the economy to international trade in assets, there are additional puzzling features of financial data, among which the international equity- and bond-premia puzzles and the Backus-Smith anomaly are notable examples.\(^{34}\) All these stylized facts imply restrictions on the stochastic discount factor that standard preferences cannot meet at the same time, and that might be all reconnected to some common misspecification.\(^{35}\) The modification of the stochastic discount factor, that our preference specification implies, is a promising tool to correct this misspecification and build macro models whose predictions are closer to the empirical implications of financial data.

\(^{33}\)See for a discussion Backus et al. (2004).

\(^{34}\)Ilut (2008) studies how ambiguity aversion can help to explain the uncovered-interest-rate puzzle.

\(^{35}\)All excess-return puzzles, for example, imply “high” lower bounds on the volatility of the equilibrium stochastic discount factor, as discussed for the equity premium by Hansen and Jagannathan (1991).
References


[34] Obstfeld, Maurice (2006), “International Risk Sharing And the Costs of Trade,” The Ohlin Lectures, unpublished manuscript.


8 Appendix

To get equation (9), we have defined

\[ A_t \equiv B_{H,t} + S_t B_{F,t} + x_{H,t} V_{H,t} + x_{F,t} S_t V_{F,t}^*, \]

and

\[ R_{p,t} = \alpha_{H,t-1} R_{H,t} + \alpha_{F,t-1} R_{F,t}^* \frac{S_t}{S_{t-1}} + \alpha_{H,t-1} R_{H,t}^e + \alpha_{F,t-1} R_{F,t}^e \frac{S_t}{S_{t-1}}, \]

with

\[ R_{H,t}^e \equiv \frac{V_{H,t} + D_{H,t}}{V_{H,t-1}}, \]

\[ R_{F,t}^e \equiv \frac{V_{F,t}^* + D_{F,t}^*}{V_{F,t-1}^*}, \]

and

\[ B_{H,t} \equiv \alpha_{H,t} A_t, \]

\[ S_t B_{F,t} \equiv \alpha_{F,t} A_t, \]

\[ x_{H,t} V_{H,t} \equiv \alpha_{H,t}^* A_t, \]

\[ x_{F,t} S_t V_{F,t}^* = \alpha_{F,t}^* A_t, \]

and analogously for the foreign country:

\[ B_{H,t}^* \equiv \alpha_{H,t}^* A_t^* S_t, \]

\[ B_{F,t}^* \equiv \alpha_{F,t}^* A_t^*, \]

\[ x_{H,t}^* V_{H,t} \equiv \alpha_{H,t}^* A_t^* S_t, \]

\[ x_{F,t}^* V_{F,t}^* = \alpha_{F,t}^* A_t^*. \]