

Problem Set 1

September 7, 2004

You should feel free to speak to your colleagues about these problems — in fact, I encourage it — but write down for yourself any answers you submit. You are welcome to compute solutions to numerical examples if you find this easier.

- (static trade) International trade provides a nice illustration of how we can use the equivalence of Pareto optima and competitive equilibria to characterize equilibria in multi-agent economies. Consider a static two-country economy with two goods, apples a and bananas b , and two agents, one representing each country. Country 1 is endowed with y_1 apples and country 2 is endowed with y_2 bananas. Each country has the same constant elasticity utility function:

$$U(a, b) = [(1 - \omega)a^\rho + \omega b^\rho] / \rho$$

with parameters $0 < \omega < 1$ and $\rho < 1$. The elasticity of substitution is $\sigma = 1/(1 - \rho)$.

- State and solve an optimum problem corresponding to an arbitrary Pareto optimal allocation. What is the implicit relative price q of apples to bananas?
 - Use the solution to (a) to find the competitive equilibrium associated with the initial endowments. Show that it satisfies the budget constraints of both agents.
 - (extra credit) Comment briefly on how your solution changes if countries have different preferences — for example, if ω differs between them.
- (trade dynamics) In international macroeconomics, the focus is on intertemporal trade, in which countries borrow and lend from each other. We can get a sense of how this works with a two-country one-good infinite-horizon world in which the “commodity space” consists of the good at date $t = 0$, the good at date $t = 1$, the good at date $t = 2$, and so on, forever. To give this some economic flavor, let us say that agents have fluctuating endowments. The agent in the first economy has endowment $y_{1t} = a$ if t is even and $y_{1t} = b$ if t is odd, where $a > b > 0$. The agent in the second economy has the reverse: $y_{2t} = b$ if t is even and $y_{2t} = a$ if t is odd. Each agent has preferences

$$U_i(c_i) = \sum_{t=0}^{\infty} \beta^t c_{it}^{1-\alpha} / (1 - \alpha),$$

with $0 < \beta < 1$ and $\alpha > 0$.

- State and solve an optimum problem for this economy. Use it to compute a competitive equilibrium. Describe qualitatively the behavior of consumption in each country over time.
- Describe the dynamics of the trade balance in country 1, $nx_1 = y_1 - c_1$. Ditto country 2.

- (c) Comment briefly on the economic logic of your answer. Why does the market lead to this pattern of consumption allocations and trade balances?
3. (risk sharing) Similar methods apply to economies with uncertainty. Consider a static two-country world economy with two “states” $s = 1, 2$ that occur with probability $\pi(s)$. To keep things simple, let us say the states are equally likely: $\pi(1) = \pi(2) = 1/2$. In state 1, the endowments are a for country 1 and b for country 2. In state two, the reverse. Agents know and agree on the probabilities and have identical expected utility functions:

$$U_i(c_i) = \sum_s \pi(s) \log c_i(s).$$

- (a) State and solve a planning problem that puts equal weight on the two agents. Describe qualitatively the behavior of consumption in each country.
- (b) Describe the trade balance in country 1 in each state, $nx_1(s) = y_1(s) - c_1(s)$. Why isn’t trade balanced in each state? If we observed many realizations of this economy (many draws of s), what do we expect the trade balance to be on average?
- (c) Comment briefly on the economic logic of your answer. Why does the market lead to this pattern of consumption allocations and trade balances?
- (d) (extra credit) Consider a market implementation of the optimal allocation, in which agents trade claims to the endowment of each country. A claim to country 1’s “equity” yields a in state 1 and b in state 2, the values of the country’s endowment in those states. Similarly, a claim to country 2’s equity yields b in state 1 and a in state 2. What allocation of these claims produces the optimal consumption allocation?