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# Problem Set 2

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*You should feel free to speak to your colleagues about these problems — in fact, I encourage it — but write down for yourself any answers you submit. You are welcome to compute solutions to numerical examples if you find this easier.*

1. (trade balance trends) The trade balance can reflect not only temporary fluctuations in output, but differences in output trends or preferences. We can get a sense of each with a two-country one-good infinite-horizon world in which agents differ in three possible ways: their discount factors, their endowments, and their utility "curvature" parameters. Preferences are characterized in each case by the utility functions

$$U_i(c_i) \;=\; \sum_{t=0}^{\infty} \beta_i^t u_i(c_{it}).$$

   (a) Version 1: Let $y_{it} = 1$, $u_i(c) = \log c$, and $1 > \beta_1 > \beta_2 > 0$. Describe qualitatively the behavior of consumption and the trade balance.

   (b) Version 2: Let $y_{1t} = 1$, $y_{2t} = \omega\gamma^t$ with $0 < \omega < 1$ and $\gamma > 1$, $u_i(c) = \log c$, and $\beta_1 = \beta_2 = \beta$. Describe qualitatively the behavior of consumption and the trade balance.

   (c) Version 3: Let $y_{it} = \gamma^t$ with $\gamma > 1$, $u_1(c) = \log c$, $u_2(c) = c$, and $\beta_1 = \beta_2 = \beta$. Describe qualitatively the behavior of consumption and the trade balance.

   (d) Comment generally on the role of preferences and endowments in generating trends in the trade balance. Is there a sense in which the trade balance returns to a "long-run equilibrium"?

2. (current accounts) International economists often talk about the current account, which equals the trade balance (net exports) plus net international flows of income (interest payments, for example). This problem illustrates the concept in a simple context and serves as an introduction to sequence representations of competitive equilibria.

   To see how this works, consider a one-good, infinite-horizon, two-country world. Endowments are $y_{1t} = \omega$ and $y_{2t} = 1 - \omega$. Each has preferences characterized by

$$U_i(c_i) \;=\; \sum_{t=0}^{\infty} \beta^t \log c_{it}$$

   with $0 < \beta < 1$. In a sequence version of this economy, we allow countries to borrow and lend. Suppose at the start of date $t$ agent $i$ has a claim of $f_{it}$ (measured in units of the good) on the other agent, with $f_{1t} + f_{2t} = 0$. Her sequence budget constraint might be written:

$$f_{i,t+1} \;=\; r_{t,t+1} f_{it} + y_{it} - c_{it},$$

where $r_{t,t+1}$ is the (gross) interest rate between periods $t$ and $t+1$. The current account is

$$ca_{it} \;=\; y_{it} - c_{it} + (r_{t,t+1} - 1) f_{it},$$

where the last term represents flows of interest income.

(a) State and solve a "date-0" planning problem that puts arbitrary positive weights of $\theta_1$ on agent 1 and $\theta_2 = 1 - \theta_1$ on agent 2. What is the interest rate?

(b) What Pareto optimum corresponds to a competitive equilibrium with a specific value of $f_{10}$?

(c) Describe the behavior over time of consumption, net exports, and the current account in country 1 if $f_{10} < 0$.

3. (terms of trade) International economists refer to the relative price of imports to exports (or sometimes its inverse) as the terms of trade. In this problem, we examine the relation between the terms of trade and the trade balance.

Consider the relation in an exchange economy based on states/histories $s^t$. Each agent is endowed with a different good: agent 1 with $y_1$ apples and agent 2 with $y_2$ bananas, each "adapted" to the history of events $s^t$. They have the same preferences:

$$U_i(a, b) \;=\; \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t)[a^\rho + b^\rho]/\rho.$$

The parameter $\rho$ governs the elasticity of substitution between goods as well as risk aversion and the intertemporal elasticity of substitution.

(a) Compute the allocations and prices associated with an arbitrary optimum.

(b) What is the terms of trade $q$ for country 1?

(c) What is the trade balance, $nx_1 = a_2 - qb_1$?

(d) What is the relation between $nx_1/y_1$ and $q$? Between $nx_1/y_1$ and $y_1$?