

Balance Sheets and Exchange Rate Policy*

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Abstract

We study the relation among exchange rates, balance sheets, and macroeconomic outcomes in a small open economy model. Entrepreneurial net worth determines the risk premium on external financing, as in Bernanke and Gertler (1989). Because debts are in foreign currency, the impact of an adverse foreign shock is magnified by the increased debt burden due to the associated real devaluation. But the devaluation also improves the asset side of the balance sheet, since it shifts demand toward domestic goods, increasing the return earned by entrepreneurs. Hence, the combination of financial imperfections and foreign currency debts need not make devaluation contractionary, contrary to conjectures in recent literature. Regardless, the fall in investment, output and employment in response to an adverse shock is larger under fixed exchange rates than under flexible rates. In fact, flexible exchange rates Pareto dominate fixed rates and maximize social welfare.

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1 Introduction

In conventional textbook accounts, expansionary monetary policy and depreciation of the currency are optimal in response to an adverse foreign shock.¹ But if an economy has a large debt denominated in foreign currency, then a weaker local currency can also exacerbate debt-service difficulties and wreck the balance sheets of domestic banks and firms. This channel may imply that devaluations are contractionary, not expansionary. As documented by Hausmann et al (2000) and Calvo and Reinhart (2002), balance sheet effects have emerged as a prime reason why many central banks are reluctant to allow their currencies to devalue in response to external shocks.

Such concerns have generated an active policy debate, but the issue is only beginning to be studied and modeled in the academic literature.² In this paper we develop a model in which liabilities are dollarized and the country risk premium is endogenously determined by domestic net worth, in the manner of Bernanke and Gertler (1989). Wages are sticky in terms of the home currency, so that monetary and exchange rate policies have real effects. And in contrast with other contributions on balance sheet effects in the open economy, ours is a dynamic general equilibrium model built from first principles, yet solvable analytically. We provide a complete characterization of the model's implications, including welfare, and address topical policy questions –in particular, whether flexible exchange rates provide useful insulation in the presence of imperfect financial markets and dollarized liabilities.

A first result is that balance sheet effects do matter for the dynamics of an economy, in that they can magnify the effects of foreign disturbances. We distinguish between a situation of high indebtedness and the resulting *financial vulnerability*, so that a real depreciation raises the country risk premium, and one of *financial robustness*, in which the opposite happens. The magnification effect is especially sharp under financial vulnerability because endogenous increases in country risk have lasting and potentially large effects on domestic variables.

Second, the policy literature has tended to emphasize that unexpected

¹This is the case for the Mundell-Fleming model and also for in state-of-the-art sticky price open economy models. See the survey by Lane (2001) for a recent exposition.

²Krugman (1999) and Aghion, Bachetta and Banerjee (2000) have considered also studied balance sheet effects in the context of exchange rate policy. However, their analysis is based on highly restrictive assumptions. Dynamics are absent, as is a systematic investigation of alternative monetary and exchange rate policies.

devaluations can adversely affect the liability side of the balance sheet of firm. The model in this paper stresses that there are also effects on the asset side that operate in the other direction. Real devaluation shifts demand toward domestic goods as in the textbook model. This in turn raises output and the return earned by entrepreneurs. It need not be the case, even with foreign currency liabilities, that depreciation lessens creditworthiness and increases the risk premium faced by an economy when borrowing abroad.

A third main result is that, in spite of financial imperfections and balance sheet effects, the conventional ranking of fixed and flexible exchanges survives, in that flexible exchange rates do play a useful insulating role against real external shocks. With the capital stock predetermined, all that matters for initial output is labor employment. An adverse shock always calls for a real devaluation. Under floating, the necessary real devaluation is accomplished by a nominal depreciation, which leaves the product real wage and hence employment unchanged; under fixing, it is accomplished by deflation, which increases the product real wage and causes a fall in employment and output. Our analysis shows that fixed rates also imply larger falls in investment and welfare. Indeed, in our model flexible exchange rates turn out to be socially optimal.

The next section lays down the basic model. Sections 3 and 4 study the workings of flexible and fixed exchange rates under sticky wages. Section 5 discusses welfare issues and characterizes optimal policy. Section 6 concludes.

2 The Model

Consider an infinite-horizon, small open economy. A single good is produced by competitive firms using labor and capital, and is exported or sold to domestic agents. Labor and capital are supplied by distinct agents called workers and entrepreneurs. These agents consume and, in the case of entrepreneurs, invest. Entrepreneurs finance investment in excess of their own net worth by borrowing from foreigners. The key assumption is that the cost of borrowing depends inversely on net worth relative to the amount borrowed. In this way the model incorporates the “net worth” or “balance sheet” effects emphasized by Bernanke and Gertler (1989) and others.

2.1 Domestic Production

Time is discrete and indexed by $t = 0, 1, 2, \dots$. Production of the home good is carried out by competitive firms, which take all prices as given, and have access to a common technology:

$$Y_t = AK_t^\alpha L_t^{1-\alpha}, \quad 0 < \alpha < 1 \quad (1)$$

where Y_t denotes home output, K_t capital input, L_t labor input, and A is a positive constant. As in Obstfeld and Rogoff (2000), workers are heterogeneous and the input L_t is an aggregate of the services of the different workers in the economy: $L_t = \left[\int_0^1 L_{it}^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$ where we have indexed workers by i in the unit interval, L_{it} denotes the services purchased from worker i , and $\sigma > 1$ is the elasticity of demand for worker i 's services.

Let W_t denote the aggregate wage, that is, the minimum cost of a unit of the L_t aggregate, expressed in terms of the domestic currency (henceforth called *peso*). Cost minimization yields the demand for worker i 's labor:

$$L_{it} = \left(\frac{W_{it}}{W_t} \right)^{-\sigma} L_t \quad (2)$$

In every period, the representative firm's problem is to maximize profits, given by $P_t Y_t - R_t K_t - W_t L_t$, subject to 1, where P_t is the price of the home good and R_t the rental rate of capital, both in pesos. Then equilibrium requires profits to be zero and the familiar input demand conditions:

$$R_t K_t = \alpha P_t Y_t \quad (3)$$

$$W_t L_t = (1 - \alpha) P_t Y_t \quad (4)$$

2.2 Workers

Since labor services are imperfect substitutes of each other, we assume the market for labor displays monopolistic competition as in Dixit and Stiglitz (1977). Worker i has preferences over consumption and labor supply in each period t given by

$$\log C_{it} - \left(\frac{\sigma - 1}{\sigma v} \right) L_{it}^v \quad (5)$$

where $v > 1$ is the elasticity of labor supply. This specification is useful since, as one can easily show, it implies that equilibrium employment would be fixed at one in the absence of nominal rigidities.

Consumption C_{it} is an aggregate of the home good (C_{it}^H) and an imported good (C_{it}^F):

$$C_{it} = (C_{it}^H)^\gamma (C_{it}^F)^{1-\gamma} / [\gamma^\gamma (1-\gamma)^{1-\gamma}] \quad (6)$$

The imported good has a fixed price, normalized to one, in terms of a foreign currency that we call the *dollar*. It is freely traded internationally and the Law of One Price holds, so that the *peso* price of a unit of imports is equal to the *nominal exchange rate* of S_t pesos per dollar.

Workers cannot save, and wage earnings constitute their only source of income. In every period t , i 's choices must respect the budget constraint

$$W_{it}L_{it} = P_t C_{it}^H + S_t C_{it}^F. \quad (7)$$

To allow monetary policy to have real effects, wages are sticky in terms of pesos: worker i must set his wage W_{it} *before* observing the realization of aggregate variables in period t , and commits to supply labor to satisfy demand as given by 2.

The solution of the worker's decision problem is standard. Given his wage income, the worker maximizes consumption 6 subject to 7, which requires

$$Q_t C_t = W_t L_t \quad (8)$$

where we have imposed symmetry (dropping the subscript i) and defined the cost of consumption or *price level* as

$$Q_t = P_t^\gamma S_t^{1-\gamma} \quad (9)$$

Expenditures on home products and imports are constant shares γ and $(1-\gamma)$ of income. Finally, optimal wage-setting requires

$${}_{t-1}L_t^\nu = 1, \quad (10)$$

where, as in the rest of the paper, the notation ${}_t N_{t+j}$ denotes the expectation of variable N_{t+j} conditional on information available at t .

2.3 Entrepreneurs

Entrepreneurs are the key players in our model: they finance investment by borrowing abroad, and borrowing is subject to frictions. These frictions can be due to informational or enforcement problems. We follow the formulation of Bernanke, Gertler, and Gilchrist (1999). The details are, however, peripheral to our line of discussion, so here we only describe the main implications for the aggregate behavior of entrepreneurs.³

At the end of any period t , entrepreneurs have *net worth* $P_t N_t$, expressed in pesos, and enjoy access to a world capital market where the *safe* interest rate for dollars borrowed between t and $t + 1$ is ρ_{t+1} . This rate fluctuates randomly but becomes known at t . Entrepreneurs invest in capital for next period, which they produce by assembling home goods and imports with the technology given by 6. Hence, the cost of one unit of capital in $t + 1$ is Q_t , as given by 9, and the entrepreneurs' budget constraint is

$$P_t N_t + S_t D_{t+1} = Q_t K_{t+1}, \quad (11)$$

where D_{t+1} denotes the amount borrowed abroad and K_{t+1} investment in $t + 1$ capital.

Entrepreneurs are risk neutral, and choose D_{t+1} and K_{t+1} so as to maximize profits. For simplicity, we assume that capital depreciates completely in production. In the absence of informational frictions, investment would be such as to equalize the world safe interest rate to the expected yield on capital, measured in dollars. Here, informational asymmetries imply a wedge between the expected return to investment and the world safe rate:

$$\frac{{}_t(R_{t+1}K_{t+1}/S_{t+1})}{Q_t K_{t+1}/S_t} = (1 + \rho_{t+1}) (1 + \eta_{t+1}), \quad (12)$$

where η_{t+1} is this wedge, which we call *risk premium* for short. Bernanke, Gertler, and Gilchrist (1999) show that

$$1 + \eta_{t+1} = F\left(\frac{Q_t K_{t+1}}{P_t N_t}\right), \quad F(1) = 1, \quad F'(\cdot) > 0. \quad (13)$$

In words, the risk premium is an increasing function of the value of investment relative to net worth.

³Detailed microfoundations for the claims in this section are provided in an appendix available on request.

At the beginning of each period, entrepreneurs collect the income from capital and repay foreign debt. Assume that they consume a portion $1 - \delta$ of the remainder, and that (in true capitalist style) they only consume imports. From now on we assume $\delta(1 + \rho) < 1$, which is required for convergence of the economy to its steady state. With this formulation, entrepreneurs' net worth is

$$P_t N_t = \delta \{R_t K_t - \Phi_t \alpha P_t Y_t - (1 + \rho_t) S_t D_t\}, \quad (14)$$

where the term $\Phi_t \alpha P_t Y_t$ reflects monitoring costs, paid in period t . Again following Bernanke, Gertler, and Gilchrist (1999) one can show that $\Phi_t = \Phi(1 + \eta_t)$, which is an increasing function of η_t . Using this result and the fact that $R_t K_t = \alpha P_t Y_t$ in equilibrium, one can write 14 as

$$N_t = \delta \{(1 - \Phi_t) \alpha Y_t - (1 + \rho_t) E_t D_t\}, \quad (15)$$

where we have defined the *real exchange rate* as $E_t = S_t/P_t$. Equation 15 emphasizes that, holding real income and the contemporaneous risk premium constant, a real devaluation (an increase in E_t) has a negative impact on net worth and, *ceteris paribus*, increases the next period's risk premium. This is a key aspect in the analysis: as noted by Calvo (1999, 2001) and others, if an entrepreneur's assets and liabilities are denominated in units of different goods, then changes in their relative prices affect creditworthiness.

2.4 Equilibrium

Given Cobb-Douglas preferences, domestic expenditure on home goods is a fraction γ of final expenditures. The home good may be also sold to foreigners; we assume that the value of home exports in dollars is exogenous and given by X .⁴ The market for home goods clears when

$$P_t Y_t = \gamma Q_t (K_{t+1} + C_t) + S_t X \quad (16)$$

A rational expectations equilibrium is defined in the usual way, after specifying the stochastic process for ρ_t and after describing monetary pol-

⁴This is justified if the foreign elasticity of substitution in consumption is one and the foreign expenditure share in domestic goods is negligible. For a similar assumption see Krugman (1999). It is straightforward to allow X_t to be stochastic: see the (2000) working paper version of the model.

icy.⁵ Before proceeding, some implications of the model are worth noting. We study approximate dynamics by log-linearizing the equilibrium equations around the non-stochastic steady state.⁶ Among the resulting expressions, the critical one concerns the risk premium, which can be written as

$$\eta'_{t+1} - \phi \eta'_t = \mu \left(\frac{1 - \lambda}{\lambda} \right) (y_t - e_t) + \mu \delta (1 + \rho) \psi [(e_{t-t-1} e_t) - (y_{t-t-1} y_t)] \quad (17)$$

Here and in the rest of the paper, lower case letters denote percentage deviations from steady state values, while ρ'_t and η'_t denote deviations from the respective steady state levels of these two variables. Also, $\lambda < 1$ is the share of investment demand in total non-consumption demand for home goods, ψ is the steady state ratio of debt to net worth, μ is the elasticity of the $F(\cdot)$ function evaluated at the steady state, and ϕ is a coefficient that depends on the financial contract problem.⁷

Equation 17 expresses the change in the risk premium as the sum of two effects. The first one, given by the first term in the RHS, is that a devaluation increases the home output value of exports; given home output, this is matched by a fall in investment, debt, and the risk premium. The last term in the RHS of 17 is the one emphasized in the current debate, since it captures the impact of *unexpected* shocks on *net worth*. An unexpected real devaluation increases the burden of inherited debt, hence reducing net worth relative to the cost of investment; an unexpected output fall, which reduces entrepreneurs' reward from previous investment, has a similar effect. In either case, the unexpected decrease in net worth pushes up the risk premium. The net effect of an unexpected devaluation depends on the relative strength of the two effects just mentioned: the risk premium increases if $\psi \delta (1 + \rho) > \left(\frac{1 - \lambda}{\lambda} \right)$. That is, the net worth effect dominates if ψ is large enough; equivalently, given the definition of ψ , if foreign debt is large enough in the steady state.

The other linearized equations characterizing equilibrium dynamics are surprisingly tractable.⁸ Denoting home output *measured in dollars* by $z_t =$

⁵The alert reader may have noted that we have not laid out the monetary side of the model –and in particular an expression for money demand– explicitly. As discussed by Woodford (2003), this is legitimate for a class of monetary policy rules that includes those we are concerned with.

⁶The appendix discusses the existence and uniqueness of the nonstochastic steady state, and characterizes dynamics in the neighborhood of that steady state.

⁷See the appendix for details.

⁸They include 22, 23, and 24 below, as well as the linearized version of 12.

$y_t - e_t$, one can show that in equilibrium,

$$z_{t+1} = \lambda^{-1} z_t + \eta'_{t+1} + \rho'_{t+1}. \quad (18)$$

In the absence of shocks ($\rho'_{t+1} = 0$), the system 17-18 determines the perfect foresight dynamics of dollar output z_t and the risk premium η'_t . Since the risk premium η'_t is predetermined at t but dollar output z_t is not, standard techniques imply that, provided 17-18 is saddle path stable, z_t can be explicitly solved for as a unique and linear function of η'_t , say $z_t = \zeta \eta'_t$.⁹ In addition, ζ is *negative*: intuitively, when the risk premium η_t is above its steady state level, dollar output z_t is below its own steady state level, and vice-versa.

More generally, 17 and 18 suffice to determine the behavior of z_t and η'_t as functions of the exogenous shocks. It follows that, in this model, monetary policy does not affect dollar output or the risk premium. But monetary policy is still powerful, since it determines how movements in dollar output z_t are split between movements in output and the real exchange rate.

3 Flexible Exchange Rates

Our focus is on the insulating properties of *fixed* versus *flexible* exchange rate regimes. To this end, this section and the next describe the economy's response, under each regime, to an unanticipated and temporary shock to the world interest rate. We obtain a complete analytical characterization of the model's dynamics and, in particular, identify the role of balance sheet effects.

Assume that the system starts from steady state and that, at $t = 0$, the world interest rate ρ_1 rises unexpectedly, but is expected to return to its steady state value after one period. Notice that, since after that single period all variables are free to adjust, from $t = 1$ on the economy settles on the saddle path converging to the steady state. Therefore, it is enough to focus on what happens in the period of the shock.

⁹The appendix establishes that 17-18 is in fact saddle-path stable if $\phi < 1 + \mu$; this is assumed in the text. Saddle path stability means, in particular, that our analysis differs from Krugman (1999) and Aghion, Bachetta, and Banerjee (2000), which focus on the possibility of multiple equilibria.

Aggregate supply behavior is easy to derive. The pre-set wage rate in any period t must be such that

$${}_{t-1}l_t = 0 \tag{19}$$

which is the linear version of 10. The linearized version of 4,

$$y_t - l_t = w_t - p_t, \tag{20}$$

determines employment. Since we start from the steady state, 19 and 20 imply $w_0 = {}_{-1}p_0 = 0$, which applied to 20 yields

$$l_0 - y_0 = \frac{\alpha y_0}{1 - \alpha} = p_0 - {}_{-1}p_0 = p_0, \tag{21}$$

after taking into account that $y_0 = \alpha k_0 + (1 - \alpha)l_0 = (1 - \alpha)l_0$. Equation 21 is a simple expectational Phillips curve.

We define a regime of *flexible* exchange rates as one in which the central bank lets the nominal exchange rate s_t adjust to market conditions. This leaves the central bank free to set monetary policy; we assume that the central bank targets the price of home output p_t , and adjusts monetary policy so as to keep $p_t = {}_{t-1}p_t = 0$ for all t .

Two implications are immediate. Since $e_t = s_t - p_t$, the price targeting rule implies that $e_t = s_t$ for all t . In other words, movements in the nominal exchange rate are equivalent to movements in the real exchange rate. In addition, 19, 20, and 21 imply that $l_t = w_t = p_t = w_t - p_t = 0$. That is, nominal and real wages are always at their steady state level, and labor supply is constant and equal to its steady state level of one.¹⁰

3.1 The IS and BP schedules

The analysis can be depicted using familiar schedules. Start with the market for home goods. After inserting 8 into 16 and linearizing the resulting expression, one obtains

$$y_t = \lambda(k_{t+1} + q_t - p_t) + (1 - \lambda)(s_t - p_t) \tag{22}$$

The linearization of 9 and 1 yield

$$q_t - p_t = (1 - \gamma)e_t \tag{23}$$

¹⁰This implies that flexible exchange rates deliver the equilibrium outcome that would obtain if there were no nominal wage rigidity.

$$y_t = \alpha k_t + (1 - \alpha)l_t \quad (24)$$

Flexible exchange rates imply $l_0 = 0$; we also have $k_0 = 0$ since the economy is assumed to start from the steady state. Hence $y_0 = 0$: the interest shock has no impact on home output. Inserting this fact and 23 into the period 0 version of 22, equilibrium in the market for home goods in period zero is given by

$$0 = \lambda k_1 + (1 - \lambda\gamma)e_0 \quad (25)$$

This equation ensures equilibrium in the market for home goods, and hence it is the *IS curve* in this model. It is always a negatively sloped curve in (e_0, k_1) space. The intuition is that, at time 0, home output is fixed, as capital is predetermined and labor is fixed because of flexible exchange rates. Hence an increase in investment must be met by an appreciation of the real exchange rate, which reduces both the relative price of capital and the value of exports.

To derive a second relation between e_0 and k_1 , start with the linearized version of the interest parity condition 12 for period 0:

$$y_1 - (q_0 - p_0) - k_1 = \rho'_1 + \eta'_1 + e_1 - e_0. \quad (26)$$

To simplify this expression, use 23 to replace $q_0 - p_0$ by $(1 - \gamma)e_0$; likewise, $y_1 = \alpha k_1$ since 24 holds and $l_1 = 0$. Observe also that, since there is perfect foresight from period 1 on, the economy must then settle on a saddle-path converging to the steady state. But from the analysis at the end of last section we know that the saddle-path is given by the relation $z_t = \zeta \eta'_t$ between dollar output and the risk premium. Hence, in period 1 the exchange rate must be $e_1 = y_1 - \zeta \eta'_1 = \alpha k_1 - \zeta \eta'_1$, which allows us to eliminate e_1 from 26.

Finally, the risk premium in period 0 must satisfy 17, which under flexible exchange rates reduces to

$$\eta'_1 = \mu \left(\psi \delta (1 + \rho) - \frac{1 - \lambda}{\lambda} \right) e_0 \equiv \varepsilon_{\eta e} e_0, \quad (27)$$

where

$$\varepsilon_{\eta e} \equiv \mu \left[\psi \delta (1 + \rho) - \frac{1 - \lambda}{\lambda} \right] \quad (28)$$

is the elasticity of the risk premium with respect to a change in the real exchange rate. This elasticity is a crucial parameter: we distinguish between a *financially robust* economy (one in which $\varepsilon_{\eta e} < 0$) and a *financially vulnerable* one (one in which $\varepsilon_{\eta e} > 0$). Clearly, whether an economy is robust or

vulnerable depends on the size of $\psi\delta(1+\rho)$ vis à vis $(1-\lambda)/\lambda$. In particular, the economy is more likely to be financially vulnerable if the steady state ratio of debt to net worth, $\psi = SD/N$, is large. This intuition is reminiscent of the work of Calvo (1999), Krugman (1999), and others.

Given these results, the interest parity condition 26 simplifies to

$$k_1 = [\gamma - (1 - \zeta)\varepsilon_{\eta e}] e_0 - \rho'_1 \quad (29)$$

We shall refer to this equation as the *BP curve*, since it captures the equilibrium conditions in the international loan market. Its slope in (k_1, e_0) space can be positive or negative, depending partly on the sign of $\varepsilon_{\eta e}$ —that is, on whether the economy is robust or vulnerable.

3.2 The financially robust economy

Under financial robustness, a devaluation in period 0 lowers the risk premium ($\varepsilon_{\eta e} < 0$). Therefore, the BP curve slopes up in (k_1, e_0) space, as in Figure 1. Intuitively, a devaluation unambiguously lowers the cost of investment: given k_1 , an increase in e_0 pushes down the RHS of 26 fall not only directly but also through a fall in the risk premium and an associated fall of e_1 ; to restore asset market equilibrium, k_1 must increase.

The unanticipated increase of the world interest rate shifts the BP curve up but leaves the IS curve undisturbed. As shown in Figure 1, the interest rate shock causes, on impact, a real depreciation and a fall in investment.

The rest of the adjustment process is easy to derive. We have seen home output cannot respond to the interest rate shock in period 0. Since k_1 falls and $y_1 = \alpha k_1$, home output must fall in period 1. Dollar output falls on impact. Financial robustness implies that $\eta'_1 = \varepsilon_{\eta e} e_0 < 0$, so the risk premium falls on impact. And since the economy jumps to the saddle path in period 1, $z_1 = \zeta \eta'_1 > 0$. Hence, the initial fall in dollar output turns out to be short-lived: that variable is above steady state levels in period 1 already. Thereafter, η rises and z falls to return to the steady state. In short, under financial robustness the response of the economy is fairly conventional, reflecting that net worth effects are relatively small.

3.3 The financially vulnerable economy

By definition, in a financially vulnerable economy a devaluation raises the risk premium on impact ($\varepsilon_{\eta e} > 0$). While the IS curve is still downward

sloping in (k_1, e_0) space, the BP curve may slope up or down, depending on the sign of $(\gamma - (1 - \zeta)\varepsilon_{\eta e})$. To understand why, consider the effect of a period 0 devaluation on the cost of investment, given by the RHS of 26: given k_1 , an increase in e_0 reduces the cost of investment directly; but under financial vulnerability it also pushes up the risk premium and e_1 , and this second effect works in the opposite direction.

If the BP slopes up, the analysis is qualitatively given by Figure 1, as in the robust case. But if financial imperfections are large enough the BP slopes down, and this alters the qualitative dynamics of the economy. This case appears in Figure 2. An unexpected increase in the world interest rate pushes the BP, causing –as in the financially robust case– a real devaluation on impact and a contraction in investment. Dollar output again falls on impact. But financial vulnerability now implies that the risk premium η'_1 increases because of the initial depreciation. As a consequence, dollar output must fall in period 1 for the economy to jump to the converging saddle path ($z_1 = \zeta\eta'_1 < 0$). This means that financial vulnerability is associated with a more persistent output effect of the interest rate shock. Gradual convergence, with dollar output rising and the risk premium falling, takes place thereafter.

Note that as the elasticity $\varepsilon_{\eta e}$ increases, the BP curve rotates in a counter-clockwise direction, which makes the response to shocks larger and more protracted. In this sense, increased financial vulnerability magnifies the economy's response to exogenous shocks. In particular, since $\varepsilon_{\eta e}$ increases with the debt ratio coefficient ψ , a larger debt ratio increases the sensitivity of the economy to movements in world interest rates.¹¹

Summarizing, with flexible exchange rates an unexpected interest rate increase has no impact on current employment or home output. But investment, and hence future output, must contract. Balance sheet effects do matter: the magnitude and the persistence of the contraction increases with financial vulnerability.

¹¹A case not discussed in the text is when the BP is downward sloping and flatter than the IS. However, this case does not seem empirically relevant: it implies, as the interested reader can check, that an adverse shock to the world interest rate would imply not only a real appreciation, but an expansion in home output and investment and a fall in the risk premium. These responses are strongly at odds with intuition and evidence, so we do not consider that case any further.

4 Fixed Exchange Rates

Focus now on a policy of *fixed* exchange rates, defined as a regime in which the monetary authority keeps the nominal exchange rate constant: $s_t = 0$ for all t . Again, we consider the effects of an unexpected and temporary increase of the world real interest rate at time 0.

With fixed exchange rates, real depreciations (appreciations) can only be accomplished through deflation (inflation): $e_t = -p_t$. Now, the Phillips curve 21 yields

$$y_0 = \left(\frac{1 - \alpha}{\alpha} \right) p_0 = - \left(\frac{1 - \alpha}{\alpha} \right) e_0, \quad (30)$$

reflecting that a real devaluation is associated with unexpected deflation and, because nominal wages are fixed, a fall in home output on impact.

The derivation of the IS schedule is the same as with flexible exchange rates, except that y_0 is no longer zero but given 30. Hence the IS is

$$y_0 = - \left(\frac{1 - \alpha}{\alpha} \right) e_0 = \lambda k_1 + (1 - \lambda \gamma) e_0. \quad (31)$$

This curve is still downward sloping, but flatter than under flexible exchange rates. This reflects the fact that a real devaluation requires price deflation and a fall in home output falls. To restore equilibrium in the home goods market, investment must fall by more than under flexible exchange rates.

The derivation of the BP curve is also the same as with flexible rates, except that 17 now implies that the response of the risk premium to a devaluation is

$$\eta'_1 = \mu \alpha^{-1} \left(\psi \delta (1 + \rho) - \frac{1 - \lambda}{\lambda} \right) e_0 = \alpha^{-1} \varepsilon_{\eta e} e_0. \quad (32)$$

In words, the elasticity of the premium with respect to the initial real exchange rate is the same as with flexible exchange rates, but scaled up by α^{-1} . This reflects that, in addition to effects previously encountered, an unexpected devaluation is now associated with a fall in output and capital income, reducing net worth and pushing up the risk premium further. The consequence is that the BP curve is

$$k_1 = [\gamma - (1 - \zeta) \varepsilon_{\eta e} \alpha^{-1}] e_0 - \rho'_1. \quad (33)$$

This is the same as before, except that $\varepsilon_{\eta e}$ is replaced by $\varepsilon_{\eta e} \alpha^{-1}$.

As with flexible rates, dynamic adjustment is easily described by the IS and BP curves. Under *financial robustness*, the IS curve slopes down and the BP curve slopes up as in the flexible rates case, although they are both flatter. This means that Figure 1 still captures the qualitative response to a shock to the world interest rate. But there is an important difference which does not appear in the figure: with fixed exchange rates, the period 0 devaluation is associated with price deflation and a *fall* in home output and domestic employment.

Since the IS and BP are both flatter, the increase in e_0 is smaller than with flexible rates, which is intuitive. Graphical analysis is less useful to compare the changes in k_1 . However, it is easy enough to solve the IS and the BP algebraically for k_1 under both exchange rate regimes. One can show that the fall in investment is greater under fixed rates.

Under *financial vulnerability*, $\varepsilon_{\eta e} > 0$ and the BP may slope up or down. If it slopes up, the analysis is similar to that under financial robustness, except that the BP curve is steeper under fixed exchange rates than with flexible rates, while the IS curve is still flatter under fixed rates. This means that the adjustment to an adverse interest rate shock requires an even deeper fall in investment under fixed rates.

If financial imperfections are sufficiently large so that the BP has a negative slope, the responses of e_0 and k_1 are still shown qualitatively in Figure 2,¹² although both the IS and the BP are both flatter than under flexible rates. This clearly implies that, in response to an interest rate shock, the increase in e_0 is smaller under fixed rates. But again this comes at the expense of a fall in home output and employment on impact. Moreover, the fall in investment turns out to be greater with fixed rates, as can be checked algebraically.

The conclusion is that, in response to a shock to the world interest rate, the impact real devaluation is smaller under fixed rates than under flexible rates. But the cost is that, under fixed rates, the devaluation requires price deflation, and hence an initial fall in home output that is absent under flexible rates. We have also argued that the fall in investment is greater under fixed rates, which reflects the fact that the cost of capital is higher.

¹²For the same reasons as in the previous footnote, we ignore the case in which the BP is downward sloping but flatter than the IS.

5 On the Optimality of Flexible Rates

While our discussion has focused on the dynamics of the model under fixed and flexible exchange rates, these comparative dynamics do reflect a welfare ranking. In fact, flexible rates are not only Pareto superior to fixed rates, but socially optimal. This is the point of this section.

Note first that, in this model, entrepreneurial welfare does not depend on monetary policy: entrepreneurial consumption in each period is a fraction of net worth in dollars, which is independent of monetary policy. It follows that optimal monetary policy must maximize the welfare of the representative worker. Note also that to analyze welfare one must consider not only one-time shocks, but stochastic equilibria under different policy regimes.

The expected welfare of the representative worker in equilibrium is given by ${}_{-1}\left\{\sum_{t=0}^{\infty}\beta\left[\log C_t - \frac{\sigma-1}{\sigma\nu}L_t^\nu\right]\right\}$, where β is the workers' discount factor. However, for any t , ${}_{-1}L_t^\nu = {}_{-1}({}_{t-1}L_t^\nu) = 1$, where the first equality follows from the Law of Iterated Expectations, and the second equality from 10. Hence optimal policy maximizes expected discounted log consumption. Taking logs in 4, 8, and 9 and simplifying yields

$$\log C_t = \gamma \log Y_t + (1 - \gamma) \log Z_t + \log(1 - \alpha). \quad (34)$$

Replacing in the preceding equation and recalling that dollar output is independent of monetary policy, it follows that optimal policy maximizes the expected discounted value of log output, ${}_{-1}\left(\sum_{t=0}^{\infty}\beta \log Y_t\right)$.

Since the production function is Cobb Douglas, $\log Y_t = \alpha \log K_t + (1 - \alpha) \log L_t$. Now, $\log K_0$ is given, while using 12 and iterating one shows that $\log K_t$ equals $\gamma(1 - \alpha)[\log L_{t-1} + \alpha \log L_{t-2} + \dots + \alpha^{t-1} \log L_0] + (\gamma\alpha)^t \log K_0$ plus a term that is independent of policy. Hence optimality requires the maximization of a linear combination of terms of the form ${}_{-1} \log L_t$, $t = 0, 1, \dots$. But now 10 and a simple application of Jensen's inequality imply that flexible exchange rates maximize each those terms and, hence, are optimal.¹³

The intuition is that there are three key distortions affecting the economy: workers have some monopoly power, financial contracts are imperfect, and wages are rigid in pesos. Monetary policy cannot counteract the effects of the first two distortions, but it can undo the effects of nominal rigidity.

¹³Note ${}_{-1} \log L_t = {}_{-1}(\log L_t^\nu) \nu^{-1}$. This is the expectation, at $t = -1$, of a concave function of L_t^ν . Moreover, ${}_{-1}(L_t^\nu) = {}_{-1}({}_{t-1}L_t^\nu) = 1$, independently of monetary policy, so the claim follows by Jensen's inequality.

6 Final Remarks

The interaction of dollarized debts and net worth complicates an economy's response to external shocks. Under financial vulnerability, a real depreciation raises the risk premium. But trying to contain real depreciation pushes domestic output down, and this is also bad for the risk premium. The net result here is that the behavior of the risk premium is independent of the exchange regime, contrary to the conjectures in much of the recent policy literature.

While the exact offset depends on special assumptions, there is a general message here: depreciation has contradictory and possibly offsetting effects on firms' balance sheets. Therefore, exchange rate policy is unlikely to have much impact on the risk premium, and financial imperfections need not render fixed rates more stabilizing than flexible rates.

For tractability, we imposed strong assumptions. Relaxing them is necessary for arriving at a quantitative evaluation of the model. But we have reason to think that our main results would survive.¹⁴ For instance, allowing for incomplete capital depreciation should strengthen the desirability of floating. In the model, the price of capital in terms of home goods is an increasing function of the real exchange rate. With incomplete depreciation, a real depreciation would raise the value of capital, thereby increasing net worth. That would constitute an additional advantage of a flexible exchange rate.¹⁵

We took the denomination of foreign bonds as given, while in reality it is endogenous and may depend on exchange rate policy. But there is no accepted view of how the menu of assets is related to the exchange rate regime.¹⁶ Simply assuming that all liabilities are dollarized is plausible, given that essentially all lending to emerging markets is denominated in a few currencies. Doing so stacks the deck against flexible exchange rates, so our policy conclusions do not hinge on this assumption.

¹⁴The working paper version explores the case in which the investment good aggregator is a general CES function. We show that this extension does not overturn the results presented here.

¹⁵But allowing for incomplete depreciation would change the quantitative implications of the model. Balance sheet effects would be much smaller, since they act through investment. We thank a referee for making this point.

¹⁶See the recent work by Burnside, Eichenbaum and Rebelo (2001), Schneider and Tornell (2004), Caballero and Krishnamurthy (2003), Jeanne (2001) and Chamon (2001).

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Figure 1: Financially Robust Flexible Exchange Rates

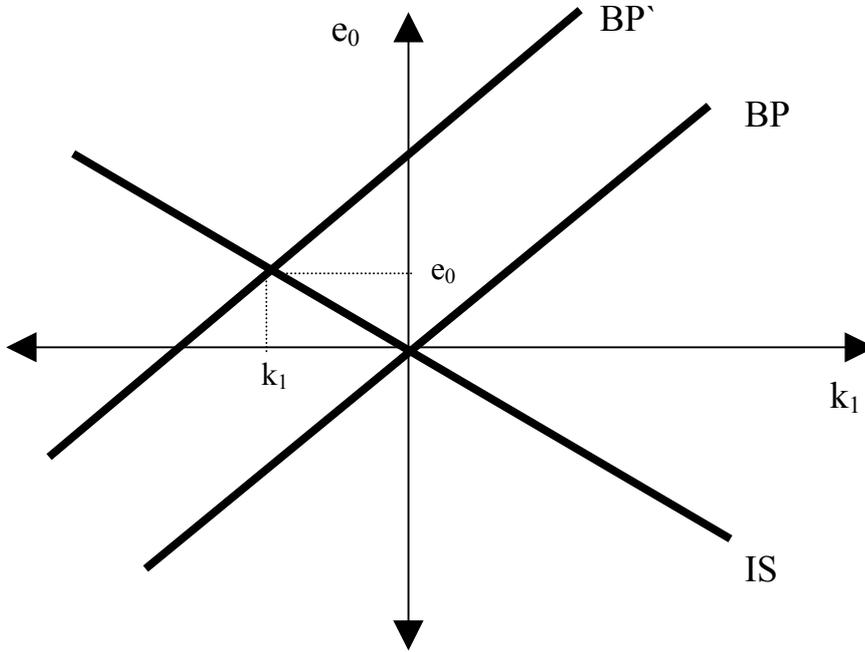
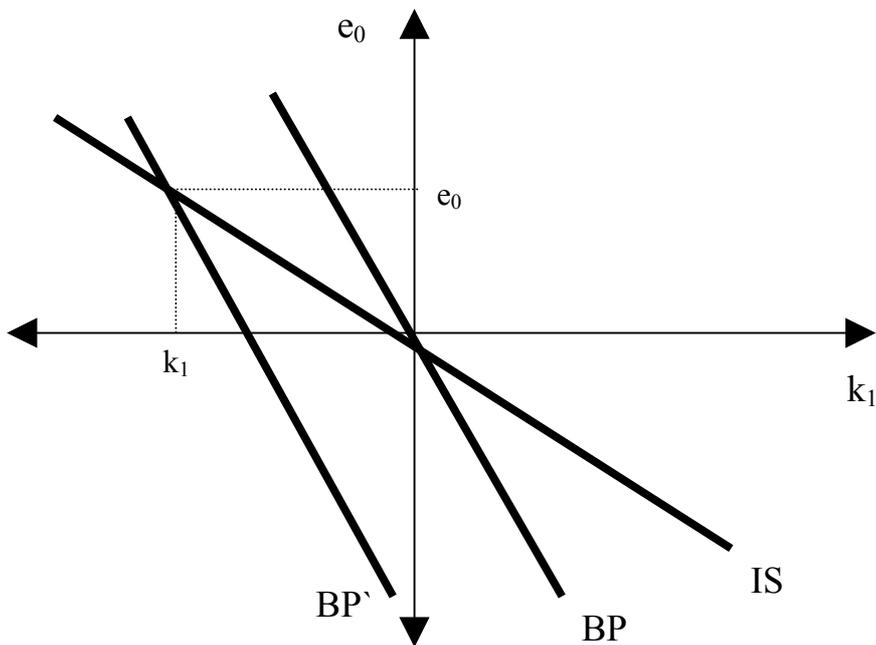


Figure 2: Financially Vulnerable Flexible Exchange Rates



Appendix to "Balance Sheets and Exchange Rate Policy"

1 Derivation of the risk premium

The purpose of this appendix is to sketch a justification for our specification of the risk premium (equations 12 and 13 in the main text). The argument outlined below closely follows Bernanke, Gertler, and Gilchrist (1999, henceforth BGG).¹

Consider the contracting problem between a single entrepreneur, indexed by j , and foreign lenders in any period t . At the time of contracting, j 's net worth ($P_t N_t^j$), the dollar interest rate (ρ_{t+1}), and prices in period t are known. For now, assume also that the period $t + 1$ rental rate on capital in dollars, R_{t+1}/S_{t+1} , is known. We shall discuss this assumption shortly.

Entrepreneurs and foreign creditors are risk neutral. Their joint problem is to choose a level of investment (K_{t+1}^j), a dollar loan (D_{t+1}^j), and a repayment schedule so as to maximize the expected return to the entrepreneur, such that creditors are paid at least their opportunity cost of funds, and subject to resource and information constraints. The latter are as follows. Investment in period t , K_{t+1}^j , yields $\omega_{t+1}^j K_{t+1}^j (R_{t+1}/S_{t+1})$ dollars next period, where ω_{t+1}^j is a random shock. The distribution of ω_{t+1}^j is public information and is such that ω_{t+1}^j is i.i.d. across j and t , and its expected value is one. Crucially, as in Townsend (1979) and Williamson (1987), we assume that the realization of ω_{t+1}^j cannot be observed by lenders unless they pay a proportional monitoring cost of $\zeta \omega_{t+1}^j K_{t+1}^j (R_{t+1}/S_{t+1})$; in contrast, ω_{t+1}^j is observed freely by the entrepreneur.

Under these conditions, it has been shown by Williamson (1987) that the optimal contract is a *standard debt contract*. Such a contract stipulates a fixed repayment, say of B_{t+1}^j dollars; if the entrepreneur cannot repay that amount, lenders monitor the outcome and seize the whole yield on the investment. Clearly, monitoring occurs only if the realization of ω_{t+1}^j is low enough. Letting $\bar{\omega}$ be such that $B_{t+1}^j = \bar{\omega} K_{t+1}^j (R_{t+1}/S_{t+1})$, monitoring occurs if and only if ω_{t+1}^j is below $\bar{\omega}$, an event interpretable as bankruptcy.

The resulting problem is formally identical to that analyzed in appendix A of BGG; for convenience, we summarize here the implications that are key for our purposes. To provide the lender with an expected return of ρ_{t+1} , it must be the case that

$$\begin{aligned} & K_{t+1}^j (R_{t+1}/S_{t+1}) \left\{ [\bar{\omega}(1 - H(\bar{\omega})) + (1 - \zeta) \int_0^{\bar{\omega}} \omega_{t+1}^j dH(\omega_{t+1}^j)] \right\} \quad (1) \\ & = (1 + \rho_{t+1}) D_{t+1}^j = (1 + \rho_{t+1}) (Q_t K_{t+1}^j - P_t N_t^j) / S_t \end{aligned}$$

where $H(\cdot)$ denotes the c.d.f. of ω_{t+1}^j . The first term of 1 gives the expected dollar yield on investment. With probability $1 - H(\bar{\omega})$ there is no bankruptcy,

¹See Cespedes (2000) for a related analysis in an open economy.

and lenders are repaid $B_{t+1}^j = \bar{\omega} K_{t+1}^j (R_{t+1}/S_{t+1})$. With probability $H(\bar{\omega})$, the entrepreneur goes bankrupt, and lenders are repaid whatever is left after monitoring costs; this is the term $(1 - \zeta) K_{t+1}^j (R_{t+1}/S_{t+1}) \int_0^{\bar{\omega}} \omega_{t+1}^j dH(\omega_{t+1}^j)$. The first equality in 1 gives the opportunity cost of the loan D_{t+1}^j . The second equality takes into account that the loan must equal the value of investment minus the entrepreneur's net worth.

The optimal contract maximizes the entrepreneur's utility

$$\left[\int_{\bar{\omega}}^{\infty} \omega_{t+1}^j dH(\omega_{t+1}^j) - \bar{\omega}(1 - H(\bar{\omega})) \right] R_{t+1} K_{t+1}^j \quad (2)$$

subject to 1. As in Williamson (1987), a key aspect of the contract is that it minimizes expected monitoring costs. Moreover, expected monitoring costs decrease with net worth, which should be intuitive.

To simplify, one can rewrite the constraint 1 as

$$\kappa_{jt} - 1 = (1 + \eta_{t+1}) \kappa_{jt} [\bar{\omega}(1 - H(\bar{\omega})) + (1 - \zeta) \int_0^{\bar{\omega}} \omega_{t+1}^j dH(\omega_{t+1}^j)] \quad (3)$$

where

$$\kappa_{jt} = Q_t K_{t+1}^j / P_t N_t^j \quad (4)$$

is the ratio of the value of investment to net worth, and

$$1 + \eta_{t+1} = \frac{R_{t+1} S_t}{Q_t S_{t+1} (1 + \rho_{t+1})} \quad (5)$$

is the risk (or "external finance") premium. Also, there is no change in the solution if the objective function 2 is multiplied or divided by positive variables known at t , so we can take the objective to be

$$\left\{ \int_{\bar{\omega}}^{\infty} \omega_{t+1}^j dH(\omega_{t+1}^j) - \bar{\omega}(1 - H(\bar{\omega})) \right\} \kappa_{jt} \quad (6)$$

Rewritten in this way, the problem is to choose the investment/net worth ratio κ_{jt} and the cutoff $\bar{\omega}$ to maximize 6 subject to 3. Since the external finance premium $1 + \eta_{t+1}$ is a parameter of this problem, BGG show that, under suitable conditions,² the optimal cutoff $\bar{\omega}$ is an increasing function of $1 + \eta_{t+1}$, or, expressing that function in inverse form,

$$1 + \eta_{t+1} = \Delta(\bar{\omega}) \quad (7)$$

where $\Delta(\cdot)$ is an increasing and differentiable function.³ Note that the optimal cutoff depends only on the external finance premium, reflecting the linearity of the problem; in particular, it is independent of j 's net worth.

²Two conditions are that (i) $\bar{\omega}$ times the hazard rate of H be increasing in $\bar{\omega}$ and (ii) that $(1 + \eta_{t+1})(1 - \zeta) < 1$.

³The conditions of the previous footnote ensure that $\bar{\omega}(1 - H(\bar{\omega})) + (1 - \zeta) \int_0^{\bar{\omega}} \omega_{t+1}^j dH(\omega_{t+1}^j)$ is maximized at some positive ω^* . BGG show that Δ is increasing and differentiable on $(0, \omega^*)$.

The optimal investment/net worth ratio, κ_{jt} , turns out to be a function of $\bar{\omega}$:

$$\kappa_{jt} = \Psi(\bar{\omega}) \quad (8)$$

where $\Psi(\cdot)$ is also increasing and differentiable.⁴ Since the cutoff $\bar{\omega}$ is independent of j , as we observed, κ_{jt} is the same for all j , and therefore the same as the *aggregate* ratio of investment to net worth:

$$\frac{Q_t K_{t+1}}{P_t N_t} = \Psi(\bar{\omega}) \quad (9)$$

Combining 7 and 9, one obtains the risk premium as an increasing function of the value of aggregate investment relative to aggregate net worth.

$$1 + \eta_{t+1} = \Delta(\Psi^{-1}(\frac{Q_t K_{t+1}}{P_t N_t})) \equiv F\left(\frac{Q_t K_{t+1}}{P_t N_t}\right) \quad (10)$$

where F is increasing and differentiable.

This completes the derivation of the optimal contract when R_{t+1}/S_{t+1} is known at the time of contracting. For most of the analysis of the paper, which deals with adjustment under perfect foresight, this is an appropriate assumption. If R_{t+1}/S_{t+1} is uncertain as of the time of contracting, matters can be considerably more complicated. In such cases, we assume that the cutoff $\bar{\omega}$ cannot depend on aggregate risk; this can be taken to be an approximation to the true optimal contract, or perhaps derived from more primitive assumptions on information and timing.⁵ Given that assumption, the preceding analysis survives intact, requiring only that R_{t+1}/S_{t+1} be replaced by its expectation at t .

Two additional details deserve comment. First, it is only a matter of accounting to show that the economy's net worth in any period t must equal aggregate capital income minus foreign debt repayment and monitoring costs, as given by equation 15 in the text. In particular, monitoring costs as a fraction of the return to capital are given by

$$\zeta \int_0^{\bar{\omega}} \omega_{t+1}^j dH(\omega_{t+1}^j) \equiv \Phi(1 + \eta_{t+1}) \quad (11)$$

since 7 implies $\bar{\omega} = \Delta^{-1}(1 + \eta_{t+1})$.

Second, our assumption in the text is that entrepreneurs consume a fraction $(1 - \delta)$ of their net worth and reinvest the rest. This can be derived from more primitive assumptions; for instance, one can assume that an individual entrepreneur j "dies" in period $t + 1$ with probability $(1 - \delta)$, and that surviving

⁴Over $(0, \omega^*)$, where ω^* is given in the previous footnote.

⁵To deal with aggregate risk, BGG assume that entrepreneurs bear all such risk, which is justified by their assumption that borrowers are risk neutral while lenders are risk averse. In contrast, we assume that both contracting sides are risk neutral.

entrepreneurs are patient enough so that they choose not to consume their wealth until death.⁶

2 Steady State and Linear Approximation

Here we sketch the proof of the existence and uniqueness of a non-stochastic steady state, describe our linear approximation to the equilibrium system, and discuss non-stochastic dynamics.

One can show that, in steady state, the Lagrange multiplier associated with 3 must equal the inverse of $\delta(1+\rho)$. The analysis of BGG shows that the Lagrange multiplier is an increasing function of $\bar{\omega}$, which goes to one as $\bar{\omega}$ goes to zero and to infinity as $\bar{\omega}$ goes to ω^* , where $\omega^* > 0$ is defined in their footnote 2. So, provided that $0 < \delta(1+\rho) < 1$ there is a unique, strictly positive, steady state solution for $\bar{\omega}$, with which one can pin down the values of η and $\frac{QK}{N}$ at the steady state.

Now we need to solve for the remaining variables, whose steady state levels are given by:

$$Y = AK^\alpha \quad (12)$$

$$Q = S^{1-\gamma} \quad (13)$$

$$\frac{\alpha Y}{QK} = (1+\rho)(1+\eta) \quad (14)$$

$$N + SD = QK \quad (15)$$

$$Y = \gamma[(1-\alpha)Y + QK] + SX \quad (16)$$

Using 12 and 13 into 16 we obtain

$$(1 - \gamma(1 - \alpha))Y - \gamma \frac{S^{1-\gamma}}{A^{1/\alpha}} Y^{1/\alpha} - SX = 0 \quad (17)$$

Now, using 12 and 13 into 14 we obtain

$$S^{1-\gamma} Y^{\frac{1-\alpha}{\alpha}} = \alpha \frac{A^{1/\alpha}}{(1+\rho)(1+\eta)} \quad (18)$$

which is a hyperbola in (Y, S) space. Using 18 in 17 we obtain

$$\left[1 - \gamma(1 - \alpha) - \frac{\alpha\gamma}{(1+\rho)(1+\eta)} \right] Y = SX \quad (19)$$

⁶To keep the number of entrepreneurs constant, one can assume that each dead entrepreneur is replaced by a newborn one. A minor problem arises since new entrepreneurs must have some initial net worth to be able to borrow. This can be remedied by assuming that new entrepreneurs are born with an exogenous and arbitrarily small endowment, or that they have a small endowment of labor (as in Carlstrom and Fuerst 1998). The effects of choosing either assumption would be negligible, and so we ignore this issue in the text.

Given that $1 - \gamma(1 - \alpha) - \frac{\alpha\gamma}{(1+\rho)(1+\eta)} = (1 - \gamma) + (\gamma\alpha - \frac{\alpha\gamma}{(1+\rho)(1+\eta)}) > 0$, this is a ray from the origin in (Y, S) space. The steady state values of Y and S must solve 18 and 19. Clearly, there is a unique positive solution. With this result in hand, it is simple to solve for the steady state values of K and Q using 12 and 13.

Log linearizing the equilibrium equations around the steady state just found, we obtain a system that includes, whether wages are flexible or sticky, equations 22, 23, and 24 in the text, the linearized version of the interest parity condition 12:

$${}_t y_{t+1} - (q_t - p_t) - k_{t+1} = \rho'_{t+1} + \eta'_{t+1} + {}_t (s_{t+1} - p_{t+1}) - (s_t - p_t) \quad (20)$$

and the following equation for the risk premium:

$$\begin{aligned} \eta'_{t+1} &= \phi \eta'_t + \mu (q_t - p_t + k_{t+1} - y_t) \\ &\quad + \mu \delta (1 + \rho) \psi [(s_t - p_t - {}_{t-1}(s_t - p_t)) - (y_t - {}_{t-1}y_t)] \end{aligned} \quad (21)$$

Here, $\lambda = \frac{\gamma QK}{\gamma QK + SX} = \frac{\alpha\gamma}{(1-\gamma+\alpha\gamma)(1+\rho)(1+\eta)} < 1$, $\mu = \frac{F'(\cdot) QK}{F(\cdot) N}$, and $\phi =$

$\delta(1 + \rho)(1 - \mu\psi) + \mu \{ \delta(1 + \rho)(1 + \psi)\eta + \delta(1 + \rho) - 1 \} \left(\frac{v}{\varepsilon_\Delta} \right)$ where v is the elasticity of $\int_0^{\bar{\omega}} \omega_{t+1}^j dH(\omega_{t+1}^j)$ with respect to $\bar{\omega}$ and ε_Δ is the elasticity of the Δ function in 10. Now 17 follows from 21 after using 22 to eliminate the term $(k_{t+1} + q_t - p_t)$ and recalling that the real exchange rate e_t equals $s_t - p_t$.

The calculation of the saddle-path $z_t = \zeta \eta'_t$ is straightforward. Without uncertainty, 17 and 18 can be written in matrix form as

$$\begin{bmatrix} z_{t+1} \\ \eta'_{t+1} \end{bmatrix} = \Phi \begin{bmatrix} z_t \\ \eta'_t \end{bmatrix} \quad (22)$$

We assume that the roots of Φ are real and that a saddle-path exists. Existence of a saddle-path requires the two roots to be on opposite sides of the unit circle; a sufficient condition is that $\phi < 1 + \mu$. The roots of Φ are real if $[(1 - \lambda)(1 + \mu) + (1 + \phi)\lambda]^2 > 4\phi\lambda$. Both sufficient conditions must hold, in particular, if μ is small enough – that is, if financial imperfections are not too stringent.

Standard techniques now yield the saddle-path coefficient $\zeta = \frac{\lambda}{\mu(1-\lambda)} \{ \xi_2 - \phi \}$, where ξ_2 is the *smaller* eigenvalue of Φ . The term in curly brackets is negative, and hence $\zeta < 0$ as stated in the text.

3 Optimality of Flexible Rates

Here we show the validity of an assertion in section 5. For all t , it is easy to show that taking logs in 12 one obtains

$$\log K_{t+1} = \log \alpha + \gamma \log Y_t + \log_t Z_{t+1} - \gamma \log Z_t - \log(1 + \rho_{t+1}) - \log(1 + \eta_{t+1})$$

So

$$\log K_{t+1} = \gamma[\alpha \log K_t + (1 - \alpha) \log L_t] + \Lambda_t$$

where the term $\Lambda_t = \log \alpha + \log {}_t Z_{t+1} - \gamma \log Z_t - \log(1 + \rho_{t+1}) - \log(1 + \eta_{t+1})$ is independent of monetary or exchange rate policy. Using the previous expression lagged one period to replace $\log K_t$, and continuing back to period 0, we get

$$\begin{aligned} \log K_{t+1} &= \gamma(1 - \alpha) \log L_t + \gamma\alpha[\gamma[\alpha \log K_{t-1} + (1 - \alpha) \log L_{t-1}] + \Lambda_{t-1}] + \Lambda_t \\ &= \gamma(1 - \alpha)[\log L_t + \alpha \log L_{t-1}] + (\gamma\alpha)^2 \log K_{t-1} + \Lambda_t + \gamma\alpha\Lambda_{t-1} \\ &= \dots \\ &= \gamma(1 - \alpha)[\log L_t + \alpha \log L_{t-1} + \dots + \alpha^t \log L_0] + (\gamma\alpha)^{t+1} \log K_0 + \Upsilon_{t+1} \end{aligned}$$

where Υ_{t+1} is, again, independent of policy, as claimed.