Rising longevity, education, savings, and growth

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Abstract

This paper examines the impact of declines in adult mortality on growth in an overlapping generations model. With public education and imperfect annuity markets, a decline in mortality affects growth through three channels. First, it raises the saving rate and thereby increases the rate of physical capital accumulation. Second, it reduces accidental bequests, lowers investment, and thereby lowers the rate of physical capital accumulation. Third, it may lead the median voter to increase the tax rate for public education initially but lower the tax rate in a later stage. Starting from a high mortality rate as found in many Third World populations, the net effect of a decline in mortality is to raise the growth rate. However, starting from a low mortality rate such as is found in most industrial populations, the net effect of a further decline in mortality is to reduce the growth rate. The findings appear consistent with recent empirical evidence.

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1. Introduction

In most industrialized countries, birth rates have been near or below the replacement level in recent decades, and life expectancy has been steadily increasing, leading to population aging. In these countries, there are growing concerns about the impacts of aging on economic growth and capital formation. Population aging has a possible adverse impact on national savings (e.g., Auerbach and Kotlikoff, 1992), and intensifies the
competition for resources between the consumption and health needs of the elderly on the one hand, and investment in children on the other (e.g., Preston, 1987). Lengthening life also means that bequests may be received later in life by the children of the elderly, and the amount of bequests may be diminished by longer consumption on the part of the elderly.

Recent empirical work indicates some economic consequences of rising longevity. Using the cross-section data set in Barro and Wolf (1989), Table 1 reports the average growth rates of per capita GDP (1960–1985) and the ratios of private investment to GDP (1970–1985) for countries with different initial life expectancy in 1960. According to this table, as the initial life expectancy rises from below 60 to 69, the investment ratio and the growth rate increase substantially; but when the initial life expectancy rises to 70 and over, the investment ratio and the growth rate drop, although their levels are still higher than those in countries with low life expectancy. These patterns of the investment ratio and the growth rate in relation to life expectancy in Table 1 are very similar to the historical movements of the investment ratio and the growth rate in Maddison (1991, 1992) that included more than 10 countries’ time series for about a century-long period in which longevity rose substantially. In a recent empirical study, Kelley and Schmidt (1995) found a positive effect of declines in mortality on growth in less developed countries from 1960 to 1990; but in developed countries, this effect diminishes over time and becomes negative in the 1980s.¹

The empirical evidence raises the possibility that the relation of economic growth to mortality decline is nonmonotonic.² Indeed, it would not be surprising if mortality decline did have different economic effects depending on its initial level, since its demographic effects are quite different. Starting from a high mortality, low life expectancy situation, mortality decline most strongly affects death rates in childhood. Population growth rates rise rapidly, making the population younger, not older (Lee, 1994). Rates of return to investment in human capital rise (Meltzer, 1992; but also see Preston, 1998, who finds that these effects are small). Eventually, parents recognize this improved child survival and may reduce fertility accordingly, which does initiate population aging. Once life expectancy has reached the mid to high 60s, however, further mortality decline occurs largely at the older ages, and population growth rates are not much affected (Lee, 1994). In this stage, the population grows older, and there is little effect on rates of return to investing in children, nor on parental fertility decisions as they seek to anticipate their children’s future mortality.

Ehrlich and Lui (1991) analyzed the earlier stage of mortality decline that is associated with the population transition. Our analysis will focus more on the later stage, in which the primary effect of mortality decline is to raise the life-cycle ratio of years lived in old age to years lived in the labor force. In an overlapping-generations model, this effect can be simply modeled as a declining adult mortality rate at the end of working age. We abstract

¹ When all countries are considered, Barro and Sala-i-Martin (1995, Ch. 12) found empirical evidence that initial life expectancy has highly significantly positive impacts on subsequent per capita growth and the ratio of investment to GDP. In both Kelley and Schmidt (1995) and Barro and Sala-i-Martin (1995), initial GDP per capita was used as an explanatory variable in addition to mortality in the growth equation, which reduces the possibility that the estimated contribution of mortality decline to growth reflects that of initial per capita income.

² This nonmonotonicity in the relationship between economic growth and mortality decline is not captured by our earlier work (see Zhang et al., 2001) that assumed altruistic bequests and perfect annuity markets. In order to produce this nonmonotonicity, we will depart from these assumptions in the present paper.
from child mortality and fertility choices since in most developed countries, mortality between birth and age 15 is around 2% or lower, and fertility rates are near or below the replacement level.

A key issue in a model with life-cycle savings and uncertain survival to retirement is how to treat savings left by agents who die before reaching old age. In this regard, we abstract from annuity markets so that these savings are given to the deceased savers’ children as bequests, as in Abel (1985, 1986), Feldstein (1990), and Huggett (1996) where the adverse selection problem in annuity markets leads to underprovision of annuity insurance and hence unintended bequests.

In general, the assumption of different capital markets alters agents’ expected returns to saving and may lead to very different results. In Ehrlich and Lui (1991), resources saved by agents who die before entering old age are neither given to their children nor to old-age survivors, which enables them to focus on a representative family. Once unintended bequests are considered, agents are heterogeneous because the amounts of their received bequests depend on their families’ entire mortality history. A fraction of accidental bequests is invested via life-cycle saving and education spending by the working-age population, whereas old-age survivors consume their savings. As rising longevity raises the ratio of dissavers to savers, considering accidental bequests with imperfect capital markets may help to understand the full effect of rising longevity on capital accumulation.3

We examine the effect of longevity on growth in a model with some realistic elements: uncertain survival to retirement at an individual’s level, imperfect capital markets, and public education. We show that a decline in mortality affects growth through three channels. First, it raises the saving rate and hence the rate of physical capital accumulation. Second, it reduces accidental bequests, lowers investment, and thereby lowers the rate of physical capital accumulation. Third, it may lead the median voter to increase the tax rate for public education at low longevity, but beyond some level of longevity, further declines in mortality may lead the median voter to lower the tax rate, so the rate of human capital accumulation may rise initially but may eventually fall. Starting from a high mortality rate, as found in many Third World populations, the net effect of a decline in mortality is to raise the growth rate. However, starting from a low mortality rate such as is found in most industrial populations, the net effect of a further decline in mortality is to reduce the growth rate. These findings justify the concerns about the possible adverse growth effect of rising longevity in developed countries, reconcile the differential mortality-growth patterns depending on initial mortality, and are consistent with the recent empirical evidence in Kelley and Schmidt (1995).

3 Kotlikoff and Summers (1981) found that bequests account for a substantial fraction of aggregate capital accumulation in the United States.

Table 1
Investment ratios, growth rates, and initial life expectancy

<table>
<thead>
<tr>
<th>Life expectancy at birth in 1960</th>
<th>&lt;60</th>
<th>60–64</th>
<th>65–69</th>
<th>≥ 70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of countries</td>
<td>41</td>
<td>8</td>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>Average growth rates (1960–1985 (%))</td>
<td>1.88</td>
<td>3.18</td>
<td>3.36</td>
<td>2.50</td>
</tr>
</tbody>
</table>
The remainder of this paper is organized as follows. Section 2 introduces the model. Section 3 derives the results. Section 4 extends the analysis to consider more rational elements in the decision on the tax rate, such as the tax effect on the future return to saving (the after-tax interest rate in the next period). Section 5 gives some concluding remarks.

2. The model

The model has a single good, an infinite number of periods, and overlapping generations of three-period-lived agents. Agents learn in school in childhood, work in young adulthood (one unit of labor time each), and live in retirement in old adulthood. Each young agent has exactly one child, who survives to young adulthood with certainty. The population of young agents is thus constant and its mass is normalized to one, implying that aggregate capital stock or saving is the same as its average per worker. Young agents save for old-age consumption, and finance public education for the next generation by a flat-rate labor income tax through voting. We will also discuss cases with taxes on all sources of income in Section 4.

There is a probability \( p \in (0, 1) \) for a young person to die before entering old age but after saving and investing in child education. As in Abel (1985), there is no annuity market, so the saving of a deceased person becomes an accidental bequest to his child. The amount of bequests in a family thus depends on the family’s mortality history. Let subscript \( t \) denote a period in time, and superscript \( j \) the type of agents in a generation. Define a type-\( j \) agent as one whose exactly \( j \) consecutive previous generations have died at the end of their young age. There are infinitely many types of agents and the number of type-\( j \) agents is \((p^j)(1-p)\). Here and in what follows, \( j \) indicates a type-\( j \) agent (exponential) if \( j \) is a superscript on a variable without (with) parenthesis.

Through public schooling, the human capital of a type-\( j \) agent, \( h^t_j \), accumulates according to

\[
\frac{h^t_j}{C_0} = Aq_{t-1}^2 h^{t-1}_{t-1}, \quad 0 < \alpha < 1
\]

where \( q_{t-1} \) is the average amount of goods invested by the preceding generation for public schooling and \( h_{t-1} \) is the average human capital of the preceding generation. The

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4 Our model is a substantial extension of Abel (1985). He assumes that an individual’s labor income and the interest rate are constant. For the purpose of our paper, we consider general equilibria and endogenous growth as pioneered by Romer (1986) and Lucas (1988).

5 According to the US Department of Education (1989), over the last 100 years, the fraction of students at the primary and secondary levels who attend public schools has never been below 79% in the United States.

6 The standard overlapping-generations model implicitly assumes that age at retirement is exogenously fixed. However, one may expect the retirement age to increase with longevity. In fact, the age at retirement has not risen in recent decades, but rather declined, and remained constant in recent years in the United States, despite the rise in longevity. See Wise (1997).

7 Other major bequest motives are altruism and exchange. While there is no consensus on which bequest motive dominates (see, e.g., Altonji et al., 1997), Hurd (1997) argues that bequests are largely accidental. In our model, altruism will be reflected in the form of intended human capital investment, perhaps the most important part of intended intergenerational transfers.
inclusion of the investment of goods in education receives empirical support from Altonji and Dunn (1996) that found a substantial positive effect of expenditures per pupil on the wages of high school graduates. Considering solely public schooling in Eq. (1) makes the analysis much easier, since agents in the same generation have the same stock of human capital from public schooling, i.e. $ht^j = h_t$.

The utility function is logarithmic and defined over life-cycle consumption ($c_{1,t}^j$ and $c_{2,t+1}^j$) and the quality of schools for children ($q_t$):

$$U_t^j = \ln c_{1,t}^j + (1 - p)\delta \ln c_{2,t+1}^j + \phi \ln q_t, \quad 0 < \delta < 1, \quad 0 < \phi < 1.$$  

In this function, subscripts 1 and 2 of consumption variables refer to young and old age, respectively; $\delta$ and $\phi$ are the subjective discount factors on the expected utility from old-age consumption and the quality of schools for children, respectively. The restriction $\phi < 1$ says agents are more concerned about their own young-age consumption than the quality of schools for their children. Here, the utility from uncertain life-cycle consumption is based on Yaari (1965) and Abel (1985, 1989), while the utility from the quality of schools for children is based on Glomm and Ravikumar (1992).

The young-age consumption of a type-$j$ agent is given by

$$c_{1,t}^j = b_{t}^j (1 + r_t) + w_{t}^j (1 - \tau_t) - s_{t}^j$$  

where $b_{t}^j$ is the bequest received at the beginning of $t$, $s_{t}^j$ the amount of saving, $r_t$ the interest rate, $w_{t}^j$ the wage rate, and $\tau_t$ the wage tax rate. The old-age consumption of a type-$j$ agent is

$$c_{2,t+1}^j = (1 + r_{t+1})s_{t}^j.$$  

As implied by the logarithmic utility function, young-age consumption and old-age consumption account for some constant fractions of lifetime resources. Since the utility-maximizing problem is the same ex ante for different agents given their different lifetime resources, these fractions are expected to be the same for all agents. Denoting young-age consumption as a fraction of after-tax young-age income by $\gamma_c$, whose value will be determined later, we rewrite Eq. (2) as

$$c_{1,t}^j = \gamma_c[b_{t}^j (1 + r_t) + w_{t}^j (1 - \tau_t)].$$  

By Eqs. (2)–(4), the ratio of the present value of old-age consumption to after-tax young-age income is $1 - \gamma_c$:

$$c_{2,t+1}^j/(1 + r_{t+1}) = (1 - \gamma_c)[b_{t}^j (1 + r_t) + w_{t}^j (1 - \tau_t)].$$  

Agents who die at the end of young age leave their savings as bequests (no bequest otherwise)

$$b_{t}^j = s_{t-1}^j, \quad j \geq 1; \quad b_0^j = 0, \quad j = 0.$$  

For type-0 young agents, labor earnings are therefore the sole source of young-age income.
The production function is

$$y_t = D k_t^\theta \left[ \sum_{j=0}^{\infty} h_t^j l_t(p)^j (1 - p) \right]^{1-\theta} = D k_t^\theta h_t^{1-\theta}, \quad D > 0, \quad 0 < \theta < 1 \quad (7)$$

where $k_t$ is the aggregate or average physical capital stock and $l_t(1)$ is the labor input per worker. Physical capital fully depreciates in one period. Factors in production receive their marginal products:

$$w_t^j = (1 - \theta) D (k_t / h_t)^\theta h_t^j = (1 - \theta) De_t^\theta h_t^j, \quad (8)$$

$$1 + r_t = \theta D (h_t / k_t)^{1-\theta} = \theta De_t^{\theta-1} \quad (9)$$

where $e = k(t)/hl = k/h$ is the physical–human capital ratio. Since $h_t^j = h_t$, we have $w_t^j = w_t$, i.e. wage rates are the same for all workers in the same generation. Markets clear when

$$k_{t+1} = \sum_{j=0}^{\infty} s_t^j (p)^j (1 - p). \quad (10)$$

With a labor income tax, the government budget constraint is

$$q_t = \tau_t \sum_{j=0}^{\infty} w_t^j (p)^j (1 - p) = \tau_t w_t. \quad (11)$$

3. Steady-state equilibrium and results

We solve the utility-maximizing problem in two stages. First, young agents choose saving subject to Eqs. (1)–(3) and (8), taking as given $\tau_t$, $b_t^j$, $e_t$ (hence $r_t$), $q_{t-1}$, and $h_{t-1}$. Second, agents choose their preferred tax rates, and the effective tax rate is determined in a political equilibrium.

The first-stage problem yields

$$\gamma_c = \frac{1}{1 + \delta (1 - p)}, \quad 1 > \gamma_c > 0 \quad (12)$$

where the ratio of young-age consumption to after-tax young-age income $\gamma_c$ is independent of the tax rate $\tau_t$ and falls with the discount factor on the expected utility from old-age consumption $\delta$. Also, $\gamma_c$ rises with $p$, that is, the fraction of income spent on young-age consumption rises with the mortality rate at which agents die before reaching old age.

Eqs. 2–6 imply

$$s_t^0 = (1 - \gamma_c) w_t (1 - \tau_t) \quad (13)$$
which determines the saving of a type-0 young agent who receives no bequest. Also, Eqs. 2–6 provide

\[ s^j_t = (1 - \gamma_c) b^j_t (1 + r_t) + s^0_t. \]  

(14)

Eq. (14) links a type-\( j \) agent’s saving to his received bequest, and to a type-0 agent’s saving. Eqs. (13) and (14) further imply that

\[ s^j_t = (1 - \gamma_c)[b^j_t (1 + r_t) + w_t (1 - \tau_t)]. \]  

(15)

In Eq. (15), \( 1 - \gamma_c \) is the saving rate. By Eq. (12), \( 1 - \gamma_c = \delta(1 - p)/[1 + \delta(1 - p)] \); that is, the saving rate rises with life expectancy, or falls with the mortality rate.

Using Eq. (10) and summing up Eq. (14) by weighting each type according to its share \( (p^j) \) for \( j \geq 0 \), we get

\[ k_{t+1} = s_t = (1 - \gamma_c)(1 + r_t) b_t + s^0_t \]  

(16)

where \( s_t \) and \( b_t \) stand for their aggregates or averages. Eq. (16) relates aggregate (average) physical capital stock in the next period to the aggregate (average) bequest at the beginning of the current period and to the current saving of type-0 agents.

Since a fraction \( p \) of agents in each type die at the end of their young age, the aggregate (average) bequest is a fraction \( p \) of the preceding period’s aggregate (average) saving. Combining this observation with Eq. (10), we get

\[ b_t = p s_{t-1} = p \sum_{j=0}^{\infty} s^j_{t-1} (p^j)/(1 - p) = p k_t. \]  

(17)

The relationship \( b_t = p k_t \) in Eq. (17) says that a fraction \( p \) of physical capital stock takes the form of unintended bequests.

Denote the growth rate of physical capital as \( 1 + g_{k,t} = k_{t+1}/k_t \). From Eqs. (16) and (17), we have

\[ k_{t+1} = s_t = \frac{1 + g_{k,t}}{1 + g_{k,t} - (1 - \gamma_c)(1 + r_t) p} s^0_t. \]  

(18)

In Eq. (18), the next period aggregate physical capital stock is related to current type-0 agents’ saving; this relationship depends positively on the saving rate and the interest rate but negatively on the growth rate. It is also interesting to observe the direct positive effect of the mortality rate \( p \) on the relationship between the next-period physical capital and the current-period savings of type-0 agents in Eq. (18). If \( p = 0 \) (i.e. survival to old age is certain), then there is no accidental bequest and \( k_{t+1} = s_t = s^0_t \) by Eq. (18). If \( p > 0 \), then there are accidental bequests that contribute to saving and physical accumulation: \( k_{t+1} > s^0_t \) must hold under \( p > 0 \). From Eqs. (8), (9), (13), and (18), the growth rate of physical capital is

\[ k_{t+1}/k_t = 1 + g_{k,t} = D(1 - \gamma_c)[\theta p + (1 - \tau_t)(1 - \theta)] e_t^{\theta - 1}. \]  

(19)
In Eq. (19), a decline in mortality has a direct negative impact on aggregate (average) physical capital accumulation that will be attributed to the response of accidental bequests later.

From Eqs. (1), (8), and (11), on the other hand, the growth rate of human capital is

\[ h_{t+1}/h_t = 1 + g_{h,t} = A\tau^x_t(1 - \theta)^x_D^x e^{\theta_0}. \] (20)

Eqs. (19) and (20) yield the evolution of the physical–human capital ratio

\[ e_{t+1} = \frac{D^{1-x}(1 - \gamma_c)[p\theta + (1 - \tau)(1 - \theta)]}{A\tau^x_t(1 - \theta)^x} e^{\theta(1 - \theta)}_t. \] (21)

Solving the steady-state ratio \( e^* = e_t = e_{t+1} \) and using it in either Eq. (19) or Eq. (20), we obtain the balanced steady-state growth rate as

\[ 1 + g = \left\{ [AD^x\tau^x(1 - \theta)^x]^{1-\theta} D[(1 - \gamma_c)]^{\theta(1 - \theta)} \frac{(1 - \theta)(1 - \gamma_c)}{(1 - \theta)^x} \right\}^{\frac{1}{1-\theta}}. \] (22)

From Eq. (22), longevity or mortality rates affect growth directly through the last factor on the right-hand side that contains \( p \), and indirectly through two possible channels: the saving rate, \( 1 - \gamma_c \), and the tax rate for public education, \( \tau \). Eq. (22) reveals the relationships between the long-run balanced growth rate and its determinants as

**Proposition 1.** \( \partial g / \partial (1 - \gamma_c) > 0; \ \partial g / \partial \tau > 0 \) if \( \tau < 1 - \theta + p\theta \).

(All proofs are relegated to Appendix A.)

From Proposition 1, a rise in the saving rate \( 1 - \gamma_c \) raises the growth rate by speeding up physical capital accumulation, while taxing labor income to finance public education is growth enhancing if the tax rate is not too high.8 Intuitively, a high tax rate represents a high degree of tax distortion which can dominate the gain in growth performance through public education spending; such conflicting contributions of public education to growth are echoed in the first factor and the third factor in the growth equation. For appropriate values of labor’s share parameter \( 1 - \theta \), the restriction on the tax rate for its positive influence on growth is certainly satisfied in practice, since the value of labor’s share parameter is about 2/3 while the ratio of public spending on education to gross income is below 10% in almost all countries. Therefore, increasing the tax rate to finance education stimulates per capita growth. Our determination of growth differs from that in Ehrlich and Lui (1991) where life-cycle saving has either no, or even a negative impact on long-run growth. The difference comes from the fact that the investment of goods in education is

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8 Barro and Sala-i-Martin (1995) found empirical evidence that the ratio of public education spending to GDP has a significantly positive effect on growth. However, the evidence on the contribution of savings or investment in physical capital relative to GDP is more mixed. In Barro and Sala-i-Martin (1995), the growth effect of the ratio of investment in physical capital to GDP is positive, but its statistical significance depends on model specifications and estimation methods. This positive effect is significant in many growth regressions as seen in Levine and Renelt (1992), Mankiw et al. (1992), and DeLong and Summers (1991). However, in some (e.g., Blomstrom et al., 1996), it is doubted as reflecting reverse causation from growth to investment.
considered in our model (i.e. \( \alpha > 0 \)), but not in their model. If \( \alpha = 0 \), the saving rate would play no role in the determination of growth in Eq. (22).

What are the growth implications of rising longevity? We look for the answers in two cases. First, we simply take the tax rate as exogenous as often seen in the literature. The advantage of doing so is to focus on the growth effects of longevity through saving which, we believe, are not well understood in the literature on longevity and growth. Then, we will determine the tax rate and see how longevity affects growth through human capital investment.

3.1. Exogenous tax rate

**Proposition 2.** For a given tax rate to finance public education, \( \frac{\partial g}{\partial p} < 0 \) unless \( \tau \geq \{1 - \Theta(1 - p)[2 + \delta(1 - p)] \}/(1 - \Theta) \). In particular, if \( p \) is sufficiently high, then \( \frac{\partial g}{\partial p} < 0 \); if \( p \) is low, then the sign of \( \frac{\partial g}{\partial p} \) is ambiguous in general and positive for sufficiently large \( \delta \), \( \Theta \), and \( \tau \).

For an exogenous tax rate, according to Proposition 2, rising longevity increases the growth rate if initial mortality is high. If initial mortality is low, rising longevity may have a positive, zero, or negative growth effect, depending on the share parameter of physical capital in the final goods production, the discount factor on the utility from old-age consumption, and the tax rate. Starting with low mortality, the effect of rising longevity on growth is more likely to be negative the larger are the discount factor, the share parameter, and the tax rate.

For a given tax rate, the longevity–growth relation depends on how longevity affects the ratio of saving (physical capital investment) to aggregate output \( s/yt \). Eqs. (8), (9), (15), and \( y_t = k(1 + r_t) + w_t \) lead to \( s/yt = (1 - \gamma_c)(\theta p + (1 - r)(1 - \Theta)) \) which corresponds to the last two factors in the growth equation. In general, there are opposing forces of rising longevity on aggregate saving: a positive one as the saving rate rises with longevity, and a negative one as the average unintended bequest falls with longevity. The saving rate effect is obvious: with higher likelihood to survive to old age, young agents save more for old-age consumption. According to Eq. (12), the saving rate, \( 1 - \gamma_c = \delta(1 - p)[1 + \delta(1 - p)] \), is concave in the rate of survival, \( 1 - p \); that is, the higher the longevity, the weaker the saving rate effect will be.

The second effect of longevity on saving via bequests is less obvious. According to Eqs. (13) and (16), \( s_f = (1 - \gamma_c)[b_f(1 + r_f) + w_f(1 - \tau_f)] \), which means that the aggregate saving \( s_f \) is positively related to the saving rate \( 1 - \gamma_c \) and the aggregate bequest \( b_f \). A rise in longevity lowers the aggregate bequest given the aggregate saving in the preceding period as seen in Eq. (17), and thereby reduces the aggregate saving or physical capital stock through Eq. (16). In the growth equation, the bequest effect takes the form of the direct growth effect of \( p \) whose strength is affected by the share parameter of physical capital in the final goods sector \( \Theta \). Intuitively, as more workers survive to old age, a larger fraction of aggregate saving will be used as old-age consumption (or dissaving), leaving a smaller fraction as bequests. Given the fact that part of the received bequests \( b_f \) is saved by young adults in Eq. (16), the reduction in bequests implies a decline in current saving \( s_t \) (hence, physical capital for the next period). The net effect is likely to be large and positive
(small or negative) when initial longevity is low (high), since the saving rate effect diminishes with longevity.

3.2. Endogenous tax rate: political equilibrium by majority voting

What happens to the longevity-growth relation if the tax rate is endogenously determined through voting? We first derive the preferred tax rate of agents in each type denoted by \( \tau_j^t \), and then determine the equilibrium tax rate preferred by a majority of voters. Note that old agents’ welfare is not affected by the labor income tax for public education which affects their grandchildren’s education. From this observation and for simplicity, we assume that old agents do not vote for the political-equilibrium labor-income tax rate since they are indifferent among the outcomes.\(^9\)

Given the solution for the first-stage optimization, the problem in the second stage is

\[
V_j^t = \max_{\tau_j^t} \{ \ln \gamma_c + [1 + \delta(1 - p)] \ln [b_j^t (1 + r_t) + w_t (1 - \tau_j^t)] \\
+ \delta (1 - p) \ln [(1 - \gamma_c) (1 + r_{t+1})] + \phi \ln \tau_j^t + \phi \ln w_t \}
\]

taking as given \( b_j^t, e_t, r_t, r_{t+1} \), and \( w_t \), where \( \gamma_c \) is independent of \( \tau_j^t \) by Eq. (12) (we will consider the effect of the tax on the next-period interest rate \( r_{t+1} \) in Section 4). This leads to

**Proposition 3.** The preferred tax rate of type-\( j \) agents is

\[
\tau_j^t = \frac{\phi}{1 + \phi + (1 - p) \delta} \left( 1 + \frac{\theta p}{1 - \theta} \frac{b_j^t}{b_j} \right).
\]

Also, \( d\tau^0/dp > 0 \).

Three features of the preferred tax rate merit some comments. First, given initial bequests, the preferred tax rates of agents in all types fall with longevity. This feature reflects the need for more old-age consumption through savings if the chance of surviving to old age is improved. Thus, rising longevity draws more resources away from investment in children’s human capital to own old-age consumption. Second, the tax rate preferred by an agent receiving a positive bequest increases with the ratio of his received bequest to the average bequest. The second feature arises from the labor income tax structure and the fact that the quality of schools for children is a normal good for young parents. Since children’s education is a normal good, young households that receive more bequests opt for a higher labor income tax to finance education. Third, type-0 agents’ preferred tax rate is time invariant.

Now, we consider the political equilibrium tax rate through voting. Since only young agents vote on the labor income tax rate, each type of voter accounts for the fraction

\(^9\) Alternatively, one can assume that old agents vote for the same labor income tax rate as they did in young age. This assumption will only increase the number of voters in each type from \((p)(1 - p)\) to \((2 - p)(p)(1 - p)\) without affecting the essence of the results.
(p)(1−p) of the whole voting generation, and no single type of agent can form a majority for p ≥ 1/2. When p < 1/2, type-0 agents represent the majority of the working population. By Proposition 3, agents who receive more bequests prefer higher labor income tax rates. In the political equilibrium, the only outcome is the median agents’ preferred tax rate because agents with median bequests and less (more)—a majority of voters—prefer this tax rate to any higher (lower) tax rates, given the fact that V′(τ̄) < 0 (see the proof of Proposition 3 in Appendix A).

More specifically, the median agents are identified by type j* such that Σi≥j∗(p)(1−p) ≥ 1/2 and Σi<j∗(p)(1−p) ≥ 1/2. As a result, Σi>j∗(p)(1−p)=0 and hence j* as a function of p is given by

\[ j^*(p) = \left\lfloor \log(1/2)/\log(p) \right\rfloor \]

where the real value in the bracket is converted to the nearest integer below it.

Given any p and hence j*(p), what will be the corresponding steady-state equilibrium tax rate? Use the balanced growth condition s^i/s^i−1 = s^i/s^i−1 and denote \( a^i = (1−\gamma)(1+r)/(1+r)/(1+g_{kt}) \). From Eqs. (9) and (19), \( a^i = \theta/[\theta p + (1−\tau)(1−\theta)] \). Rewrite \( s^i = (1−\gamma)/(1+r)/(1+g_{kt}) \) and denote \( s^i = (1−\gamma)/(1+r)/(1+g_{kt}) \) as \( s^i = a s^i−1 + s^i_0 \) according to Eqs. (6) and (14). Thus, for \( a_i < 1 \), we have

\[ s^i = [1−(a_i)^{j+1}]s^i_0/(1−a_i). \]

Eqs. (6), (17), (18), and (24) imply

\[ b^i/b_i = s^i−1/(ps^i−1) = [1−(a_i)^{j}](1−a_i p)/(p(1−a_i)). \]

Eq. (25), Proposition 3, and \( \tau_* = \tau_{i}^{j*} \) lead to the equilibrium tax rate

\[ \tau_{i}^{j*} = \frac{\phi}{1 + \phi + \delta(1−p)} \{ 1 + \theta[1−(a_i)^{j*}](1−a_i p)/(1−\theta)(1−a_i) \} \]

and

\[ a_i = \theta/[\theta p + (1−\tau_{i}^{j*})(1−\theta)]. \]

Eqs. (26) and (27) then jointly determine the solution for \( a_i \) and \( \tau_{i}^{j*} \). Given any value of p and hence \( j^* \), both \( a_i \) and \( \tau_{i}^{j*} \) are in fact time invariant: \( a_i = a \) and \( \tau_{i}^{j*} = \tau^* \). The impact of a change in longevity on balanced steady-state growth can now be examined.

**Proposition 4.** With the tax rate for public education endogenously determined by a majority of voters, \( dg/dp < 0 \) if p is large enough; if p is small enough, then \( dg/dp > 0 \) when \( \delta \) and \( \theta \) are sufficiently large.

With an endogenous tax rate, there are three forces of rising longevity on per capita growth: a negative one via the fraction of income invested publicly in human capital (\( \tau^* \)),

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10 Let \( \tau^* \) be the equilibrium tax rate. Since \( V'((\tau^*))/0 < 0 \) for voters whose preferred tax rates \( \tau_i \), at which \( V'((\tau^*))/0 = 0 \), are above (below) the equilibrium tax rate \( \tau^* \); namely, such agents will vote rationally for the highest (lowest) possible tax rate that can be supported by a majority so long as \( \tau_i < \tau^* \) (≥).
and the others via saving (as discussed earlier). It is interesting to see whether the negative effect of rising longevity on human capital investment has any empirical support. Cutler et al. (1993) listed some empirical studies that found significant negative effects of the elderly share of the population on public education spending per pupil at state, county, town, or education district levels in the United States. A more recent study of Poterba (1997) also found that raising the fraction of elderly residents in a state is associated with a significant reduction in per-child education spending by using panel data for the states of the United States over the 1960–1990 period.  

The net impact of rising longevity on growth is complicated and nonmonotonic. When adult mortality is very high, expected old-age consumption (hence saving) is low, and thus the positive growth impact of a decline in mortality through increasing saving dominates. However, when adult mortality falls further, the positive saving effect diminishes, and the negative human capital investment effect and the negative bequest effect may eventually dominate. With sufficiently low mortality, a negative growth effect of a decline in mortality is guaranteed given plausible values for physical capital’s share parameter and the discount factor (e.g., \( \delta \) is above 0.9 and \( \phi \) above 1/3).

Given plausible values for the share parameters and the discount factor, what is the likely long-run longevity-growth relationship for the whole range of the mortality rate? As reported in Table 2, when the mortality rate falls from 0.98, the ratio of aggregate saving to aggregate income, \( s/y_\tau = (1 - \gamma_c)[\theta p + (1 - \tau^\phi)(1 - \theta)] \), rises first (substantially initially) and declines slightly at the final stage, whereas the ratio of public education investment to aggregate income, \( q/y_\tau = \tau^\phi (1 - \theta) \), declines. As a result, the net effect of rising longevity on growth is first positive and then negative. These experiments indicate that longevity stimulates growth in countries with high mortality rates, but depresses growth in countries with low mortality rates.

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\[ \text{Table 2} \]

Growth effects of rising longevity with labor income taxes (I)

\[ x = 0.15, \quad \delta = 0.995, \quad \theta = 0.36, \quad \phi = 0.06, \quad \text{and} \quad A = D = 3.0 \]

<table>
<thead>
<tr>
<th>Adult mortality ((p))</th>
<th>Saving/income ((s/y_\tau %))</th>
<th>Public education/income ((q/y_\tau %))</th>
<th>Annual growth ((g^{1/25} %))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.98</td>
<td>1.83</td>
<td>5.58</td>
<td>2.18</td>
</tr>
<tr>
<td>0.90</td>
<td>8.25</td>
<td>5.29</td>
<td>2.63</td>
</tr>
<tr>
<td>0.80</td>
<td>14.59</td>
<td>4.87</td>
<td>2.77</td>
</tr>
<tr>
<td>0.72</td>
<td>18.62</td>
<td>4.47</td>
<td>2.80</td>
</tr>
<tr>
<td>0.60</td>
<td>23.31</td>
<td>3.72</td>
<td>2.77</td>
</tr>
<tr>
<td>0.50</td>
<td>26.07</td>
<td>3.53</td>
<td>2.77</td>
</tr>
<tr>
<td>0.40</td>
<td>28.44</td>
<td>2.32</td>
<td>2.56</td>
</tr>
<tr>
<td>0.30</td>
<td>29.81</td>
<td>2.19</td>
<td>2.54</td>
</tr>
<tr>
<td>0.20</td>
<td>30.64</td>
<td>2.07</td>
<td>2.52</td>
</tr>
<tr>
<td>0.04</td>
<td>31.04</td>
<td>1.91</td>
<td>2.48</td>
</tr>
<tr>
<td>0.02</td>
<td>30.99</td>
<td>1.89</td>
<td>2.47</td>
</tr>
</tbody>
</table>

It should be noted that the effect of population aging on education spending is a controversial subject, and that conflicting empirical results have been reported.
It is interesting to see that the simulated ratio of saving to income in Table 2 moves in a way similar to that in Table 1 for the groups of countries differentiated by initial life expectancy. It is also interesting to see that the net effect on growth turns from positive to negative around $p = 0.72$. This pattern of changes in the growth rate caused by rising life expectancy is consistent with the observed one in Table 1 and the empirical evidence in Kelley and Schmidt (1995).

We can also consider a uniform income tax on labor income $w_t$, unearned income $b_t(1 + r_t)$, and capital income $r_t s_t$. With the uniform income tax individuals’ budget constraints become $c_{t, t+1} = [b_t(1 + r_t) + w_t](1 − r_t) − s_t$ and $c_{t+1, t+1} = [1 + r_t + (1 − r_t + 1)] s_t$. The optimal tax rate is then $r_t = \tau^* = \phi / [1 + \phi + \delta(1 − p)]$. This uniform income tax rate is preferred by all young adults (the majority of voters), and is lower at a lower mortality rate. The saving rate is the same as that with the labor income tax. It can be verified that investments in both human and physical capital respond to mortality decline in ways similar to those with labor income taxation. Thus, Proposition 4 holds with uniform income taxation. A numerical example is given in Table 3 using the same parameterization as in Table 2. The numerical results in Table 3 are similar to those with labor income taxes in Table 2.

4. Equilibrium with more rational voting

In the preceding sections, atomistic voters only considered the direct impacts of taxes. There are two additional channels through which taxes affect individual voters, however. One of them is that voters could rationally take into account how the tax affects investment decisions of other individuals, and thus the aggregate physical–human capital ratio in the next period ($c_{t+1}$), which in turn determines the return to their own saving ($r_{t+1}$). The other is that the tax rate by the next generation ($r_{t+1}$) will reduce the after-tax return to current savings via $1 + r_{t+1}(1 − r_{t+1})$ in the case with uniform income taxation. Rational
voters may have an incentive to institutionize a stationary tax rate that takes into account this latter tax effect by setting $\tau_t = \tau_{t+1}$.

It is unclear to what extent voters will have these rational considerations or whether they have sufficient information for such rational choices. We expect the realistic voting outcome to lie somewhere between those with or without such considerations. In this section, we consider these rational elements. Unfortunately, when these rational voting motives are incorporated in the model, there is no analytical solution. We therefore only provide simulation results in steady-state balanced equilibrium.

4.1. A labor income tax

With a labor income tax and with the consideration of the tax effect on the physical–human capital ratio, the utility-maximizing problem in the first-stage gives the same saving rate as before. Using Eqs. (9) and (21), the next-period interest rate, $1 + r_{t+1}$, is an increasing function of the current tax rate $s_t$ through $[\theta p + (1 - \tau_t)(1 - \theta)]^{\phi/\gamma} = 1/\tau_t^{\phi(1-\theta)}$. Intuitively, workers may have the incentive to benefit from the fact that a higher tax rate now improves the return to savings in the next period. The second-stage problem, max$_{s_t}$\{ln $c_t + (1 + \delta)(1 - \rho)$ln $c_t + \phi \ln \tau_t + \phi \ln w_t$\}, is then equivalent to:

$$\max_{s_t} \left\{ [1 + \delta(1 - \rho)]\ln \left[ b_t^\prime (1 + r_t) + w_t (1 - \tau_t) \right] + \delta(1 - p)\ln \left[ (1 - \gamma_t)(1 + r_{t+1}) \right] + \phi \ln \tau_t + \phi \ln w_t \right\}.$$

The first-order condition of this problem is:

$$\frac{w_t [1 + \delta(1 - \rho)]}{b_t^\prime (1 + r_t) + w_t (1 - \tau_t)} = \frac{\phi \tau}{\phi + \delta(1 - p)(1 - \theta)} \left( \frac{\phi}{\theta p + (1 - \tau_t)(1 - \theta)} + \frac{\delta(1 - p)(1 - \theta)}{\tau_t} \right).$$

The second and third terms in the right-hand side capture the benefits of the tax effect on the physical–human capital ratio $e_{t+1}$.

According to Eqs. (25) and (27), $b_t^\prime / b_t = (1 - (a)\theta)/(\theta p (1 - a))$ and $a = \theta/\theta p + (1 - \tau)(1 - \theta)$ in steady-state growth equilibrium. The median voter $j^*$ is the same as that in Eq. (23). We can then simulate the model for steady-state solutions that correspond to different mortality rates. The simulation results are reported in Table 4 which uses the same parameterization as in Table 2.

In Table 4, the ratio of saving to output responds to mortality decline in a way similar to that in Table 2, but the ratio of investment in human capital to output behaves quite differently. When the tax effect on the next-period interest rate is considered in the tax rate decision, the human capital investment ratio rises when mortality is initially high; it even rises when mortality is initially moderate or low, so long as the median type $j^*$ remains the same. This rise in human capital investment is driven by the realization that a higher tax rate now leads to higher human capital relative to physical capital later, and hence a higher return to saving later. However, human capital investment to output tends to fall with longevity at moderate or low mortality rates when the median type $j^*$ switches to a lower
value, for the same reason as was given in previous sections. As a result, the growth rate of per capita output rises initially in response to mortality decline when saving and human capital investment are all rising sharply. In a later stage, the growth rate may rise or fall and tends to be lower than its high level reached at the middle stage.

4.2. A uniform income tax

Again, the saving rate $1 - \gamma_c$ from the first-stage problem is the same as before. The uniform income tax is chosen by solving

$$\max_s \{[1 + \delta(1 - p)]\ln(1 - \tau_s) + \delta(1 - p)\ln[1 + r_{t+1}(1-\tau_{t+1})] + \phi \ln \tau_t\} \text{ subject to } \tau = \tau_t = \tau_{t+1}, \ r_{t+1} = \theta D e_{t+1}^\theta - 1 \text{ from Eq. (9), and} \ e_{t+1} = D(1 - \gamma_c)[p\theta + 1 - \theta]e_t^\theta /\{A \tau_t^\theta [D e_t^\theta - (1 - p)e_t^\theta] e_t^{\phi - 2}\}.\text{ Considering both } \tau_t \text{ and } \tau_{t+1} \text{ simultaneously, voters would gain from recognizing the tax effects on the return to saving that arise indirectly through the physical–human capital ratio } e_{t+1} \text{ or the interest rate } r_{t+1}, \text{ and directly through } \tau_{t+1}. \text{ The optimal tax rate should satisfy}

$$\frac{\delta(1 - p)r_{t+1}}{1 + r_{t+1}(1 - \tau)} + \frac{1 + \delta(1 - p)}{1 - \tau} = \frac{\phi}{\tau} + \frac{\delta(1 - p)\theta D(\theta - 1)(1 - \tau)e_t^\theta}{1 + r_{t+1}(1 - \tau)} \left(\frac{\partial e_{t+1}}{\partial \tau}\right)$$

(29)

where $\partial e_{t+1}/\partial \tau = -D(1 - \gamma_c)(p\theta + 1 - \theta)e_t^\theta [x(1 - x)\tau /\{A [D e_t^\theta - (1 - p)e_t^\theta] \tau + x\}] < 0$. The first term in the left-hand side of Eq. (29) is the cost of raising the tax rate $\tau_{t+1}$ that lowers the after-tax return to saving, while the second term in the right-hand side is the benefit of raising the tax rate that raises the interest rate $r_{t+1}$ (or lowers the physical–human capital ratio $e_{t+1}$). The optimal tax rate, implicitly given in this equation, is the same for all agents. Using the same parameterization as in Tables 2–4, we simulate the model and report the results in Table 5.

Table 5 shows that the response to mortality decline of the saving rate is similar to that shown in Tables 2–4. However, the response of human capital investment as a fraction of output to mortality decline is hump-shaped, peaking at an intermediate level.
of mortality. As mortality falls from an initially high level, the human capital investment–output ratio first rises quickly, and then decelerates and may even fall as mortality falls farther.

The results with more rational voting on either the wage income tax rate or the uniform income tax rate show positive impacts of early mortality decline on human capital investment, which differ from those in previous cases. The reason for such a rise in human capital investment is the realization that higher taxes for education now mean higher human capital relative to physical capital later, and thus higher rates of return on saving later. Our model with only one working period of life allows full strength of this tax effect on the future interest rate. In more complicated models with multi-working periods, such a positive force of mortality decline on human capital investment and growth through the future interest rate will be weakened because current taxes contribute to the education of current young agents who will be only part, not all, of the labor force in the next period. We thus expect that results of more realistic models would fall between the earlier results and those in this section: investments in both kinds of capital and growth itself would respond positively to mortality decline initially, but at some point switch to negatively as the mortality decline proceeds.

5. Conclusion

We have investigated the impacts of exogenous declines in adult mortality rates on per capita growth in a two-sector endogenous growth model without annuity markets. When agents face uncertain survival to old age, an imperfect annuity market leads to accidental bequests and heterogeneity across agents. With public education, both exogenous and endogenous tax rates are assumed respectively. In the latter case, the tax rate is determined by the majority of co-existing agents of the adult population in a political equilibrium. The model thus differs in many important ways from the existing ones on this topic, and provides some new insights.
It is found that with imperfect annuity markets, increasing longevity exerts opposing forces on growth. There is a negative bequest effect as accidental bequests fall with rising longevity; there is a positive saving rate effect as higher life expectancy implies more old-age consumption. As for human capital investment, mortality decline exerts opposing effects. Increased consumption needs in later life tend to reduce investment in human capital, but operating through the influence of the tax rate on interest rates, there may also be a positive effect. The net outcome depends on the initial level of mortality, the discount factor, and the share parameters. When initial life expectancy is low, the positive effects dominate and hence rising longevity stimulates growth. When initial life expectancy is high, however, the negative effects tend to dominate and thus rising longevity tends to hinder growth. These results thus lead to a humpy pattern of the growth effect of mortality decline. While some of the effects identified here may arise from details of the specification of our model, we believe our analysis also captures some of the major impacts of rising longevity on economic growth.

Although we mentioned empirical evidence on how public education spending and investment in physical capital affect growth, and on how longevity affects investment in physical capital and growth, there is a lack of empirical evidence on how longevity influences bequests and human capital investment and on how bequests contribute to growth (data for both bequests and private human capital investment are largely unavailable at this stage). The positive contributions of investments in human and physical capital to growth in our model are in line with the spirit of recent literature on endogenous growth, and consistent with the evidence. However, the different channels through which rising longevity affects growth are novel, and await examination in future research.

Rising longevity requires higher consumption in old age. Our analysis shows that longer life may reduce human capital investment, and may thus have a negative effect on growth. For some countries, the negative influence of declining mortality on human capital investment may be dominated for some time by positive effects. Such positive effects could derive from falling child mortality, as has been analyzed in other models, or by the positive tax effect on the interest rate. However, once child mortality has fallen to very low levels, as in most industrial nations and some Third World countries, further mortality declines occurring primarily at older ages may actually cause decreased investments in education. This suggests that governments in these countries should be cautious in cutting public expenditures on education if growth is their prime concern.

Finally, further research may extend the exploration in several dimensions. First, as the impact of rising longevity on savings depends crucially on the retirement age, it is interesting to treat the retirement age as an endogenous variable. Second, with rising longevity, births have tended to occur later in the life cycle. To capture this timing change of births, one may add another adult stage of life before retirement. Third, one may also treat mortality as an endogenous variable which responds to income, wealth, or human capital.

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Appendix A

Proof of Proposition 1. Differentiating $g$ in Eq. (22) with respect to $1 - \gamma_e$ and $\tau$, respectively, provides the results.

Proof of Proposition 2. Note that for a given $\tau$, $p$ affects $g$ only through $(1 - \gamma_e)[(\theta p + (1-\tau)(1-\theta)]$ according to Eq. (22), where $\gamma_e$ is given in Eq. (12). Thus, $\partial g/\partial p$ is signed by $f(p) = \theta(1-p)[1+\delta(1-p)]-\theta p - (1-\tau)(1-\theta)$ and $f(p)<0$. The restriction on $\tau$ for signing $\partial g/\partial p$ follows. Evidently, $f(p)<0$ and hence $\partial g/\partial p<0$ if $p$ is sufficiently high. If $p$ is low, the sign of $\partial g/\partial p$ is ambiguous but positive for sufficiently large $\delta, \theta$ and $\tau$.

Proof of Proposition 3. First $V'[\tau_j]=0$, $V''(\tau_j)<0$, and the relation among $w_t, r_j, k_t$, and $b_t$. First $V'(\tau_j) = - \frac{w_j(1+(1-p)\delta)}{h_j(1+r_j)+w_j(1-\tau_j)} + \phi/\tau_j = 0$, which can be rearranged as

$$\tau_j = \frac{\phi}{1 + \phi + (1-p)\delta} \left[ 1 + \frac{b_t/(1+r_j)}{w_t} \right].$$

From Eqs. (8), (9), and (17), $(1+r_j)/w_t=\theta/[(1-\theta)k_t]=p\theta/[(1-\theta)b_t]$. Then $b_t/(1+r_j)/w_t=\theta p b_t/[(1-\theta)b_t]$. Furthermore, $V''(\tau_j)<0$ comes from differentiating $V'(\tau_j)$ with respect to $\tau_j$ given $b_t$, $r_j$, and $w_t$. Obviously, $\tau_0=\phi/[1+\phi+\delta(1-p)]$ and $d\tau_0/dp>0$. Proposition 3 follows.

Proof of Proposition 4. From Eq. (22), the sign of $dg/dp$ is determined by the first-order derivative (with respect to $p$) of $\Phi(p) = \tau = 1 - 1/[1 + \phi + \delta(1-p)]$. Let $\tau = \tau^*$. If $p \rightarrow 1$, then $j^* \rightarrow \infty$ by Eq. (23), $a<1$ by Eq. (27), and $b_j^*/b_j \rightarrow 1$ by Eq. (25). Note that as $p \rightarrow 1$, $\phi<1$ implies $\tau^*<1$. Combining these with Eq. (26) gives that $d\tau^*/dp$ is finite. As $p \rightarrow 1$, $\Phi'(p)$ is determined only by one term $-\theta \tau^{*1-\delta}[(1+\phi(1-p))(1-\theta) - \tau^*[(1-\theta)]^0/(1+\delta(1-p))]^2$ that approaches negative infinity, with other terms approaching zero.

When $p \leq 1/2$, type-0 young agents form a majority, and hence $\tau^* = \tau_0 = \phi/[1+\phi+\delta(1-p)]$. Then, as $p \rightarrow 0$, $\Phi(p)'$ is signed by $\delta(1-\theta)(1+\delta)[(1-\theta)(1+\delta)-\delta(1-\theta)]\theta(1+\delta)\phi(1+\delta)]^0(2+\phi+\delta) \cdot (1+\phi+\delta)[(1+\phi+\delta)] = 0$ which is positive when $\delta$ and $\theta$ are large enough.

References

