Demography and Low-Frequency Capital Flows*

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Abstract
We consider the role of demographic trends in driving international capital flows in a multi-country overlapping generations model in which saving decisions are tied to agents’ life expectancy. Capital flows reflect differences between saving and investment across countries. Demographic changes affect the aggregate accumulation of assets in two ways: by changing life expectancy which changes individual household saving behavior, and by changing the age distribution of the population by which individual household decisions are aggregated. We use a quantitative version of the model to illustrate the impact of demography on capital flows and net foreign assets in China, Germany, Japan, and the United States.

JEL Classification Codes: F21, J11.

Keywords: current account balance, net foreign asset position, life expectancy, mortality rates, population aging.

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1 Introduction

The movement of capital from less productive to more productive uses is a story repeated over and over again throughout history and around the world. Whether capital moves within a country — say from Massachusetts to North Carolina — or between countries — say from the US to Mexico — its flow addresses imbalances between local sources of funds (savings) and uses (investment). Certainly capital could flow for other reasons, but it’s not hard to believe that market forces could account for substantial flows on their own. The period from 1880 to 1913 is often described as the golden era of capital mobility. Bordo (2002) describes this earlier period: “The fifty years before World War I saw massive flows of capital from Western Europe to (mainly) the Americas and Australasia. At its peak, the outflow from Britain reached nine percent of GNP and was almost as high in France, Germany, and the Netherlands.” Over this period, Great Britain accumulated claims on the rest of the world equal to about one year’s GDP. Among the recipients were Australia, Canada, Sweden, and the United States.

Through much of history, the major capital flows were from rich countries to poorer ones. A narrow view of these flows is that capital should flow from rich countries to poor countries because the returns to capital should be higher in the latter. But, this view is challenged by the evidence. Ohanian and Wright (2010) have shown that the direction of capital flows was not always consistent with the pursuit of higher returns as they measured them.

The recent history of capital flows also challenges the traditional view. The most notable importer of capital has been the richest country, the United States. Australia, and the UK have also been importers of capital. Germany, Japan, and China, have been significant exporters of capital. Moreover, these capital flows have been persistent - countries experiencing capital flows, in or out, are likely to do so for long periods of time. These low frequency net capital flows are collectively referred to as “global imbalances.”

If capital flows are persistent, the question is why. In this paper we study the role of demography. Demographic trends are persistent and changes in demographics are evident worldwide. What is important, however, is that countries exhibit enough heterogeneity in these changes to make capital flows a plausible consequence. We show that differences in demographics, affecting both decisions and composition, can have a big impact on capital
flows and can account for the pattern of flows between the U.S. and Japan as well as other flows in the data. We also show that these changing demographics imply increased savings and a persistent decline in the rate of return on capital.

To study the connection between demography and capital flows we use a calibrated general equilibrium model with a rich set of demographics. We ask to what extent can net foreign asset positions be accounted for over time (and hence capital flows) by demography. Differential demographic trends drive savings and investment differences as workers adapt to increased life expectancy and decreased fertility. In such a world capital flows to countries with more favorable demographics and these demographics account well for the differences in net foreign asset positions observed in the data.

The idea of using an overlapping model to study the impact of demography on international capital flows is not new. Attanasio, Kitao, and Violante (2007), Börsch-Supan, Ludwig, and Winter (2006) Brooks (2003), Feroli (2003), Ferrero (2010), Henriksen (2009), and Krueger and Ludwig (2007) all took this approach. Others before them, among others Taylor and Williamson (1994) and Taylor (1995), expressed similar ideas without the formal structure of a model. This paper differs from the literature cited above in that we have a more parsimonious and transparent analytical framework, use a richer demographic structure that allows us to include more countries and more carefully parametrize the model. The long-run quantitative results do not rely on implausible long-run interest-rate and wage paths.¹

We begin by relating some important facts about capital flows, demography, and capital output ratios for the subset of economies that we focus on. We then describe a one-good model and some equilibrium concepts that we use to explore the role of demography. In each country, households have power utility and firms have identical constant elasticity aggregate production functions. Countries differ only in their demography: the mortality rates and life expectancies faced by households and the age distribution of their populations. The question is how much variation in capital flows we can generate across countries and over time from these differences alone.² We show, using steady state calculations, that

¹The challenge with this kind of model are the dozens of decisions we must make about details, far more than we would have in a representative agent model. Did households foresee the large drop in mortality we’ve seen around the world? How do they deal with uncertain lifetimes? Bequests? Are pensions substitutes for private saving or something more?

²We think the time interval is very important. Most of the data, and most of the work based on it, uses
demographic changes affecting decisions as well as the composition of the population can, in principle, have large effects on capital outflows or inflows and thus on net foreign asset positions. We go on to simulate paths for capital flows for China, Germany, Japan, and the United States, countries with large capital flows, both in or out.

2 Facts

We start with some facts about international capital flows and stocks, facts about demography, and facts about capital-output ratios. In describing these facts, we look at four countries: China (ISO country code CHN), Germany (DEU), Japan (JPN), and the United States (USA). These countries account for a substantial fraction of net capital flows in the world, and they have striking, and different, demographics. We show — for these countries anyway — that capital flows are persistent. Since capital flows determine net foreign asset positions, it follows that they are persistent as well. Demography, of course, is inherently persistent. We describe changes in the age distribution of the population, in life expectancy, in old-age dependency, and in retirement ages. All but the retirement age have changed dramatically over the last few decades. Finally, we look at capital-output ratios, a central component of the modeling exercise that follows.

2.1 Capital flows and stocks

Global capital flows are most often depicted by plotting the current account — aggregate investment minus aggregate savings — as a fraction of GDP. Countries like the US that tend to have current account deficits, have them for a long time as do countries that have current account surpluses - e.g. Germany, Japan and China. Although countries do reverse from surplus to deficit and vice versa they seem to do so infrequently.

The net foreign asset positions of these countries are shown in Figure 2 below.\(^3\) In principle these should represent the same phenomena since the current account should simply represent the change in Net Foreign Assets. In practice they do not line up and the

\(^3\) Net foreign asset positions were computed by Lane and Milesi-Ferretti (2007).
difference is the subject of exploration. Hausmann and Sturzenegger (2006) have described
this difference as “dark matter” that is explained by unmeasured flows of liquidity services,
knowledge capital and insurance. They have argued that when this is taken into account
the current account flows look very different. McGrattan and Prescott (2010) have argued
that technology capital and plant specific intangible capital can account for much of the
difference between current account flows and net foreign asset positions and accounts for
much of the measured differences in asset returns between countries. For our purposes the
precise nature of “dark matter” is a secondary issue and we will focus primarily on Net
Foreign Assets and will treat changes in Net Foreign Asset positions as the capital flows of
interest.

Although much attention has been directed at the flows of capital as represented by
changes in net foreign assets, not enough has been directed at the question of why these
data display the characteristics that they do. The features of the data belie the frequently
voiced worry about sudden reversal of capital flows.

We see, for a start, that Japan has had a growing net foreign asset position since the
1980s. These correspond to capital outflows and amounted to 50 percent of GDP in 2007,
the last date in the available data. The US has had the opposite experience and now has
a negative net foreign asset position. In both cases, the direction of net capital flows has
been the same for almost three decades. Germany and China have had more variation, but
there is a great deal of persistence in their capital flows as well. China, for example, has
had capital inflows for almost twenty years. Germany has had the same for ten. Although
there is a clear cyclical component in capital flows, the bulk of fluctuations operate at a
lower frequency. Henriksen and Lambert (2012) make the same argument more formally
for a broader range of countries.

2.2 Demographics

Could demography play a role in these capital flows? The current account for a country is
simply the difference between domestic savings and domestic investment. It is natural to
look at life-cycle considerations as primary drivers of domestic savings. For a given country
the key drivers are demographic variables affecting decisions and composition. The former
is to a large extent determined by changes in mortality, whereas the latter is, in addition to
mortality, determined by fertility and immigration. As fertility and mortality decline, the population distribution will shift and the average age will tend to increase. Immigration tends to affect the population distribution in the opposite direction.

Formally, let \( x_t \in \mathbb{R}^I \) be the vector of number of members in each cohort in period \( t \). The demographic structure of the population changes through changes in fertility, mortality and immigration. According to time and age specific fertility rates \( \varphi_{i,t} \), in each period these individuals give birth to a certain number of new individuals, and the number of newborns in period \( t + 1 \), \( x_{1,t+1} \), is the product of \( x_t \) and the vector of fertility rates \( \varphi_t \). Then the law of motion of a population with survival rates determined by changing mortality and life expectancy, but with deterministic fertility, can be described by a simple \( (I \times I) \) matrix\(^4\):

\[
\hat{\Gamma} = \begin{bmatrix}
\varphi_1 & \varphi_2 & \varphi_3 & \cdots & \varphi_I \\
s_1 & 0 & 0 & \cdots & 0 \\
0 & s_2 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & s_{I-1} & 0
\end{bmatrix}
\]

where the diagonal elements \( (s_1, \ldots, s_{I-1}) \) are the conditional survival probabilities.

Let \( m_t \in \mathbb{R}^I \) be a vector with each element representing the cohort specific number of net immigrants at time \( t \). Denoting \( \hat{\Gamma}_t \) the matrix of deterministic fertility and mortality rates at time \( t \), the law of motion for the population may be written

\[
x_{t+1} = \hat{\Gamma}_t x_t + m_t.
\]

2.2.1 Mortality and Decisions

Changes in mortality \( \{s_i\}_{i=1}^I \) and life expectancy are crucial to understanding households’ decision over the life cycle. Increases in life expectancy reflect decreases in mortality at all ages. We document this with data from the WHO’s Global Health Observatory. In Figure 5 we see log of age-specific mortality rates for the United States, Japan, China and India for the year 2011. We see the common pattern and that countries with highest life expectancy

\(^4\)The largest eigenvalue of the matrix \( \Gamma \) is the rate of growth of the population in steady state regardless of the initial condition. The eigenvector corresponding to this eigenvalue describes the share of each age group in the steady state population.
at birth tend to have lower mortality across all ages. The drop at higher ages is larger in absolute terms, because the underlying rates are larger. The differences reflect differences in life expectancy: Japan has the lowest mortality rates and China the highest.

2.2.2 Composition and aggregation

The age composition of households \( x_t \) is crucial for the aggregation of households’ decisions at different ages. Consider the evidence reported in the UN’s *World Population Prospects*, the 2010 revision. We see in Figure 3 that UN data and projections for the future show significant increases in the median age of the populations of China, Germany, Japan, and the U.S. Japan’s aging is the most pronounced in this group, with more than a third of the population expected to be over 70 by 2040. Germany is also aging quickly while the US, in this group, is aging the most slowly. This aging reflects, in part, a continuing increase in life expectancy; see Figure 4. The levels of life expectancy differ, but we see the same pattern of increase in all four countries. In each of them, life expectancy has increased almost a decade since 1970 and is projected to increase another decade by the end of the century.

The other side of population dynamics is fertility. Fertility rates in Germany and Japan have been low, lower even than in China with its one-child policy. The lower input of young people into the population reinforces the impact of reduced mortality on the aging of their populations. The US has the highest fertility of the four countries, where the average number of children that would be born per woman if all women lived to the end of their childbearing years is 2.06. The same number for China is 1.55, for Germany 1.42, and for Japan 1.39.

We see, in short, gradual but significant aging of the populations of all four countries, but also significant differences in the age composition among them. None of these facts are new. Bongaarts (2004) provides a comprehensive analysis and a good summary of related work.

2.2.3 Retirement

The calibration of an overlapping-generations model is conditional on a given retirement age. We show in Figure 6 how the retirement age has changed with time. The retirement age
comes from the OECD’s *Statistics on average effective age of retirement* and is computed from labor market participation rates of older workers. We see in the figure that retirement ages differ across countries but show little variation over time over the period 1980-2011. Evidently, increases in life expectancy are leading to longer periods of retirement.

### 2.2.4 Modeling Mortality

Mortality and life expectancy affect individuals’ decision, but conditional mortalities used in current studies are specific for particular countries at particular points in time, and are often reported as five-year cohorts/intervals.

A precise formula for mortality at all ages is, obviously, impossible. In order to analyze the effect of aging, it is, however, necessary to have a parsimonious representation of how age-specific mortality evolves with life expectancy at birth. Using the observation that the logarithm of mortality rates are almost linear in age Lee and Carter (1992) proposed a principal-components-based model, which has become the “leading statistical model of mortality [forecasting] in the demographic literature” (Deaton and Paxson, 2004).

Henriksen (2013) propose a transparent method for computing representative age-dependent survival probabilities as functions of life expectancy based on Lee and Carter (1992). We use that method here because it provides a straightforward way to compute representative sequences of mortality at annual frequency, including for countries like China where such data are not immediately available, but which do report current and projected life expectancies. This permits a more transparent economic analysis of aging in terms of survival probabilities at every age.

### 2.3 Capital stocks

We look at one last variable, the capital-output ratio, which plays a central role in our model. We compute capital stocks by standard methods from the Penn World Table, version 7.1. We take data on investment, estimate an initial capital stock value from a steady state approximation, and update by the perpetual inventory method using an annual depreciation rate of 6 percent. Caselli (2005) is one of many to describe the approach.
We show in Figure 7 how the ratio has evolved in our four countries. Over the period 1980 to 2010, the capital-output ratio has been between two and three in the US and China, about three in Germany, and has risen above four in Japan. China is the most surprising. Between 1990 and 2010, its real investment share of GDP averaged 37 percent, significantly higher than the 22 percent experienced by the US, yet its capital-output ratio is similar. The reason it’s not higher, of course, is that output has been growing so quickly. Holz (2006) constructs similar estimates directly from company balance sheets.

3 An overlapping generations model

We study the impact of demographic changes in a model with overlapping generations of agents who spread their consumption over their lifetimes and supply labor inelastically. There is a common technology for producing goods from capital and labor. Countries differ primarily in their demographics, including their age distribution and mortality rates.

We use the structure of competitive equilibrium as the organizing framework to analyze the effects of demographic change. In particular:

1. Capital supply:
   (a) Individuals at every age solve their optimization problems and make decisions given aggregate prices and conditional life expectancy.
   (b) The decisions of individuals of different ages are aggregated by the composition of cohorts

2. Capital demand: Firms employ all individuals of working age in each country and choose optimal quantities of capital demanded given prices.

3. Prices are determined internationally.
   If the supply of capital is greater than demand for capital given prices the country is a capital exporter, and vice versa

In the following sections we will consider variations on this definition of equilibrium that illustrate the role of differences in mortality in the steady state and the nature of capital flows given a path for interest rates and prices. Finally, we will compute a general equilibrium.
3.1 Households

Individual households take prices of labor and capital as given. Demographic changes affect households individual decision problems by changing their life expectancy at every age. They affect aggregate decisions through the change in the age composition of cohorts.

3.1.1 Individual households’ decisions

We refer to households as agents or cohorts. Households work, save, and consume.

Consumption starts at age $I_w$ and continues until death. Utility has a time-additive power form, which we express recursively by

$$U_{it} = c_{it}^{1-\sigma} / (1 - \sigma) + \beta s_{it} U_{i+1,t+1}$$

for $i = I_w, \ldots, I$. Here $U_{it}$ is utility from date $t$ forward for an agent of age $i$, $c_{it}$ is date-$t$ consumption for the same agent, and $\beta$ is the discount factor. The intertemporal elasticity of substitution is $1/\sigma$. The limiting case $\sigma = 1$ corresponds to log utility. The use of the survival probability $s_{it}$ follows the now-familiar application of expected utility to uncertain lifetimes proposed by Yaari (1965).

Labor is supplied inelastically once agents reach working age. Formally, the agent begins to work at age $I_w$ ($w$ for work), supplying one unit of labor every year until retirement. At age $I_r$ ($r$ for retirement), the household stops working.

We build productivity into labor. Each individual of working age supplies one unit of labor. For an agent of age $i$ at date $t$, that unit has efficiency $e_{it}$. Efficiency is zero for children and retirees: $e_{it} = 0$ for $i < I_w$ and $i > I_r$. If the wage per efficiency unit is $w_t$, the agent earns labor income $e_{it}w_t$. Differences in $e_{it}$ across time and countries will lead to level effects on the economies but will not matter much for the dynamics of capital flows. They can however be used to capture differences in productivity levels across countries.

Consumption and income are connected to changes in net worth through the budget constraint. Let $a_{it}$ be financial assets or net worth owned by agents of age $i$ at the start of the period $t$. The sequence budget constraint for an agent of age $i$ is

$$a_{i+1,t+1} = (1 + r_t)a_{it} + e_{it}w_t - c_{it} + b_{i+1,t+1},$$
where \( r_t \) is the real return between \( t \) and \( t+1 \). We have one of these constraints for each age \( i = I_w, \ldots, I \), plus boundary conditions

\[
a_{I_w,t} = a_{I+1,t} = 0. \tag{3}
\]

Bequests \( b_{it} \) are a necessary ingredient here, because we need to distribute the accidental bequests of agents who die before age \( I \) (see, among many others, Hansen and Imrohoroglu, 2008; Ríos Rull, 2001; Yaari, 1965). The simplest method is to spread the assets of those who die among the living of the same generation. Other alternatives are to assume an annuity system, to distribute accidental bequests equally to all individuals, to distribute accidental bequests to individuals of the assumed offspring, or to let them be lost. Since we are calibrating preference and technology parameters to match certain moments conditional on retirement age and retirement system, how we treat accidental bequests is not that critical.

The household’s Euler condition for any time \( t \) and any age \( i \) is

\[
c_{it}^{-\sigma} = s_{it}\beta c_{i+1,t+1}^{-\sigma}(1 + r_t).
\]

In addition to the constant, age-independent discount factor \( \beta \), and the interest rate \( r_t \), the slope of the lifetime consumption profile is governed by the sequence of conditional survival probabilities.

Another key assumption about household decisions in defining an equilibrium is what they assume about future demographic changes and the impact these have on prices. We could assume that they either have perfect foresight over all future demographic changes or that they are myopic and assume that current life expectancies (and hence prices) are a good proxy for what they should expect in the future. Clearly this has an important impact on savings and hence on prices. We assume the latter. There is no obviously correct stance to take on this issue but the evidence is that official statistics have consistently underestimated the increase in longevity in many populations.

### 3.1.2 Aggregation of individual decisions

Capital supply \( K^s \), equivalently asset demand, at any country \( j \) at any given time \( t \) is given by aggregation of households’ decisions by the given cohort composition

\[
K^s_{j,t} = \sum_i a_{ij,t} x_{ij,t}
\]
Each of the remaining individual or cohort variables has an aggregate analog. Aggregate consumption is the sum across generations: $C_t = \sum_i c_{it}x_{it}$. The total supply of labor at date $t$ is the sum over all agents of working age: $N_t = \sum_i e_{it}x_{it}$.

### 3.2 Firms

Firms in aggregate combine capital $K_t$ and efficiency units of labor $N_t$ to produce output $Y_t$. Demographic change affects firms’ demand for capital through changes in the number of efficiency units of labor supplied by the households.

We give their technology a constant elasticity form:

$$Y_t = F(K_t, N_t) = \left[ \omega K_t^{1-\nu} + (1 - \omega)N_t^{1-\nu} \right]^{1/(1-\nu)}.$$  \hfill (4)

The elasticity of substitution between capital and labor is $1/\nu$. The limiting case $\nu = 1$ corresponds to Cobb-Douglas. The law of motion for capital is the usual

$$K_{t+1} = (1 - \delta)K_t + I_t,$$  \hfill (5)

where $I_t$ is gross investment in new capital and $\delta$ is the rate of depreciation.

A representative firm with this technology facing prices $(r_t, w_t)$ chooses capital and labor equate marginal products to prices:

$$\frac{\partial F(K_t, N_t)}{\partial K_t} = r_t + \delta$$  \hfill (6)

$$\frac{\partial F_n(K_t, N_t)}{\partial N_t} = w_t.$$  \hfill (7)

With the constant elasticity function (4), the marginal product of capital takes the form

$$\frac{\partial F(K_t, N_t)}{\partial K_t} = \omega(K_t/Y_t)^{-\nu},$$

a decreasing function of the capital-output ratio. Capital demand is

$$K^d_{j,t} = \left( \frac{(r+\delta)^{-\frac{1}{\nu}} - \omega}{\nu - \omega} \right)^{\frac{1}{\nu-1}} N_t.$$

Hence capital demand depends on the size and the composition of the population.
3.3 Equilibrium and capital flows

At any time $t$, each country takes the international rate of return on capital as given. For given prices of capital and labor, households make their capital-supply decisions and firms make their capital-demand decisions. If the supply of capital is larger than the demand for capital for given prices, a country is a capital exporter – and vice versa if demand is larger than supply.

International capital markets must clear and the rate of return be such that the sum of capital demanded across all countries equals the sum of capital supplied.

$$R_t : \sum_j K_{j,t}^d = \sum_j K_{j,t}^s$$

4 Demography and steady states

First we consider the importance of demographics for long-run capital flows by analyzing an overlapping-generations structure in steady state. The demographic inputs give us a stationary age distribution for each country. If mortality rates decrease, life expectancy increases at every age and changes the age distribution, making it older. This change in the age distribution has consequences for the aggregation of all of the variables in the model: consumption, labor supply, aggregate net worth, the capital stock, the wage, and the interest rate. In a closed economy, an increase in life expectancy raises aggregate net worth and the stock of capital. This reduces the marginal product of capital and hence the interest rate. This is an important consequence of demographic change that has been discussed by others (see eg. Geanakoplos, Magill, and Quinzii, 2004).

In an open economy, with a given, exogenous interest rate, the increase in aggregate net worth shows up as an increase in net foreign assets. We describe these effects in a supply and demand diagram, where the demand for capital comes from firms’ first-order condition and the supply comes from households accumulation of assets.

4.1 Parameter values

We review the inputs to the model, starting with demography. As described in Section 2.2.4, for years and countries where annual survival probabilities are not available we use
estimates based on WHO data to compute representative sequences of mortality rates, given life expectancies at birth. Demographers will recognize this as a simplified version of Lee and Carter (1992).

We see stylized results in Figure 8, where we plot the resulting survival probabilities. The logarithmic form means that the greatest impact is on the largest mortality rates: those of the young and old. With these mortality rates, stylized stationary age distribution can be computed.

When calibrating the model we take as given a retirement age, a retirement systems and other conventions in place. The model is not very sensitive to these assumptions because the composition reflects existing institutions. This does not mean it will be insensitive to changes in these institutions, indeed they could have serious effects. We do not explore such changes in this paper, but the way we model households’ expectations of future factor prices makes the model robust to potential institutional changes.

The next input is the technology. We set \( \delta = 0.06 \), which we used to generate the data. We also set \( \nu = 1 \), which corresponds to an elasticity of substitution of one, and choose \( \omega \) to set capital’s share equal to one-third at a capital-output ratio of three. The capital share in general is

\[
\frac{\partial F(K, N)}{\partial K} \cdot \frac{(K/Y)}{K/Y} = \omega (K/Y)^{1-\nu}.
\]

With \( \nu = 1 \), the capital share is one-third when \( \omega = 1/3 \). With other values of \( \nu \), we adjust \( \omega \) appropriately. The interest rate is the marginal product of capital minus depreciation:

\[
r_t = \omega (K_t/Y_t)^{-\nu} - \delta.
\] (8)

Evaluated at a steady state with \( K_t/Y_t = \) and \( \nu = 1 \), we have \( r = 0.0511 \).

A typical household’s problem includes the interest rate and labor income as inputs and generates paths for consumption and net worth. We choose labor efficiencies \( e_{it} = 1 \) for agents of working age and zero otherwise. Working age starts at age \( I_w = 21 \) and ends at retirement age \( I_r = 65 \). Finally, we set \( \sigma = 1 \) (log utility) and choose \( \beta \) to match the steady state ratio of aggregate net worth to output of three. Since net worth and the capital stock are the same, net foreign assets is zero in the benchmark case.
4.2 Steady states

The interaction between the supply of capital by households and the demand for it by firms takes its cleanest form in a steady state, where we can capture its properties in a supply-and-demand diagram.

Demand is relatively simple. The demand for capital comes from the first-order condition (6). If we express capital as a ratio to output, the inverse demand function is equation (8). This equation holds at every date, as well as in a steady state.

Supply requires calculations that go beyond what we can show in an equation or two. But suppose we have a steady state age distribution for the population. Then we can compute the ratio of aggregate net worth to output for any constant interest rate. The overlapping generations structure is essential here. In a representative agent model, supply in a steady state is horizontal at the discount rate \((1 - \beta)/\beta\). Here there is some slope, which depends on intertemporal elasticity.

The results of these two sets of calculations are pictured in Figure 9. The downward-sloping line is demand, the upward-sloping one is supply. They cross by design at our steady state point: \(K/Y = A/Y = 3\) and \(r = 0.0511\). We show two examples of each. The solid downward-sloping line is the demand curve for \(\nu = 2/3\) and the dashed line the demand curve for \(\nu = 3/2\). As we might guess from (8), the line is flatter when \(\nu\) is smaller. The supply curves depend in a less obvious way on household decisions, but they have a similar form. The solid line corresponds to \(\sigma = 1/2\) and the dashed line to \(\sigma = 2\). Evidently the line gets flatter as we increase \(\sigma\). In what follows, we compute steady states for the intermediate values \(\nu = \sigma = 1\) (log utility and Cobb-Douglas production).

In a closed economy, we get an increase in the capital-output ratio and a decline in the interest rate. These effects of aging are well-known features of overlapping-generations economies. Here the impact combines two effects. One is a composition effect – we have more households at ages associated with high net worth. The other is that households have more wealth at all ages. The mechanism is one noted by Bloom, Canning, and Graham (2003): with longer life expectancy, households save more.

In an open economy facing a fixed interest rate, the impact of increased life expectancy falls entirely on aggregate net worth. The demand for capital, and therefore the capital-
output ratio, doesn’t change, but with aggregate net worth rising, the result is a positive steady state net foreign asset position. If we take a general equilibrium perspective, we might imagine a world with two countries, one with longer life expectancy than the other. The equilibrium interest rate will split the difference, leading the country with longer life expectancy to lend to the other — forever, if this situation continues.

It is important to recognize how different these results are from what one would find in a representative agent model. In the latter the supply of capital is perfectly elastic and capital will flow only temporarily in response to shocks to the marginal rate of substitution of the representative agent. In our model with its rich demographics the supply of capital from households is not perfectly elastic. Changes in life expectancy and composition will alter the supply curves giving rise to long term differences in net foreign assets and persistent capital flows.

5 Country dynamics

We now compute time paths for net foreign assets and other variables for our four countries. In each case, all firms and all households in each country take interest rates and wage rates as given at any time. The interest rate is computed endogenously as the rate that period-by-period clears the capital market between the United States, Japan and Germany. We emphasize that this is not an implicit assumption that these three countries constitute the global capital market. Obviously, there are countries – China, the commodity exporters – that are big contributors to capital flows.

The set of countries included in the definition of equilibrium will not change the relative results. As we saw from the definition of the equilibrium, households are making decisions given factor prices and life expectancy, and firms are making investment decisions given factor prices and labor supply. Here we study the evolution of capital flows for one reasonable endogenous interest rate path as an example to see how effective demographic differences might be in accounting for observed capital flows.

Conditional on, among other things, no changes in retirement age and retirement system, the period-by-period market-clearing interest path is in line with the findings of other work: the interest rate declines steadily from about 2005 and onwards (see eg. Krueger and Ludwig,
A major difference between this paper and previous papers is, however, that the quantitative results are more robust and transparent because we do not rely on households perfectly foreseeing the entire future interest path predicted by the model, and which is conditional on the retirement age and retirement system remaining unchanged despite large predicted gains to longevity. Instead here households are making their decisions based on the expectation that future factor prices will be equal to the market clearing factor prices at the time of decision.

The other parameters are similar to those in our steady state calculations. We use log utility ($\sigma = 1$) and a Cobb-Douglas production function ($\nu = 1$). The discount factor is chosen to match steady state net worth for a benchmark economy and is the same in all countries.

Another difference is in the demographics. We take data for life expectancy at birth and compute annual survival probabilities using the method described earlier. This gives us the survival probabilities that enter household consumption and saving decisions. Since mortality rates change with time, every generation has different ones. The age distributions are adapted from the UN’s *World Population Prospects*. They report distributions every five years from 1950 to 2100 for five-year cohorts. We interpolate them to get annual numbers.

The last input is the initial values of household asset positions. We compute initial asset positions from their steady state values. From that point on, asset positions are computed recursively, starting in 1950. We report capital stocks and flows starting in 1980, with the hope that the effect of the initial conditions has worn off.

### 5.1 Capital Flows

Figure 10, shows the change in net foreign assets as a percentage of GDP implied by the model with the endogenous interest rate path. This picture can be usefully compared to Figure 1 which plots the Current Account in the data for these four countries. The ratio of Net Foreign Assets to GDP implied by these flows is shown in Figure 11.
5.1.1 U.S. and Japan

It is useful to begin by looking at the flows between the U.S., Japan, and the rest of the World; an exercise that is close to the work in Henriksen (2009). In Figures 12 and 13, the capital flows predicted by the model for the three last decades are compared the historic current account for the same time period that we saw in Figure 1. The difference between the two, in the model, is due entirely to differences in life expectancy and differences in the age composition of the population.

The model accounts very well for the facts that the US has experienced capital inflows and Japan capital outflows, and for the persistence of these flow. The model also accounts remarkably well for the magnitude of Japanese flows. The model also accounted for the magnitude of the U.S. flows for the first 20 years, but during the last 10 years the prediction of the model is roughly half of what we observed. Obviously other factors have an influence, too, but these results strongly suggests that demography is an important component.

We also see that Japan’s three decades of capital outflows is projected to reverse course a bit over the next several years and then increase again. One issue for Japan is that its capital-output ratio has risen significantly over the last twenty years;see Figure 7. That’s inconsistent with our model unless either Japan has a different technology or faces, for some reason, a different interest rate than the other countries. This increase in capital tends to offset what would otherwise be an even larger increase in net foreign assets.

5.1.2 Germany and China

Figures 14 and 15 also shows the pattern of flows implied by the model for German and China compared to the historical current account.

The capital flows for Germany predicted by the model differ from the German experience. The model predicted capital outflows in the 1990s whereas in the ten years after the reunification with East Germany, the country experienced capital inflows. For the last decade it reversed: the model predicted capital inflows while the country experienced outflows.

Chinese demographics of course are striking and the changes in life expectancy as well as age composition of the population are dramatic. These factors alone account for significant
capital flows given world interest rates. As shown in Figure 15, with the exception for the
spike in capital outflows China experienced about five years ago, the model does remarkably
well accounting for the magnitude and persistence of capital flows from China.

Further inspecting these results, the model is hard challenged to account for China’s
sky-high saving rate. Clearly, the simple overlapping generations structure cannot account
for a lot of the Chinese experience and the role of state directed savings. We have seen
that the capital-output ratio is not out of line, but it strikes us as unlikely that a model
of this sort will account for what others have failed to account for; but see, among others,
Chamon and Prasad (2010), Coeurdacier, Guibaud, and Jin (2012), Wei and Zhang (2011),
and Yang, Zhang, and Zhou (2011). The results indicate demographic factors may be
important in order to account for the savings rate, potentially in combination with other
modeling features.

6 Concluding Comments

In recent years, international capital flows have been large and persistent. Many view these
so called “global imbalances” as a threat the stability of the international financial system.
One reason for the significant angst expressed in the economics literature over international
capital flows may be the inability of the standard representative agent economy of account
for the persistence of these flows.

We argue that persistent phenomena have persistent causes and the one of the most
important and neglected drivers is demographic change. From a theoretical point of view,
we show that whereas the supply of capital is perfectly elastic in a representative agent
economy, in an heterogeneous-agent overlapping-generations economy capital supply is not
perfectly elastic. This feature is essential for overlapping-generation economies in accounting
for the persistence of capital flows.

We have shown that analytically, from the point of view of international capital flows,
the two most important features of demographic change are changes in life expectancy,
which affect decisions, and changes in the age composition, which affect the aggregation of
those decisions. Transparent modeling of annual survival probabilities have allowed annual
predictions as well as the inclusion of countries like China where survival probabilities at
annual frequency are not publicly available.
We have deliberately kept this model simple to highlight the important role of demographics for capital flows. Clearly there are other important differences across countries that drive capital flows - productivity differences, tax rates on capital and retirement policies that affect savings and investment. These are all candidates for inclusion in a richer model.
References


Figure 1

Current account balances
Figure 2
Net foreign asset positions
Figure 3
Median age of populations

![Median age of populations diagram](image-url)
Figure 4
Life expectancy at birth

Source: UN, World Population Prospects, 2010 revision
Figure 5
Mortality by age: estimates for 2011

Source: WHO, Global Health Observatory
Figure 6
Effective retirement ages
Figure 7
Capital-output ratios

Source: Penn World Table and authors calculations
Figure 8
Representative survival probabilities
Figure 9
Steady state supply and demand for capital

- Marginal product of capital net of depreciation
- Capital/wealth-to-output ratio
- Demand for capital given $\nu = 0.67$
- Demand for capital given $\nu = 1.00$
- Demand for capital given $\nu = 1.50$
- Supply of capital, representative agent
- Supply of capital given $\sigma = 0.5$
- Supply of capital given $\sigma = 1.0$
- Supply of capital given $\sigma = 2.0$
Figure 10
Dynamics of capital flows in the model
Figure 11
Implied Net Foreign Assets from the model
Figure 12
Dynamics of capital flows in the model: United States

Change in net-foreign-assets-to-output ratios

Model 1980−2013
Model 2013−2030
Data 1980−2013
Figure 13
Dynamics of capital flows in the model: Japan

![Graph showing dynamics of capital flows in the model: Japan. The graph plots the change in net-foreign-assets-to-output ratios from 1980 to 2030, with data and model projections for the periods 1980-2013 and 2013-2030.](image-url)
Figure 14
Dynamics of capital flows in the model: Germany
Figure 15
Dynamics of capital flows in the model: China