Calculating Life Expectancy at Birth

Suppose there are 21 intervals of age, with interval start and end points:

\[ x_0 = 0, \ x_1 = 1, \ x_2 = 5, \ x_3 = 10, \ldots, \ x_{21} = 100 \]

Let \( M = 21 \times 1 \) vector of mortality probabilities which denote the probability of dying between ages \( x_{i-1} \) and \( x_i \), for \( i = 1, \ldots, 21 \).

1 Basic Calculation

To find the life expectancy at birth, we must calculate total years lived in each interval. Let us denote this as \( L_i \), where \( i \) takes values from 1 to 21.

\[ L_i = (x_i - x_{i-1})p_i + a_id_i \]

From earlier, \( x_i \) and \( x_{i-1} \) are the ending and starting points of each interval. We define \( p_i \) to be the percentage of total population that lives on to the \( i+1 \) interval. We also define \( a_i \) to be the average number of years lived in an interval by an individual who passes away (in the same interval). Finally, \( d_i \) is defined to be the percentage of total population that dies in the interval \((x_{i-1}, x_i)\).

We can calculate \( p_i \) and \( d_i \) as the following:

\[ p_i = \prod_{j=1}^i (1 - M_j) \] with \( p_0 = 1 \)

\[ d_i = p_{i-1}M_i \]

From here we calculate life expectancy at birth to be:

\[ LE = \sum_{i=1}^{21} L_i \]

2 Estimating \( a_i \)

Estimating the average years lived in an interval by those that have died is a difficult task. It would be simple to assume the following.

\[ a_i = \frac{x_i - x_{i-1}}{2} \]

This implies that on average, people die in the middle of the interval. Obviously this is a bit presumptuous and leads to small errors in calculation versus WHO data.
3 Some Examples

Using this methodology and the previous estimate for $a_i$, we calculate life expectancy for US in 1990, 2000, and 2006:

In 1990 US Life Expectancy was 75.3. Using mortality probabilities, we receive a life expectancy of 75.38.

In 2000 and 2006, US Life Expectancy was 76.9 and 78.0. Using mortality probabilities, we receive life expectancies of 77.00 and 78.08, respectively.