Comment on the Campbell-Cochrane Habit Model

Lars Ljungqvist  Harald Uhlig*

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Abstract

Campbell and Cochrane (1999) formulate a model that successfully explains a wide variety of asset pricing puzzles, by augmenting the standard power utility function with a time-varying “external habit”, that adapts nonlinearly to current and past average consumption in the economy. We demonstrate that their preference specification has the unusual implications that habit can move negatively with consumption, and that the social marginal utility can be negative. As a result, government interventions that occasionally destroy part of the endowment can lead to substantial welfare improvements.

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*Ljungqvist: Stockholm School of Economics and New York University (email: lars.ljungqvist@hhs.se); Uhlig: University of Chicago (email: huhlig@uchicago.edu). We thank Fernando Alvarez and John Cochrane for criticisms and suggestions on our earlier exploration of the properties of the Campbell-Cochrane preference specification. The present exposition has benefitted much from the comments of the editor and three anonymous referees. Ljungqvist’s research was supported by a grant from the Jan Wallander and Tom Hedelius Foundation. Uhlig’s research has been supported by the NSF grant SES-0922550.
1 Introduction

Campbell and Cochrane (1999, hereafter denoted C-C) formulate a model that successfully explains a wide variety of asset pricing puzzles, including a high equity premium, procyclical variation of stock prices, countercyclical variation of stock market volatility, and a low and smooth riskfree rate. These remarkable results are achieved by augmenting the standard power utility function with a time-varying subsistence level, or “external habit”, that adapts nonlinearly to current and past average consumption in the economy. Given the breakthrough in matching key asset pricing facts as well as the successful adoption of the C-C preferences in a number of other applications, it is all the more important to fully understand the implications of these modeling choices.

We show that the assertions by Campbell and Cochrane (1999, p. 212, p. 246) that “habit moves nonnegatively with consumption everywhere,” and that “more consumption is always socially desirable” are incorrect. As a result, government interventions that occasionally destroy part of the endowment can lead to substantial welfare improvements. Large interventions are not required: welfare improves already with the rather tiny one-time destruction of less than one tenth of a percent of aggregate consumption. Hence, Campbell and Cochrane’s (1999, pp. 245–247) attempt to map their results into a version of the model with internal habit formation must be reconsidered. Households faced with such an internal habit would themselves choose to periodically destroy endowments.

Our results are true for the specific formulation of Campbell and Cochrane, and due to their particular and nonlinear specification of the evolution for the habit. They are not true for habit specifications in general. Indeed, for a more conventional linear habit formulation, one can show that welfare must decrease along the balanced growth path, if parts of consumption are destroyed: the utility gain later is outweighed by the initial utility loss.

2 The model

The utility function of the representative agent is

\[ E_0 \sum_{t=0}^{\infty} \delta^t \frac{(C_t - X_t)^{1-\gamma} - 1}{1 - \gamma}, \]
where $\delta$ is the subjective time discount factor and $X_t$ is the level of external habit. A conventional linear external habit formulation specifies that

$$X_{t+1} = \mu X_t + \alpha C^a_t,$$  

(2)

where $C^a$ denotes average consumption by all agents in the economy, and $\mu$ and $\alpha$ are parameters.

Campbell and Cochrane (1999) proceed differently. They postulate a process for the economy’s surplus consumption ratio, $S^a_t \equiv (C^a_t - X_t)/C^a_t$. Using lowercase letters to indicate logarithms, they assume that the logarithm of the surplus consumption ratio evolves as a heteroscedastic AR(1) process,

$$s^a_{t+1} = (1 - \phi)\bar{s} + \phi s^a_t + \lambda(s^a_t) (c^a_{t+1} - c^a_t - g),$$  

(3)

where $\phi \in [0, 1)$, $g$ and $\bar{s}$ are parameters, and the function $\lambda(s^a)$ is given by

$$\lambda(s^a) = \begin{cases} S^{-1}\sqrt{1 - 2(s^a - \bar{s})} - 1, & s^a \leq s_{\text{max}}; \\ 0, & s^a \geq s_{\text{max}}; \end{cases}$$  

(4)

with $s_{\text{max}} = \bar{s} + (1 - \bar{S}^2)/2$. The parameter $\bar{s}$ is the logarithm of the steady-state surplus consumption ratio $\bar{S}$, and Campbell and Cochrane set $g$ equal to the logarithm of the mean consumption gross growth rate $G$. It can be shown that the C-C formulation and the conventional linear habit formulation in equation (2) share the same steady state if $\mu = G\phi$ and $\alpha = G(1 - \phi)(1 - \bar{S})$.

Campbell and Cochrane consider a pure endowment economy. Let $Y_t$ be the per capita endowment in period $t$. Endowment growth is modeled as an i.i.d. lognormal process,

$$\Delta y_{t+1} = g + \nu_{t+1}, \quad \nu_{t+1} \sim \text{i.i.d. } \mathcal{N}(0, \sigma^2).$$  

(5)

The equilibrium outcome in a laissez-faire market economy is that consumption equals endowment, $c^a_t = y_t$, since the private marginal utility under the C-C preference specification is strictly positive. But, contrary to Campbell and Cochrane’s assertion, we shall demonstrate that the social marginal utility can be negative. Specifically, we investigate the welfare effects when a social planner destroys some of the endowment.

Let $\psi_t$ denote the logarithm of the fraction of the endowment that the representative agent gets to consume in period $t$, i.e., $\psi_t \equiv c^a_t - y_t \leq 0$. The welfare of the representative
agent can then be written as

\[
E_0 \sum_{t=0}^{\infty} \delta^t \left( \frac{(S^a_t C^a_t)^{1-\gamma} - 1}{1 - \gamma} \right) = E_0 \sum_{t=0}^{\infty} \delta^t \exp \left( \frac{(1 - \gamma)(s^a_t + \psi_t + y_t)) - 1}{1 - \gamma} \right),
\]

and the law of motion for the log surplus consumption ratio can be expressed as

\[
s_{t+1}^a = (1 - \phi)\bar{s} + \phi s_t^a + \lambda(s_t^a)(\psi_t + \nu_{t+1}),
\]

where we have used equation (5) to substitute out for \( g \). From now on, we will leave out the superscript \( a \) since outcomes for the representative agent and economy-wide averages are the same in an equilibrium.

## 3 Social marginal utility can be negative

If Campbell and Cochrane had been correct when asserting that the social marginal utility is always positive in their model, the highest welfare would be attained by setting \( \psi_t = 0 \) in all periods. However, Campbell and Cochrane only prove that an infinitesimal destruction of the endowment leads to a welfare loss. To illustrate our surprising finding that noninfinitesimal destructions can increase welfare under the C-C habit formulation, it is instructive to consider a one-time perturbation from a steady state.

Consider an economy in a non-stochastic steady state with endowment and consumption growing at a constant growth rate \( G \geq 1 \). The parameter restriction that ensures a bounded objective function is

\[
\delta G^{1-\gamma} < 1.
\]

Suppose now that the social planner destroys part of the endowment in one single period, denoted period 0. Thus, we have \( \log(C_0/Y_0) = \psi < 0 \), and the sequences of the logarithms of consumption and the surplus consumption ratio evolve as follows

\[
\begin{align*}
c_0(\psi) &= y_0 + \psi < y_0, \\
c_t(\psi) &= y_t, \quad \text{for all } t \geq 1; \\
s_0(\psi) &= \bar{s} + \bar{\lambda}\psi < \bar{s}, \\
s_t(\psi) &= \bar{s} - \phi^{t-1}\psi \left[ \lambda(\bar{s} + \bar{\lambda}\psi) - \phi\bar{\lambda} \right] > \bar{s}, \quad \text{for all } t \geq 1;
\end{align*}
\]
where $\bar{\lambda} \equiv \lambda(\bar{s})$. Evidently, the representative agent’s utility falls in period 0 because both his consumption level and the surplus consumption ratio decline relative to the steady state. But the utilities in all future periods increase due to a higher surplus consumption ratio that asymptotically returns to its steady-state value. The question is whether the discounted sum of these changes in utilities produce a welfare gain or a welfare loss.

After eliminating the constant terms involving $-1/(1-\gamma)$ in the preference specification and dividing through by $\exp((1-\gamma)y_0)$, the discounted life-time utility of the described perturbation can be expressed as

$$W(\psi) \equiv \exp\left(\frac{(1-\gamma)(s_0(\psi) + \psi)}{1-\gamma}\right) + \sum_{t=1}^{\infty} \delta^t \exp\left(\frac{(1-\gamma)(s_t(\psi) + tg)}{1-\gamma}\right)$$

(10)

with the derivative

$$W'(\psi) = (1 + \bar{\lambda}) \exp\left(\frac{(1-\gamma)(s_0(\psi) + \psi)}{1-\gamma}\right) + \sum_{t=1}^{\infty} \delta^t \left[\phi^{t-1}s'_1(\psi)\right] \exp\left(\frac{(1-\gamma)(s_t(\psi) + tg)}{1-\gamma}\right),$$

(11)

where

$$s'_1(\psi) = -\left[\lambda(\bar{s} + \psi \bar{\lambda}) - \phi \bar{\lambda}\right] - \psi \bar{\lambda} \lambda'(\bar{s} + \psi \bar{\lambda}) < 0.$$ 

(12)

The derivative $s'_1$ is negative since $\psi < 0$ and $\lambda$ is a decreasing function. Thus, whether welfare marginally increases or decreases at negative values of $\psi$ depends on whether the first or the second term in equation (11) dominates numerically. It can be shown that welfare is globally increasing with a conventional linear habit formulation: there, endowment destruction in a steady state always leads to a welfare loss. This is also true locally along the steady-state path for the C-C preferences.\(^1\) This should come as no surprise since Campbell and Cochrane (1999, p. 246) prove that the social marginal utility is positive in their model. More specifically, they show that the social marginal utility is positive for infinitesimal perturbations when the endowment follows a random walk. When setting growth equal to

\(^1\)We verify that an infinitesimal endowment destruction decreases welfare under the C-C preferences;

$$W'(0) = (1 + \bar{\lambda})S^{1-\gamma} - \sum_{t=1}^{\infty} \delta^t \phi^{t-1}(1-\phi)\bar{\lambda}\left[S^t\right]^{1-\gamma} = \frac{1 - \phi \delta S^{1-\gamma} + (1- \delta G^{1-\gamma})\bar{\lambda}}{1 - \phi \delta G^{1-\gamma}} S^{1-\gamma} > 0,$$

where the convergence of the infinite sum and the strict inequality follow from $\phi \in [0,1)$ and parameter restriction (8). Thus, in the neighborhood around the steady-state growth path, welfare is strictly increasing in the fraction of the endowment that is consumed.
zero in our calculations, we have a constant endowment level or a degenerate random walk.

However, the local result for the C-C preferences fails to hold globally. Given Campbell and Cochrane’s parameterization,\(^2\) we compute the representative agent’s welfare associated with one-time endowment destructions between 0 and 25 percent. Figure 1 shows clearly that there exist endowment destructions that do raise welfare in the C-C habit model,\(^3\) while welfare decreases globally in the standard habit model. To understand the mechanisms at work, consider the example of a five-percent endowment destruction in Figure 2. Under the conventional linear habit formulation in equation (2), habit responds with a one-period lag to the endowment destruction in period 0 and, as can be seen in Figure 2, the resulting decline in habit is much less than under the C-C habit model. In addition, under the C-C habit formulation in equation (3), habit moves contemporaneously with consumption changes and according to the solid curve in Figure 2, the habit level now falls in response both to the endowment destruction (period 0) and to the subsequent increase in consumption (period 1). Hence, the loss of utility in the period with endowment destruction is mitigated under the C-C habit formulation because of the contemporaneous drop in habit, and future utility gains are magnified by the additional habit decline that is triggered by the consumption hike after the endowment destruction.

The effects on agents’ welfare in the C-C habit model depend on how the distance between consumption and habit change in different periods as illustrated in Figure 3 where

\[
\begin{align*}
C_0(\psi) - X_0(\psi) &= \exp (y_0 + \psi + s_0(\psi)) ; \\
C_t(\psi) - X_t(\psi) &= \exp (y_t + s_t(\psi)) ,
\end{align*}
\]

for all \(t \geq 1\).

For a given \(\psi \leq 0\), it is informative to study the marginal change in \((C_t - X_t)\) when perturbing \(\psi\),

\[
\frac{d}{d \psi} [C_0(\psi) - X_0(\psi)] = (1 + \bar{\lambda}) \exp (y_0 + \psi + s_0(\psi)) ;
\]

\(^2\)Following Campbell and Cochrane, all numerical analyses are performed at a monthly frequency. The parameter values are as follows with annualized values in parantheses, which are taken from Campbell and Cochrane (1999, Table 1); \(\gamma = 2, \delta = 0.9909 \ (0.89), \phi = 0.9885 \ (0.87), \bar{S} = 0.057, g = 0.1575\% \ (1.89\%),\) and \(\sigma = 0.4330\% \ (1.50\%).\) While the latter standard deviation of endowment growth is used in the stochastic welfare calculations by Ljungqvist and Uhlig (2009), as we report on in section 5, the deterministic calculations of the present paper set \(\sigma\) equal to zero.

\(^3\)One cannot discern in the upper panel of Figure 1 that endowment destructions lead to welfare losses locally around the steady state under the C-C habit model, as proven in footnote 1. But after magnifying the scale in the lower panel of Figure 1, we see that welfare does indeed decrease for endowment destructions less than 0.07 percent.
\[
\frac{d}{d \psi} \left[ C_t(\psi) - X_t(\psi) \right] = \phi^t - s'_1(\psi) \exp \left( y_t + s_t(\psi) \right), \quad \text{for all } t \geq 1.
\]

We can see that the derivative in period 0 gets muted at low values of \( \psi \), i.e., at higher levels of endowment destruction, while the opposite is true for the corresponding derivatives in future periods. In fact, the multiplicative term \( s'_1 \) as given in equation (12) becomes arbitrarily large and negative when \( \psi \) is driven to ever lower values and therefore, the associated loss in \( (C_t - X_t) \) for \( t \geq 1 \), becomes arbitrarily large when reducing the amount of endowment destruction in period 0. This in turn implies that \( (C_t - X_t) \) for \( t \geq 1 \), must take on arbitrarily large values when computed at ever lower values of \( \psi \). Figure 3 depicts the exploding outcome for \( (C_1 - X_1) \) when increasing the amount of endowment destruction in period 0. Behind the exploding outcome for \( (C_1 - X_1) \) in Figure 3 lies a critical property of the C-C preference specification: habit can move negatively with consumption.

4 Habit can move negatively with consumption

In Figure 2, the consumption hike in period 1 does not increase but rather decreases the habit level in the C-C habit model. Hence, Campbell and Cochrane’s claim that habit moves nonnegatively with consumption everywhere is incorrect. In fact, habit can fall contemporaneously with a rise in consumption even locally around the steady state. After differentiating the law of motion for the surplus consumption ratio in equation (3), we obtain

\[
\frac{dx_{t+1}}{dc_{t+1}} = 1 - \frac{\lambda(s_t)}{\exp(-s_{t+1}) - 1}.
\]

In the steady state, \( s_t = s_{t+1} = \bar{s} \), so the parameterization of the function \( \lambda(s) \) in equation (4) guarantees that \( dx/dc = 0 \) at the steady state. Next, we calculate the second derivative,

\[
\frac{d^2 x_{t+1}}{dc_{t+1}^2} = -\frac{\lambda(s_t)^2}{[\exp(-s_{t+1}) - 1]^2} \exp(-s_{t+1}) \leq 0,
\]

and the expression is strictly negative at the steady state. This establishes that there is a region around the steady state in which habit moves negatively with consumption.\(^4\)

Based on Campbell and Cochrane’s parameterization (see our footnote 2), Figure 4 maps out the relationship between consumption changes and movements in the habit level. In particular, for a given value of last period’s surplus consumption ratio, the figure depicts

\(^4\)We are thankful to John Cochrane for suggesting this exposition of our argument.
how contemporaneous habit responds to percentage changes in consumption relative to last period’s levels. As a reference point, the steady-state surplus consumption ratio is 0.057. It can be seen that the habit level moves negatively with consumption for a wide range of consumption increases. This property is central to the numerical findings in Ljungqvist and Uhlig’s (2009) simulations of the C-C framework, which reveal substantial welfare gains of big as well as small periodic endowment destructions.

5 Concluding remarks

Ljungqvist and Uhlig (2009) calculate that a society of agents with the preferences and stochastic endowment process of Campbell and Cochrane (1999) would experience a welfare gain equivalent to a permanent increase of nearly 16% in consumption, if the government enforced one month of fasting per year, reducing consumption by 10 percent then. The large welfare improvements associated with the cyclical destruction of endowments can be understood as “investments” in a lower habit level. That is, a period of endowment destruction is most likely to be followed by a rebounce in consumption next period and this consumption growth will often be associated with the strange effect of lowering the habit level.

If the Campbell-Cochrane preferences were embedded in an economy with storage or production, it would rationalize outcomes of consumption bunching either chosen by households themselves under internal habit formation or through destabilizing policies by a benevolent government under external habit formation. In calculations not reported here, using the stochastic endowment process of Campbell and Cochrane, we find large welfare gains from storing roughly 10 percent of the endowment and consuming the savings in a consumption binge every other month. To make the households in a laissez-faire economy that consumes the endowment indifferent to such a policy, consumption would have to be raised by more than 30 percentage points for the indefinite future.

5By contrast, Ljungqvist and Uhlig (2000) report on how welfare can be improved through policies of consumption stabilization under catching-up-with-the-Joneses preferences, i.e., the conventional linear external habit formulation. In a productivity-shock driven economy, it is shown that such a consumption externality calls for an optimal tax policy that affects the economy countercyclically via procyclical taxes, i.e., “cooling” down the economy with higher taxes in booms and lowering taxes in recessions to stimulate the economy.
References


Figure 1: Welfare gain associated with a one-time endowment destruction, measured by the permanent percentage increase in consumption needed to attain the same utility without any destruction. In the upper (lower) panel, along a non-stochastic steady-state growth path, a fraction between 0 and 25 percent (0 and 0.09 percent) of the endowment is destroyed in one single period. The solid and dashed curve depict the welfare gain associated with such a destructive policy including the utility loss of the initial endowment destruction, in the C-C habit model and the standard habit model, respectively. Note that utility is not defined in the standard habit model for endowment destructions that exceed the surplus consumption ratio in the steady state, $\bar{S} = 0.057$. (The parameterization is the same as that of Campbell and Cochrane (1999) except that there is no uncertainty, as reported in our footnote 2.)
Figure 2: Detrended consumption and habit level associated with a five-percent endowment destruction in period 0. In the upper panel, the dash-dotted curve depicts the consumption time series that bounces back in period 1, and the solid and dashed curve show the habit time series for the C-C habit model and the standard habit model, respectively. In the lower panel, the solid and dashed line depict the difference between the consumption and habit time series for the C-C habit model and the standard habit model, respectively. (The parameterization is the same as that of Campbell and Cochrane (1999) except that there is no uncertainty, as reported in our footnote 2.)
Figure 3: Difference between consumption and habit level in the C-C habit model in response to a one-time endowment destruction. The solid curve depicts the difference in the period of the endowment destruction, \( C_0 - X_0 \), and the dashed curve depicts the detrended difference in the next period, \( C_1 - X_1 \). (The parameterization is the same as that of Campbell and Cochrane (1999) except that there is no uncertainty, as reported in our footnote 2.)
Figure 4: How contemporaneous habit is affected by a consumption change relative to last period’s levels, for different values of last period’s surplus consumption ratio. (The parameterization is the same as that of Campbell and Cochrane (1999), as reported in our footnote 2.)
Appendix

We show that welfare cannot increase by destroying part of the endowment along a steady-state growth path, given that the external habit level is governed by a conventional linear law of motion;

\[ X_t = \mu X_{t-1} + \alpha C_{t-1} = \alpha C_0^\gamma \sum_{j=0}^{t-1} \mu^j G^{t-1-j} + \mu' X_0, \]

where the second equality would hold along the constant growth path. In a steady state, habit is ensured to be less than consumption if the parameters satisfy

\[ G > \mu + \alpha, \quad (13) \]

and habit would then grow at the rate \( G \) and result in a steady-state ratio \( X_t/C_t = \alpha/(G-\mu) \).

Let \( \{C_t, X_t\} \) denote the sequence of consumption and habit levels in the steady state, and consider a one-time perturbation where a fraction \( \Delta \in [0, 1 - \alpha/(G-\mu)) \equiv \Gamma \) of the endowment is destroyed in period 0: \( \tilde{C}_0 = (1 - \Delta)C_0, \tilde{C}_t = C_t \) for all \( t \geq 1 \); \( \tilde{X}_0 = X_0, \tilde{X}_t = X_t - \mu^{t-1} \alpha \Delta C_0 \) for all \( t \geq 1 \). Let \( \Omega(\Delta) \) denote the welfare associated with a perturbation \( \Delta \), i.e., the preferences in (1) are evaluated at the allocation \( \{\tilde{C}_t, \tilde{X}_t\} \). Since \( \Omega''(\Delta) < 0 \) for all \( \Delta \in \Gamma \), it suffices to show that \( \Omega'(0) < 0 \) in order to establish that \( \Omega'(\Delta) < 0 \) for all \( \Delta \in \Gamma \). We can compute

\[ \Omega'(0) = -(C_0 - X_0)^{-\gamma}C_0 + \sum_{t=1}^{\infty} \delta^t (C_t - X_t)^{-\gamma} \mu^{t-1} \alpha C_0. \]

After substituting in for the steady-state allocation, a condition for \( \Omega'(0) < 0 \) is

\[ -1 + \sum_{t=1}^{\infty} \delta^t G^{-\gamma} \mu^{t-1} \alpha < 0 \quad \Rightarrow \quad G^\gamma \delta^{-1} > \mu + \alpha, \]

which is guaranteed to hold under our parameter restrictions (8) and (13).