Risk and Ambiguity in Models of Business Cycles

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Carnegie-Rochester-NYU Conference

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The “Great Recession” and its aftermath

Real Gross Domestic Product
Percentage change from previous peak, Seasonally Adjusted

Quarters from previous peak

Source: U.S. Bureau of Economic Analysis

Backus, Ferriere, & Zin (NYU)
What happened?

- What we see
  - Magnitude: deeper recession than usual
  - Persistence: longer recovery — maybe slower, too

- Like Kydland-Prescott with productivity shocks?
  - Relative magnitudes look right
  - Comovements look right, too
  - But... measured productivity didn’t fall very much
Was it uncertainty?

Great Recession

From Wikipedia, the free encyclopedia

This article is about the global economic downturn during the early 21st century. For background on financial market events dating from 2007, see financial crisis of 2007–08.

The Great Recession[^3][^4] (also referred to as the Lesser Depression[^5], the Long Recession[^6] or the global recession of 2009[^7][^8]) was a global economic decline in the late 2000s. According to aggregated national data, a worldwide recession began in Q3-2008 and ended in Q1-2009. It is widely believed that the severity and length of this recession was the direct consequence of an increase in macroeconomic uncertainty.

It is related to a liquidity crisis, commonly being dated to have started when several central banks had to step in with liquidity lending to the interbank lending market on 9 August 2007. This was a response to a situation where BNP Paribas temporarily had to block money withdrawals from three hedge funds—citing a "complete evaporation of liquidity".[^9] The bursting of the U.S. housing bubble[^10] where the median price for real estate home sales in US started to decline after its peak in July 2006,[^11] had caused the values of securities tied to U.S. real estate pricing to plummet, which damaged financial institutions globally—to a degree ultimately resulting in the subsequent interbank credit crisis.[^12][^13] The first sign
Was it uncertainty?

- Marco Buti, Director General of the European Commission

*Economic theory suggests that uncertainty has a detrimental effect on economic activity by giving agents the incentive to postpone investment, consumption and employment decisions until uncertainty is resolved, and by pushing up the cost of capital through increased risk premia.*
What happened?

- Nick Bloom

*The onset of the Great Recession was accompanied by a massive surge in uncertainty. The size of this uncertainty shock was so large it potentially accounted for around one third of the 9% drop in GDP versus trend during 2008-2009.*
What we do

- Take a streamlined business cycle model
- Ask: How does **uncertainty** affect the **dynamics** of output, consumption, and investment?
  - Magnitude: Does uncertainty magnify fluctuations?
  - Persistence: Can it reduce the speed of recovery?
Modeling ingredients

- **Streamlined** business cycle model
  - Recursive preferences
  - Unit root in productivity
  - Fixed labor supply

- **With fluctuations in** uncertainty
  - *Risk* (stochastic volatility)
  - *Ambiguity* (unobservable long-term growth)
Preview of results

Fluctuations in uncertainty have **limited impact**

- **Persistence**
  - Separation property: internal **dynamics independent of risk and risk aversion**
  - Persistence must be in the shock

- **Magnitude**
  - Impact typically small, but magnified by **risk aversion**

Business cycle properties governed by IES
Risk and uncertainty

- **Recursive references**

\[
U_t = V[c_t, \mu_t(U_{t+1})] = [(1 - \beta)c_t^\rho + \beta \mu_t(U_{t+1})^\rho]^{1/\rho}
\]

\[
\mu_t(U_{t+1}) = [E_t(U_{t+1}^\alpha)]^{1/\alpha}
\]

\(V, \mu_t\) homogeneous of degree one, \(RA = 1 - \alpha\), \(IES \equiv \sigma = 1/(1 - \rho)\)

- **Stochastic structure** of productivity \(a_t\)

\[
\log g_t = \log(a_t/a_{t-1}) = \log g + e^\top x_t \quad \text{("productivity growth")}
\]

\[
x_{t+1} = Ax_t + \nu_t^{1/2} Bw_{1t+1} \quad \text{("news")}
\]

\[
\nu_{t+1} = (1 - \varphi_v)\nu + \varphi_v \nu_t + \tau w_{2t+1} \quad \text{("risk")}
\]

\((w_{1t}, w_{2t}) = \text{iid standard normals}\)
Scaling

- Bellman equation

\[ J(k_t, x_t, v_t, a_t) = \max_{c_t} V\{c_t, \mu_t[J(k_{t+1}, x_{t+1}, v_{t+1}, a_{t+1})]\} \]
\[ \text{s.t.} \quad k_{t+1} = f(k_t, a_{t n}) - c_t \]

- Assume \( f \) \( hd1 \): \( f(k, a_n) = k^\omega a_n^{1-\omega} + (1 - \delta) k \)

- **Rescaled** Bellman equation \([\tilde{k}_t = k_t/a_t, \tilde{c}_t = c_t/a_t]\)

\[ J(\tilde{k}_t, x_t, v_t) = \max_{\tilde{c}_t} V\{\tilde{c}_t, \mu_t[\tilde{g}_{t+1}J(\tilde{k}_{t+1}, x_{t+1}, v_{t+1})]\} \]
\[ \text{s.t.} \quad g_{t+1}\tilde{k}_{t+1} = f(\tilde{k}_t, n) - \tilde{c}_t \]

- Numerical solution
## Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
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<td><strong>Preferences</strong></td>
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<td>$0.74 \times 10^{-5}$</td>
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Model is essentially loglinear
Insights from loglinearization I

- Goal: loglinear decision rule for capital
  \[ \log \tilde{k}_{t+1} = h_k \log \tilde{k}_t + h_x^\top x_t + h_v v_t - \log g_{t+1} \]

- Dynamic programming version of Campbell (JME, 1994)

- Loglinearization around the **stochastic** steady-state
Loglinearize **capital’s marginal product** and **law of motion**

\[
\log f_{kt} = \lambda_r \log \tilde{k}_t + \lambda_0 \\
\log \tilde{k}_{t+1} = \lambda_k \log \tilde{k}_t - \lambda_c \log \tilde{c}_t + \lambda_1 - \log g_{t+1}
\]

where \((\lambda_k, \lambda_c, \lambda_r)\) are steady-state objects.

**Guess loglinear value function and derivative**

\[
\log J_t = p_k \log \tilde{k}_t + p_x^\top x_t + p_v \nu_t + p_0 \\
\log J_t^{\rho-1} J_{k,t} = q_k \log \tilde{k}_t + q_x^\top x_t + q_v \nu_t + q_0
\]
Seperation property

**Claim**

Consider the loglinear approximation of capital’s law of motion,

\[ \log \tilde{k}_{t+1} = h_0 + h_k \log \tilde{k}_t + h_x^T x_t + h_v v_t - \log g_{t+1} \]

If we hold constant the stochastic steady state:

1. \(h_k\) is independent of properties of all shocks and risk aversion
2. \(h_x\) is independent of properties of uncertainty shocks and risk aversion

\[
\begin{align*}
    h_k &= \lambda_k + \sigma \lambda_c (q_k - \lambda_r), \quad h_x^T = \sigma \lambda_c q_x^T \\
    q_k &= q_k [\lambda_k + \sigma \lambda_c (q_k - \lambda_r)] + \lambda_r \\
    q_x &= -(\sigma^{-1} + q_k) e^T A [(1 - \sigma q_k \lambda_c) I - A]^{-1}
\end{align*}
\]
The claim is informative
Risk aversion magnifies uncertainty shocks.
## Business cycles governed by IES

### Risk Aversion

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<tr>
<th>US Data</th>
<th>Benchmark</th>
<th>Cst. vol.</th>
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<tbody>
<tr>
<td>Risk Aversion</td>
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<td>10</td>
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### Standard deviations (%)

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<td>0.82</td>
<td>0.82</td>
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<td>0.55</td>
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<td>2.79</td>
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### Correlations with output growth

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**Intertemporal elasticity of substitution:** 0.5
## Business cycles governed by IES

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**Risk aversion: 10**
Risk and ambiguity

- Divide the state in two: \( s_t = (s_{1t}, s_{2t}) \)

- **Ambiguity** (Klibanoff, Marinacci, & Mukerji; Ju & Miao)

  \[
  \text{risk} = p_{1t}(s_{1t+1}|s_{2t+1}, I_t) \\
  \text{ambiguity} = p_{2t}(s_{2t+1}|I_t)
  \]

- Two-part certainty equivalent

  \[
  \mu_{1t}(U_{t+1}) = \left[ E_{1t}(U_{t+1})^{\alpha} \right]^{1/\alpha} \\
  \mu_{2t}[\mu_{1t}(U_{t+1})] = \left\{ E_{2t}[\mu_{1t}(U_{t+1})^{\gamma}] \right\}^{1/\gamma}
  \]

  \( \alpha \) controls risk aversion, \( \gamma < \alpha \) controls ambiguity aversion
A second certainty equivalent

- Rule of thumb: associate ambiguity with unobservables
- Consider three stochastic processes
  - $x_t = \text{mean growth rate}$ (not observable)
  - $\log g_t = \text{realized growth rate}$ (observable)
  - $\nu_t = \text{"stochastic volatility"}$

$$\log g_t = \log g + x_t + \nu_t^{1/2} w_{1,t}$$

$$x_{t+1} = \varphi_x x_t + \nu_t^{1/2} w_{2,t+1}$$

$$\nu_{t+1} = \varphi \nu + (1 - \varphi_v) \nu_t + \tau w_{3,t+1}$$

- Kill learning ($\varphi_x = 0$)
- Magnitudes small, separation property holds — as before
## Calibration

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