Uncertainty, Default Risk and the Effects of Monetary Policy*

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Abstract

During 2009 and 2010, lending to non-financial businesses in the US decreased by 22%, an unprecedented drop in the US post war economy. An explanation that has been proposed is that this is the natural consequence of banks’ capital losses during the Great Recession. I argue that this explanation may be incomplete. In fact, banks have accumulated excess reserves balances at the Federal Reserve that exceeded the drop in lending. As an alternative explanation of the reduction in lending, I provide evidence of increased riskiness in the US non-financial business sector. I develop a model of financial intermediation where lending to firms is risky since they can default on their liabilities. I use the model to argue that the increase in firms’ riskiness could be an explanation of the observed drop in lending. Finally, I analyze the types of policies that can be successful in restoring lending in a situation of firms’ increased riskiness.

*PRELIMINARY AND VERY INCOMPLETE. Click here for updates. Comments are welcome Email: gaston.navarro@nyu.edu. I am greatly indebted to Ricardo Lagos for his continuous guidance and advice. I am also grateful to Anmol Bhandari, Andres Blanco, Mark Gertler, Anna Gumen, Boyan Jovanovic, and Thomas Sargent for helpful discussions and comments.
1 Introduction

During 2009 and 2010, lending to non-financial businesses in the US decreased by 22%, a drastic contrast with the 3.3% average annual growth experienced during the previous 60 years.\(^1\) One proposed explanation of this fact is that, during the 2007-2009 crisis, banks suffered large capital losses, which limited their ability to raise funds and consequently impaired their lending capacities. During this period, the Federal Reserve pursued large-scale asset purchases programs, with the goal of preserve financial stability and stimulate lending. However, simultaneously with the expansion of the Federal Reserve’s balance sheet, banks accumulated reserves that were more than 15 times greater than those required by the Federal Reserve.\(^2\) Naturally, this casts doubt on the argument that banks lacked capacity to lend.\(^3\)

In this paper, I argue that the observed decrease in lending can be the result of banks’ unwillingness rather than inability to lend. During periods of deep recessions, default risk at firms’ level increases, to which banks respond by reducing lending. As evidence in firms’ increased riskiness during the last few years, Figure 2 plots firms’ inter-quartile range (IQR) annual sales growth and the delinquency rate on loans to non-financial businesses.\(^4\) These series show that non-financial business were severely hit by the crisis: it took more than two years for these measures of riskiness to return to their pre-crisis values.

I develop a quantitative model that incorporates both of these arguments, banks’ inability and unwillingness to lend, to assess their relative importance in explaining the fall in lending. The main finding is that banks’ unwillingness to lend played a larger role in deterring lending during the last crisis. Intuitively, this comes from the observed shift in banks’ portfolio towards the risk-free asset (reserves). I argue that these findings have important implications for policy design. In particular, if banks are unwilling to lend because of firms increased risk, policies that improve banks’ net worth

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\(^1\)This series correspond to the entry "Commercial and industrial loans, all commercial banks, seasonally adjusted" in Table H.8 from Flow of Funds.

\(^2\)The source for the Federal Reserve balance sheet, banks’ reserves and required reserves come from Table H.3 from Flow of Funds.

\(^3\)For a more general analysis of capital requirements, see Lambertini and Uysalz (2013).

\(^4\)The IQR of sales growth was computed using firms on Compustats as follows. I kept only firms for which I have more than 80 observation (20 years of data). For each firm \(i\), I computed sales growth in period \(t\) as \(\Delta s_{it} = \log(s_{it}/s_{it-4})\), where \(s_{it}\) are sales deflated by CPI inflation. Delinquency rate corresponds to "Commercial and industrial loans, all commercial banks, seasonally adjusted" from Flow of Funds.
will be ineffective in stimulating lending.

The model has two key components. First, banks obtain deposits from households and lend to firms, in presence of capital requirement constraints. After periods of capital losses, this induces banks to decrease their size and cut down lending. I refer to this as a leverage channel that reflects banks’ inability to lend. The second key component is modeling default risk at the firm level. Firm’s idiosyncratic productivity is drawn from a distribution with stochastic volatility. After an increase in this volatility, firms’ default rate increases and consequently banks respond by reducing lending. I refer to this as a volatility channel that reflects banks’ unwillingness to lend.

I argue that the banks’ portfolio decision is key to distinguish the leverage channel and the volatility channel. The leverage channel induces banks to uniformly decrease all of their asset holdings. However, in presence of the volatility channel, banks rebalance their portfolio away of firms’ debt and towards the risk-free asset. These dynamics for banks’ portfolio decision resembles the ones observed in Figures 1 and 2, which the model interprets as evidence in favor of the volatility channel.

The importance of the volatility channel can explain why the efforts exerted by the Federal Reserve have fallen short to fully restore lending. Policies that provide banks with lending facilities, or that directly enhance bank’s net worth, will have very little success in restore lending as banks will hoard these resources without increasing lending. In Section 4, I describe a set of policies that can effectively stimulate lending in the presence of the volatility channel. These take the form of government’s purchases of firms’ defaulted bonds (bailouts). This increases banks’ incentives to lend, since it offers insurance against the increased riskiness in firms.

1.1 Literature Review

This paper is related to two lines of research: (i) macroeconomic models with financial frictions; and (ii) models with stochastic volatility and/or uncertainty shocks. I briefly discuss how this paper relates to each topic and comment as well on other works in the intersection of these topics.

Following Bernanke and Gertler (1989), a large body of work has been devoted to understanding the relationship between financial markets and overall macroeconomic performance. The two
Figure 1: FED balance sheet, bank’s reserves and Lending

Notes: FED Assets are holdings of Treasuries, mortgage-backed securities and agency debt. Reserves and required come from Flow of Funds, Table H.3. Loans correspond to non-financial corporate lending from commercial banks from Flow of Funds, Table 8.

Figure 2: IQR Sales Growth and Delinquency Rates

Notes: IQR Sales Growth is based on Compustats firms. Delinquency rate is on non-financial corporate lending from commercial banks.
canonical references are Kiyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999). More recently, Gertler and Kiyotaki (2009) extended a real business cycle model to incorporate frictional financial intermediation. I build-up on their environment to incorporate default risk in a non-financial sector. This allows me to separately discuss the leverage channel from the volatility channel and assess the importance of each one during the last couple years.

Since the seminal paper of Bloom (2009), there is a large and increasing literature incorporating uncertainty shocks in macroeconomic models. In these models, uncertainty is typically introduced as stochastic volatility on firms' idiosyncratic productivity. I borrow this modeling structure and incorporate it in a model of corporate default risk.

This paper is more closely related to the recent line of research that incorporates volatility shocks in models with financial frictions. In a recent paper, Arellano, Bai, and Kehoe (2011) use an environment with default risk to argue that fluctuations in uncertainty can have large impacts on employment. Similarly, Gilchrist, Sim, and Zakrajsek (2013) use an environment with default risk and costly adjustment of capital to argue that changes in volatility are a key determinant of investment dynamics. Finally, Christiano, Motto, and Rostagno (2013) extend the canonical framework of Bernanke, Gertler, and Gilchrist (1999) to argue that volatility shocks are the main driving force of business cycle fluctuations. This paper contributes to this line of research in two dimensions. First, I incorporate financial intermediation and show how banks' portfolio decision is affected by changes in volatility. Second, I explain how stochastic volatility can affect the outcomes of different government policies. Carefully modeling financial intermediation is key for the policy discussion.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 provides the quantitative analysis. Section 4 discuss policy recommendations. Section 5 concludes.

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5 For more recent contributions, see: Adrian and Boyarchenko (2012), Bigio (2012), Brunnermeier and Sannikov (2012), Jermann and Quadrini (2012) and Kiyotaki and Moore (2012) among others.
6 See also Bloom, Floetotto, Saporta-Eksten, and Terry (2012) and citations therein.
2 Model

Time is discrete and indexed by \( t = 0, 1, \ldots \). The economy is populated by a continuum of firms, banks, capital producers, a representative household and a government. Firms have access to a constant returns to scale technology in capital and labor given as

\[
y = (axk)^{\alpha} n^{1-\alpha}
\]

where \( k \) and \( n \) stand for capital and labor, respectively. Firm’s productivity is given by an aggregate component \( x \) and an idiosyncratic component \( a \). The idiosyncratic productivity \( a \) is independently distributed across time and firms, and its drawn from a time-varying distribution \( a \sim H(\sigma_a) \) which variance \( \sigma_a \) stochastically evolves over time.\(^7\) The volatility component of the idiosyncratic productivity is the way I introduce the volatility channel discussed in the Introduction.

I assume that the idiosyncratic productivity component is independent over time to reduce the dimensionality of the state space. This type of assumption has extensively been used in the literature of macroeconomic models with financial frictions.\(^8\)

In order to fund her activities, firms can issue debt and equity.\(^9\) However, since firms may decide to default, debt is risky. I further assume that debt has long maturity: every period, a fraction \( \lambda \) of the firm’s outstanding liabilities must be repaid in order to avoid default.\(^10\) Upon default the firm disappear and a fraction \( \Xi \) of her capital is lost. Finally, I assume that the household owns all of these firms.

Banks intermediate resources between households and firms. They can take short-term deposits from households and lend to firms by purchasing her risky long-term debt. Since deposits must always be repaid, banks are engaged both in maturity and risk transformations. They also have two additional investment alternatives. On one hand, banks can purchase risk-free government debt;

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\(^7\)For other work about the effect of stochastic volatility in models with financial frictions, see Arellano, Bai, and Kehoe (2011), Christiano, Motto, and Rostagno (2013) and Gilchrist, Sim, and Zakrajsek (2013)

\(^8\)See for instance Gertler and Kiyotaki (2009), Kiyotaki and Moore (2012) and Bigio (2012) where they assume this for investment opportunities.

\(^9\)In this setup, we understand equity injections as negative dividend payments.

\(^10\)Similar assumptions are used in Arellano and Ramanarayanan (2012), Gomes, Jermann, and Schmid (2013) and Blanco and Navarro (2014). As in those papers, I think of \( 1/\lambda \) as the average maturity of the debt.
and on the other hand, they can invest in a risky project with a stochastic return $\xi$. Since only banks can invest in the risky project, I think of realizations of $\xi$ as financial shocks that randomly affects bank’s net worth.\footnote{In other words, banks have access to a "Lucas tree" (Lucas, 1978) that affects their wealth. An alternative would have been to endow banks with a stochastic stream. I opted for using a "Lucas tree" instead because it maintains the homogeneity structure of the bank’s problem. See Section 2.2.}

Household preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta U(C_t, N_t)$$

where $C_t$ and $N_t$ stand for household consumption and hours work, respectively. The utility function $U(\cdot)$ satisfies standard assumptions.

**Timing** within a period is as follows. At the beginning of period $t$, every firm starts with debt $b$ and capital $k$; and banks start with holdings of firms debt $\{b\}$, government bonds $R$ and deposits from households $D$. Then the aggregate and idiosyncratic shocks $\{a, x, \sigma, \xi\}$ are realized. After this, each firm decides whether to default or not. In case of no default, she can hire labor and produce output. Simultaneously, bank’s collect returns from her assets - firm’s bonds, government debt and the risky project - and pays deposits back to the household. At the same time, households make saving, consumption and labor supply decisions. Finally, firms and banks make new portfolio decisions and period $t + 1$ begins.

Next, I formally describe the problem faced by firms, banks, capital producers and the household and define a recursive competitive equilibrium for this economy. I will denote $S$ to the aggregate state of the economy and characterize explicitly later on.\footnote{"Finding the state is an art".}

### 2.1 Firms

If a firm did not default, she can hire workers and produce output that period. This is a static problem described as follows

$$\pi(a, k, S) = \max_n \left\{ (axk)^{\alpha} n^{1-\alpha} - w(S)n \right\}$$
where \( w(S) \) is wages when the aggregate state of the economy is \( S \). The following lemma characterizes firm’s solution

**Lemma 1** Firm’s profits are linear in capital

\[
\pi(a, k, S) = ak\psi(S)
\]

where

\[
\psi(S) = xw(S)^{-\frac{1-\alpha}{\alpha}} \left[ (1 - \alpha)^{\frac{1-\alpha}{\alpha}} - (1 - \alpha)^\frac{1}{\alpha} \right]
\]

(All proofs are contained in the Appendix) The linearity of the profit function is an implication from the constant returns to scale technology assumption. The function \( \psi(S) \) determines the average return to capital in the economy and does not depend on idiosyncratic variables. Total return to capital is \( a\psi(S) \), and those firms with a higher idiosyncratic productivity \( a \) will experience a higher return on production. I will use the function \( \psi(S) \) next in describing the firm’s intertemporal decision.

Let \( E(a, b, k, S) \) be the value of a non-defaulting firm with productivity \( a \), outstanding debt \( b \) and capital \( k \) when the aggregate state of the economy is \( S \). Then

\[
E(a, b, k, S) = \max_{d, b', k'} \left\{ d + \mathbb{E}_{S', a'} \left[ m(S, S') \max_{d, n} \{ 0, E(a', b', k', S') \} \right] | S \right\}
\]

subject to

\[
P(a, b, k, S) = (1 - \tau) \left[ -\lambda b + \psi(S)ak \right]
\]

\[
d + Q^k(S)k' \leq P(a, b, k, S) + Q^k(S)(1 - \delta)k + Q(l', S) \left[ b' - (1 - \lambda)b \right]
\]

\[
S' = \Gamma(S)
\]

where \( l = b/k \) denotes leverage and \( \delta \) is the depreciation rate of capital.

The amount \( P \) in the first line of the feasible set accounts for the firm’s profits after production, debt payment and taxes. As stated in lemma 1, if the firm did not default, she obtains \( ak\psi(S) \) as her returns on production. Similarly, in order to avoid default, she has to pay a fraction \( \lambda \) of her
outstanding debt $b$. Finally, a fraction $\tau$ of the profits are paid in taxes. The second line in the feasible set is the firm’s budget constraint. Her expenses are composed by dividend payments $d$ plus new capital purchases $k'$ at a price $Q^k(S)$. Her available resources this period are profits $\mathcal{P}$, the value of remaining capital $Q^k(S)(1-\delta)k$; and the inflow of new debt $b' - (1-\lambda)b$ which is sold at a market price $Q(l', S)$. The last line is the law of motion for the state of the economy.

The firm’s problem is then to maximize the present discount value of her dividend payments. In doing so, she takes into account that she may optimally decide to default in the future. Also, since households own the firm, their stochastic discount factor $m(S, S')$ is used by the firm to discount the stream of dividend payments.

Firm’s capital structure responds to different incentives. On one hand, since debt payments are tax deductible, debt has a tax benefit. This generates incentives at the firm level to issue debt. However, the price of debt $Q(l', S)$ depends on the firm’s next period leverage position $l' = b'/k'$. The reason why debt prices depend on leverage $l'$ is simply because default probabilities depend on this, and prices reflect the likelihood of this scenario. In making her capital structure decision, firm’s internalize both the benefits and costs of issuing more debt. This same logic is found in default models like Arellano (2008), Eaton and Gersovitz (1981) or Gomes and Schmid (2010). Note as well that the firm may desire to issue new debt ($b' > (1-\lambda)b$) or decrease her liabilities ($b' < (1-\lambda)b$), in which case I assume that the firm is repurchasing old debt. Finally, firms can finance her activities with equity injections, since I allow for $d$ to be negative.

The following lemma characterizes firm’s optimal policies.

**Lemma 2 (Firm’s Problem)** The value function of the firm is linear in capital

$$ E(a, b, k, S) = \mathcal{P}(a, b, k, S) + e(l, S)k $$

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13 Defaults by itself breaks Modigliani-Miller irrelevance result. In particular, the firm would strictly prefer to use equity only. Thus, the corporate tax benefit of debt is necessary for obtaining debt issuance in equilibrium. See Gourio (2013), Gomes and Schmid (2010) or Gomes, Jermann, and Schmid (2013) for similar assumptions.

14 One difference in this set up is that, since the idiosyncratic shock has no persistence, the likelihood of default next period does not depend on the current idiosyncratic productivity. This is why the price of debt does not depend on the firm’s productivity, although it does on the aggregate state of the economy $S$, which includes the aggregate productivity $x$ and the volatility $\sigma_a$. 

9
where \( e(l, S) = E(a,l,1,S) - \mathcal{P}(a,l,1,S) \). Policies for capital and debt are linear in capital as well

\[
k'(a,b,k,S) = \iota(l,S)k
\]

\[
b'(a,b,k,S) = \ell'(l,S)\iota(l,S)k
\]

Default is optimal only for firms with productivity below a threshold \( a(l,S) \), given as

\[
a(l,S) = \frac{1}{\psi(S)} \left[ \lambda l - \frac{e(l,S)}{1-\tau} \right]
\]

Finally, in equilibrium, the value \( e(l,S) \) is given as follows

\[
e(l,S) = Q^k(S)(1-\delta) - Q(\ell'(l,S,S))l
\]

Lemma 2 explains how to compute firm’s policies and also contains much of the model’s intuition. First of all, firm’s policies are linear in capital which makes aggregation easy. Furthermore, if all firms start with the same leverage level \( l \) this period, they will all have the leverage \( \ell'(l,S) \) next period. Thus, the distribution of leverage degenerates in a time-varying point every period. Second, the firm’s default decision follows a threshold decision: firm’s with idiosyncratic productivity below \( a(l,S) \) will optimally decide to default. In turn, this threshold reflects static and dynamic components that affects firm’s default decision. On one hand, the default threshold positively depends on leverage \( l \) and thus firm’s with a higher leverage are more likely to default. This is a standard component where highly leveraged firms are more risky and likely to default. On the other hand, the higher value of capital \( e(l,S) \) the lower the default threshold. This is a forward looking component: periods in which the firms find the future value of capital more profitable will coincide with periods of low default. As we argue below, the forward component in the default decisions allows to have procyclical leveraged and countercyclical default, as observed in data.

\[\text{In general, if the objective is homogeneous of some degree and the feasible set is homogeneous of degree one, policies are linear in some measure of wealth. See Angeletos (2007).}\]
2.2 Banks

I borrow many of the bank’s modeling assumptions from Gertler and Kiyotaki (2009). In particular, I assume a stochastic industry equilibrium structure (Cooley and Quadrini, 2001) in which banks exit the market every period with a constant probability $1 - \sigma$. While in the market, they can take deposits from households and invest in firm’s bonds, as well as in government bonds and in the risky project. However, in any given period the banker can divert a fraction $\theta$ of her balance sheet and liquidate the bank. In order to prevent this, banks face a portfolio constraint such that, in equilibrium, they decide not to divert assets. I also assume that bank’s retain all of their earnings until they exit, moment in which the pay all of their net worth as a dividend to the household.\footnote{The actual assumption needed is that banks cannot pay negative dividends. Assuming that they only pay dividends upon exit is done for simplicity. Similar assumptions are made in Gertler and Kiyotaki (2009)} Banks always repay their deposits, even upon exit. Finally, a fix flow $1 - \sigma$ of new banks enter every period which keeps the measure of banks fixed. New banks receive a one time equity injection from the household to begin operations (see Section 2.4).

Let $V(\omega, S)$ be the value to a bank that has wealth $\omega$ when the aggregate state of the economy is $S$. Then

\[
V(\omega, S) = \max_{(b(l'))_{l',A',R',D'}} \left\{ \mathbb{E}_S \left[ m(S, S') \left\{ (1 - \sigma)\omega' + \sigma V(\omega', S') \right\} | S \right\} \right\}
\]  

subject to

\[
\int Q(l', S)b(l')dl' + Q^A(S)A' + Q^R(S)R' \leq \omega + Q^D(S)D'
\]

\[
\omega' = \int \left[ 1 - H(a(l', S)) \right] \left[ \lambda + (1 - \lambda)Q(l'(l', S'), S') \right] b(l')dl'
\]

\[
+ \left[ \xi_A' + Q^A(S') \right] A' + R' - D'
\]

\[
V(\omega, s) \geq \theta \left[ \int Q(l', S)b(l')dl' + Q^A(S)A' + Q^R(S)R' \right]
\]

The first line of the feasible set is the bank’s budget constraint. The bank’s expenses have three components. First, the bank can buy debt $b(l')$ of a firm with leverage $l'$ at a market price $Q(l', S)$. Adding up over all possible leverages, we obtain the amount of lending from bank to firms $\int Q(l', S)b(l')dl'$. Second, the bank can purchase government debt $R'$ which pays one unit
tomorrow and has a price $Q^R(S)$ today. Third, the bank can buy $A'$ units of the risky project, which pays $\xi'$ next period and has a price $Q^A(S)$. The bank’s available resources are her wealth $\omega$, and the new deposits she takes from the household $D'$ which have a price $Q^D(S)$. The second line describes the realization of bank’s wealth $\omega'$ next period. First of all it includes the returns from lending to banks: from lending to firms with leverage $l'$, a fraction $[1 - H(a(l', S))]$ repays, in which case the banks obtains the repayment fraction $\lambda$ plus the market value of the remaining debt $(1 - \lambda)Q(l'(l', S'), S')$. Finally, the bank obtains the returns on the risky project, which includes the new price of the asset, plus his savings in the government bond minus deposits repayments.

Equation (10) reflects the portfolio constraint that banks face. At the end of the period, the bank can divert a fraction $\theta$ of the value of her assets: $\int Q(l', S)b(l')dl' + Q^A(S)A' + Q^R(S)R'$. In turn, the constraint is such that, in equilibrium, the value of staying in the market $V(\omega, S)$ is larger than the value of diverting funds.

The bank’s problem is the to maximize the expected dividend payment upon exit. Next period, with a probability $1 - \sigma$ the bank will exit and pay $\omega'$, and with the remaining probability she survives and gets the value $V(\omega', S')$. Same as with the firm, since the household owns the bank, his stochastic discount factor $m(S, S')$ is used to value dividend payments.

Next lemma characterizes bank’s optimal decisions

**Lemma 3 (Bank’s Problem)** The value function of the bank is linear in wealth $\omega$

\[ V(\omega, s) = \omega v(s) \]  

(11)

where $v(s)$ is bank’s marginal value of wealth. Bank’s portfolio decisions are also linear in wealth

\[ b'(l', \omega, s) = \alpha_B(l', S)\omega, \quad R'(\omega, S) = \alpha_R(S)\omega \]

\[ A'(\omega, s) = \alpha_A(S)\omega, \quad D'(\omega, S) = \alpha_D(S)\omega \]

\[ \text{I am implicitly assuming that, when purchasing debt } b(l'), \text{ the bank is diversifying across a continuum of firms with leverage } l', \text{ which allows her to eliminate firms’ idiosyncratic risk.} \]
Finally, the marginal value of wealth $v(S)$ satisfies the following recursion

$$v(S) = \frac{1}{Q^D(S)} \mathbb{E}_{S'} \left[ m(S, S') \{ (1 - \sigma) + \sigma v(S') \} \right] \frac{1}{1 - \mu(S)}$$

(12)

where $\mu(S)$ is the Lagrange multiplier associated with (10).

Lemma 3 shows that aggregation at the bank level is also simple: as a fraction of their wealth, every bank makes the same portfolio decisions. By introducing equation (11) in (10) we have the following

$$\int Q(l', S)b(l')dl' + Q^A(S)A' + Q^R(S)R' \leq \frac{v(S)}{\theta}$$

(13)

The left hand side on equation (13) is bank’s leverage. Then, the portfolio constraint on the bank results in a leverage constraint: in any given period, bank’s portfolio must achieve a leverage lower than $v(S)/\theta$. Therefore, in periods of capital losses, banks will have lower net worth and consequently their borrowing capacities will be limited. This is the way I obtain the leverage channel discussed in the Introduction.

For future reference, let $\Omega(S) = \int \omega_i di$ be the sum of all of the banks’ net worth.

### 2.3 Capital Producers

There is a continuum of one-period lived capital producers under perfect competition. They can transform $i$ units of output good into $\Phi \left( \frac{i}{K} \right) K$ of capital goods, where $\Phi' > 0$, $\Phi'' < 0$ and $K$ is aggregate capital in the economy. Their problem is static and given as follows

$$\Pi(S) = \max_i \left\{ Q^k(S) \Phi \left( \frac{i}{K} \right) K - i \right\}$$

(14)

---

18 Notice that, if $\theta = 0$, the banks cannot divert funds and there is no bound to leverage.

19 Introducing capital producers to the model is an artifact to obtain a state dependent price of capital. Similar assumption have been used in Bernanke, Gertler, and Gilchrist (1999), Brunnermeier and Sannikov (2012) and Bianchi and Mendoza (2010) among many others.
The optimal condition for capital producers reads

\[ Q^k(S) = \frac{1}{\Phi'(\frac{I(S)}{K})} \]  

(15)

where \( I(S) \) is optimal investment is state \( S \). Since \( \Phi \) is concave, equation (15) states that, in periods of low investment, capital prices will be low as well.

The relevance of capital producers in this model comes from inspecting Lemma 2. During recessions, investment will be low and thus capital prices will be low as well. Equation (8) then states that the value of capital for firms \( e(l, S) \) will decline as well. Furthermore, from equation (7), this will induce higher default and depress investment even further.

2.4 Household

The problem of the household is standard. Every period the household starts with deposits at the bank \( D \), can decide how many hours to work, how much to consume and how much to save. Let \( V^H(D, S) \) be the value to a household with \( D \) deposits when the aggregate state of the economy is \( S \). Then

\[
V^H(D, S) = \max_{C,N,D'} \{ U(C, N) + \beta E_{S'} [V^H(D', S') | S] \} \\
\text{subject to} \\
C + Q^D(S)D' \leq D + w(S)N - T_H(S) + d^F(S) + d^B(S) + \Pi(S) \\
d^F(S) = \int_{a_i \geq a(l, S)} d_i + \int_{a_i < a(l, S)} (1 - \Xi)Q^k(S)k_idi \\
d^B(S) = (1 - \sigma)\Omega(S) - (1 - \sigma)\bar{\omega}\Omega(S) \\
S' = \Gamma(S)
\]

The first line of the feasible set is the household’s budget constraint. Household’s expenses are consumption \( C \) and deposits \( D' \) at a price \( Q^D(S) \). He also pays lump-sum taxes \( T_H(S) \) to the government. His available income is given by holding of deposits \( D \) and labor earnings \( w(S)N \). He also obtains dividend payments from firms \( d^F(S) \), which account for all dividends paid by non-
defaulting firms plus the fraction of capital capital \((1 - \Xi)\) that survives from defaulting ones. He also receives dividends from exiting banks \((1 - \sigma)\Omega(S)\) and injects equity \(\varpi\Omega(S)\) to the \((1 - \sigma)\) new banks created, the sum of both terms which accounts for bank dividends \(d^B(S)\). Finally, the household also receives profits made by capital producers \(\Pi(S)\).

2.5 Government and Aggregate Feasibility

The government’s budget constraint is given by

\[
R = Q^R(S)R' + T_H(S) + \tau \int [-\lambda b_i + \psi(S)a_i k_i] \mathbb{I}(a_i > a(l, S)) di
\]

(17)

In order to repay his debt \(R\), the government can issue new debt \(R'\) at a price \(Q^R(S)\) or use his tax revenues. Notice that only non-defaulting firm pay taxes.

I assume that the government has control over the price of the risk-free debt \(Q^R(S)\). In particular, in any given period the government sets \(Q^R(S) = \mathcal{F}^R(S)\) for some function \(\mathcal{F}^R(\cdot)\) to be specified later on.\(^{20}\) Then, at given price, the government must supply the amount of risk-free debt demanded by banks. Although there is no money neither nominal rigidities, I will think of \(\mathcal{F}^R(S)\) as conventional monetary policy. The reason is that, by setting the interest rate, the government affects banks’ investment incentives in the way we typically think of monetary policy. I come back to this in Section 3.2.

I assume that the risky asset is in fixed supply at \(\bar{A}\). Thus, bank’s total investment in the risky asset must satisfy \(\int A(\omega_i, S) di = \bar{A}\). Similarly, let \(K'(S)\) be the total capital held by firms for next period \(K'(S) = \int_{a_i \geq a(l, S)} k'(a_i, b_i, k_i, S) di\). Then, investment \(I(S)\) must satisfy \(\Phi \left( \frac{I(S)}{K} \right) K = K'(S) - (1 - \delta) [1 - \Xi H(a(l, S))] K\). Finally, feasibility requires \(Y(S) + \xi \bar{A} = C(S) + I(S)\), where \(Y(S)\) is the output produced by the non-defaulting firms.

Next I summarized this discussion in a formal equilibrium definition.

**Definition 1** A **recursive competitive equilibrium** for this economy is given by: value functions \(\{E, V, V^H\}\); policies for the firm \(\{d, b', k'\}\); policies for the bank \(\{b'(l'), A', R', D'\}\); policies for the government \(\{\mathcal{F}^R, \mathcal{F}^H\}\); policies for the household \(\{\omega, \varpi, \Pi\}\). The only constraint I impose is that the function \(\mathcal{F}^R\) must induce stable dynamics in the economy. In this set-up this implies that, on average, the price of government debt must be higher that the effective discount factor of the bank. The intuition of this can be found in Chamberlain and Wilson (2000).
cies for the household \( \{C, N, D'\} \); such that, given prices \( \{w(S), Q(l', S), Q^k(S)\} \) and government policies \( \{Q^R(S), T_H(S)\} \): (i) Firms, banks and household optimize and achieve value \( E, V, \) and \( V^H \) respectively; (ii) Government policies satisfy his budget constraint; (iii) Labor market clears: \( \int_{a_i \geq \Phi(l, S)} n(a_i, k_i, S) \, di = N(D, S) \); (iv) Bonds market clears: \( \int_{a_i \geq \Phi(l, S)} b'(a, b, k, z) = b(l', \omega, S) \forall l' \); (v) Deposits market clears: \( \int D'(\omega_i, S) \, di = D'(D, S) \); (vi) Risky market clears: \( \int A'(\omega_i, S) \, di = \bar{A} \); (vii) Risk-free market clears: \( \int R'(\omega_i, S) \, di = R' \); (viii) Goods market clears: \( Y(z) + \xi \bar{A} = C(z) + I(z) \); (vi) The law of motion for the state \( S' = \Gamma(S) \) is consistent with individual policies and market clearing.

3 Quantitative Analysis

3.1 Calibration

I calibrate the model to perform the quantitative evaluation. Some of the model parameters are standard and I borrow values from previous literature. Other parameters are calibrated within the model in order to match certain moments. I explain the calibration next.

A period in our model is a quarter. I set the discount factor \( \beta = 0.99 \), the exponent of labor in the production function to \( \alpha = 0.66 \) and the depreciation rate of capital to \( \delta = 0.025 \). I assume \( U(C, N) = \frac{[C - \chi N^{1+\varphi}](1-\gamma)}{1-\gamma} \), and set \( \gamma = 2, \varphi = 2.5 \) and \( \chi = 2.3 \) so that average employment is about 70%. I borrow the capital production function from Guvenen (2009) and set \( \Phi(x) = \phi_0 x^{1-\phi_1} + \phi_2 \), with \( \phi_1 = 2.5 \), and \( \phi_0 \) and \( \phi_2 \) so that on average the price of capital is one, and the investment to output ratio is 20%. I choose tax rate \( \tau = 0.13 \) using evidence from the U.S. Government Accountability Office. The maturity of debt is set to \( \lambda = 1/16 \) so that the average maturity of debt is 4 years. The fraction of assets than banks can divert, as well as their exit probability and the equity injection they receive upon entry, are borrowed from Gertler and Kiyotaki (2009). The average level of firms’ idiosyncratic volatility is chosen to obtain an annual default rate of 2%. Finally, I assume that the productivity \( x \), the volatility \( \sigma_a \) and the financial shock \( \xi \) follow the same AR(1) process in logs. Table 1 summarizes all of the parameter values.

I solved the model using linear perturbation methods. In particular, I linearize the economy
around a non-stochastic steady-state in which banks’ leverage constraint (10) is always binding.21

### 3.2 Financial Shocks and Volatility Shocks

As discussed in the Introduction, the core motivation of the paper is to argue the economies performance will be very different if the leverage channel is active (ξ shocks) than if volatility channel is at play (σa shocks). In this section, I use the model described in Section 2 to formalize the argument.

For the discussion in this section I assume that \[ Q^R(S) = Q^R_{ss} - \varsigma_R(R(S) - R_{ss}), \] where \( \varsigma_R > 0 \) and \( Q^R_{ss} \) and \( R_{ss} \) stand for the values of \( Q^R \) and \( R \) in a non-stochastic steady state. I set \( \varsigma_R = 0.00001 \), which is small enough to not significantly affect outcomes, but induces stable dynamics in the model.

Figure 3 shows the economy response to a negative one standard innovation in ξ. As argued above, this shock can be understood as a financial shock because it directly decreases the banks’ net worth. Since the leverage constraint of the bank is binding, the decrease in banks’ net worth implies that the they must reduce their balance sheet in order to meet the leverage constraint. Thus, banks decrease their holdings of both firm’s debt as well as risk-free bonds. This financial shock also has real consequences. The decrease in banks’ borrowing pushes down firms’ bond prices. In turn, this increases firms’ incentives to default and decreases investment and labor demand. As a consequence, output declines.

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21 A global solution of this model is the next step of this project.
The logic just discussed corresponds to what I referred in the Introduction as a leverage channel. In periods of financial distress, banks’ capacity to raise funds is impaired, which decreases banks’ capacity to lend and ultimately induces a recession. This line of thought has been used as an explanation of the slow recovery in lending during the last couple of years. However, as I argued before, banks in the US have been hoarding reserves, which makes the leverage channel explanation less likely. I argue next that a volatility shock is a more appealing explanation.

Figure 4 shows the economy response to a positive one standard innovation in \( \sigma_a \). This shock is an increase in the dispersion of firms’ idiosyncratic productivity that pushes many firms below the default threshold. This results in a sharp sudden increase of the default rate, which remains high for many periods thereafter. Consequently, firms’ bond prices decline. Banks respond to this shock in two ways. First, since firms’ debt performs worst, the balance sheet of the bank decreases. In turn, the banks needs to decline her overall asset holdings. However, in doing so, banks significantly cut down borrowing to firms and actually increase holdings of the risk-free asset. In other words, faced to an increase in the riskiness of the real sector of the economy, banks decide to shift their portfolio towards a safer asset. This is in line with the dynamics observed in Figure 1 and 2 in the Introduction.

The economy response to the volatility shock is what I referred before as a volatility channel. In response to the increase in firms riskiness, banks decide to cut down lending and rebalance their portfolio towards the risk-free asset. The decline in lending is then not the result of a tighter leverage constraint, but the outcome of banks' portfolio decision. A policy that aims to facilitate banks’ capacity to raise funds will fail to restore lending, since this is not the initial problem to deal with. I believe this shed lights on why, despite of the efforts pursued by the Federal Reserved, lending to firms was only restored after measures of volatility returned to pre-crisis levels.

4 Conventional Monetary Policy and Bailouts

The main message of the previous section is that the nature of the shock disrupting the economy can have significant implications for policy design. In particular, if there is an increase in the volatility of firm’s idiosyncratic productivity, banks react by lending less an accumulating the government
Figure 3: Impulse response function to a 1 std $\xi$ negative innovation.

Figure 4: Impulse response function to a 1 std $\sigma_a$ positive innovation.
debt. Consequently, a policy that aims to stabilize the economy in this scenario, must fight against
the banks’ incentives to accumulate risk-free assets. The aim of this section is to discuss how this
could be achieved.

I discuss two alternative policies that can help to stabilize the economy and stimulate lending
in the presence of increased riskiness at the firms. The first one accounts for cutting down risk-free
interest rates (i.e., increasing $Q^R(S)$) in response to an increase in $\sigma_a$. I think of this as conventional
monetary policy since it accounts to decrease interest rates when facing adverse macroeconomic
conditions. The second one is to systematically purchase firms’ defaulted bonds (bailouts). Both of
these policies make lending to firms more attractive and consequently decreases banks’ incentives
to accumulate government debt. Finally, I show an equivalence result between these two type of
policies: the equilibrium allocations achieved by a set of conventional monetary policies can be
obtained with a bailout policy. This is of particular interest in conventional monetary policy faces
certain limits, as in the case of a zero-lower bound scenario.

4.1 Conventional Monetary Policy

In this section I assume that the governments sets the price of the risk-free debt by following
a "Taylor rule" type of policy, as described in equation (18). In particular, the government
increases the price of the risk-free debt in states where output is low or when volatility is high. It
also responds to fluctuations in $R$ in order to keep model dynamics stable.

\[
Q^R(S) = Q^R_{ss} - \varsigma_Y (Y(S) - Y_{ss}) + \varsigma_{\sigma_a} (\sigma_a - \sigma_{a,ss}) + \varsigma_R (R - R_{ss})
\] (18)

Figure 5 shows the model response to a volatility shock with and without policy intervention.
In the case with policy, the government responds to the volatility shock by increasing the price
of risk-free debt. In turn, bank’s have less incentives to accumulate government debt and decide
to lend more to firms. This implies a smaller drop in firm’s bond prices and a smaller increase in

\footnote{In equation (18), $x_{ss}$ stands for the non-stochastic steady state value of $x$.}
default rates.\textsuperscript{23} As a result, the recession is not as severe as in the case with no policy intervention.

Although actually pursued with fiscal policy, the logic described above is similar to the one obtained with conventional monetary policy.\textsuperscript{24} In presence of lower returns in the risk-free asset, banks find it more profitable to lend to firms’. Consequently, even if firms are still riskier than on average, the government policies results in a smaller contraction of the economy.

4.2 Bailouts

In this section I discuss the effect of governments purchases of firms defaulted debt. I think of this as a debt guarantee program that increases bank’s incentives to lend to firms since, in case of default, the government buys a fraction of the non performing bonds.

A bailout policy is composed by three elements. First, $\eta(a, l, S) \in [0, 1]$ that stands for the fraction of defaulted debt that the government wants to purchase. I allow it to be a function of the

\textsuperscript{23}This last thing is hard to see in the plot, but default rates increases by half at the period of the shock.

\textsuperscript{24}There is a subtlety to this policy. In this economy, Ricardian equivalence does hold. However, the government is trading bonds with banks but taxing households to pay for the interest on these bonds. This is the reason the government can actually affect outcomes by changing the price of debt. If the bank were paying the lump-sum taxes required to repay the governments debt, it would be impossible for the government to affect equilibrium outcomes.
aggregate state of the economy \( S \), as well as of the firm’s idiosyncratic variables \( a \) and \( l \). Second, \( \varpi(a, l, S) \geq 0 \) is the price that the government pays to the bank for each defaulted bond. I restrict it to be non-negative, since banks can always choose not to participate in the bailout program. Third, \( \chi(a, l, S) \leq 1 \) which is a debt relief that the government offers to the defaulted firm. Thus, if a firm accepts this offer, she only pays a fraction \( \chi(a, l, S) < 1 \) of her maturing liabilities this period.

**Definition 2** A bailout policy is a 3-tuple \( \{ \eta, \varpi, \chi \} \).

(The Appendix describes firms’ and banks’ problems once the bailout policy is introduced.) As an illustration, I focus on the following bailout policy

\[
\begin{align*}
\varpi(a, l, S) &= [\lambda + (1 - \lambda)Q(\ell'(l, S), S)] \\
\chi(a, l, S) &= \frac{1}{\lambda l} [\psi(S)a + \frac{e(l, S)}{1 - \tau}] \\
\eta(a, l, S) &= -\eta_Y (Y(S) - Y_{ss})
\end{align*}
\]

Equation (19) states that the government pays for a defaulted bond the market value of a non-defaulted one. Equation (20) states that the government offers the minimum necessary relief to the firm so that default is avoided.\(^{26}\) Finally, equation (21) states that the government buys more defaulted debt during recessions (\( \eta_Y > 0 \)).

Figure 6 shows the model response to a volatility shock with and without bailouts. Bailouts result in a smaller decrease of firm’s bond price. This is because banks have more incentive to buy firm’s debt, since, in case of default, they still receive the government bailout. In turn, the drop in firms’ borrowing is smaller, which avoids a severe contraction in investment, labor and output.

### 4.3 An Equivalence Result

The Federal Funds rate has typically been one of the most important instruments the Federal Reserve had in conducting monetary policy. In December of 2008, the Federal Funds Rate achieved its

\(^{25}\) In order to keep the rescaling property of the economy, I need bailout policies that also rescale in the firms’ level of capital. That is why I allow the policies to depend on leverage \( l \), but not on debt \( b \) and capital \( k \) separately.

\(^{26}\) See Appendix for details.
minimum possible of zero and left the Federal Reserve without this policy instrument. If possible, the Federal Reserve would have probably wanted to keep on cutting down interest rates in order to stimulate the economy. The following lemma shows how to achieve the same stimulus with bailouts.

Lemma 4 Assume a government that sets the price of debt as $Q^R(S) = F^R(S)$. Let $Y^*(S)$, $C^*(S)$, $N^*(S)$ and $I^*(S)$ denote the equilibrium outcomes for output, consumption, labor and investment. Next, assume a government that sets the price of debt as $Q^R(S) = \min\{F^R(S), 1}\$. It exist a bailout policy $\{\eta, \varpi, \chi\}$ that obtain the equilibrium outcomes $Y^*(S)$, $C^*(S)$, $N^*(S)$ and $I^*(S)$.

5 Conclusions

During 2009 and 2010, lending to non-financial businesses sharply declined. I argued that this decline is unlikely to have reflected a diminished lending capacity from the financial sector. Instead, I provided evidence of increased riskiness at the non-financial sector as an alternative explanation. In turn, I offered a model that rationalizes the decrease in lending as the result of the banks’ optimal portfolio decision. I argued that the increase riskiness in firms may be on the reasons the efforts
exerted by the Federal Reserve have fallen short to fully restore lending
References


