Risk and Ambiguity in Models of Business Cycles

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The “Great Recession” and its aftermath

Real Gross Domestic Product
Percentage change from previous peak, Seasonally Adjusted

Source: U.S. Bureau of Economic Analysis
Cooley-Rupert Economic Snapshot: www.econsnapshot.com

Quarters from previous peak

1973 cycle
1981 cycle
1990 cycle
2001 cycle
Current cycle
The “Great Recession” and its aftermath

Real Personal Consumption Expenditures
Percentage change from previous peak, Seasonally Adjusted

- 1973 cycle
- 1981 cycle
- 1990 cycle
- 2001 cycle
- Current cycle

Quarters from previous peak

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The “Great Recession” and its aftermath

Real Private Nonresidential Fixed Investment
Percentage change from previous peak, Seasonally Adjusted

1973 cycle
1981 cycle
1990 cycle
1991 cycle
2001 cycle
Current cycle
What happened?

- What we see
  - Magnitude: deeper recession than usual
  - Persistence: longer recovery — maybe slower, too

- Like Kydland-Prescott with productivity shocks?
  - Relative magnitudes look right
  - Comovements look right, too
  - But... measured productivity didn’t fall very much
What happened?

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- What’s missing?
What we do

- Take a streamlined business cycle model
- Ask: How does uncertainty affect the dynamics of output, consumption, and investment?
  - Magnitude: Does uncertainty magnify fluctuations?
  - Persistence: Can it reduce the speed of recovery?
- Compute solutions with
  - Transparent loglinear approximation
  - Accurate numerical method
Modeling ingredients

- Streamlined **business cycle model**
  - Recursive preferences
  - Unit root in productivity
  - Fixed labor supply

- With fluctuations in **uncertainty**
  - *Risk* (stochastic volatility)
  - *Ambiguity* (unobservable long-term growth)
What we find

Fluctuations in uncertainty have **little impact**

- **Persistence**
  - Separation property: internal *dynamics independent of risk and risk aversion*
  - Persistence must be in the shock

- **Magnitude**
  - Impact typically small, but magnified by *risk aversion*

Business cycle properties governed by IES
Risk

- Recursive references

\[
U_t = V[c_t, \mu_t(U_{t+1})] \\
= [(1 - \beta)c_t^\rho + \beta\mu_t(U_{t+1})^\rho]^{1/\rho} \\
\mu_t(U_{t+1}) = [E_t(U_t^\alpha)]^{1/\alpha}
\]

\(V, \mu_t\) homogeneous of degree one, \(RA = 1 - \alpha, IES \equiv \sigma = 1/(1 - \rho)\)

- Productivity \(a_t\)

\[
\log g_t = \log(a_t/a_{t-1}) = \log g + e^T x_t \\
x_{t+1} = Ax_t + \nu_t^{1/2} Bw_{1t+1} \text{ ("news")} \\
\nu_{t+1} = (1 - \varphi_v)\nu + \varphi_v \nu_t + \tau w_{2t+1} \text{ ("risk")} \\
(w_{1t}, w_{2t}) = \text{iid standard normals}
\]
Scaling

- **Bellman equation**

\[
J(k_t, x_t, v_t, a_t) = \max_{c_t} V\left\{ c_t, \mu_t [J(k_{t+1}, x_{t+1}, v_{t+1}, a_{t+1})]\right\}
\]

s.t. \( k_{t+1} = f(k_t, a_t n) - c_t \)

\( f \) hold 1: eg, \( f(k, an) = k^\omega (an)^{1-\omega} + (1-\delta)k \)

- **Rescaled** Bellman equation \([\tilde{k}_t = k_t / a_t, \tilde{c}_t = c_t / a_t]\)

\[
J(\tilde{k}_t, x_t, v_t) = \max_{\tilde{c}_t} V\left\{ \tilde{c}_t, \mu_t [g_{t+1} J(\tilde{k}_{t+1}, x_{t+1}, v_{t+1})]\right\}
\]

s.t. \( g_{t+1} \tilde{k}_{t+1} = f(\tilde{k}_t, n) - \tilde{c}_t \)
### Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>$-1$</td>
<td>intertemporal substitution $\sigma = 1/(1 - \rho) = 1/2$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$-9$</td>
<td>risk aversion $1 - \alpha = 10$</td>
</tr>
<tr>
<td>$\beta$</td>
<td></td>
<td>chosen to hit $k/y = 10$ (quarterly)</td>
</tr>
<tr>
<td><strong>Technology</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega$</td>
<td>$1/3$</td>
<td>Kydland and Prescott (1982, Table I), rounded off</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$0.025$</td>
<td>Kydland and Prescott (1982, Table I)</td>
</tr>
<tr>
<td><strong>Productivity growth</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log g$</td>
<td>$0.004$</td>
<td>Tallarini (2000, Table 4)</td>
</tr>
<tr>
<td>$e$</td>
<td>$1$</td>
<td>normalization</td>
</tr>
<tr>
<td>$A$</td>
<td>$0$</td>
<td>no predictable component (“news”)</td>
</tr>
<tr>
<td>$B$</td>
<td>$1$</td>
<td>normalization</td>
</tr>
<tr>
<td>$\nu^{1/2}$</td>
<td>$0.015$</td>
<td>Tallarini (2000, Table 4), rounded off</td>
</tr>
<tr>
<td>$\varphi_\nu$</td>
<td>$0.95$</td>
<td>arbitrary</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$0.74 \times 10^{-5}$</td>
<td>makes $\nu$ three standard deviations from zero</td>
</tr>
</tbody>
</table>
Model is essentially loglinear
Loglinearization I

- Goal: loglinear decision rule for capital
  \[ \log \tilde{k}_{t+1} = h_k \log \tilde{k}_t + h_x^\top x_t + h_v v_t - \log g_{t+1} \]

- Dynamic programming version of Campbell (JME, 1994)

- Loglinearization around the **stochastic** steady-state
Loglinearization II

- Loglinearize **capital’s marginal product** and **law of motion**

\[
\log f_{kt} = \lambda_r \log \tilde{k}_t + \lambda_0 \\
\log \tilde{k}_{t+1} = \lambda_k \log \tilde{k}_t - \lambda_c \log \tilde{c}_t + \lambda_1 - \log g_{t+1}
\]

where \((\lambda_k, \lambda_c, \lambda_r)\) are steady-state objects.

- Guess **loglinear value function and derivative**

\[
\log J_t = p_k \log \tilde{k}_t + p_x^T x_t + p_v v_t + p_0 \\
\log J_t^{p-1} J_{kt} = q_k \log \tilde{k}_t + q_x^T x_t + q_v v_t + q_0
\]
Separation property

**Claim (Tallarini)**

Consider the loglinear approximation of capital’s law of motion,

\[
\log \tilde{k}_{t+1} = h_0 + h_k \log \tilde{k}_t + h_x^T x_t + h_v v_t - \log g_{t+1}
\]

*If we hold constant the stochastic steady state:*

1. \(h_k\) is independent of properties of all shocks and risk aversion
2. \(h_x\) is independent of properties of uncertainty shocks and risk aversion

\[
h_k = \lambda_k + \sigma \lambda_c (q_k - \lambda_r), \quad h_x^T = \sigma \lambda_c q_x^T
\]

\[
q_k = q_k [\lambda_k + \sigma \lambda_c (q_k - \lambda_r)] + \lambda_r
\]

\[
q_x = -(\sigma^{-1} + q_k) e^T A [(1 - \sigma q_k \lambda_c) I - A]^{-1}
\]
Loglinearization III

The graph illustrates the comparison between the numerical solution and the loglinear approximation for a specific function. The x-axis represents \( \log k(t) \), while the y-axis shows the density measure. The numerical solution is represented by a solid blue line, and the loglinear approximation is indicated by a dashed magenta line. The graph highlights the close alignment between the two methods, especially in the range of \( 3.4 \) to \( 3.6 \) on the x-axis.
Risk aversion magnifies uncertainty

- Shock in volatility (+1 std)
- Response in consumption
- Response in capital

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## Business cycles and risk aversion

<table>
<thead>
<tr>
<th>Risk Aversion</th>
<th>US Data</th>
<th>Model w/ RA =</th>
<th>Cst. vol.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>10</td>
<td>50</td>
<td>10</td>
</tr>
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</table>

### Standard deviations (%)

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<tr>
<td>Output growth</td>
<td>1.04</td>
<td>0.82</td>
<td>0.82</td>
</tr>
<tr>
<td>Consumption growth</td>
<td>0.55</td>
<td>0.75</td>
<td>0.76</td>
</tr>
<tr>
<td>Investment growth</td>
<td>2.79</td>
<td>1.03</td>
<td>1.06</td>
</tr>
</tbody>
</table>

### Correlations with output growth

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<tr>
<td>Consumption growth</td>
<td>0.52</td>
<td>0.99</td>
<td>0.97</td>
</tr>
<tr>
<td>Investment growth</td>
<td>0.65</td>
<td>0.98</td>
<td>0.97</td>
</tr>
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</table>

**Intertemporal elasticity of substitution: 0.5**

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# Business cycles and IES

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<td><strong>IES</strong></td>
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<td>0.5</td>
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**Correlations with output growth**

|               |       |       |       |
| Consumption growth | 0.52   | 0.99  | 0.98  |
| Investment growth  | 0.65   | 0.97  | 0.93  |

**Risk aversion: 10**
Risk and ambiguity

- Divide state in two: $s_t = (s_{1t}, s_{2t})$ *(ask about Stan’s story)*

- **Smooth ambiguity**

  \[
  \text{risk} = p_{1t}(s_{1t+1}|s_{2t+1}, I_t) \\
  \text{ambiguity} = p_{2t}(s_{2t+1}|I_t)
  \]

- **Two-part certainty equivalent**

  \[
  \mu_{1t}(U_{t+1}) = \left[ E_{1t}(U_{t+1}^\alpha) \right]^{1/\alpha} \quad \text{ (“risk”)} \\
  \mu_{2t}[\mu_{1t}(U_{t+1})] = \left\{ E_{2t}[\mu_{1t}(U_{t+1})^\gamma] \right\}^{1/\gamma} \quad \text{ (“ambiguity”)}
  \]

  $\alpha$ controls risk aversion, $\gamma < \alpha$ controls ambiguity aversion
Ambiguity about what?

- Rule of thumb
  - Risk about observables
  - Ambiguity about unobservables

- Example: observe productivity growth $g_t$ but not its mean $x_t$

  \[
  \text{Risk: } \log g_{t+1} | x_{t+1} \sim \mathcal{N}(\log g + x_{t+1}, b)
  \]
  \[
  \text{Ambiguity: } x_{t+1} \sim \text{AR}(1)
  \]

- Filtering gives us (say)

  \[
  x_{t+1} | \mathcal{I}_t \sim \mathcal{N}(\hat{x}_{t+1}, h_{t+1}), \quad \mathcal{I}_t = g^t
  \]
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- But: **none of this has much impact**
Summary

- Uncertainty fluctuations have intuitive appeal
- But they add little to standard business cycle model
  - Magnitude: impact is small with common parameter values
  - Persistence: they add nothing to internal dynamics, just the persistence of the shocks themselves
Summary

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- But they add little to standard business cycle model
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- Where next?
  - Uncertainty about parameters?
  - Endogenous uncertainty? (Veldkamp, Schaal)
  - Micro uncertainty with financial frictions? (Arellano, Bai, & Kehoe)
  - Cause or effect? (Alessandria, Choi, Kaboski, & Midrigan)
Related work (some of it)

- Recursive business cycles
  - Campanale, Castro, & Clementi; Tallarini

- Approximation methods
  - Anderson, Hansen, McGrattan, & Sargent; Campbell; Kaltenbrunner and Lochstoer; Malkhozov

- Risk and business cycles
  - Basu & Bundick; Caldara, Fernandez-Villaverde, Rubio-Ramirez, & Wen; Justiniano & Primiceri; Liu & Miao

- Ambiguity and business cycles
  - Klibanoff, Marinacci, & Mukerji; Ju & Miao; Ilut & Schneider; Jahan-Parvar & Miao
Intertemporal substitution and uncertainty

**Shock in volatility (+1std)**

- Volatility
- Response in capital
- Response in consumption

IES=0.5
IES=1.5

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Productivity

Output Per Hour of All Persons
Percentage change from previous peak, Seasonally Adjusted, Nonfarm Business

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