TEMPTATION AND TAXATION

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PREFERENCE REVERSALS

• Kirby and Herrnstein (Psychological Science, 1995): “Of 36 subjects, 34 reversed preference from a larger, later reward to a smaller, earlier reward as the delays to both rewards decreased.”

• This evidence is not consistent with the standard model of geometric discounting.

• Two theoretical responses:

  1. The Strotz/Phelps-Pollak/Laibson model of hyperbolic, or quasi-geometric, discounting. (Assume that the slope of the discount function is a decreasing function of time.)

  2. The Gul-Pesendorfer model of temptation and self-control. (Assume that utility depends not only on the choice but also on the set from which it is chosen.)
PREFERENCE REVERSALS
IN THE LAIBSON MODEL

Preferences of self 0: \(c_0 + \beta \delta c_1 + \beta \delta^2 c_2\)

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Late reward chosen if \(\beta \delta a < \beta \delta^2 b\).

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Early reward chosen if \(a > \beta \delta b\).

Preference reversal if \(\beta \delta b < a < \delta b\).
THE LAIBSON MODEL:
QUASI-GEOMETRIC DISCOUNTING

Preferences:

Self 0: \[ U_0 = u_0 + \beta \left( \delta u_1 + \delta^2 u_2 + \delta^3 u_3 + \cdots + \delta^T u_T \right) \]

Self 1: \[ U_1 = u_1 + \beta \left( \delta u_2 + \delta^2 u_3 + \cdots + \delta^T u_T \right) \]

Self 2: \[ U_2 = u_2 + \beta \left( \delta u_3 + \cdots + \delta^T u_T \right) \]

Behavior:

- The consumer cannot commit to future actions.
- The consumer is “sophisticated”: he realizes that his preferences will change and makes the current decision taking this into account.
- The decision-making process is viewed as a dynamic game, with the agent’s current and future selves as players.
MARKOV EQUILIBRIA IN THE LAIBSON MODEL

- **Environment**: A simple (finite-horizon) consumption-savings problem.

- **Intrapersonal equilibrium**: Iterate backwards (the current self correctly anticipates the decisions of his future selves).

- The period $T - t$ self solves:

  $$\max_{k_{T-t}} u(f(k_{T-t}) - k_{T-t+1}) + \beta \delta W_{T-t+1}(k_{T-t+1}).$$

  This problem determines the period $T - t$ decision rule:

  $$k_{T-t+1} = g_{T-t}(k_{T-t}).$$

- The “value function” of the period $T - t$ self is:

  $$W_{T-t}(k_{T-t}) = u(f(k_{T-t}) - g_{T-t}(k_{T-t})) + \delta W_{T-t+1}(g_{T-t}(k_{T-t})).$$
• Perceptions: The consumer perceives that future savings decisions are determined by $k_{t+1} = g(k_t)$.

• The current self solves the “first-stage” problem:

$$\max_{k'} u(f(k') - k') + \beta\delta W(k').$$

• $W$ is an “indirect” utility function: it must satisfy the “second-stage” functional equation

$$W(k) = u(f(k) - g(k)) + \delta W(g(k)).$$

• A Markov equilibrium obtains if $g(k)$ also solves the first-stage problem.
DRAWBACKS OF THE LAIBSON MODEL

• Difficult to do welfare analysis:
  1. Lack of axiomatic foundation.
  2. When we evaluate policy, which self’s utility function do we use? (Krusell, Kuruşçu, and Smith (2000, 2001a) study time-consistent government policy in the Laibson model.)


• Computation: Multiplicity makes computation difficult (recent progress: perturbation methods).
AN ALTERNATIVE APPROACH:  
GUL AND PESENDORFER’S MODEL

• This recent approach is axiomatically-based decision theory.

• It emphasizes temptation and self-control.

• It can address the experimental evidence.

• There is a dynamic version of the GP model that seems potentially useful for macroeconomic analysis.
BRIEF SUMMARY OF RESULTS

• Neoclassical growth analysis: We characterize steady states and dynamics. In general, the model with temptation is not observationally equivalent to a model without temptation. In addition, the curvature of the utility function plays a role in determining the steady state.

• Connection to Laibson: We develop a formulation in which the temptation is “quasi-geometric discounting”. If this temptation is strong enough, our model coincides with the Laibson model. This view of the Laibson model says that period $t$ utility should be evaluated from the perspective of self $t - 1$!

• Taxation: Our policy analysis suggests that there should be a subsidy to investment.

• Asset Pricing: Krusell, Kuruşçu, and Smith (2001b) study equilibrium asset prices in a Mehra-Prescott model with GP consumers, some of whom have an “urge to save” rather than an “urge to consume”. These compulsive savers play a dominant role in asset markets, driving down the risk-free rate.
THE GUL-PESENDORFER MODEL:  
A QUICK-AND-DIRTY INTRODUCTION

• “Second-period” preferences defined over ordered pairs \((A, x)\), where \(A\) is a choice set and \(x \in A\).

• Definition: \(y\) tempts \(x\) if \((\{x\}, x)\) is preferred to \((\{x, y\}, x)\).

• Assumptions:
  1. Removing temptations cannot make the consumer worse off.
  2. If \(y\) tempts \(x\), then \(x\) does not tempt \(y\).
  3. Adding \(y\) to \(A\) does not make the consumer worse off unless \(y\) tempts every element in \(A\).

• These assumptions imply that “tempts” is a preference relation. Moreover, the utility of a fixed choice is affected by the choice set only through its most tempting element.
PREFERENCES OVER CHOICE SETS

• Second-period preferences induce “first-period” preferences over choice sets themselves: $A \succeq B$ if and only if there is an $x \in A$ such that $(A, x)$ is preferred to $(B, y)$ for all $y \in B$.

• The above assumptions imply set betweenness:

$$A \succeq B \text{ implies that } A \succeq A \cup B \succeq B.$$  

Choice sets cannot be compared simply by looking at their best elements.
PREFERENCE FOR COMMITMENT, SELF-CONTROL, AND SUCCUMBING TO TEMPTATION

Assume $A \succ B$. “Set betweenness” allows three possibilities:

1. Standard decision maker:
   
   $A \sim A \cup B \succ B$

2. Preference for commitment and self-control:
   
   $A \succ A \cup B \succ B$

3. Preference for commitment and succumbing to temptation:
   
   $A \succ A \cup B \sim B$
A REPRESENTATION THEOREM FOR PREFERENCES OVER SETS

• Set betweenness (together with standard axioms) implies the following representation of preferences over sets:

\[ W(A) = \max_{x \in A} \{ U(x) + V(x) \} - \max_{\tilde{x} \in A} V(\tilde{x}). \]

• Second-period preferences are represented by:

\[ W^*(A, x) = U(x) + V(x) - \max_{x \in A} V(\tilde{x}). \]

Interpretation:

• \( U \) determines the commitment ranking (i.e., the utility of singleton sets).

• \( V \) determines the temptation ranking (i.e., \( V \) gives higher values to more tempting elements).

• The second-period choice (given \( A \)) maximizes \( W^*(A, x) \). That is, actual behavior maximizes \( U(x) + V(x) \).

• \( V(x) - \max_{\tilde{x} \in A} V(\tilde{x}) \) is the disutility of self-control.
A SIMPLE EXAMPLE

• Two alternatives: $x$ and $y$.
• $x$ maximizes the commitment ranking: $U(x) > U(y)$.
• $y$ maximizes the temptation ranking: $V(y) > V(x)$.

\[
W^*(\{x\}, x) = U(x) + V(x) - V(x) = U(x)
\]
\[
W^*(\{x, y\}, x) = U(x) + V(x) - V(y)
\]
\[
W^*(\{x, y\}, y) = U(y) + V(y) - V(y) = U(y)
\]
\[
W^*(\{y\}, y) = U(y) + V(y) - V(y) = U(y)
\]

• The consumer has a preference for commitment.
• The consumer has self-control if

\[
U(x) + V(x) - V(y) > U(y).
\]

In this case, $W(\{x\}) > W(\{x, y\}) > W(\{y\})$.

• The consumer succumbs to temptation if

\[
U(x) + V(x) - V(y) < U(y).
\]

In this case, $W(\{x\}) > W(\{x, y\}) = W(\{y\})$.
THE TWO-PERIOD CONSUMPTION-SAVINGS MODEL

- Consumption today and tomorrow.
- Neoclassical production.
- Standard budget set (borrowing and lending at \( r \)).
- General equilibrium.
- With \( \tilde{u}(c_1, c_2) \) playing the role of \( U \) and \( \tilde{v}(c_1, c_2) \) the role of \( V \), let the temptation function \( \tilde{v} \) have a stronger preference for present consumption. For example, let
  \[
  \tilde{u}(c_1, c_2) = u(c_1) + \delta u(c_2)
  \]
  and
  \[
  \tilde{v}(c_1, c_2) = \gamma \left( u(c_1) + \beta \delta u(c_2) \right),
  \]
  with \( \beta < 1 \).
THE CONSUMER’S PROBLEM

• The consumer’s budget set is:

\[ B(k_1, \bar{k}_1, \bar{k}_2) \equiv \{(c_1, c_2) : \exists k_2 : \]
\[ c_1 = r(\bar{k}_1)k_1 + w(\bar{k}_1) - k_2 \]
\[ c_2 = r(\bar{k}_2)k_2 + w(\bar{k}_2)\} \]

• The consumer solves:

\[ \max_{c_1, c_2} \{(1 + \gamma)u(c_1) + \delta(1 + \gamma/\beta)u(c_2)\} - \max_{\tilde{c}_1, \tilde{c}_2} \{\gamma u(\tilde{c}_1) + \gamma/\beta \delta u(\tilde{c}_2)\} \]
subject to: \((c_1, c_2) \in B(k_1, \bar{k}_1, \bar{k}_2), (\tilde{c}_1, \tilde{c}_2) \in B(k_1, \bar{k}_1, \bar{k}_2).\)

• The consumer’s first-order condition:

\[ \frac{1 + \gamma}{\delta(1 + \gamma/\beta)} \frac{u'(c_1)}{u'(c_2)} = r(\bar{k}_2) \]

• Compare to:

\[ \frac{1}{\delta u'(c_2)} u'(c_1) = r(\bar{k}_2) \quad \text{and} \quad \frac{1}{\beta \delta u'(c_2)} u'(c_1) = r(\bar{k}_2) \]

• Note that:

\[ \frac{1}{\beta \delta} \geq \frac{1 + \gamma}{\delta(1 + \gamma/\beta)} \geq \frac{1}{\delta} \]
COMPETITIVE EQUILIBRIUM VS. AUTARKY

• In equilibrium, \( k = \bar{k} \) and \( r(\bar{k}) = f'(\bar{k}) \).

• Consumer’s first-order condition becomes:

\[
\frac{1 + \gamma}{\delta(1 + \gamma \beta)} \frac{u'(\bar{c}_1)}{u'(\bar{c}_2)} = f'(\bar{k}_2).
\]

• This is the same first-order condition as in autarky. (In this case, the “budget set” is the production possibility set determined by \( c_1 = f(k_1) - k_2 \) and \( c_2 = f(k_2) \).)

• BUT: the consumer is better off in autarky because the temptation is weaker (the disutility of self-control is higher in competitive equilibrium).
POLICY IN THE TWO-PERIOD MODEL

• Command policy: The government chooses for the consumer, eliminating self-control problems. The command policy is therefore first-best: it maximizes $u(c_1) + \delta u(c_2)$ and there is no disutility of self-control.

• Taxation policy in competitive equilibrium: The gov’t taxes income and investment (savings) in the first period. It chooses the tax rates to maximize welfare given a budget-balancing constraint. Result: The gov’t subsidizes investment and taxes income. Given “log-Cobb” assumptions, the optimal allocation is the same as under the command policy. But welfare is lower because of the self-control cost.
OPTIMAL PROPORTIONAL TAXES

• The consumer’s budget set is:

\[
\{(c_1, c_2) : \exists k_2 : \\
c_1 = [r(\bar{k}_1)k_1 + w(\bar{k}_1)](1 - \tau_y) - (1 + \tau_i)k_2 \\
c_2 = r(\bar{k}_2)k_2 + w(\bar{k}_2)\}
\]

• The government budget constraint is:

\[
\tau_y f(\bar{k}_1) + \tau_i \bar{k}_2 = 0.
\]

• Should investment be subsidized? Yes! The representative consumer’s (indirect) utility is a decreasing function of \(\tau_i\) at \(\tau_i = 0\).
WHY SUBSIDIZE INVESTMENT?

• At $\tau_i = 0$, there is no first-order effect on max $U + V$ of changing $\tau_i$.

• So $\tau_i$ should be decreased (from 0) if doing so decreases temptation utility (i.e., if doing so decreases max $V$).

• The effect of increasing $\tau_i$ on temptation utility is twofold:
  (i) $\tilde{c}_1$ increases, by the amount $\bar{k}_2 - \tilde{k}_2$;
  (ii) $\tilde{c}_2$ decreases, by the amount $(\bar{k}_2 - \tilde{k}_2)r'(\bar{k}_2)\frac{dk_2}{d\tau_i}$.

In utility terms, this means that an increase in $\tau_i$ increases temptation utility if

$$(\bar{k}_2 - \tilde{k}_2) \left(1 - (\text{MRS})r'(\bar{k}_2)\frac{d\bar{k}_2}{d\tau_i}\right) > 0,$$

i.e., if

$$(\text{MRS})r'(\bar{k}_2)\frac{d\bar{k}_2}{d\tau_i} < 1.$$  

• Equilibrium requires: $\text{MRS}(\tau_i)r(\bar{k}_2(\tau_i)) = 1 + \tau_i$.

This means that:

$$(\text{MRS})r'(\bar{k}_2)\frac{d\bar{k}_2}{d\tau_i} + r(\bar{k}_2)\frac{d\text{MRS}}{d\tau_i} = 1.$$  

Since $\frac{d\text{MRS}}{d\tau_i} > 0$ (increasing $\tau_i$ lowers savings, thereby decreasing $c_2$ and increasing $c_1$), it must be that $(\text{MRS})r'(\bar{k}_2)\frac{d\bar{k}_2}{d\tau_i} < 1$. 

QUASI-GEOMETRIC TEMPTATION

• Idea: temptation can only occur if it involves the immediate present.

• The two-period model determines preferences over two-period choice problems.

• Longer-horizon choice problems are defined recursively: every choice problem requires choosing today’s consumption and tomorrow’s choice problem.

• Iterating backwards, one obtains:

\[
W_{T-t}(k_{T-t}) = \max_{k_{T-t+1}} \left\{ u(f(k_{T-t}) - k_{T-t+1}) + \delta W_{T-t+1}(k_{T-t+1}) + V_{T-t+1}(k_{T-t}, k_{T-t+1}) \right\} \\
- \max_{k_{T-t+1}} \left\{ V_{T-t+1}(k_{T-t}, k_{T-t+1}) \right\},
\]

where

\[
V_{T-t}(k_{T-t}, k_{T-t+1}) \equiv \gamma \left\{ u(f(k_{T-t}) - k_{T-t+1}) + \beta \delta W_{T-t+1}(k_{T-t+1}) \right\}.
\]

• Notice:

1. When \( \gamma = 0 \) or \( \beta = 1 \), the consumer does not have self-control problems: standard model.

2. When \( \beta = 0 \): temptation by immediate consumption as in Gul and Pesendorfer (2000b).
THE LAIBSON LIMIT CASE

• If $\beta \neq 1$ and $\gamma$ goes to infinity, we move toward the Laibson case: (i) the agent puts so much weight on the temptation that he succumbs to $\beta \delta$ behavior; and (ii) he views the future period utils as being compared with $\delta$’s alone. (Gul and Pesendorfer (2001) also study this case.)

• Focusing on the Laibson limit case, this approach tells us how to evaluate policy (which “self’s” utility function to use): the current self maximizes $V$, but $W$ corresponds to his utility over sets. This is effectively utility as perceived by his most recent self.
PREFERENCE REVERSALS
IN THE GUL-PESENDORFER MODEL

Let $u(c) = c$ and $v(c) = \gamma c$. Set $\beta = 0$ for simplicity.

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Late reward chosen if $\delta a < \delta^2 b$. (No self-control problems since both rewards occur after the current period.)

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Early reward chosen if $a > \delta b + \gamma(0 - a)$.

Preference reversal if $\delta b - \gamma a < a < \delta b$.  

23
EULER EQUATIONS

• There is a pair of Euler equations, one for realized behavior and one for temptation behavior:

\[ u'(c_t) = \delta \frac{1 + \beta \gamma}{1 + \gamma} f'(k_{t+1}) \left\{ u'(c_{t+1}) + \gamma [u'(c_{t+1}) - u'(^{\sim}c_{t+1})] \right\} \]

\[ u'(^{\sim}c_t) = \delta \beta \gamma f'(k_{t+1}) \left\{ u'(^{\sim}c_{t+1}) + \gamma [u'(c_{t+1}) - u'(^{\sim}c_{t+1})] \right\} \]

• These are functional equations in a “realized” decision rule \( k' = g(k) \) and a “temptation” decision rule \( ^{\sim}k' = ^{\sim}g(k) \):

• Compare and contrast with the generalized Euler equation in the Laibson model:

\[ u'(c_t) = \beta \delta u'(c_{t+1}) \{ f'(k_{t+1}) + (1/\beta - 1)g'(k_{t+1}) \}. \]
MACROECONOMIC APPLICATIONS

• We consider long horizons: the limit of the finite-horizon problems.

• We study competitive equilibrium under two kinds of parametric restrictions:

  1. Isoelastic utility and no restrictions on technology: characterization and existence in the neighborhood of a steady state.

  2. Logarithmic utility, Cobb-Douglas production, and full depreciation: full analytical solution of recursive competitive equilibria.
BARRO ANALYSIS: COMPETITIVE EQUILIBRIUM

• The consumer takes as given: factor prices and a law of motion $\bar{k}' = G(\bar{k})$.

• The consumer’s problem in recursive form:

$$W(k, \bar{k}) = \max_{k'} \{u(r(\bar{k})k + w(\bar{k}) - k') + \delta W(k', \bar{k}') + \gamma (u(r(\bar{k})k + w(\bar{k}) - k') + \beta \delta W(k', \bar{k}'))\} - \gamma \max_{\tilde{k}'} \{u(r(\bar{k})k + w(\bar{k}) - \tilde{k}') + \beta \delta W(\tilde{k}', \bar{k}')\},$$

given $\bar{k}' = G(\bar{k})$.

• This problem determines:

1. A “realized” savings rule $k' = g(k, \bar{k})$.
2. A “temptation” savings rule $\tilde{k}' = \tilde{g}(k, \bar{k})$.

• Equilibrium requires $g(\bar{k}, \bar{k}) = G(\bar{k})$. 
THE LOG-COBB MODEL

Parametric assumptions: logarithmic $u$, full depreciation, Cobb-Douglas production.

1. Autarky

Realized savings rule:

$$ g(k) = \frac{\alpha \delta}{\alpha \delta + (1 - \alpha \delta) \frac{1+\gamma}{1+\beta \gamma}} Ak^\alpha $$

Temptation savings rule:

$$ \tilde{g}(k) = \frac{\alpha \delta \beta}{1 - \alpha \delta + \alpha \delta \beta} Ak^\alpha $$

2. Competitive Equilibrium

Realized savings rule:

$$ g(k, \bar{k}) = \frac{\delta}{\delta + (1 - \delta) \frac{1+\gamma}{1+\beta \gamma}} r(\bar{k})k $$

Temptation savings rule:

$$ \tilde{g}(k, \bar{k}) = \frac{\delta \beta}{1 - \delta + \delta \beta} (r(\bar{k})k + w(\bar{k})) - \frac{\varphi (1 - \delta)}{1 - \delta + \delta \beta} G(\bar{k}) $$
One can show that the following properties hold:

\[
g(k, \bar{k}) = \lambda(\bar{k})k + \mu(\bar{k})
\]

\[
\tilde{g}(k, \bar{k}) = \tilde{\lambda}(\bar{k})k + \tilde{\mu}(\bar{k})
\]

where \((\lambda(\bar{k}), \mu(\bar{k}), \tilde{\lambda}(\bar{k}), \tilde{\mu}(\bar{k}))\) solves the following functional equations:

\[
\mu(\bar{k}) + \frac{w(\bar{k}') - \mu(\bar{k}')}{r(\bar{k}') - \lambda(\bar{k}')} = \frac{w(\bar{k}) - \mu(\bar{k})}{r(\bar{k}) - \lambda(\bar{k})}\lambda(\bar{k})
\]

\[
\tilde{\mu}(\bar{k}) + \frac{w(\bar{k}') - \tilde{\mu}(\bar{k}')}{r(\bar{k}') - \tilde{\lambda}(\bar{k}')} = \frac{w(\bar{k}) - \tilde{\mu}(\bar{k})}{r(\bar{k}) - \lambda(\bar{k})}\tilde{\lambda}(\bar{k})
\]

\[
\frac{1 + \gamma}{\delta (1 + \beta \gamma) r(\bar{k}')} =
\left\{(1 + \gamma) \left[\frac{(r(\bar{k}') - \lambda(\bar{k}'))\lambda(\bar{k})}{r(\bar{k}) - \lambda(\bar{k})}\right]^{-\sigma} - \gamma \left[\frac{(r(\bar{k}') - \tilde{\lambda}(\bar{k}'))\tilde{\lambda}(\bar{k})}{r(\bar{k}) - \lambda(\bar{k})}\right]^{-\sigma}\right\}
\]

\[
\frac{1 + \gamma}{\delta \beta r(\bar{k}')} =
\left\{(1 + \gamma) \left[\frac{(r(\bar{k}') - \lambda(\bar{k}'))\tilde{\lambda}(\bar{k})}{r(\bar{k}) - \lambda(\bar{k})}\right]^{-\sigma} - \gamma \left[\frac{(r(\bar{k}') - \tilde{\lambda}(\bar{k}'))\lambda(\bar{k})}{r(\bar{k}) - \lambda(\bar{k})}\right]^{-\sigma}\right\}
\]
STEADY STATE

• The steady-state interest rate is unique and given by:

\[
\frac{1 + \gamma}{r(k_{ss})\delta(1 + \beta\gamma)} = 1 + \gamma - \gamma \left(1 - \frac{1 - \left(\frac{\beta(1+\gamma)}{1+\beta\gamma}\right)^{1/\sigma}}{r(k_{ss})}\right)^{\sigma}.
\]

Table 1:
Steady-State Interest Rate

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\beta = 0.4$</th>
<th>$\sigma = 1$</th>
<th>$\sigma = 2$</th>
<th>$\sigma = 3$</th>
<th>$\sigma = 5$</th>
<th>$\sigma = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = 0.4$</td>
<td>8.724%</td>
<td>7.519%</td>
<td>7.123%</td>
<td>7.012%</td>
<td>6.930%</td>
<td>6.872%</td>
</tr>
<tr>
<td>$\beta = 0.7$</td>
<td>6.303%</td>
<td>6.192%</td>
<td>6.142%</td>
<td>6.127%</td>
<td>6.114%</td>
<td>6.105%</td>
</tr>
</tbody>
</table>

• As $\gamma \to \infty$, the steady state converges to that of the Laibson model (and $\sigma$ no longer matters).

• When $\beta = 0$ (myopic temptation), $\bar{g}(k, \bar{k}) = \frac{-w(k)}{r(k)-1}$ and the steady state interest rate is given by:

\[
\frac{1}{\delta r(k_{ss})} = 1 - \frac{\gamma}{1 + \gamma} \left[\frac{r(k_{ss})}{r(k_{ss}) - 1}\right]^{-\sigma}.
\]

• The linearity of the savings rules implies that the steady-state wealth distribution is indeterminate (contrast with Gul and Pesendorfer (2000b)).
DYNAMICS: NUMERICAL RESULTS

• Given isoelastic utility, dynamics can be computed using numerical methods.

• On observational equivalence: varying $\beta$ and $\sigma$, while adjusting $\delta$ to keep the steady-state interest rate constant:

<table>
<thead>
<tr>
<th></th>
<th>$\beta = 0.25$</th>
<th>$\beta = 0.5$</th>
<th>$\beta = 0.75$</th>
<th>$\beta = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = 0.5$</td>
<td>0.79093</td>
<td>0.79757</td>
<td>0.80155</td>
<td>0.80477</td>
</tr>
<tr>
<td>$\sigma = 1$</td>
<td>0.86039</td>
<td>0.86039</td>
<td>0.86039</td>
<td>0.86039</td>
</tr>
<tr>
<td>$\sigma = 3$</td>
<td>0.93254</td>
<td>0.93075</td>
<td>0.92854</td>
<td>0.92643</td>
</tr>
</tbody>
</table>
POLICY IN THE INFINITE-HORIZON MODEL

For the log-Cobb model:

- The first-best is (again) the command policy: give the consumer the consumption path that he would choose given no self-control problems and a discount rate equal to $\delta$.

- If the government chooses tax rates to maximize the welfare of the representative agent in a competitive equilibrium, then it will subsidize investment: $\tau^*_y > 0$ and $\tau^*_i < 0$.

The realized savings decision in equilibrium is:

$$G(\bar{k}, \tau^*) = \alpha \delta A \bar{k}^\alpha g(k, \bar{k}, \tau^*) = \delta r(\bar{k}) k.$$  

This is the same allocation as under the command policy, but with lower welfare because of the self-control cost.

- When $\gamma > 0$, the savings rate is higher in competitive equilibrium than in autarky. This is a dynamic response to the larger temptation faced by a consumer in competitive equilibrium.

- The gap between the two savings rates is increasing in $\gamma$. Consequently, for low values of $\gamma$, autarky is better than a laissez-faire competitive equilibrium (without taxation); for high values of $\gamma$, competitive equilibrium dominates.
PRELIMINARY CONCLUSIONS

- The Gul-Pesendorfer framework is in some ways more attractive as a vehicle for addressing preference reversals and a “bias toward the present”.

- We develop the Gul-Pesendorfer model toward non-standard discounting and connect it to the Laibson model.

- The Laibson model appears as a limit case. This case implies that utility should be interpreted as that perceived by one’s previous self.

- In a neoclassical growth setup, we characterize steady states and local dynamics. Observational equivalence does not hold in general.

- We characterize optimal policy: the government should restrict the agent’s choices as much as possible subject to not eliminating those choices that are “good”.
  1. Informed command policy is best.
  2. Taxation policy in a competitive equilibrium involves subsidizing investment.
  3. If the government can influence the extent of price-taking behavior, then perhaps it should.

- In separate but related work, we show how to compute asset prices in a Mehra-Prescott economy with GP consumers. This model can help to explain the low risk-free rate.
Figure 4
Figure 5
\[ W_1 \quad W_2 \]

\[ k_2' \quad k_1' \quad k_1' = k_2' \quad k', \tilde{k}' \]

Figure 6