Why Does the Stock Market Fluctuate?
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WHY DOES THE STOCK MARKET FLUCTUATE?*

ROBERT B. BARSKY AND J. BRADFORD DE LONG

Major long-run swings in the U. S. stock market over the past century are broadly consistent with a model driven by changes in current and expected future dividends in which investors must estimate the time-varying long-run dividend growth rate. Such an estimated long-run growth rate resembles a long distributed lag on past dividend growth, and is highly correlated with the level of dividends. Prices therefore respond more than proportionately to long-run movements in dividends. The time-varying component of dividend growth need not be detectable in the dividend data for it to have large effects on stock prices.

I. INTRODUCTION

The prevailing model of asset pricing is the efficient markets present-value model, which implies that stock market fluctuations reflect revisions in expected future cash flows or discount rates.1 However, the work of Shiller [1981], LeRoy and Porter [1981], and many others2 appears to provide strong evidence against this model as applied to American broad stock market indices over the past century. Volatility in stock market averages is not matched by volatility in the present value of dividends paid ex post, for the realized present value of dividends is close to being a smooth trend.

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1. This corresponds to Kleidon’s [1986] benchmark case, in which log dividends follow a random walk. This relative success of a constant price-dividend ratio model as a first approximation is a central point stressed in Mankiw, Romer, and Shapiro [1985, 1991].

2. Of whom Mankiw, Romer, and Shapiro [1985, 1991]; Campbell and Shiller (see Shiller [1989]); and Flavin [1983] have perhaps been the most influential. For good surveys see West [1988] and LeRoy [1989].

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The market is also smoother than present values of dividend forecasts from ARMA or ARIMA models estimated using the past century’s data. The actual price-dividend ratio is correlated with, but fluctuates much more widely than, does a “warranted price-dividend ratio” computed as the present value of dividend growth rates as forecast from a vector autoregression (see Campbell and Shiller [1988] and Shiller [1989]).

This paper stresses two features of the data that lead to a more sanguine view of the plausibility of the present-value model. First, as previously stressed by Mankiw, Romer, and Shapiro [1985, 1991] and Kleidon [1986], the log dividend process is to a rough approximation a random walk. For such a process the warranted stock price does move proportionately with dividends. This by itself is in marked contrast to the warranted price process implied by Shiller’s stationary models, in which future dividends move less than one-for-one with the current dividend and in which the warranted price is substantially damped compared with the dividend series and the actual price series.

The second feature of the data that we stress in this paper is that not only are stock prices not “smoother” than dividends, prices appear to “overreact” to long swings in dividends. A long-run 1 percent increase in the level of dividends is associated with an approximately 1.5 percent increase in equity values. Figure I plots, on a log scale, the real S&P index of the U. S. stock market over the past century alongside twenty times the value of the

![Figure I](image-url)

**Figure I**
current dividends paid on the index.\textsuperscript{3} It shows that the stock market index has exhibited wider, not narrower, fluctuations than has the "warranted" price implied by the very simple valuation rule of estimating fundamentals to be twenty times dividends.

Our explanation of this apparent anomaly is a consequence of dropping the assumption that the dividend growth rate has a mean that is constant over time and known to the agents throughout the sample. Instead, we see investors as having to estimate period by period a growth rate that is nonstationary. Heuristically, the model can be thought of in terms of the traditional Gordon [1962] valuation model:

\begin{equation}
\frac{P_t}{D_t} = \frac{1}{r - g_t},
\end{equation}

where $P_t$ is the price, $D_t$ is the current dividend, the real required rate of return $r$ is a constant, and $g_t$, the "permanent" dividend growth rate in the sense of Friedman's "permanent income," is unknown, changing over time, and must be reestimated each period. Normally, the estimate $g_t$ will closely resemble a distributed lag on past one-period dividend growth rates, with slowly declining weights. Thus, it will in turn resemble the level of dividends. Since $D_t$ and $g_t$ are closely correlated and affect $P_t$ in the same direction, $P_t$ will mirror $D_t$ with an elasticity greater than unity.

Therefore, in addition to the one-for-one response of log prices to dividends coming from the numerator of (1) stressed in Mankiw, Romer, and Shapiro [1985, 1991], there is an additional response reflecting the dependence of prices on dividend growth rates and the resemblance between dividend levels and long-term expected dividend growth. The magnitude of this second effect depends inversely on $r - g_t$. Since $r - g_t$ is a fairly small number on average, the appropriate multiplier will be rather large. The combined effect of these two departures from the model underlying the work of Shiller and others arguing that major market movements are excessive or even inexplicable is a shift toward the view that changes in current and expected future dividends can account for the bulk of long-run stock price fluctuations, although much less so for short-term price movements.

\textsuperscript{3} The S&P composite is taken from Standard and Poors Securities Price Index Record and from Cowles et al. [1939]. The data series from 1871 up to the late-1980s is printed in Shiller [1989]. Stock prices are real values for January. Dividends are totals for the year divided by the year's average producer price level. In calculating perfect-foresight fundamentals, the present value of post-sample dividends is assumed to be equal to the terminal price.
II. THE PRICE-DIVIDEND RATIO AND EXTRAPOLATED DIVIDEND GROWTH

A. Dividends and Earnings

Figure II plots real dividends and earnings for the U. S. stock market from 1880 to the present. Decades that see rapidly rising dividends see rising earnings as well. Table I quantifies the long-swing relationship between dividends and earnings, reporting regressions of multiyear changes in log earnings on corresponding changes in log dividends. For twenty-year changes such a regression yields a coefficient of 1.00 (with an estimated standard error, correcting for the overlapping nature of the data using a Hansen-Hodrick procedure, of 0.21) and a correlation between twenty-year dividend and earnings changes of 0.70. Thus, long swings in dividends significantly reflect long swings in earnings and profitability, not merely shifts in payout ratios.4

4. Figure II also shows that year-to-year dividend changes become substantially less volatile after World War II. Large year-to-year changes in dividends become rare, as if the amount of dividend smoothing has substantially increased. The standard deviation of annual log dividend changes falls from 0.147 for 1880–1939 to 0.081 for 1940–1981. Under the assumption that annual changes are independent, this difference produces an \( F(59, 51) \)-statistic of 3.29. The 0.01 level is 1.90. Earnings measures as well are substantially smoother in the second half of the sample. Such shifts in the variance of earnings and dividends suggest a possible structural break at World War II. Such a shift in the process generating fundamentals would make data from the distant past less relevant for investors seeking to forecast future dividends.
**WHY DOES THE STOCK MARKET FLUCTUATE?**

**TABLE I**

**Regression of Multiyear Changes in Log Earnings on Changes in Log Dividends**

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Hansen-Hodrick Standard Error</th>
<th>$R^2$</th>
<th>SEE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-year changes</td>
<td>-0.014</td>
<td>0.160</td>
<td>0.001</td>
<td>0.214</td>
</tr>
<tr>
<td>5-year changes</td>
<td>0.772</td>
<td>0.203</td>
<td>0.271</td>
<td>0.068</td>
</tr>
<tr>
<td>10-year changes</td>
<td>0.729</td>
<td>0.182</td>
<td>0.407</td>
<td>0.028</td>
</tr>
<tr>
<td>20-year changes</td>
<td>1.005</td>
<td>0.210</td>
<td>0.487</td>
<td>0.016</td>
</tr>
<tr>
<td>30-year changes</td>
<td>1.086</td>
<td>0.260</td>
<td>0.464</td>
<td>0.011</td>
</tr>
</tbody>
</table>

**B. Excess Volatility and the Price-Dividend Ratio**

Figure I showed that the long swings in stock prices are roughly in phase with, but somewhat larger in proportional terms than, long swings in real dividends. From 1920 to 1929 the log real stock index rises by 1.45, while the log of dividends rises by 1.08.

**Figure III**

Long Twenty-Year Swings in Log Prices Regressed on Twenty-Year Swings in Log Dividends, 1890–1991
From 1949 to 1969 the log real stock index rises by 1.63, while the log of the real dividends paid on the index rises by only 0.72. From 1969 to 1982 the log real stock index falls by 0.91, while log dividends paid fall by only 0.26.

Figure III illustrates, and Table II quantifies, this relationship between long swings in prices and dividends. Table II regresses multiyear changes in log real stock index prices on multiyear swings in log real dividends. For changes in the ten- to thirty-year range, the regression slope is about 1.50. The highest coefficient is for a twenty-year horizon: each 1 percent increase in dividends is then accompanied by a 1.61 percent change in the same direction in prices (with a Hansen-Hodrick standard error of 0.212).

This high elasticity imposes restrictions on the pattern of growth rate expectations that must be implicit in market prices if the present value model is to hold. A greater than unit elasticity is difficult to make consistent with the combination of rational expectations and a dividend-generating process containing a long-run component mean-reverting in levels, as implicitly assumed in Shiller [1989, Ch. 5; a reprint of Shiller, 1981]. Indeed, Shiller [1989, Ch. 1; a reprint of Shiller, 1984] regards this high responsiveness of prices to dividends as an alternative way of stating the “excess volatility” puzzle.5

The conditions under which such a high elasticity of price with respect to dividends is consistent with a present-value interpretation are easily calculated. For the moment, define $g_t$ as the expected “permanent” dividend growth rate that makes equation (1) hold.6 Fixing discount rates, using $\partial$ to denote a partial derivative, and using lowercase “$p$” and “$d$” for the logs of prices and dividends, then the elasticity of prices with respect to dividends is

$$\frac{\partial p_t}{\partial d_t} = 1 + \left[ \frac{1}{r - g_t} \right] \frac{\partial g_t}{\partial d_t}. \tag{2}$$

For the elasticity of prices with respect to dividends $\partial p_t/\partial d_t$ to be greater than one, expected future growth rates $g_t$ must be

5. Froot and Obstfeld [1991] attempt to account for this high elasticity of prices with respect to dividend changes not in terms of shifts in dividend growth rate expectations but in terms of a state-dependent speculative “bubble” term in prices that is positively correlated with fundamentals.

6. The dividend process in our explicit basic model below will have such an unambiguously defined $g_t$ at every point in time.
Why Does the Stock Market Fluctuate?

Table II
Regressions of Multiyear Changes in Log Prices on Changes in Log Dividends

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Hansen-Hodrick standard error</th>
<th>$R^2$</th>
<th>SEE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-year changes</td>
<td>0.865</td>
<td>0.106</td>
<td>0.361</td>
<td>0.142</td>
</tr>
<tr>
<td>5-year changes</td>
<td>1.143</td>
<td>0.150</td>
<td>0.602</td>
<td>0.050</td>
</tr>
<tr>
<td>10-year changes</td>
<td>1.496</td>
<td>0.176</td>
<td>0.758</td>
<td>0.027</td>
</tr>
<tr>
<td>20-year changes</td>
<td>1.608</td>
<td>0.212</td>
<td>0.703</td>
<td>0.016</td>
</tr>
<tr>
<td>30-year changes</td>
<td>1.424</td>
<td>0.236</td>
<td>0.609</td>
<td>0.010</td>
</tr>
</tbody>
</table>

positively correlated with dividends.\(^7\) Moreover, the relationship between expected future growth rates and past dividend changes must be strong. The $r - g$, term in the denominator of equation (2) is on the order of 0.04. To fit the 1.5 regression coefficient of multiyear log price changes on dividend changes, a 10 percent increase in the level of dividends—an increase of 0.5 percent per year in average dividend growth—over a twenty-year period must carry with it a shift in the expected $g$, of 0.2 percent per year. Thus, to account for the actual correlations of multiyear price and dividend changes, a little less than half of any shift in average dividend growth rates over a twenty-year period must be expected to persist indefinitely.

C. Extrapolative Expectations of Growth Rates

Expectations of long-run dividend growth will be correlated with recent past changes in dividends if agents estimate the "permanent" growth rate by taking a distributed lag of past dividend growth rates. We call such forecasting rules "extrapolative" expectations of the growth rate. Such forecasts may be rational expectations. The classic paper of Muth [1961] presents the case in which a distributed lag with exponentially declining weights on past observations generates the optimal forecast of the future: this is so if the variable to be estimated is the sum of a random walk and a transitory white-noise error. Such

7. Alternatively, the rate of discount $r$ could vary. For a recent attempt at explanation based on variation in discount rates rather than anticipated growth rates, see Cecchetti et al. [1990]. Campbell and Shiller (see Shiller [1989]) and Mankiw, Romer, and Shapiro [1991] find that under two common specifications of fluctuating real discount rates the magnitude of the excess volatility puzzle is increased.
forecasts may be reasonable ones in the sense of Herbert Simons, in that an exact solution to a simple model of a problem is likely to be a good solution to the original problem. Such forecasts may be suboptimal, although “optimality” should be assessed ex ante, not ex post with hindsight. Certainly, the possibility that the long-run dividend growth rate may shift over time with long-run movements in productivity and profitability should not be dismissed ex ante: some economies do turn into Argentinas, where growth stalls for generations, or into Germanys, Japans, and Spains, where growth accelerates to levels that were far outside the bounds of what was previously expected.

A simple univariate model of log dividend growth that captures the idea that the long-run growth rate of dividends is uncertain and changing is

\[ \Delta d_t = \epsilon_t + \sum_{i=1}^{t-1} (1 - \theta)\epsilon_{t-i} + g_0. \]

In equation (3), \( g_0 \) is the permanent growth rate of dividends as of time 0. The \( \epsilon_t \)'s are stochastic shocks to dividend growth that have not only a once-and-for-all permanent effect on the level of dividends (the lead \( \epsilon_t \) term in equation (3)) but also permanent, albeit attenuated, effects on the “permanent” growth rate of dividends (the \( (1 - \theta)\epsilon_{t-i} \) terms under the summation sign in equation (3)), which we denote by \( g_t \):

\[ g_t = g_{t-1} + (1 - \theta)\epsilon_t. \]

Since \( g_t \) itself is a random walk, the change in log dividends is thus an IMA(1,1).

For \( \theta \) near one, the process examined over relatively short time intervals is close to a random walk. But this near random walk process has a rate of drift that is itself slowly time varying. As a result, observations of dividend changes in the past will, over time, slowly become less and less relevant to determining the current underlying permanent rate of dividend growth.

From equation (3) it follows that for all horizons the expected dividend growth rate expected as of time \( t \) is the same:

\[ E_t[\Delta d_{t+j}] = g_t = \sum_{i=1}^{t-1} (1 - \theta)\epsilon_{t-i} + g_0. \]

The permanent dividend growth can be written as a geometric
average of past dividend changes:

\[ g_t = (1 - \theta) \sum_{i=0}^{t} \theta^i \Delta d_{t-i} + \theta^t g_0. \]  

Equation (6) implies that investors extrapolate past dividend growth into the future. They take an average of recent past dividend changes with geometrically declining weights, and project that growth rate forward. The range of relevant \( \theta \)'s in our case is close to unity. Most of the year-to-year variation in dividend growth is transitory, so the optimal filter for \( g_t \) involves slowly declining weights on past dividend growth.

If \( g_t \) were the deterministic "permanent" growth rate of dividends, then the log present value of future dividends would be simply the logarithmic transformation of equation (1):

\[ p_t = d_t - \ln (r - g_t). \]  

Equation (7) does not hold exactly in a stochastic model for the dividend process given by (3): equation (7) does not allow prices today to be influenced by investors' knowledge that they will be revising their estimate of \( g \) in the future. But for simplicity we ignore these higher-order corrections here and use (7) as our formula for the log of the warranted stock index price.  

8. Alternatively, instead of treating (7) as an approximation to the pricing rule, we could treat (3) as an approximation to the dividend process. Equation (7) is the expected present value of future dividends if log dividends are generated by the continuous time process given by

\[ g_t = g_0 + (1 - \theta)\sigma_t W_t - \int_0^t (1 - \theta)^2 \sigma_t^2 \frac{d\sigma_t^2}{r - g_t} dt \]

and

\[ d_t = d_0 + \int_0^t g_t dt + \sigma_t W_t - \int_0^t \sigma_t^2 \frac{d\sigma_t^2}{2} dt, \]

where \( W_t \) is a standard unit Brownian motion. The higher-order terms are then canceled out by the last terms in equations (8) and (9). The last integral in equation (9) incorporates a downward adjustment to the rate of drift of the log dividend \( d_t \) necessary because for a drifting continuous-time stochastic process with standard deviation per unit time \( \sigma_t \), the expected growth rate is higher than the drift by \( \sigma_t^2/2 \).

The last integral in equation (8) incorporates a downward adjustment to the growth rate \( g \) that becomes very strong as \( g \) approaches \( r \). That this term is necessary can be seen by the fact that in models with a constant discount rate \( r \) the asset price becomes infinite if ever the permanent growth rate \( g \) reaches \( r \); and the asset price becomes infinite even if \( g \) today is less than \( r \) if there is a finite probability that \( g \) will reach \( r \) in the future. The last term in (8) ensures that \( g \) will never reach \( r \), and is of just the right magnitude to balance capital gains from upward market revaluations on possible future increases in the permanent growth rate against capital losses from downward market revaluations on possible future decreases in \( g_t \), and so make equation (7) hold.

For the processes considered in this paper, these "nuisance" terms are on the order of 0.5 percent per year in the level equation and 0.01 percent per year in the growth rate equation, and so are quantitatively insignificant.
The dividend process (3) thus produces equation (6) as its rational forecast of the future dividend growth rate \( g_t \). The basic point is that of Muth [1960]. The underlying dividend growth rate is an unknown and time-varying parameter that has to be reestimated and updated each year. Faced with such a forecasting problem, rational investors will extrapolate recent past dividend growth into the future according to equation (6).

III. FITTING LONG SWINGS IN STOCK PRICES

A. Long Swings and Extrapolated Growth Rates

The simple rule (6) for estimating the current long-run dividend growth rate and the simple present-value formula (7) together do a good job of fitting the bulk of low-frequency variation in real stock index prices over the twentieth century. Moreover, they do so for values of \( \theta \)—the parameter determining the proportion of each year's dividend shock that is a transitory and not a permanent shock to the growth rate—close to one. Figures IV and V plot actual stock prices and “warranted” values constructed according to equations (6) and (7) for the parameter values \( \theta = 0.95 \) and \( \theta = 0.97 \).

9. Figures IV and V assume a constant real discount rate of 6 percent per year. In calculating the current value of \( g_t \) from equation (6), all values of dividend growth before our data begin in 1870 are for simplicity set equal to the 1870–1991 average rate of dividend growth. In earlier versions (see Barsky and De Long [1989]) we truncated the series of past growth rates at 1870, assuming that investors in the 1880s and 1890s had no information about dividend growth before 1870. This generated unrealistically large fluctuations in estimated future growth rates and warranted prices in the 1880s, but by 1900 it had little effect on warranted prices. For \( \theta = 0.95 \), for example, as of 1900 only 22 percent of the weight in calculating \( g_t \) is
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Because \( \theta \) is near one, only a negligible part of year-to-year variance in dividend growth is the result of revisions in the long-run growth rate of dividends \( g_t \). An investor's estimate of future years' dividend growth is revised upward by only 0.05 (for \( \theta = 0.95 \)) or 0.03 (for \( \theta = 0.97 \)) times the unexpected component of this year's dividend growth. Shifts in \( g_t \) thus account for only 1/400 of the variance of year-to-year dividend growth about its one-period-ahead forecast in the case shown in Figure IV, and for only 1/1000 in the case shown in Figure V.

Yet even though such revisions have little effect on year-to-year forecasts, the implications for warranted values are dramatic. In Figure IV long swings in warranted prices are substantially greater than, and in Figure V long swings are about the same magnitude as, actual low-frequency long swings in the stock market. ¹⁰

One way to assess the success of the model of equations (5) and (6) at fitting long-run movements in stock prices is to compare the variance of twenty-year changes in actual and "warranted" prices. The variance of twenty-year price changes is 0.353, but the

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10. The hypothesis advanced in this paper about the causes of low-frequency overreaction is similar in structure to an interpretation advanced for the nineteenth century Gibson paradox. Investors determine warranted prices by marking up dividends by a multiple that depends on a growth rate, which is calculated as a distributed lag of past growth. Since long-run distributed lags of past growth are correlated with present levels, movements in warranted prices appear an amplified version of long swings in dividends, just as in the Gibson paradox nominal interest rates appeared more closely correlated with the price level than the inflation rate. See Friedman and Schwartz [1982], Sargent [1973], and Shiller and Siegel [1977] (reprinted as chapter 14 of Shiller [1989]).
TABLE III
SHARE OF VARIANCE IN MULTIYEAR CHANGE IN LOG STOCK INDEX ACCOUNTED FOR BY DIFFERENT FUNDAMENTAL ESTIMATES

<table>
<thead>
<tr>
<th>Alternative fundamentals measure</th>
<th>1-year changes</th>
<th>5-year changes</th>
<th>10-year changes</th>
<th>20-year changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant multiple of dividends</td>
<td>36%</td>
<td>59%</td>
<td>67%</td>
<td>60%</td>
</tr>
<tr>
<td>Figure IV warranted price ($\theta = 0.95$)</td>
<td>$-33%$</td>
<td>15%</td>
<td>60%</td>
<td>65%</td>
</tr>
<tr>
<td>Figure V warranted price ($\theta = 0.97$)</td>
<td>9%</td>
<td>47%</td>
<td>76%</td>
<td>72%</td>
</tr>
</tbody>
</table>

Variance of twenty-year price changes relative to shifts in the "warranted" price series of Figure V is only 0.102.

Table III shows the proportion of the variances in multiyear changes in prices accounted for by three alternative estimates of fundamental values for one-, five-, ten-, and twenty-year horizons. The last row of Table III shows that changes in warranted valuations using the parameter values of Figure V accounts for fully 72 percent of changes in log prices relative to ex post fundamentals at a twenty-year horizon, and for 76 percent of changes at a ten-year horizon. But such shifts in warranted prices calculated by extrapolating past dividend growth into the future can account for only 47 percent of five-year changes and for only 9 percent of yearly changes. Dividend growth extrapolation is far from a complete model of movements in the real S&P index. It fits the long swings, not the year-to-year movements, in prices.

Note from Table III that a simple formulation that takes the future dividend growth rate to be a constant and marks real dividends up by a constant multiple does not do at all badly in accounting for long-run stock price movements over the past century. The constant dividend multiple model accounts for more than three-fifths of twenty-year price changes, although it does leave a residual variance 42 percent larger than does our model.11 The amount of "excess volatility" in stock prices is an order of magnitude smaller assessed in terms of the variability of the price-dividend ratio (as in Mankiw, Romer, and Shapiro [1991]) than when assessed in terms of the variability of prices relative to some more sluggish benchmark like a constant times a moving average of past dividends (as in Shiller [1990a]).

11. This paper assumes a constant discount rate throughout, and does not deal with the possibility that stock market fluctuations reflect changes in real discount rates.
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Figure VI plots variation in the price-dividend ratio over time alongside variation in the "warranted" price-dividend ratio from Figure V. Variation in the "warranted" price-dividend ratio calculated from our basic model accounts for only one-third of even long-run movements in the price-dividend ratio. Our model, however, does fit some major historical episodes: the fall in the price-dividend ratio from 1910 to 1920, the rapid rise in the price-dividend ratio in the 1920s, and a substantial component of the upward swing in the price-dividend ratio in the 1960s and its subsequent fall in the 1970s.12

B. Alternative Extrapolative Formulations

Note that there is little special or unusual about the specific form of equation (6). It is simple and analytically tractable, but the element that allows expectations of \( g_t \) based on (6) to produce long swings in stock prices as large as those seen in the past century is the positive correlation of \( g_t \) with past dividend growth rates. This element is shared by many other alternative formulations under

12. An alternative metric to use to assess the fit of our model to actual long-run movements in stock index prices is to examine the implied long-run pattern of co-movements between prices and dividends. For \( \theta = 0.97 \) and \( r = 0.06 \), twenty-year changes in log warranted prices are 1.62 times twenty-year change in log dividends. As Table II above showed, this is almost exactly the 1.61 coefficient estimated by regressing twenty-year changes in actual log prices on log dividends.
which estimates of mean future growth are also made by averaging past growth.

As an extreme case, even an investor certain that the dividend growth rate is fixed would forecast warranted values by extrapolating past dividend growth into the future if he used the past sample average of dividend growth to estimate an unknown constant parameter $g$. It is hard to see how investors could, in a sense, do anything other than extrapolate. If the “permanent” dividend growth rate $g_t$ is unknown—even if it is certain that it is a fixed unchanging parameter of the economy—then investors must obtain an estimate of $g_t$ from somewhere. The most natural place to obtain it is from the mean of past dividend growth rates.

Figure VII plots the log of actual prices, twenty times dividends, and “warranted” valuations as they would be calculated by an investor using the past sample average (since 1870) of dividend growth to estimate $g$. The series performs very poorly in the 1880s, as a period of high dividend growth generates extremely high but unstable warranted values.

After the 1880s, however, taking the estimated future rate of long-run dividend growth to be simply the past sample average can more than account for long swings in stock prices, and can account for the more than unit elasticity of pre-World War II price with respect to dividend changes.\textsuperscript{13}

\textsuperscript{13} This point is also stressed by Timmermann [1992]. As Mankiw, Romer, and Shapiro [1985] note, however, there is less sign of a greater than unit elasticity of price with respect to dividend changes before than after World War II.
After World War II the warranted valuation as plotted in Figure VII is close to a constant multiple of dividends: the existence of a century of past data estimates the assumed constant $g$ relatively precisely. Warranted prices do not rise as much as stock prices in the booms of the 1950s and 1960s, or fall as much as in the 1970s. The possibility that extrapolation is due to investors’ need to estimate a fixed but unknown dividend growth rate parameter $g$ cannot account for post-World War II fluctuations. At least, it cannot account for post-World War II fluctuations if investors believe that $g$ is the same after as before World War II.

However, World War II is the single most obvious candidate for a structural break. Economic growth rates in a large number of European countries appear to have taken a permanent upward jump in the years after World War II: European industrial nations that had been slowly losing ground relative to the United States began to catch up and converge, and the world economy as a whole grew at a previously unheard-of pace for more than a quarter century. It would have been very rash for an investor in the late 1950s to argue that future dividend growth rates would be low because pre-World War II growth rates had been low. If World War II is admitted as a structural break, and pre-World War II evidence that the value of $g$ is discounted to some degree, then it might be possible to fit at least some of the long swings in the post-World War II stock market as due to investors estimating an unknown $g$ that is fixed in the post-World War II regime.

Closely related to estimation of a fixed mean is a rolling average of past dividend growth. If a dividend growth forecasting rule that took expected future growth to be the average of the past twenty years of growth, then warranted stock prices would exhibit long-run fluctuations as large as or larger than those seen in the major bull and bear markets of this century. However, the sharp cutoff two decades in the past would impart a jagged character to the year-to-year fluctuations in warranted prices that is not present in Figures IV and V.\textsuperscript{14}

These are extreme examples. Nevertheless, they serve to

\textsuperscript{14} In the late 1930s, for example, warranted prices would jump suddenly and sharply because the real dividend declines of World War I cross the cutoff and disappear from the sample used to estimate $g$. As another example, note that in the late 1960s the warranted price is halved in two years as the rapid dividend rises of the late 1940s disappear from the rolling sample. We do not advocate that investors use such a forecasting rule: we merely observe that it is another example of a rule that extrapolates past dividend growth into the future, and that manages to generate fluctuations in warranted prices at least as great as those in actual stock index prices.
illustrate the point that a wide range of different forecasting rules—each optimal under different assumptions about the process generating dividends—will generate extrapolation of past dividend growth into the future, and a correlation between past changes in dividends and expected future growth.

C. The Magnitude of Permanent Growth Rate Shocks

Is there in fact positive evidence of the permanent shocks to dividend growth rates that we invoke to rationalize investors’ extrapolation of past dividend growth into the future? If our argument depended on the existence of a “large” unit root in the dividend growth process—a unit root that made significant contributions to the year-to-year variance of dividend growth—it would be refuted, for many have found a random walk with constant drift to be a good first approximation to the U. S. dividend process [Mankiw, Romer, and Shapiro, 1985; Kleidon, 1986]. However, the values of $\theta$ equal to 0.95 and 0.97 required to generate Figures IV and V correspond to a unit root in dividend growth that contributes only a very small share of year-to-year dividend growth volatility. Such a small variance of permanent growth rate shocks is very hard to estimate empirically.

Information about the parameters of the dividend process is limited today. Our estimates today of the IMA(1,1) of equation (3) produces a maximum likelihood estimate of $\theta$ equal to 0.989 with an (asymptotic) estimated standard error of 0.023. However, the most important point is not that the likelihood is maximized for $\theta = 0.989$. The most important point is that the data do not speak strongly about the value of $\theta$.

Complicating inference is the existence of small-sample bias. Convergence to the asymptotic distribution is very slow for $\theta$ near one. Shephard and Harvey [1990] investigate the small sample behavior of estimates of the Muth-type IMA(1,1) process considered here. They find a disturbingly large probability of calculating a maximum likelihood estimate of $\theta$ equal to 1.00 even when $\theta$ is less than one and there are permanent shocks. Thus, even a finding of a maximum likelihood estimate of $\theta$ equal to one would not be evidence that there are no permanent growth rate shocks.

Shephard and Harvey [1990] report that for a sample size of 50 and for a true $\theta$ of 0.90, there is one chance in three that the
maximal likelihood estimate will be at \( \theta = 1.00 \). Our model has more than twice the number of observations available as does Shephard and Harvey’s Monte Carlo study, but it requires an underlying \( \theta \) only one-third as far from 1.00, so our model possesses even less power to resolve differences of 0 from 1.00 as does Shephard and Harvey’s. Our own Monte Carlo simulations with a sample size of 120 and a true underlying \( \theta \) of 0.97 find that 36 out of 100 times the maximum of the likelihood is at 1.00.

Thus, the sample size and the relative magnitude of permanent growth rate shocks are too small for estimation of equation (3) to be informative about values of \( \theta \) in the range needed to produce figures like IV and V. An individual with a point belief that \( \theta \) was 0.95, or 0.97, or 0.99, or 1.00 in 1871, who decided then to hold that belief until evidence forced a statistically significant rejection, would today still hold to his original prior opinion.

The natural conclusion is that a century is not long enough to estimate the parameter \( \theta \) in equation (3) with sufficient precision. An investor could find reason to believe that permanent shifts in the rate of mean dividend growth are relatively large, and that the “true” \( \theta = 0.95 \). In which case, as Figures IV and V showed, the warranted fluctuations in the stock market were noticeably more volatile than the actual fluctuations of stock indices. An investor could perhaps believe that there are no permanent shifts in the rate of mean dividend growth—that the “true” \( \theta = 1.00 \)—if such were his prior. With respect to such a prior the stock market has been too volatile: it has been anticipating permanent shifts in dividend growth that have never occurred, and will never occur.

We have shown that an econometrician attempting in 1992 to detect a small unit root in dividend growth could not obtain precise estimates, even with 120 years of data. A fortiori an investor in the

15. Under the procedure providing the least chance of incorrectly estimating \( \theta = 1.00 \), which begins estimation with a diffuse prior on the initial state. See Shephard and Harvey [1990].

16. Little hinges on the null/alternative framework. Rephrased in Bayesian terms, the likelihood function for the IMA(1,1) process with normal innovations is sufficiently flat over the parameter \( \theta \) that an individual who in 1871 held a uniform prior for \( \theta \) over \([0, 1]\), would today hold a posterior with a relatively large variance. Since with a uniform prior the posterior is proportional to the likelihood, such an investor would hold a subjective distribution for \( \theta \) with a mean of 0.959, and a standard deviation of 0.048.

Such an investor would think that there was a 90 percent chance that \( \theta \) was in \([0.91, 1]\), a 60 percent chance that \( \theta \) was in \([0.96, 1]\), and a 15 percent chance that \( \theta \) was in \([0.99, 1]\). His posterior distribution would still be relatively diffuse.
past (in 1929, 1933, or 1963) operating with the smaller sample of data then available could not determine whether \( \theta \) was really 1.00 or 0.95. Information about the form and the parameters of the dividend process is limited today, and was even more limited at the beginning of this century. Even if we were lucky and could precisely estimate \( \theta \), no investor in 1870 or 1929—lacking the data that we possess—had any chance of doing so.

How should an investor uncertain whether permanent shocks to the rate of dividend growth are relatively common or nonexistent forecast dividend growth? Clearly, he should do so by splitting the difference: if uncertain whether there are no permanent shocks or some permanent shocks, forecast as if you know that there are a few. And so any maximizing investor, except for one certain back in 1870 that the dividend growth rate is a constant of nature, will weight the recent past more heavily than the distant past and so extrapolate recent past dividend growth into the future.

A similar argument holds for an investor who does not believe that there are frequent small permanent shocks to dividend growth rates, but who does believe in changes of economic regime that are accompanied by changing dividend growth rates. Such beliefs are certainly reasonable. Output growth rates both in the world as a whole and in individual countries are not constants of nature, but vary with the economic environment and the politico-economic regime. Profit shares and dividend payout rates are also variables that might well shift.

An investor believing in such regime changes will attempt to use experience from the present regime to estimate the regime-specific trend dividend growth rate, and will downweight or ignore experience from past economic regimes.

Such a forecasting procedure will, like equation (6), tend to make expected future dividend growth an extrapolation of recent past dividend growth. In such a context, especially if the investor is not confident of his ability to conclusively identify regime breakpoints, a forecasting rule like equation (6) might be a reasonable rough approximation of the investor’s forecasts.

\[ \text{IV. Conclusion} \]

We have couched our account of stock price fluctuations in a rational-expectations framework. Nevertheless, it bears considerable resemblance to a number of alternative frameworks in which price rises engender “positive feedback” in investors’ demands
through less than fully rational processes. Shiller [1984, 1990] has proposed “naive” models in which investors forecast capital gains by extrapolating past price changes. When augmented to allow prices to depend on current dividends as well as extrapolations of past price changes into the future, Shiller’s model behaves in a fashion extremely close to ours. In particular, the elasticity of price with respect to the dividend is always greater than unity, and is substantially so if the weight on the extrapolative component is significant. De Long, Shleifer, Summers, and Waldmann [1990] have presented a model in which the trading of rational speculators reinforces such positive-feedback behavior: anticipating that positive-feedback traders will buy in the future, rational investors buy today, increase prices, and so increase the magnitude of positive-feedback behavior. Such models could well produce the same patterns of price and dividend movements as our model, but the interpretation they place on these patterns is very different from the rational-expectations language in which we have written this paper.

Interpretations of stock price fluctuations that focus on rates of changes can be roughly divided into three categories. First, there is the present-value model, in which the price-dividend ratio is a good forecast of the present value of future dividend growth rates. Second, there is an “irrational” present-value model, in which stock price movements are driven by inappropriate shifts in expected fundamentals: for example, if investors believe that it is rational to extrapolate past dividend growth into the future, but it is not in fact rational to do so. Finally, there are “fads” and “irrational bubble” models, in which demands are largely determined by market expectations of short-term capital gains that are inconsistent with long-run fundamentals and that are grossly falsified when bubbles “burst.”

One major difficulty with interpreting our model as a rational-expectations model—thus classifying it in the first of these three alternative categories—is that our IMA(1,1) process does not have predictive power for ex post dividend growth. However, the counter-argument that agents who did not have the econometrician’s access to hindsight might have been rational ex ante to choose the extrapolative forecast is one that we find very powerful. There was no a priori information that would have led investors to foreclose the possibility of time variation or even nonstationarity in dividend growth. In Barsky and De Long [1989] we discussed, in a different context, the problem in implementing rational-expectations mod-
els of attributing to investors ex ante the expectations implied in a
model imposed by an econometrician ex post. Even when all the
information in the sample is processed, the IMA(1,1) distributed
lag dividend model is not sharply at odds with the data. Even with
the benefit of hindsight, it is not clear that it was irrational for
investors to extrapolate.

The second alternative that agents estimate fundamentals by
extrapolation, even though they "ought" to know better than to do
so is less refutable because it is less restrictive. The secure point in
this paper is that the observed price-dividend behavior is what one
would see if agents view the dividend process as containing a
persistent growth component. The interpretation is much more
difficult: exactly how is one to test whether a set of expectations
were or were not rational ex ante to investors uncertain of the
structure of the economy?

The third set of models, in which prices can in the short run be
maintained above fundamentals by "castles in the air" promising
further short-run capital gains, may have an essential role to play
in accounting for short-run price dynamics. But looking at the long
swings in stock prices in terms of such "castles in the air" seems
unjustified in light of the strong link between dividends and prices,
and between dividends, earnings, and productivity. The present
value model thus retains considerable power.

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