First Order Risk Aversion, Aggregation, and Asset Pricing

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Abstract

We study the aggregation properties of an economy with multiple agents who have first-order risk aversion preferences, and their implications to asset pricing. Motivated by either psychological evidence or axiomatic argument, people have used models with a representative agent who has first order risk aversion preference in asset pricing research. Such models can successfully explain many asset pricing phenomenon such as equity premium puzzle etc. We show that when individual agents have first-order risk aversion preferences, the degree of first-order risk aversion of the representative agent will be much smaller. Specifically, we solve numerically the optimal consumption allocation in a dynamic general equilibrium model with two agents, each having disappointment aversion (DA) preferences. Then we decompose the economy to solve for the prices of stock and bond from individual agent’s optimization problem. From this we simulate returns of stock and bond. Finally we assume there is a representative agent with similar DA preference and estimate the preference parameters from simulated returns. We show that the resulting parameter for DA preferences will be much smaller. In other words, the resulting preferences are much closer to second order risk aversion preferences. The intuition for this is that multiple agents may result in a representative agent whose utility function has many “kinks”. Thus the utility function of the representative agent will become smoother. We show that only a few agents (actually two) will have a dramatic effect. However, the asset pricing implications of first-order risk aversion preferences on asset prices, such as high equity premium and low risk-free rate, still hold. So in essence, the intuition of using first order risk aversion preferences still hold at individual level, but the current usage of one representative agent is not correct.

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1 Introduction

The past decade has witnessed enormous development in applying behavior finance models to solve the problems in asset pricing, such as equity premium puzzle (Mehra and Prescott (1985)) and its associated challenges. Such models are built upon either solid psychological evidences (for example, Kahneman and Tverseky (1979)) or axiomatic consideration (for example, Gul (1991)). One of most populous and thus fruitful approach is to assume that agents have so-called “first-order” risk aversion utility function. The applications of such type of models have generated significant results in asset pricing and portfolio research.

To assume that agents have first-order risk-averse utility function basically assumes that agents’ utility functions are not smooth functions of consumption or wealth. As shown in Epstein and Zin (1990), such utility functions can generate significant equity premium without the need of large risk-aversion coefficient. However, many critiques on behavior models remain. Among them, one of the most significant ones is the aggregation problem. While people may be sympathetic to the assumption about individual preferences \(^1\), in the applications so far people generally assume a representative agent approach. As Zin argues (Zin (2002)), “. . . having a better understanding of aggregation will be an essential component in obtaining a consensus on reasonableness.”

This paper tries to explore this front. We study the aggregation properties of multiple agents with first-order risk aversion. Specifically we study the aggregation of disappointment-aversion utility function. We choose this utility function because it has a natural and consistent extension to dynamic case, so that the results are not dependent upon the problems associated with how to get a reasonable dynamic version of behavior finance model. As one will see from the arguments in the following, we believe that our results are in fact general to other first-order risk version preferences.

The intuition for using first-order utility function to generate large equity premium is as follows. Instead of smooth preferences, first-order preferences will put more weights on the “downside risk”. Thus there is a “kink” in the utility function. For the popular loss-aversion (LA) utility function, the weight is more on the loss comparing to some reference points. For disappointment-aversion (DA) utility function, the weight is more on the disappointment comparing to the certainty equivalent. Since agents are more aversion to loss, they will be more “risk-averse” than the standard smooth utility function. Thus to induce them holding equity, there must be a higher risk premium associated with the equities.

But this brings the question whether or not the “kink” will survive aggregation. Imagine two agents both with first-order risk aversion utility function. Then if there is a representative agent, in general the utility function of this representative agent will have two kinks. If there are very many individual agents, or in the extreme there are a continuum agents, the utility function of the representative agent will be quite “smooth”. Thus it may be the case that at the aggregate level, the preferences may not have enough risk aversion to account for the high risk premium and the low risk-free rate.

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2There are also other critiques on this front. For example, one may argue that results from experiments in a laboratory may not be applied to real life investment environment. Or there are so many different behavior biases that one needs some kinds of “discipline” to gauge them uniformly and use them systematically.
With this intuition we study a two-agent exchange economy in this paper. We assume that aggregate consumption growth follow a finite state Markovian process as in Mehra and Prescott (1985). We further assume that each agent has DA preference in the dynamic case using the framework presented in Epstein and Zin (1989). The security market is complete. Then we ask if there is a representative agent with a DA preference, what kind of parameters for the first-order risk aversion she will have?

The challenge to this problem is to do it in a rigorous and consistent way. We proceed in three steps. First we obtain efficient consumption allocation for each agent at each state. Then we decompose the economy to obtain equity (actually price-dividend ratios) and bond prices from individuals’ optimization problem. Lastly, we assume there is a representative agent with DA preference, and we estimate the preference parameters from the simulated returns.

The last two steps are quite straight forward and standard. The main problem is the first step. In the research on recursive utility functions, there has been a long history to study the efficient allocation of consumption (see for example, Lucas and Stokey (1979), Duffie, Geoffard and Skiadas (1992), Kan (1995), Dumas, Uppal and Wang (2002), etc.) While the method in Dumas and Uppal and Wang (2002) is quite natural in our setup, there is no easy way to extend it to DA case. So we use the method presented in Lucas and Stokey (1979) and Kan (1995). In this approach, one solves for efficient consumption allocation as well as future value function in each state of the next time period. The original maximization problem becomes a min-max problem: the future value functions must satisfy a minimization constraint. Since analytic solutions are not available, we use numerical techniques to get our results through out dynamic case.

We find that first-order risk aversion can not survive the aggregation process. The estimated aggregate “kink” parameter is much smaller than those of individuals. However, using DA preferences at the individual levels can still obtain desirable asset pricing implications, such as high risk premium and low risk-free rate. This implies that using behavior models is a plausible way to study asset returns. But the current ways of doing it in terms of representative agents is not correct.

Furthermore, we find that aggregation over the standard Epstein-Zin preferences also increase risk-premium. The intuition is similar to that in Constantinides and Duffie (1996) for incomplete markets. However, the risk-free rate will increase as well.

The paper goes as follows. In the next section we start by showing analytically that aggregating multiple-agent DA preferences will result in a representative agent with multiple kinks in the utility function. Then we will show the theoretical foundation of our numerical approach. Then we will present and analyze the results. Last we conclude.

## 2 Static Case

We start by studying a static model with aggregation to see the intuition. In each case, we will first present the familiar CRRA case, then DA case. To explore the properties of DA preferences and aggregation, we will in turn study the following problem:

1. Optimal portfolio choice for single agent
2. Equilibrium asset pricing with representative agent

3. Equilibrium asset pricing with two agents

2.1 Setup

Let there be two dates \( t = 0, 1 \). Agents are initially endowed with \( X_0 \) wealth and want to maximize next period (expected) utility (there is no consumption at \( t = 0 \)). There are two assets, stock and bond with return \( R^S \) and \( R^f \). Since agents only maximize expected wealth, one can without loss of generality fix the return of the riskless bond. Denote the proportion of wealth invested in the stocks to be \( \alpha \). So the wealth at time \( t = 1 \) is given by:

\[
X_1 = X_0(R^f + \alpha(R^S - R^f)).
\]  

(1)

Agents’ utility functions can be CRRA or DA. Let the CDF of the distribution of wealth \( R^S \) be \( F(R^S) \). Denote

\[
V(W) = \frac{W^{1-\gamma}}{1-\gamma}, \quad \gamma > 0.
\]  

(2)

For CRRA, it is just

\[
U_0 = EV(X_1)
\]  

(3)

While for DA, it is:

\[
U_0 = U(\mu) = V(\mu)
\]  

(4)

\[
= E(\chi_\mu V(X_1)),
\]  

(5)

where

\[
\chi_\mu = \begin{cases} 
A/K & \text{if } X_1 \leq \mu \\
1/K & \text{if } X_1 > \mu 
\end{cases}
\]  

(6)

\[
A > 1, \quad \text{constant}
\]  

(7)

\[
K = A\Pr(X_1 \leq \mu) + \Pr(X_1 > \mu).
\]  

(8)

2.2 Optimal Portfolio Choice

2.2.1 CRRA utility

First order condition is the following:

\[
E(X_1^{-\gamma}(R^S - R^f)) = 0.
\]  

(9)

Generally this has no analytic solution so that one has to solve it numerically. EXAMPLE
2.2.2 DA utility

First order condition is the following\(^3\):

\[
E(\chi_\mu X_1^{-\gamma}(R^S - R^f)) = 0. \tag{10}
\]

Again, one has to solve this numerically. More importantly, the certainty equivalent has to be satisfied. Namely when one search for optimal \(\alpha\) one has to make sure that the certainty equivalent is determined by the above DA utility function.

2.3 Equilibrium asset pricing with representative agent

As we said above, in the current setting one can only determine one variable namely equity premium. If one fix bond returns, then stock price can be determined. As a representative agent, it is just that \(\alpha = 1\). Namely

\[
X_1 = X_0 R^S \tag{11}
\]

2.3.1 CRRA utility

From the first order condition, one has

\[
E(R^S - R^f) = -\frac{1}{E((R^S)^{-\gamma})} \text{cov}(R^S, (R^S)^{-\gamma}). \tag{12}
\]

Or if we let \(P^S\) denote the stock price at \(t = 0\) and \(D\) as the (random) dividend of stocks at \(t = 1\). We have the price function:

\[
P^S = \frac{1}{R^f} E(D) + \frac{1}{R^f E(D^{-\gamma})} \text{cov}(D, D^{-\gamma}) \tag{13}
\]

2.3.2 DA utility

From the first order condition, one has

\[
E(R^S - R^f) = -\frac{1}{E(\chi_\mu (R^S)^{-\gamma})} \text{cov}(R^S, \chi_\mu (R^S)^{-\gamma}). \tag{14}
\]

Or if we let \(P^S\) denote the stock price at \(t = 0\) and \(D\) as the (random) dividend of stocks at \(t = 1\). We have the price function:

\[
P^S = \frac{1}{R^f} E(D) + \frac{1}{R^f E(\chi_\mu D^{-\gamma})} \text{cov}(D, \chi_\mu D^{-\gamma}) \tag{15}
\]

\(^3\)Note that since \(\mu\) is endogeneously determined, when taking derivatives, one has to take this into account. However, the results are such that there is no effect from this endogeneity. See Ang, Bekaert and Marshall (2000).
2.4 Equilibrium Asset pricing with two agents

We assume that the market is complete (although this can not be satisfied by only two assets in general). And we use $a, b$ superscript to denote two agents (to avoid the confusion between agent indicator and square). So a social planner maximizes (for some weight $\omega$):

$$\max_{X^a, X^b} (1 - \omega)U^a + \omega U^b$$

subject to

$$X^a + X^b = D.$$  \hspace{1cm} (16)

2.4.1 CRRA utility

Since it is an exchange economy and the utility functions are state separable, social planner ends up maximizing state by state (complete market assumption). So for some realization of $D$ at time $t = 1$, central planner wants to do the following:

$$\max_{X^a, X^b} (1 - \omega)\left(\frac{X^a}{1 - \gamma^a}\right)^{1 - \gamma^a} + \omega \left(\frac{X^b}{1 - \gamma^b}\right)^{1 - \gamma^b}$$

subject to

$$X^a + X^b = D.$$  \hspace{1cm} (17)

Denote the Lagrangian multiplier to be $\xi$. One then have the following:

$$(1 - \omega)(X^a)^{-\gamma^a} = \omega(X^b)^{-\gamma^b} = \xi$$  \hspace{1cm} (18)

To obtain $\xi$, one just solve for $X^a, X^b$ and substitute back to the budget constraint. Here one can immediately see that if $\gamma^a \neq \gamma^b$, in general one can not obtain analytic solution for the individual consumption (here wealth) distribution (Jiang Wang (1996) use $\gamma^a = 1, \gamma^b = 1/2$, so that one just solve a quadratic equation). But if they are the same, then the consumptions (here wealth) of the two agents must be proportional to each other.

To get $\omega$, note that from the above result, one can obtain the utility of the representative agent and thus the price kernal. One can substitute back to the individual budget contraint for, say agent $a$, to obtain $\omega$.

2.4.2 DA utility

Social planner maximizes the following:

$$\max_{X^a, X^b} (1 - \omega)\mathbb{E}_{X^a} \left(\frac{X^a}{1 - \gamma^a}\right)^{1 - \gamma^a} + \omega \mathbb{E}_{X^b} \left(\frac{X^b}{1 - \gamma^b}\right)^{1 - \gamma^b}$$

subject to the budget constraints for each realization of $D$. As shown in the appendix, when taking derivatives with respect to $\mathbb{E}(\chi_\mu g(\cdot))$, it is as if only to take derivative with respect to $g(\cdot)$. So for some realization of $D$ at time $t = 1$, central planner wants to:

$$V = \max_{X^a, X^b} (1 - \omega)\mathbb{E}_{X^a} \left(\frac{X^a}{1 - \gamma^a}\right)^{1 - \gamma^a} + \omega \mathbb{E}_{X^b} \left(\frac{X^b}{1 - \gamma^b}\right)^{1 - \gamma^b}$$  \hspace{1cm} (19)
subject to

\[ X^a + X^b = D. \]  

(23)

One immediately sees the difference. Even if \( \gamma^a = \gamma^b = \gamma \), so the consumptions (here wealth) of the two agents is proportional to each other, the ratio of the two consumption (wealth) may be different at different regions (\( A^a \neq A^b \)). The total value function thus will be a multi-kinked utility function (if \( \mu^a \neq \mu^b \) and \( A^a \neq A^b \), one can see that there may be 2 kinks for the total value function.)

Specifically, let the Lagrandian multiplier be \( \xi \). Then one has the first order condition:

\[(1 - \omega)\chi_{\mu^a} (X^a)^{-\gamma} = \omega \chi_{\mu^b} (X^b)^{-\gamma} = \xi \]  

(24)

So

\[ X^a = \left( \frac{\xi}{(1 - \omega)\chi_{\mu^a}} \right)^{\frac{1}{\gamma}} \]  

(25)

\[ X^b = \left( \frac{\xi}{\omega \chi_{\mu^b}} \right)^{\frac{1}{\gamma}} \]  

(26)

Substitute back into the budget constraint, we have:

\[ \xi = D^{-\gamma} \left( ((1 - \omega)\chi_{\mu^a})^{\frac{1}{\gamma}} + (\omega \chi_{\mu^b})^{\frac{1}{\gamma}} \right)^{\gamma} \]  

(27)

and

\[ V = \frac{1}{1 - \gamma} E \left( (1 - \omega)\chi_{\mu^a} (X^a)^{1-\gamma} + \omega \chi_{\mu^b} (X^b)^{1-\gamma} \right) \]  

(28)

\[ = \frac{1}{1 - \gamma} E \left\{ \left[ ((1 - \omega)\chi_{\mu^a})^{\frac{1}{\gamma}} + (\omega \chi_{\mu^b})^{\frac{1}{\gamma}} \right] \xi^{\frac{1 - \gamma}{\gamma}} \right\} \]  

(29)

\[ = \frac{1}{1 - \gamma} E \left\{ \left[ ((1 - \omega)\chi_{\mu^a})^{\frac{1}{\gamma}} + (\omega \chi_{\mu^b})^{\frac{1}{\gamma}} \right]^{\gamma} D^{1-\gamma} \right\} \]  

(30)

And

\[ (\mu^a)^{1-\gamma} = (1 - \omega)^{\frac{1 - \gamma}{\gamma}} E \left[ D^{1-\gamma} \left( \frac{\chi_{\mu^a}^{\frac{1}{\gamma}}}{\left( ((1 - \omega)\chi_{\mu^a})^{\frac{1}{\gamma}} + (\omega \chi_{\mu^b})^{\frac{1}{\gamma}} \right)^{1-\gamma}} \right) \right] \]  

(31)

\[ (\mu^b)^{1-\gamma} = \omega^{\frac{1 - \gamma}{\gamma}} E \left[ D^{1-\gamma} \left( \frac{\chi_{\mu^b}^{\frac{1}{\gamma}}}{\left( ((1 - \omega)\chi_{\mu^a})^{\frac{1}{\gamma}} + (\omega \chi_{\mu^b})^{\frac{1}{\gamma}} \right)^{1-\gamma}} \right) \right] \]  

(32)

These two equations give the value of certainty equivalent for two agents given \((\omega, A^a, A^b)\). To actually get the value \( \omega \) since it is not an exogenous variable, we can do the following. Note the discounted wealth of any agent can not be more than her initial wealth. The price
of a unit at each consumption will be the marginal utility of the representative agent in the above central planner’s problem, namely $\xi$. So from individual agent’s budget contraint, we have:

$$E(\xi X^a) \leq X^a_0, \quad (33)$$

$$E(\xi X^b) \leq X^b_0. \quad (34)$$

This will gives us the relationship between $\omega$ and initial wealth distribution.

There are some observations about the above results:

- Even if agents have the same initial wealth, different $A$ will render them to have different certainty equivalent.

- The representative agent’s utility function will have two kink points instead of one if the two certainty equivalents of the two agents are not the same.

A simple thought will make one believe that it can be easily generated to the multiple-agent case. And what this says is that the utility function of the representative agent will exhibit multi-kink points if the individual agents’ $A$ are different. Specifically, suppose we have a continuous number of agents, the utility function of representative agents will be smoothed out.

## 3 Formal Model

### 3.1 Setup of the model

There is a probability space $(\Omega, \mathcal{F}, \mathbb{Q})$ and the filtration $\mathcal{F} = \{\mathcal{F}_t : \mathcal{F}_t = \sigma(X_1, \ldots, X_t), \ t \in \{1, \ldots\}\}$ of sub-sigma fields generated by a time-homogeneous Markov chain with the state $S_t$ taking values in a finite set $S = \{1, 2, \ldots, S\}$ and the transition probability matrix is $\Pi$. Such setup of uncertainty includes, for example, that in Mehra and Prescott (1985).

There is one commodity in the economy. Consumption space is $l^1_\mathbb{R} \times \mathbb{R}^S \times \cdots$, while an agent’s consumption process is an element from the nonnegative cone $l^\infty_\mathbb{R}^+$. There are $N$ agents in the economy. Agents’ preferences are described by recursive utility, which includes an aggregator $W(\cdot, \cdot)$ and a certainty equivalent $u(\cdot)$.

**EZ Preference**

$$V(S_t) = W(c_t, u_t; S_t) = \frac{1}{1 - \gamma} \left( c_t^{1 - \rho} + \beta((1 - \gamma) u_t)^{1 - \rho} \right)^{\frac{1}{1 - \rho}}, \quad (35)$$

where

$$u_t(S_t) = E_t(V(S_{t+1}))$$

There are several points about the format I am using here:

- Here the parameters values are: $\gamma > 0, \gamma \neq 1, \rho > 0, \rho \neq 1, \beta < 1$. The usual setup in the asset pricing is $(1 - \gamma, \theta)$, with $\theta = (1 - \gamma)/(1 - \rho)$ and $\gamma > 0, \rho > 0$. 

Note that the definition of $V(\cdot)$ here is not the certainty equivalent. Instead it is already raised to a power of $\gamma$. To see that what this really means, let $\rho = \gamma$. Presumably we will go back to the time separable case and it is indeed so:

$$V_t = \frac{e_t^{1-\gamma}}{1-\gamma} + \beta E_t(V_{t+1})$$

$$= E_t \left( \sum_{s=t}^{\infty} \beta^{s-t} e_s^{1-\gamma} \right).$$

The recursive form makes it complicated. As we will see later on, using *felicity utility* we can make the above utility form quasi-separable, which greatly simplifies the problem.

**DA preferences** same $W$ but different $u$. Specifically, denote $\mu$ as the certainty equivalent of the expectation. Then $u(\cdot)$ is defined as follows:

$$u(\mu) = \frac{\mu^{1-\gamma}}{1-\gamma}$$

$$= \frac{1}{K} (EV' + (A-1)E(V'|V' < u(\mu))),$$

where $K = \text{Prob}(V' > u(\mu)) + A\text{Prob}(V' \leq u(\mu))$, $A \geq 1$. In other worlds, agents put more weights on the lower end of the certainty equivalent. They do not want to be disappointed.

### 3.1.1 Theoretical Consideration

We use the following approach to solve the aggregation of Disappointment-Aversion agents. First, we use a representative agent approach to get the optimal allocation of consumptions for each individual agent. Then we solve the security prices (price-dividend ratios) from individual agent’s maximization problem. We then simulate series of security prices (returns) according to the results. Last we assume that there exist a single representative agent and assume that she also has a Disappointment-aversion utility function. We estimated DA parameters from the security prices.

### 3.1.2 Efficient Allocation

The algorithms we use are from those in Lucas & Stockey (1984), Kan (1995)). This algorithm possesses some nice properties. First of all, it is robust to any forms of certainty equivalent which satisfy some basic assumptions. Second, it can be put into a stationary form.

Let us start by assume that the uncertainty follows an $S$–state markov regime switching process (as we will see, we will let the growth rate of aggregate endowment follow S-state Markov switching process). This is to ensure that the value function of the agent is well-behaved. It is also necessary for our numerical implementation since we have to discretize the state space anyway.

Given the current state of economy with the aggregate endowment to be $x_t$, a Pareto-optimal allocation is a consumption process $c$ such there is no other $u_i$ will be greater than...
\( V_i(c) \) and other agents will be no less better off. We define the support function of \( V \) as follows:

\[
v(x, \alpha) = \sup_u \sum_{n=1}^{N} \alpha^n u^n, \tag{40}\]

where \( \sum_n \alpha^n = 1 \). Since the consumption space we consider is compact, the supremum is actually achieved.

With this definition, we will have the following problem:

\[
v(x, \alpha) = \max_{c, u'} \sum_n \alpha^n W(c^n, \mu^n(u')) \tag{41}\]

subject to

\[
\sum_n c^n \leq x \tag{42}
\]

\[
v(x', \theta) - \sum_n \theta^n u^n \geq 0, \quad \forall x', \alpha'. \tag{43}\]

The second equation is necessary since we want the choices of \( u' \) to be feasible. And one can easily show the following:

**Lemma 1.** \((u^n)_{n=1,...,N} \) is feasible if and only if the above constraint is satisfied.

Proof: see Lucas and Stokey (1979) and Kan (1995) for the recursive utility functions in the certainty and uncertainty cases. Here it is similar process.

If we denote the solution to the last constraint be:

\[
v'(x') = \min_{\theta} v(x', \theta) - \sum_n \theta^n u^n \tag{44}\]

\[
\alpha' = \arg \min v(x', \theta) - \sum_n \theta^n u^n . \tag{45}\]

Then basically given current \((x, \alpha)\) we obtain from above the consumption \( c \), next period \( u \) and weights \( \alpha' \). In the following we will give an example, which we will concentrate later on.

**Epstein-Zin Preferences** Suppose we have two agents with EZ preferences. Furthermore, we will assume that they have the same \( 1 - \gamma \) while they may have different \( \beta, \rho \). So the social planner maximizes the following:

\[
v(x_0, \alpha_0; S_0) = \max_{c_1, \alpha_1} (1 - \alpha_0) W^1(c^1_0, EV^1_0) + \alpha_0 W^1(c^2_0, EV^2_1) \tag{46}\]

subject to

\[
c^1_0 + c^2_0 = x_0(S_0) \tag{47}
\]

\[
v(x_1, \theta; S_1) - ((1 - \theta) V^1_1(S_1) + \theta V^2_1(S_1)) \geq 0, \forall S_1 \tag{48}\]
Let $\alpha_1 \equiv \arg \max(...)$. It is then the weights for the next period. Furthermore, since they have the same $\gamma$, we may assume that the value function is of the form

$$v(x_0, \alpha_0; S_0) = \frac{x_0^{1-\gamma}}{1-\gamma} f(\alpha_0; S_0)$$  \hspace{1cm} (49)$$

so that we can use the growth rate $x'/x$ and consumption-aggregate consumption ration $c^1/x$ as the variable. The state $S_0$ is of course now the state of growth rate. If we use the say 3-state markov chain, the value function $v$ is then of dimension $3 \times \dim(\alpha)$. The argument is then $c^1/x$ plus $\theta$, 3 $V^1$ and 3 $V^2$. The algorithm for calculate the optimal consumption and value functions are then:

- Guess the form of $v(x, \alpha)$
- Iterate through $(V^1, V^2, c)$. For each $(V^1, V^2)$:
  - Check equation (48) is satisfied for all $\theta \in [0, 1]$. Pick $\theta$ that minimizes the LHS of equation (48) and denote it as $\alpha'$.
  - Choose $c_0^1 (c_0^2 = x_0 - c_0^1)$ to maximize the RHS of equation (46).
- Choose $(V^1, V^2)$ (and associated $c_0^1, \alpha'$) that maximizes the RHS of equation (46) and let the new value be $v(x_0, \alpha_0; S_0)$
- Iterate through $v(x_0, \alpha_0)$ until it converges.

**DA Preferences** For DA preference, there is nothing change except the expectation is taken to be the certainty equivalent. Now the complication arises here. Note the certainty equivalent for DA preferences is determined endogenously. Here one has to make sure that $(V^1, V^2)$ is chosen such that one has to satisfy the DA requirement. Namely in the second step above, one has to iterate again to make sure that certainty equivalent is satisfied.

### 3.1.3 The Price-Dividend Ratios

Here we decompose the economy. Namely we use the first order condition of one agent’s optimization problem to get the price-dividend ratio and price function for risk-free bond. Specifically, the first order condition for DA preferences are:

$$\frac{P^S}{D} = E \left\{ x_{\alpha_i} \beta_i \left( \frac{C^*_i}{C^*_i} \right)^{\frac{1-\rho_i}{1-\rho_i}} \left( \frac{W_i}{W_i - C_i} \right)^{\frac{1-\gamma_i}{1-\rho_i} - 1} \left( 1 + \frac{P^S}{D'} \right) \left( \frac{D'}{D} \right)^{1-\gamma_i} \right\}$$ \hspace{1cm} (50)$$

$$P^B = E \left\{ x_{\alpha_i} \beta_i \left( \frac{C^*_i}{C^*_i} \right)^{\frac{1-\rho_i}{1-\rho_i}} \left( \frac{W_i}{W_i - C_i} \right)^{\frac{1-\gamma_i}{1-\rho_i} - 1} \left( \frac{D'}{D} \right)^{1-\gamma_i} \right\}$$ \hspace{1cm} (51)$$

Since we know the consumption and wealth allocation from the first step we can solve for $P^S/D$ and $P^B$ as a function of state variable (in this case the weights and dividend growth rate).
3.1.4 Estimation of DA Parameter for Representative Agent

After getting the price-dividend ratio and bond price, we simulate a sequence of stock and bond returns. From the simulated returns we use GMM to actually estimate preference parameters such as $A$ and $\rho$ ($\gamma$) from the simulated series of stock and bond returns, assuming that we have a representative agent with DA preference in recursive form. The following are first order conditions for the representative agent with DA preference:

\[ 0 = E \left\{ \chi_{A} \left( \frac{C'}{C} \right)^{\frac{(\rho\gamma)}{1-\rho}} (R^S)^{\frac{1-\gamma}{1-\rho}} \left( R^S - R^B \right) \right\} \]  \hspace{1cm} (52)

\[ 1 = E \left\{ \chi_{A} \beta^{\frac{1-\gamma}{1-\rho}} \left( \frac{C'}{C} \right)^{\frac{(-\rho\gamma)}{1-\rho}} (R^S)^{\frac{1-\gamma}{1-\rho}} \right\} \]  \hspace{1cm} (53)

3.2 Numerical Calibration For CRRA Case

We consider two agent economy with 2 state Markovian switching process for the aggregate endowment. The endowment follows Mehra-Prescott (1985). Suppose two agents have CRRA utility and have the same risk aversion coefficient $\gamma$.

In this case, one can solve for the analytic solution for different weight $\lambda$. Thus we can check if the algorithm is correct or not before we start out to aggregate for the general case.

We assume that the weights are discretized into $0 : 0.1 : 0.9$. So basically the state variables are the product of two growth states and 9 weights. We need to find the consumption of agent 1 and the next period value function for the two agents and the weights at each future growth state.

As the above algorithm shows, we calculate $V$ for 18 states and substitute into the RHS and iterate. The criterion for convergence is the following: we get the old $V$ value and the new calculated $V$. The error term is the average change ratio.

For the price-dividend ratio, we solve for first order condition for individual agents’ optimization problem.

Then we simulate 100 series of consumption growth, price-dividend ratio, and bond prices with length 10,000. In the estimation stage we delete the first 1,000 data points to get rid of the effects from initial conditions.

**Result**

Table 1 is the results from the estimation of CRRA case. As one can see, with the increase in risk aversion, the return of stock and bond increases too. And the equity premium increases too.

The estimated $A$s are generally not one, but they are insignificant different from one except for the case when $\gamma = 10$.

**Robustness of the algorithm**

We want to make sure that the algorithms are robust to different initial condition. The nice thing about CRRA case is that we can obtain closed form solution at each stage of algorithm. So we can compare the results with the closed form solutions.

Table 2 shows the average percentage differences between calculated values and closed form solutions.
In the optimal consumption allocation cases, one can see that the errors are generally quite small except for the case when $\gamma = 2$. This is because $\gamma$ is very small so the value functions are quite flat. By iterating long enough and imposing smaller convergence criteria, we can get lower error. But for the purpose of comparison, we keep the same convergence criteria. For the case of aggregate value function, individual consumption allocation and weights changes, the errors are almost none. Note that we start with different initial conditions for different $\gamma$. So the algorithms are robust to initial conditions. The significant differences come from the individual value functions at the next period. However, the exact values of these two functions are not needed in the following price-dividend ratio and riskless bond price calculations. What we need is the fact that we can calculate the certainty equivalent to determine the cutoff point for disappointment aversion. In the two states model, this error will not introduce much difference.

As one can see for the price-dividend ratio and riskless bond prices, there are almost no errors from calculated value and closed form solution.

4 Results

4.1 $A = 1, \gamma = 5$ with Homogeneous Agents

As stated above, we choose $\gamma = 5$ (or relative risk aversion is 5) to get a value function with reasonable curvature. First we still keep $A = 1$ and aggregate over different $\rho$. Now to simplifying the calculation, we only use 5 weights: 0.1 : 0.2 : 0.9.

Table 3, Table 3B and Table 3C show the aggregation results.

For simplicity we start with two individual agents having the same preferences. However, one should note that even with same utility function, the aggregate preferences might not be the same form when we have recursive utility.

First let us look at the returns, when we hold the risk aversion coefficient to be 5 ($\gamma = 5$), and increase $\rho$ (namely decreasing elasticity of intertemporal rate of substitution(EIS)), risk-free rate and equity premium increases. In the mean time, mean stock return, and volatility of stocks and bond increase too.

That risk free rate increases with the decrease of EIS is consistent with previous results obtained from representative agent with EZ preferences, as seen from Campbell (1993) and Bansal and Yaron (2004) etc. However, the increase in equity premium is different. In Campbell (1993), he showed that under log normal distributed returns, equity premium will not be affected by the change of EIS, only risk aversion matters; while in Bansal and Yaron (2004), they showed that equity premium decreases with EIS decreases.

The intuition for why equity premium increases in our case is the following. In a complete market with time separable preferences, multi-agent will not affect prices comparing with representative agent case. Since the marginal rate of substitution of consumption is equalized across the agents. Individual agents’ consumption are no more volatile than the representative agent’ consumption. In the case of recursive agents, that is not the case. Individual agents’ consumption will be more volatile than the aggregate consumption. This can be seen from the weights in the representative agent’s utility function. In the time separable preferences case, the weights on each individual agent’s utility function will be a
constant when there is complete market. In the recursive utility case, the weights will be time-variant (See Dumas, Uppal and Wang (2002)).

This is similar to the case in Constantinides and Duffie (1996) for the aggregation of agents in incomplete markets. There shocks are permanent but they are not correlated with returns (so that agents can not trade away the risks). The shocks affect the marginal utility, however, thus affect equity premium. The weights in the presentative agent’s preference is just the inverse of individual marginal utility function.

Table 3 shows that estimated aggregate $A$ is close to one, while estimated $\rho$ increase monotonically with individual $\rho$. We will discuss more about $A$ in the DA case later. For now, it is comforting for us to know that aggregating two smooth utility function will not bring many bias toward kink-utility function.

For $\rho = 5 = \gamma$, we are back to power utility function. This can be seen from the $A$ and $\rho$ value we estimate from simulated data.

For $\rho$ close to $\gamma$, the results still holds. However, when $\rho$ is significant higher or lower than $\gamma$, the distortion begins to appear. As one can see for $\rho = 10$, the estimated $A$ are significantly lower than 1 and estimated aggregate $\rho$ deviates from its original value significantly.

What if we fix $A = 1$ in the estimation? In this case, we assume that the aggregate preference is still a recursive form, which as we pointed before, should not hold. But as we can see from table 3B, the estimated $\rho$ increases monotonically, even for the case of individual $\rho = 10$.

A more complete picture can be seen if we simultaneously estimate both $\rho$ and $\gamma$ for the representative agent. Table 3C shows the results. It is comforting to see that the estimated aggregate $\gamma$ is around the individual $\gamma = 5$, except for the case $\rho_i = 10$. This confirms our assumption initially, namely the representative agent has the form $C^{1-\gamma}/(1-\gamma)$. And our previous concentration on estimating aggregate $A$ and $\rho$ seems to be right. Again, estimated aggregate $\rho$ increases monotonically as well.

4.2 $\gamma = 5$, heterogenous $\rho$

Here we study the case when we fix $\gamma = 5$ for both agents. Yet, one agent has power utility with $\rho = 5$ and we change the $\rho$ for the other agent.

The results are in table 4. The results for this are mixed. First of all, the returns are not monotonic. The convergence of algorithm (not shown here) is also not quite satisfactory. So it is not straight forward to see the intuition on the estimated aggregate $A$ and $\rho$ here.

There are potentially several reasons for the results. First of all, from the previous results on aggregation of heterogenous agents with different risk aversion under CRRA utility, we know that there are no stationary distribution for the consumption allocation and wealth if agents have different risk aversion. One of the agents (the less risk averse one) will hold all the stocks over time while the other only hold risk-free asset. In our model, we assume that presentative agent have a preference with the form $C^{1-\gamma}/(1-\gamma)$ multiplying other terms. This does not mean we will have a stationary distribution when agents have different $\rho$. In the first step of our calculation, efficient consumption allocation, we try to keep error term as small as possible. But this does not guarantee that the resulting distribution is stationary.

The second reason may be the following. Even if we do have a stationary distribution with heterogenous agents, the resulting representative agent might have a preference with
the aggregate \( \rho \) to be a very nonlinear function of individual \( \rho \). Thus the resulting returns (and the estimated aggregate \( \rho \)) is not monotonic.

In any case, aggregation of heterogenous agent with different \( \rho \) remains a very interesting case. However, deeper discussion of this is beyond this paper. In the following, we will concentrate on the aggregation of homogenous agents case.

4.3 \( \gamma = 5, \rho = 6 \)

Table 5 and Table 5B are the main results of the paper. For two homogeneous agents with individual \( \gamma_i = 5 \) and \( \rho_i = 6 \), but \( A \neq 1 \), the tables show the estimated aggregate results for \( A \).

First let us look at the returns. When individual \( A \) increases, we see that risk-free rate decreases and equity premium increases, while equity returns, volatility of equity and bond decrease.

That the returns of risk-free bond decrease as individual \( A \) increases is not surprising. As we would expect from DA preferences, agents now do not want to hold stock because they are more “risk-averse” than case in the usual Epstein-Zin preferences. So the returns of bond decrease as individual agents become more and more “disappointment averse”. That equity premium increase as \( A \) increases is again not surprising. This is because when agents become more “disappointment averse”, equity is not a good investment comparing to bond. It must offer higher premium for agents to hold stock. The surprising part is the return of stock decreases as \( A \) increases. Regarded separately, one would expect stock return to increase when \( A \) becomes larger. However, here we do not know the exact forms of stock returns except for the results on equity premium.

Let us now turn to the estimation of aggregate \( A \). One can see that the value of aggregate \( A \) is much smaller than individual \( A \). This result should not surprise us. From discussion in static case, we know that for two agents with kinks in their utility function, the resulting aggregate utility function will have two kinks. If we want to use preference with only one kink to approximate this, the aggregate \( A \) will be smaller. But how to absorb the increasing in equity premium and decreasing in risk-free rate with less \( A \)? This has to be in increasing \( \rho \) or \( \gamma \)! We can see from table 6 that \( \rho \) has to increase enormously to compensate for this, as well as \( \gamma \) from table 6B.

So increase individual \( A \) will indeed increase equity premium and decrease risk-free rate. More generally, from discussion in the aggregation of Epstein-Zin preferences, one can introduce higher risk premium through two aspect: multi-agent with Epstein-Zin preference (increasing \( \rho \)) or with DA preferences (increasing \( A \)). Yet increasing \( \rho \) will increase risk-free rate, while increasing \( A \) will decrease risk-free rate.

Yet using a representative agent with large \( A \) is NOT a valid argument in promoting DA preferences (or other first-order risk aversion utility functions, such as loss-aversion utility). The first-order risk aversion itself will not survive the aggregation. Just as the parameter \( A \) shows here, when there are more and more agents in the economy, the resulting representative agent’s utility function will become “smoother” and “smoother”. One needs to impose a very large individual \( A \) for it to survive at the aggregate level.

To summarize our findings, using first order risk aversion at the individual level, one can indeed get higher equity premium and lower risk free rate. Yet using a representative agent
with a first order risk aversion utility is not a reasonable assumption.

4.4 $\gamma = 5, \rho = 5$

Here we take a quick look at the case in which individuals have $\gamma = \rho = 5$, but $A \neq 1$. Or in other words, individual agents have time-separable preferences, but the certainty equivalent is of the DA form. Table 6 shows the results.

For the returns, one can see that again, as individual $A$ increases, equity premium increases. Yet this increase in equity premium increasing is not from change in equity returns, which are virtually constant. Instead, it is from decreasing in risk-free rate. Furthermore, there is a sharp decreasing of equity premium when $A > 1$. We further calculate the case for $A = 1.01, 1.001$ (which are not shown here), the results are robust. The main reason comes from the drop in equity returns.

As such, the estimated aggregate $A$ and $\rho$ are quite dramatic. Again aggregate $A$ is much smaller than individual $A$, but the aggregate $\rho$ is quite different. It is constant for small individual $A$ but increases sharply as individual $A$ becomes larger.

The reason for this might be two folds. One is that calculation errors cause the problem. Because now the preferences are time separable, there might be a singular point in the algorithm that cause the problem. The other is that the singular point is a genuine property. As long as $A$ is different from one so that individual agents’ utility function becomes non-smooth, the aggregation will cause the drop. Further investigation for this issue is needed to differentiate these two possibilities.

5 Conclusion

We have studied an exchange economy with heterogenous agents, who have first order risk aversion utility functions. We find that that at the aggregate level, the degree of first-order risk aversion decreases significantly. In other words, if we want to apply a representative agent model to this economy, the agent’s utility function becomes much “smoother”. This is in sharp contrast to the popular models trying to get asset pricing and portfolio implications by using first-order risk aversion utility. However, we show that the general results obtained by those models still holds, but it is applied at individual level instead of aggregate level.
References


This table shows the results for the CRRA case. We assume two agents with the utility function:

$$V = \frac{1}{1 - \gamma} \left( c_t^{1-\rho} + \beta((1 - \gamma)u_t)^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\rho}},$$

where

$$u_t(S_t) = E_t(\chi_A V(S_{t+1})).$$

and $\chi_A$ is the indicator function\(^4\). This is the case where $\gamma = \rho$ and $A = 1$. We simulated 100 series with 10000 year. Then we delete the first 1000 to avoid the effects from initial conditions.

The standard deviations in the brackets are the \textit{per series} standard deviations, namely they are the standard deviations cross series divided by $\sqrt{100} = 10$.

\(^4\)Compare this with the form used by say Campbell (1993):

$$W(c, z) = \left\{ (1 - \delta)c^{\frac{1-\gamma}{1-\theta}} + \delta z^{\frac{1-\gamma}{1-\theta}} \right\}^{\frac{1}{1-\theta}}$$

where $z$ is the certainty equivalent value for consumption and $\theta = \frac{1-\gamma}{1-1/\psi}$.

The correspondence between the one we use above and this are the following: $\gamma \leftrightarrow \gamma$, $\rho \leftrightarrow 1/\psi$, $\gamma = \rho \leftrightarrow \theta = 1$. 

---

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$A$ (std)</th>
<th>$E(R^*)$ (std)</th>
<th>$\sigma(R^*)$ (std)</th>
<th>$E(R^f)$ (std)</th>
<th>$\sigma(R^f)$ (std)</th>
<th>$E(R^f) - R^f$</th>
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<td>0.0423 (0.0000)</td>
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Table 2
CRRA Case: Robustness Check

<table>
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<tr>
<th>(\gamma)</th>
<th>(V) Consumption</th>
<th>Weights</th>
<th>Next Period (v_1)</th>
<th>Next Period (v_2)</th>
<th>(P/D)</th>
<th>(P^J)</th>
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</tr>
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<td>0.0000</td>
<td>0.0506</td>
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<td>0.0000</td>
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<tr>
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<td>0.0000</td>
<td>0.0426</td>
<td>0.0433</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

This table shows the errors for the CRRA case in the stage of optimal allocation. We assume two agents with the utility function:

\[ V = \frac{1}{1 - \gamma} \left( c_t^{1-\rho} + \beta ((1 - \gamma) u_t)^{1-\gamma} \right)^{1-\gamma}, \]

where

\[ u_t(S_t) = E_t(\chi_A V(S_{t+1})) \]

and \(\chi_A\) is the indicator function\(^5\). This is the case where \(\gamma = \rho\) and \(A = 1\). We simulated 100 series with 10000 year. Then we delete the first 1000 to avoid the effects from initial conditions.

We start from an arbitrary initial conditions and solve the aggregate value function, individual agent’s consumption allocation, next period weights at each states, next period individual agents at two states. The above error terms are the average percentage differences from the closed form solution.

The last two columns show the average percentage differences between calculated price-dividend ratio, riskless bond prices and their closed form solutions.

\(^5\)Compare this with the form used by say Campbell (1993):

\[ W(c, z) = \left\{ (1 - \delta) c^{1-\gamma} + \delta z^{1-\gamma} \right\}^{\frac{1}{1-\gamma}} \]

where \(z\) is the certainty equivalent value for consumption and \(\theta = \frac{1-\gamma}{1-1/\psi}\).

The correspondence between the one we use above and this are the following: \(\gamma \leftrightarrow \gamma\), \(\rho \leftrightarrow 1/\psi\), \(\gamma = \rho \leftrightarrow \theta = 1\).
Table 3
EZ Case: Results Estimating both $\rho$ and $A$

<table>
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<tr>
<th>$\rho_1(\rho_1 = \rho_2)$</th>
<th>$A$</th>
<th>$\hat{\rho}$</th>
<th>$E(R^*)$</th>
<th>$\sigma(R^*)$</th>
<th>$E(R^f)$</th>
<th>$\sigma(R^f)$</th>
<th>$E(R^*) - R^f$</th>
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This table shows the results for the Epstein-Zin case. We assume two agents with the utility function:

$$V = \frac{1}{1 - \gamma} \left(c_t^{1-\rho} + \beta((1 - \gamma)u_t)^{\frac{1-\gamma}{1-\rho}}\right)^{\frac{1-\gamma}{1-\rho}},$$

where

$$u_t(S_t) = E_t(\chi_A V(S_{t+1}))$$

and $\chi_A$ is the indicator function\(^6\). This is the case where $\gamma = \rho$ and $A = 1$. We simulated 1 series with 500000 year. Then we delete the first 1000 to avoid the effects from initial conditions.

standard deviations, namely they are the standard deviations cross series divided by $\sqrt{100} = 10$.

In this table, we choose $\gamma = 5$ (or the relative risk aversion is 5). We assume two agents having the same $\rho$ (the elasticities of intertemporal substitutions are $1/\rho$). For this table we estimate $\rho$ and $A$ simultaneously.

---

\(^6\)Compare this with the form used by say Campbell (1993):

$$W(c, z) = \left\{(1 - \delta)c^{\frac{1-\gamma}{\varphi}} + \delta z^{\frac{1-\gamma}{\varphi}}\right\}^{\frac{1}{1-\varphi}},$$

where $z$ is the certainty equivalent value for consumption and $\theta = \frac{1-\gamma}{1-1/\psi}$. The correspondence between the one we use above and this are the following: $\gamma \leftrightarrow 1/\psi$, $\varphi \leftrightarrow 1/\psi$, $\gamma = \rho \leftrightarrow \theta = 1$. 

21
Table 3B

EZ Case: Results Estimating $\rho$ While Fixing $A=1$

<table>
<thead>
<tr>
<th>$\rho_1 (\rho_1 = \rho_2)$</th>
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<th>$\sigma(R^n)$</th>
<th>$E(R^f)$</th>
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<td>1.1407</td>
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<td>0.0123</td>
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<td>0.0935</td>
<td>1.1708</td>
<td>0.0521</td>
<td>0.0142</td>
</tr>
<tr>
<td>9.0</td>
<td>1.0000</td>
<td>11.8694</td>
<td>1.2361</td>
<td>0.1153</td>
<td>1.2183</td>
<td>0.0699</td>
<td>0.0178</td>
</tr>
<tr>
<td>10.0</td>
<td>1.0000</td>
<td>16.2351</td>
<td>1.3282</td>
<td>0.1635</td>
<td>1.3138</td>
<td>0.1134</td>
<td>0.0143</td>
</tr>
</tbody>
</table>

This table shows the results for the Epstein-Zin case. We assume two agents with the utility function:

$$V = \frac{1}{1 - \gamma} \left( c_t^{1-\rho} + \beta ((1 - \gamma)u_t)^{\frac{1-\gamma}{1-\rho}} \right)^{\frac{1-\gamma}{1-\rho}},$$

where

$$u_t(S_t) = E_t(\chi_A V(S_{t+1}))$$

and $\chi_A$ is the indicator function$^7$. This is the case where $\gamma = \rho$ and $A = 1$. We simulated 1 series with 500000 year. Then we delete the first 1000 to avoid the effects from initial conditions.

In this table, we choose $\gamma = 5$ (or the relative risk aversion is 5). We assume two agents having the same $\rho$ (the elasticities of intertemporal substitutions are $1/\rho$). For this table we estimate $\rho$ while fixing $A = 1$.

$^7$Compare this with the form used by say Campbell (1993):

$$W(c, z) = \left\{ (1 - \delta)c^{\frac{1-\gamma}{1-\theta}} + \delta z^{\frac{1-\gamma}{1-\theta}} \right\}^{\frac{1}{1-\psi}},$$

where $z$ is the certainty equivalent value for consumption and $\theta = \frac{1-\gamma}{1-1/\psi}$.

The correspondence between the one we use above and this are the following: $\gamma \leftrightarrow \gamma$, $\rho \leftrightarrow 1/\psi$, $\gamma = \rho \leftrightarrow \theta = 1$. 

22
Table 3C
EZ Case: Results Estimating $\gamma$ and $\rho$ While Fixing $A=1$

<table>
<thead>
<tr>
<th>$\rho_1 (\rho_1 = \rho_2)$</th>
<th>$\hat{\gamma}$</th>
<th>$\hat{\rho}$</th>
<th>$E(R^\gamma)$</th>
<th>$\sigma(R^\gamma)$</th>
<th>$E(R^\rho)$</th>
<th>$\sigma(R^\rho)$</th>
<th>$E(R^\gamma) - R^\rho$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.0337</td>
<td>1.0256</td>
<td>0.0025</td>
<td>0.0051</td>
</tr>
<tr>
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<td>0.0421</td>
<td>1.0481</td>
<td>0.0100</td>
<td>0.0065</td>
</tr>
<tr>
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<td>3.9283</td>
<td>1.0884</td>
<td>0.0548</td>
<td>1.0799</td>
<td>0.0207</td>
<td>0.0085</td>
</tr>
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<td>1.1072</td>
<td>0.0621</td>
<td>1.0976</td>
<td>0.0267</td>
<td>0.0096</td>
</tr>
<tr>
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<td>0.0804</td>
<td>1.1407</td>
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<td>1.1850</td>
<td>0.0935</td>
<td>1.1708</td>
<td>0.0521</td>
<td>0.0142</td>
</tr>
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<td>5.0113</td>
<td>11.8732</td>
<td>1.2361</td>
<td>0.1153</td>
<td>1.2183</td>
<td>0.0699</td>
<td>0.0178</td>
</tr>
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<td>0.1635</td>
<td>1.3138</td>
<td>0.1134</td>
<td>0.0143</td>
</tr>
</tbody>
</table>

This table shows the results for the Epstein-Zin case. We assume two agents with the utility function:

$$V = \frac{1}{1-\gamma} \left( c_t^{1-\rho} + \beta((1-\gamma)u_t)^{\frac{1-\gamma}{\gamma}} \right)^{\frac{1-\gamma}{\gamma}},$$

where

$$u_t(S_t) = E_t(\chi_A V(S_{t+1}))$$

and $\chi_A$ is the indicator function$^8$. This is the case where $\gamma = \rho$ and $A = 1$. We simulated 1 series with 500000 year. Then we delete the first 1000 to avoid the effects from initial conditions.

In this table, we choose $\gamma = 5$ (or the relative risk aversion is 5). We assume two agents having the same $\rho$ (the elasticities of intertemporal substitutions are $1/\rho$). For this table we estimate $\rho$ while fixing $A = 1$.

---

$^8$Compare this with the form used by say Campbell (1993):

$$W(c, z) = \left\{ (1-\delta)c^{\frac{1-\gamma}{\gamma}} + \delta z^{\frac{1-\gamma}{\gamma}} \right\}^{\frac{1}{1-\gamma}}$$

where $z$ is the certainty equivalent value for consumption and $\theta = \frac{1-\gamma}{1-1/\psi}$.

The correspondence between the one we use above and this are the following: $\gamma \leftrightarrow \gamma$, $\rho \leftrightarrow 1/\psi$, $\gamma = \rho \leftrightarrow \theta = 1$. 

23
Table 4
EZ Case, Heterogeneous Agents: Results Estimating both $\rho$ and $A$

<table>
<thead>
<tr>
<th>$\rho_2(\rho_1 = 5)$</th>
<th>$A$</th>
<th>$\hat{\rho}$</th>
<th>$E(R^*)$</th>
<th>$\sigma(R^*)$</th>
<th>$E(R^\dagger)$</th>
<th>$\sigma(R^\dagger)$</th>
<th>$E(R^*) - R^\dagger$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.9268</td>
<td>3.5164</td>
<td>1.1071</td>
<td>0.0592</td>
<td>1.1589</td>
<td>0.0379</td>
<td>-0.0519</td>
</tr>
<tr>
<td>2.0</td>
<td>0.9794</td>
<td>2.2567</td>
<td>1.0663</td>
<td>0.0400</td>
<td>1.0641</td>
<td>0.0261</td>
<td>0.0023</td>
</tr>
<tr>
<td>3.0</td>
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<td>2.5211</td>
<td>1.0657</td>
<td>0.0467</td>
<td>1.0878</td>
<td>0.0246</td>
<td>-0.0221</td>
</tr>
<tr>
<td>4.0</td>
<td>1.1058</td>
<td>2.3146</td>
<td>1.0651</td>
<td>0.0294</td>
<td>1.0951</td>
<td>0.0199</td>
<td>-0.0300</td>
</tr>
<tr>
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<td>1.0002</td>
<td>5.0066</td>
<td>1.1072</td>
<td>0.0621</td>
<td>1.0976</td>
<td>0.0267</td>
<td>0.0096</td>
</tr>
<tr>
<td>6.0</td>
<td>0.9637</td>
<td>2.3562</td>
<td>1.0735</td>
<td>0.0584</td>
<td>1.0973</td>
<td>0.0288</td>
<td>-0.0238</td>
</tr>
<tr>
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<td>2.3062</td>
<td>1.0709</td>
<td>0.0609</td>
<td>1.0970</td>
<td>0.0315</td>
<td>-0.0261</td>
</tr>
<tr>
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<td>0.9692</td>
<td>2.3043</td>
<td>1.0708</td>
<td>0.0614</td>
<td>1.0970</td>
<td>0.0310</td>
<td>-0.0262</td>
</tr>
<tr>
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<td>0.9539</td>
<td>2.4594</td>
<td>1.0792</td>
<td>0.0567</td>
<td>1.0970</td>
<td>0.0309</td>
<td>-0.0178</td>
</tr>
<tr>
<td>10.0</td>
<td>0.9619</td>
<td>2.3713</td>
<td>1.0742</td>
<td>0.0566</td>
<td>1.0970</td>
<td>0.0310</td>
<td>-0.0228</td>
</tr>
</tbody>
</table>

This table shows the results for the Epstein-Zin case. We assume two agents with the utility function:

$$V = \frac{1}{1 - \gamma} \left( c_t^{1-\rho} + \beta((1 - \gamma)u_t)^{1-\gamma} \right)^{\frac{1-\gamma}{1-\rho}},$$

where

$$u_t(S_t) = E_t(\chi_A V(S_{t+1}))$$

and $\chi_A$ is the indicator function\(^9\). This is the case where $\gamma = \rho$ and $A = 1$. We simulated 1 series with 500000 year. Then we delete the first 1000 to avoid the effects from initial conditions.

standard deviations, namely they are the standard deviations cross series divided by $\sqrt{100} = 10$.

In this table, we choose $\gamma = 5$ (or the relative risk aversion is 5). We assume one agent having CRRA utility while the other has EZ utility with changing $\rho$ (the elasticities of intertemporal substitutions are $1/\rho$). For this table we estimate $\rho$ and $A$ simultaneously.

\(^9\)Compare this with the form used by say Campbell (1993):

$$W(c, z) = \left\{ (1 - \delta)c^{\frac{1-\gamma}{\theta}} + \delta z^{\frac{1-\gamma}{\theta}} \right\}^{\frac{1}{1-\theta}}$$

where $z$ is the certainty equivalent value for consumption and $\theta = \frac{1-\gamma}{1-1/\psi}$.

The correspondence between the one we use above and this are the following: $\gamma \leftrightarrow \gamma$, $\rho \leftrightarrow 1/\psi$, $\gamma = \rho \leftrightarrow \theta = 1$.  

24
Table 4B
EZ Case, Heterogenous Agents: Results Estimating $\rho$ While Fixing $A=1$

<table>
<thead>
<tr>
<th>$\rho_1$ ($\rho_1 = 5$)</th>
<th>$A$ (fixed)</th>
<th>$\rho$</th>
<th>$E(R^s)$</th>
<th>$\sigma(R^s)$</th>
<th>$E(R^f)$</th>
<th>$\sigma(R^f)$</th>
<th>$E(R^s) - R^f$</th>
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<td>1.0711</td>
<td>0.0592</td>
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<td>0.0379</td>
<td>-0.0519</td>
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<td>0.0400</td>
<td>1.0641</td>
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<td>0.0023</td>
</tr>
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<td>1.0878</td>
<td>0.0246</td>
<td>-0.0221</td>
</tr>
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</tr>
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<td>1.0976</td>
<td>0.0267</td>
<td>0.0096</td>
</tr>
<tr>
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<td>1.0000</td>
<td>3.0114</td>
<td>1.0735</td>
<td>0.0584</td>
<td>1.0973</td>
<td>0.0288</td>
<td>-0.0238</td>
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<td>1.0970</td>
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<td>-0.0261</td>
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<tr>
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<td>1.0708</td>
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<td>1.0970</td>
<td>0.0310</td>
<td>-0.0262</td>
</tr>
<tr>
<td>9.0</td>
<td>1.0000</td>
<td>3.3708</td>
<td>1.0792</td>
<td>0.0567</td>
<td>1.0970</td>
<td>0.0309</td>
<td>-0.0178</td>
</tr>
<tr>
<td>10.0</td>
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<td>3.0694</td>
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<td>1.0970</td>
<td>0.0310</td>
<td>-0.0228</td>
</tr>
</tbody>
</table>

This table shows the results for the Epstein-Zin case. We assume two agents with the utility function:

$$V = \frac{1}{1 - \gamma} \left( c_t^{1-\rho} + \beta((1 - \gamma)u_t)^{\frac{1-\gamma}{1-\rho}} \right)^{\frac{1-\gamma}{1-\rho}},$$

where

$$u_t(S_t) = E_t(\chi_A V(S_{t+1}))$$

and $\chi_A$ is the indicator function\(^{10}\). This is the case where $\gamma = \rho$ and $A = 1$. We simulated 1 series with 500000 year. Then we delete the first 1000 to avoid the effects from initial conditions.

In this table, we choose $\gamma = 5$ (or the relative risk aversion is 5). We assume one agent having CRRA utility, while the other having EZ with changing $\rho$ (the elasticities of intertemporal substitutions are $1/\rho$). For this table we estimate $\rho$ while fixing $A = 1$.

\(^{10}\)Compare this with the form used by say Campbell (1993):

$$W(c, z) = \left\{ (1 - \delta)c^{\frac{1-\gamma}{\theta}} + \delta z^{\frac{1-\gamma}{\theta}} \right\}^{\frac{1-\gamma}{\theta}}$$

where $z$ is the certainty equivalent value for consumption and $\theta = \frac{1-\gamma}{1-1/\psi}$.

The correspondence between the one we use above and this are the following: $\gamma \leftrightarrow \gamma$, $\rho \leftrightarrow 1/\psi$, $\gamma = \rho \leftrightarrow \theta = 1$. 25
Table 5
Estimating Aggregation of DA Preferences with Individual
$\gamma = 5, \rho = 6$: Aggregate A and $\rho$

<table>
<thead>
<tr>
<th>$A_1 = A_2$</th>
<th>$A$</th>
<th>$\hat{\rho}$</th>
<th>$E(R^*)$</th>
<th>$\sigma(R^*)$</th>
<th>$E(R^1)$</th>
<th>$\sigma(R^1)$</th>
<th>$E(R^2) - R^1$</th>
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</thead>
<tbody>
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<td>1.0</td>
<td>1.0002</td>
<td>5.0066</td>
<td>1.1072</td>
<td>0.0621</td>
<td>1.0976</td>
<td>0.0267</td>
<td>0.0096</td>
</tr>
<tr>
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<td>1.1061</td>
<td>0.0327</td>
<td>0.0135</td>
</tr>
<tr>
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<td>15.9000</td>
<td>1.1119</td>
<td>0.0690</td>
<td>1.0960</td>
<td>0.0319</td>
<td>0.0159</td>
</tr>
<tr>
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<td>1.0870</td>
<td>0.0311</td>
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<tr>
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<td>0.0674</td>
<td>1.0788</td>
<td>0.0303</td>
<td>0.0198</td>
</tr>
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<td>1.0648</td>
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<td>0.0227</td>
</tr>
<tr>
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<td>19.9000</td>
<td>1.0826</td>
<td>0.0651</td>
<td>1.0586</td>
<td>0.0280</td>
<td>0.0239</td>
</tr>
<tr>
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<td>0.0273</td>
<td>0.0253</td>
</tr>
<tr>
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<td>0.0637</td>
<td>1.0478</td>
<td>0.0266</td>
<td>0.0261</td>
</tr>
</tbody>
</table>

This table shows the results for the case in which $\gamma_i = 5, \rho_i = 6$. We assume two agents with the utility function:

$$V = \frac{1}{1 - \gamma} \left( c_t^{1-\rho} + \beta((1 - \gamma)u_t)^{\frac{1-\gamma}{1-\rho}} \right)^{\frac{1}{1-\rho}},$$

where

$$u_t(S_t) = E_t(\chi_A V(S_{t+1}))$$

and $\chi_A$ is the indicator function$^{11}$. This is the case where $\gamma = \rho$ and $A = 1$. We simulated 100 series with 10000 year. Then we delete the first 1000 to avoid the effects from initial conditions.

---

$^{11}$Compare this with the form used by say Campbell (1993):

$$W(c, z) = \left\{ (1 - \delta)c^{\frac{1-\gamma}{1-\rho}} + \delta z^{\frac{1-\gamma}{1-\rho}} \right\}^{\frac{1}{1-\rho}}$$

where $z$ is the certainty equivalent value for consumption and $\theta = \frac{1-\gamma}{1-1/\psi}$.

The correspondence between the one we use above and this are the following: $\gamma \Leftrightarrow \gamma$, $\rho \Leftrightarrow 1 - 1/\psi$, $\gamma = \rho \Leftrightarrow \theta = 1$.
Estimating Aggregation of DA Preferences with Individual
\( \gamma = 5, \rho = 5: \) Aggregate A and \( \gamma \)

<table>
<thead>
<tr>
<th>( A_1 = A_2 )</th>
<th>( A )</th>
<th>( \hat{\gamma} )</th>
<th>( E(R^a) )</th>
<th>( \sigma(R^a) )</th>
<th>( E(R^l) )</th>
<th>( \sigma(R^l) )</th>
<th>( E(R^a) - R^l )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
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<td>5.0066</td>
<td>1.1072</td>
<td>0.0621</td>
<td>1.0976</td>
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<td>0.0096</td>
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<td>0.0135</td>
</tr>
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<td>0.0690</td>
<td>1.0960</td>
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<td>0.0159</td>
</tr>
<tr>
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<td>1.0648</td>
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</tr>
<tr>
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<td>1.0826</td>
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</tr>
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<td>0.0637</td>
<td>1.0478</td>
<td>0.0266</td>
<td>0.0261</td>
</tr>
</tbody>
</table>

This table shows the results for the case in which \( \gamma_i = 5, \rho_i = 6 \). We assume two agents with the utility function:

\[
V = \frac{1}{1 - \gamma} \left( c_t^{1-\rho} + \beta((1 - \gamma)u_t) \right) ^{\frac{1-\gamma}{1-\rho}},
\]

where

\[
u_t(S_t) = E_t(\chi_A V(S_{t+1}))
\]

and \( \chi_A \) is the indicator function\(^\text{12}\). This is the case where \( \gamma = \rho \) and \( A = 1 \). We simulated 100 series with 10000 year. Then we delete the first 1000 to avoid the effects from initial conditions.

\(^{12}\)Compare this with the form used by say Campbell (1993):

\[
W(c, z) = \left\{ (1 - \delta)c^{\frac{1-\gamma}{1-\psi}} + \delta z^{\frac{1-\gamma}{1-\psi}} \right\} ^{\frac{1}{1-\psi}}
\]

where \( z \) is the certainty equivalent value for consumption and \( \theta = \frac{1-\gamma}{1-1/\psi} \).

The correspondence between the one we use above and this are the following: \( \gamma \Leftrightarrow \gamma, \ \rho \Leftrightarrow 1 - 1/\psi, \ \gamma = \rho \Leftrightarrow \theta = 1.\)

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Estimating Aggregation of DA Preferences with Individual
\( \gamma = 5, \rho = 5: \text{Aggregate } A \text{ and } \rho \)

<table>
<thead>
<tr>
<th>( A_1 = A_2 )</th>
<th>( A )</th>
<th>( \rho )</th>
<th>( E(R^s) )</th>
<th>( \sigma(R^s) )</th>
<th>( E(R^f) )</th>
<th>( \sigma(R^f) )</th>
<th>( E(R^s) - R^f )</th>
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<td>0.0575</td>
<td>1.0422</td>
<td>0.0218</td>
<td>0.0238</td>
</tr>
</tbody>
</table>

This table shows the results for the case in which \( \gamma_i = \rho_i = 5 \). We assume two agents with the utility function:

\[
V = \frac{1}{1 - \gamma} \left( c_t^{1-\rho} + \beta((1 - \gamma)u_t) \right)^{\frac{1-\gamma}{1-\rho}}
\]

where

\[
u_t(S_t) = E_t(\chi_A V(S_{t+1}))
\]

and \( \chi_A \) is the indicator function$^{13}$ This is the case where \( \gamma = \rho \) and \( A = 1 \). We simulated 100 series with 10000 year. Then we delete the first 1000 to avoid the effects from initial conditions.

$^{13}$Compare this with the form used by say Campbell (1993):

\[
W(c, z) = \left\{ (1 - \delta) c^{\frac{1-\gamma}{1-\theta}} + \delta z^{\frac{1-\gamma}{1-\theta}} \right\}^{\frac{1-\theta}{1-\gamma}}
\]

where \( z \) is the certainty equivalent value for consumption and \( \theta = \frac{1-\gamma}{1-1/\psi} \).

The correspondence between the one we use above and this are the following: \( \gamma \leftrightarrow \gamma, \rho \leftrightarrow 1/\psi, \gamma = \rho \leftrightarrow \theta = 1 \).

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