PRODUCTION, GROWTH AND BUSINESS CYCLES
II. New Directions

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This paper outlines new directions for investigations of real business cycle models: consideration of stochastic growth of exogenous and endogenous forms, analysis of suboptimal outcomes arising due to externalities or distorting taxes, and implications of labor market heterogeneity.

1. Introduction and summary

The frontiers for the real business cycle research program involve altering aspects of the preferences, technology, and endowments of the basic neoclassical model of capital accumulation that we explored in the first essay in this volume [King, Plosser and Rebelo (1988)]. Indeed, the earliest real business cycle models – those of Kydland and Prescott (1982) and Long and Plosser (1983) – were much richer than the basic neoclassical model. Kydland and Prescott (1982) incorporated time-to-build investment technology, inventories as a factor of production, temporal dependence in labor supply decisions via non-time-separable utility, and signal extraction problems involving permanent and temporary components of technological change. Long and Plosser (1983) developed an economy with multiple consumption and investment goods to explore sectoral interactions. Elsewhere in this volume, the range of perturbations on preferences, technologies and endowments includes inventories as a factor of production [Christiano (1988)], endogenous capacity util-

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This essay develops four main ideas that we believe will play a central role in the real business cycle research program over the next few years. Since these ideas are new, we are, in most cases, providing hints about the path of future developments rather than detailed answers.

In the introductory essay we found that exogenous technical progress had to be relatively persistent in character for the outcomes of the basic neoclassical model to resemble macroeconomic time series. Thus, the first two ideas involve altering the specification of technological progress. First, we investigate the implications for the business cycle – including transient movements in labor input – of stochastic technology shifts that are exogenous but permanent. Second, we study the implications for economic time series when technology is permitted to be endogenous.

Real business cycle analysis is frequently criticized for focusing primarily on technology shocks, on competitive equilibria that are optimal, and on models that abstain from heterogeneity of agents. The third idea we pursue is to extend the methods of real business cycle analysis to include the study of suboptimal dynamic equilibria. As a sample application, we consider whether policy rules – which make variations in government spending or taxes responsive to economic conditions – substantially alter the dynamic characteristics of the basic neoclassical model. The fourth modification we explore incorporates agent heterogeneity into real business cycle models to determine whether such heterogeneity alters the characteristics of macroeconomic time series.

The detailed organization of the paper is as follows. We begin by examining exogenous stochastic growth for two reasons. First, stimulated by Nelson and Plosser (1982), there is an accumulation of recent work that provides evidence that macroeconomic time series contain important stochastic trends. Second, the extent of serial correlation required to generate empirically reasonable business cycles raises the possibility that stochastic growth may be at the heart of observed economic fluctuations. Section 2 discusses the effects of permanent (random walk) shifts in productivity in the basic neoclassical model developed in the companion essay. The key result is that a permanent shift in productivity raises the desired path of capital accumulation, initiating transitional responses in labor supply that are analogous to those considered in the earlier paper. The presence of non-stationary stochastic processes for technology introduces some statistical problems since the data must be transformed to achieve stationarity if an analysis of moments is to be meaningful. Pursuing two appropriate data transformations for post-war U.S. time series, we conclude that a common stochastic trend plays a potentially major role in neoclassical explanations of fluctuations.
Most prior real business cycle research treats the determination of long-run growth as exogenous, while constraining preferences and production possibilities to produce steady state growth. In section 3, we consider extensions of the basic neoclassical model that incorporate endogenous steady state growth. This discussion draws heavily on work by King and Rebelo (1986, 1987), which in turn builds on the developments in growth theory due to Romer (1986) and Lucas (1988). King and Rebelo (1986) describe production possibilities for new human capital (endogenous labor augmenting technical progress) as depending on allocations of physical and human capital. To generate steady state growth, it is necessary that the human and physical capital production functions be constant returns to scale. Restricting overall possibilities so that endogenous growth is feasible has important implications for the economic system’s response to various disturbances. For example, temporary shocks can have permanent effects on the level of economic activity [King and Rebelo (1986)], and permanent changes in policies can alter the growth rate of economic activity [Rebelo (1987a)].

In section 4, we develop extensions of our analytical methods to handle suboptimal dynamic competitive equilibria so that distorting taxes, public goods and productive externalities can be investigated. The key point is that suboptimal equilibria can generally be represented as the outcome of a difference equation system of the same form as that encountered in environments with optimal outcomes. But the quantitative outcomes of environments with taxes, public goods and productive externalities can be very different. We discuss some concrete examples that illustrate this principle. Second, drawing on recent work by Rebelo (1987b), section 5 discusses how our steady state models can be extended to incorporate heterogeneity in a tractable manner. Section 6 provides some concluding remarks and directions for future research.

2. Stochastic growth

Our prior analysis takes the growth path as exogenous and deterministic, focusing on the response of the basic neoclassical model to temporary shocks. In this section and the next, we consider models in which the growth path is stochastic. These models can be studied with minor extensions of the methods that are developed in King, Plosser and Rebelo (1988) because models with steady states under certainty can readily be modified to accommodate stochastic growth [King and Rebelo (1986, 1987)].

\[1\] This discussion builds on suggestive research by Romer (1983, 1986, 1987), which analyzes productive externalities and monopolistic competition.
We begin by briefly summarizing the preferences, technology and endowments of the basic neoclassical environment we studied in the introductory essay.\(^2\)

**Preferences.** The economy is populated by many identical infinitely-lived individuals with preferences over goods and leisure represented by

\[
U = \sum_{t=0}^{\infty} \beta^t u(C_t, L_t), \quad \beta < 1,
\]

where \(C_t\) is commodity consumption in period \(t\) and \(L_t\) is leisure in period \(t\). Consumption and leisure are assumed throughout to be goods, so that utility is increasing in \(C_t\) and \(L_t\).

**Production possibilities.** The single good is produced according to a constant returns to scale neoclassical production technology given by

\[
Y_t = A_t K_t^{1-\alpha} (N_t X_t)^\alpha, \quad 0 < \alpha < 1,
\]

where \(K_t\) is the predetermined capital stock (chosen at \(t - 1\)) and \(N_t\) is the labor input in period \(t\). The economy's initial capital stock, \(K_0\), is given. We permit temporary changes in total factor productivity through \(A_t\). Permanent technology variations, \(X_t\), are restricted to be in labor productivity.

**Capital accumulation.** The capital stock evolves according to

\[
K_{t+1} = (1 - \delta_K) K_t + I_t,
\]

where \(I_t\) is gross investment and \(\delta_K\) is the rate of depreciation for capital.

**Resource constraints.** In each period, an individual faces two resource constraints given by

\[
L_t + N_t \leq 1, \quad C_t + I_t \leq Y_t,
\]

which restrict allocations of commodities and time. There are also non-negativity constraints associated with \(I_t, N_t, C_t\) and \(K_t\).

2.1. **Deterministic and stochastic trends**

In our previous development of the basic neoclassical model, we considered the implications of labor augmenting technological change for the basic

\(^2\)See King, Plosser and Rebelo (1988) for more detailed discussion.
neoclassical model. In particular, under the assumption that the index of labor augmenting technical conditions ($X_t$) grows at a constant proportional rate (i.e., $X_{t+1}/X_t = \gamma_X$), all quantity variables possess a common deterministic trend. That is, consumption, investment and output are each the sum of a common deterministic trend and a stationary stochastic component. In the terminology of Nelson and Plosser (1982), the basic neoclassical model implies that consumption, investment and output time series are trend stationary.

But Nelson and Plosser (1982) present evidence that many macroeconomic time series – including consumption, investment, output, and measures of technology change constructed along Solow (1957) lines – may not be trend stationary. In particular, using tests developed by Dickey and Fuller (1979), Nelson and Plosser show that one cannot reject the hypothesis that many macroeconomic time series are integrated stochastic processes whose first differences are stationary and invertible.

Within the class of linear time series models of the autoregressive-moving average form [Wold (1938), Box and Jenkins (1970)], the assumption that series are integrated implies a particular class of parametric forms. For example, if the log of technology is a differenced stationary stochastic process, then it has the general form

$$\varphi(B)(1-B)\log(X_t) = \log(\gamma_X) + \theta(B)\varepsilon_t, \quad (2.1)$$

where $B$ is the backshift operator and $\varphi(B)$ and $\theta(B)$ are polynomials in $B$ whose roots are outside the unit circle. Within this class of integrated stochastic processes, Beveridge and Nelson (1981) show that the time series $\log(X_t)$ can be decomposed into two components: a permanent component that is a random walk with drift (a stochastic trend) and a stationary stochastic process. The stochastic trend can be defined as

$$\log(X^p_t) = \log(X_0) + t \log(\gamma_X) + \sum_{i=0}^{t} \varepsilon^p_{t-i}, \quad (2.2)$$

where $\varepsilon^p_t$ is proportional to $\varepsilon_t$. From our standpoint, the key feature of the stochastic trend is that the shocks to the stochastic trend at time $t$, $\varepsilon^p_t$, result in a permanent shift in the level of $\log(X^p_t)$ and thus in $\log(X_t)$.

2.2. Stochastic trends in the neoclassical model

The approach to analyzing stochastic trends in the neoclassical model follows the procedures outlined in the previous essay. First, we transform the economy to obtain a stationary system. The transformation involves dividing all variables in the system by the growth (permanent) component $X^p_t$. This
The capital accumulation equation must be written as \( \gamma_\Delta k_{t+1} = (1 - \delta_\Delta)k_t + i_t \), where lower case letters indicate ratios (e.g., \( k_t = K_t/X_t^P \)) and \( \gamma_\Delta \) is one plus the mean rate of growth of technical change. Transforming consumption may alter the effective rate of time preference in the preference specification in that the pure discount rate \( \beta \) must be modified to \( \beta(\gamma_\Delta)^{1-\sigma} \). Each of these modifications is discussed in the first essay. With stochastic growth added, however, the initial conditions of the individual's choice problem will generally be affected by growth shocks, since the initial value of transformed capital is \( k_0 = (K_0/X_0^P) \).

As discussed at length in part one of this pair of essays, our solution method is to linearize the first-order conditions of the choice problem about the steady state. This yields a set of linear difference equations which are solved for the approximately optimal path of capital accumulation.

If the labor augmenting technology is a logarithmic random walk with drift [i.e., \( \log(X_t) = \log(X_0^P) \)] then the solution to the transformed economy's problem is particularly simple. The only impact of technical progress is to reset the transformed economy's capital stock relative to its long-run stationary level. A positive 1% technological innovation in the untransformed economy leads to a negative 1% decline in the transformed economy's capital stock, since \( k_t = (K_t/X_t^P) \).

Using the baseline model of the companion essay, we can undertake a quantitative analysis of exogenous stochastic growth under the random walk assumption.\(^3\)\(^4\) In particular, consider a one percentage point increase in labor augmenting technical conditions at date \( t \). In the long run, this shock will lead to one percent increases in consumption, output and investment once the transitory dynamics have been worked off. From the perspective of the transformed economy, these transitory dynamics are a result of the resetting of the initial conditions of the transformed economy implied by \( k_t = (K_t/X_t^P) \). Since we have previously discussed the transition path from a low initial capital stock in the companion essay, it is straightforward to discuss the dynamic responses in the transformed economy. A lower capital stock in the transformed economy lowers consumption and raises investment and effort. These effects continue until the stationary capital stock is reached, which is a lengthy process since the half-life of the adjustment is about 14 quarters in the baseline parametric specification of the neoclassical model. In fig. 1, the effects

\(^3\) These results are based on the parameter values of the baseline model employed in our first essay. Specifically, we interpret the time interval as a quarter and at \( \alpha = 0.58, \delta_k = 0.025, \sigma = 1.0, \beta = 0.988 \) and \( \gamma_\Delta = 1.004 \). For a discussion of these values see King, Plosser and Rebelo (1988).

\(^4\) See also Hansen (1986). Christiano (1986) studies economies where there are non-zero higher-order terms in the \( \phi(B) \) and \( \phi(B) \) polynomials the representation (4.1). For his transformed economy, he shows that this involves a combination of analyzing temporary technology shifts and shifts in the initial capital stock.
DYNAMIC ADJUSTMENT TO A PERMANENT SHOCK

Fig. 1

of a unit impulse in labor productivity are presented for the untransformed economy. In the untransformed economy, consumption and output rise gradually to a 1% higher level. On the other hand investment is above its new long-run steady state while hours are temporarily above its unchanged steady state.

Notably, in an economy with permanent shifts in production possibilities, there are general equilibrium mechanisms that induce intertemporal substitution in consumption and work effort. A positive permanent technology shock raises the real rate of return, inducing additional hours relative to an unchanged steady state level. Further, there is a postponement of consumption along the transition path.

Formally, when we assume that technology follows a random walk with drift so that $\log(X_t) = \log(X_0) + \sum_{i=1}^{t} \varepsilon_i$, the linear system that describes the joint evolution of output, consumption, investment, capital, hours and technology is as follows. First, each time series (except for hours) is the sum of a

---

5 Note that with a Cobb–Douglas production function, one percent impulse in labor productivity is equivalent to an $\alpha < 1$ percent increase in total factor productivity.
common stochastic trend, \( \log(X_P^t) \), and a stationary component. That is, we begin by specifying that

\[
\begin{align*}
\log(Y_t) &= \log(X_P^t) + \log(Y) + \hat{y}_t, \\
\log(C_t) &= \log(X_P^t) + \log(c) + \hat{c}_t, \\
\log(I_t) &= \log(X_P^t) + \log(i) + \hat{i}_t, \\
\log(K_t) &= \log(X_P^t) + \log(k) + \hat{k}_t,
\end{align*}
\]

where the hats (\( \hat{\cdot} \)) denote percent deviations from steady state paths. Second, the approximate solution to the economy's optimum problem implies that the transformed capital stock is driven by the technology shock in a one-for-one manner, reflecting the fact that the marginal product of capital depends on the ratio \( K/X \), i.e.,

\[
\hat{k}_t = \mu_1 \hat{k}_{t-1} - \varepsilon_t. \tag{2.3}
\]

Third, all other stationary variables of the system respond only to the position of the transformed capital stock, since there are no transitory components of technology under the random walk assumption. That is, the equations of the linear system take the simple forms in (2.4), although these now involve deviations from a common stochastic rather than deterministic trend:

\[
\begin{align*}
\hat{c}_t &= \pi_{ck} \hat{k}_t, \\
\hat{N}_t &= \pi_{Nk} \hat{k}_t, \\
\hat{i}_t &= \pi_{ik} \hat{k}_t, \\
\hat{y}_t &= \pi_{yk} \hat{k}_t, \\
\hat{w}_t &= \pi_{wk} \hat{k}_t, \\
\hat{r}_t &= \pi_{rk} \hat{k}_t.
\end{align*} \tag{2.4}
\]

In these linear system expressions, the \( \pi \) coefficients are implied by parameters of preferences and technology of the transformed economy.

2.3. Stochastic trends: Empirical issues and observations

In line with our discussion in section 5 of the first essay, we use the stochastic growth structure (2.3)–(2.4) as the point of departure for our empirical analysis. The stochastic growth model indicates, however, that the simple removal of deterministic components will not render the variables of interest stationary as is necessary for the computation of moments.

One common approach to dealing with economic time series with stochastic trends is to first difference the series [see, e.g., Box and Jenkins (1970)]. Under our assumptions, first differencing achieves stationarity, but it also emphasizes
high frequency implications of the model. Using the structure (2.3)–(2.4), the first differences of output are

$$\Delta \log(Y_t) = \log(\gamma_X) + (1 - \pi_{yk})e_t + \pi_{yk}(\mu_1 - 1)\hat{k}_{t-1}. \quad (2.5)$$

Similarly, the growth rates of consumption and investment are governed by the following expressions:

$$\Delta \log(C_t) = \log(\gamma_X) + (1 - \pi_{ck})e_t + \pi_{ck}(\mu_1 - 1)\hat{k}_{t-1}, \quad (2.6)$$

$$\Delta \log(I_t) = \log(\gamma_X) + (1 - \pi_{ik})e_t + \pi_{ik}(\mu_1 - 1)\hat{k}_{t-1}. \quad (2.7)$$

Hours are stationary, but persistent according to the theory, since $\hat{N}_t = \pi_{NK}\hat{k}_t$ and $\hat{k}_t = \mu_1\hat{k}_{t-1} - e_t$ with $\mu_1 < 1$. For $\mu_1$ close to one, hours will exhibit substantial serial correlation and an unstructured empirical investigation might difference hours as well. The first difference is readily shown to be

$$\Delta \hat{N}_t = -\pi_{NK}e_t + \pi_{NK}(\mu_1 - 1)\hat{k}_{t-1}. \quad (2.8)$$

When $\mu_1$ is close to one, the stochastic properties of these growth rates are determined principally by the term involving $e_t$ rather than $\hat{k}_{t-1}$. In particular, when $\mu_1$ is close to 1.0 eqs. (2.5)–(2.7) imply that the growth rates of $C_t$, $I_t$ and $Y_t$ will exhibit little serial correlation in individual time series and relatively minor cross-correlations between time series at lags other than zero.

The model and sample first differenced statistics are reported in table 1 and provide useful information about the basic neoclassical model with a stochastic trend. First, the model implies substantial contemporaneous cross-correlations (near unity) for the changes in all variables since (i) there is a single common shock and (ii) the first difference filter results in series with almost no serial correlation. Second, the model fails to reproduce the positive serial correlation in growth rates of output, investment and hours present in the data, suggesting that there are missing dynamic elements to our baseline model. One possible remedy for this is to alter the serial correlation of driving processes [i.e., choose $\phi(B)$ and/or $\theta(B) \neq 1.0$ in (2.1)], implying a gradual diffusion of technological change.

There are alternative approaches to coping with the implied non-stationarity of the untransformed series that are also consistent with the theoretical model. As noted by King, Plosser, Stock and Watson (1987), the time series of $\log(Y_t)$, $\log(C_t)$ and $\log(I_t)$ are co-integrated in the terminology of Engle and Granger (1987). In particular, certain ratios of key variables are stationary in the model when there is a common source of growth. We are lead, therefore, to
Table 1
Stochastic growth: Implications for first differences.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Std. dev.</th>
<th>Relative std. dev.</th>
<th>Autocorrelations</th>
<th>Cross-correlations with $\Delta \log(Y_{t-j})$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$\Delta \log(Y)$</td>
<td>0.75</td>
<td>1.00</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>$\Delta \log(C)$</td>
<td>0.39</td>
<td>0.52</td>
<td>0.13</td>
<td>0.12</td>
</tr>
<tr>
<td>$\Delta \log(I)$</td>
<td>1.63</td>
<td>2.17</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\delta \hat{N}$</td>
<td>0.30</td>
<td>0.40</td>
<td>-0.02</td>
<td>-0.02</td>
</tr>
<tr>
<td>$\hat{N}$</td>
<td>0.97</td>
<td>1.29</td>
<td>0.95</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Panel A: Population moments: Baseline model ($\sigma = 1.0$)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Std. dev.</th>
<th>Relative std. dev.</th>
<th>Autocorrelations</th>
<th>Cross-correlations with $\Delta \log(Y_{t-j})$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$\Delta \log(Y)$</td>
<td>0.30</td>
<td>0.40</td>
<td>-0.02</td>
<td>-0.02</td>
</tr>
<tr>
<td>$\Delta \log(C)$</td>
<td>0.41</td>
<td>0.30</td>
<td>0.06</td>
<td>-0.08</td>
</tr>
<tr>
<td>$\Delta \log(I)$</td>
<td>0.16</td>
<td>0.16</td>
<td>0.14</td>
<td>-0.08</td>
</tr>
<tr>
<td>$\Delta \hat{N}$</td>
<td>0.25</td>
<td>0.19</td>
<td>0.01</td>
<td>-0.18</td>
</tr>
<tr>
<td>$\hat{N}$</td>
<td>0.84</td>
<td>0.74</td>
<td>0.12</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Panel B: Sample moments: 1948.1-1986.4

\(^a\)Relative standard deviation is $\sigma(z)/\sigma(\Delta \log(Y))$.
\(^b\)All variables are taken from the National Income Accounts.
investigate the properties of

\[
\begin{align*}
\log(C_t) - \log(Y_t) &= \hat{c}_t - \hat{y}_t = (\pi_{ck} - \pi_{yk}) \hat{k}_t, \\
\log(I_t) - \log(Y_t) &= \hat{i}_t - \hat{y}_t = (\pi_{ik} - \pi_{yk}) \hat{k}_t, \\
\hat{N}_t &= \pi_{nk} \hat{k}_t.
\end{align*}
\]  

(2.9) \hspace{1cm} (2.10) \hspace{1cm} (2.11)

In these expressions, the common stochastic trend is imposed to achieve stationarity of the data investigated. While these relations are valid quite generally, the disadvantage of this procedure is that we cannot make statements about the independent variation in components of ratios, e.g., in consumption and gross national product separately.

There is a stochastic singularity in the expressions (2.9)–(2.11), since the consumption–output, investment–output, and hours ratios are all just functions of the contemporaneous deviation of the capital stock from its stationary level \(\hat{k}_t\). Thus, as shown in panel A of table 2, these variables are perfectly correlated contemporaneously and share the same autocorrelation and cross-correlation structures. Since \((\pi_{ck} - \pi_{yk}) = (0.62 - 0.25) = 0.37\) and \((\pi_{ik} - \pi_{yk}) = (-0.63 - 0.25) = -0.88\), it follows that the (log of the) investment–output ratio is much more volatile than the (log of the) consumption–output ratio. Further, since \(\pi_{nk} = -0.29\), it follows that (the log of) our hours measure is less volatile than both ratios. This ranking is sustained in the sample moments of post-war U.S. data in panel B of table 2. Comparison of panels A and B of table 2 also indicates that there is less serial correlation in empirical than theoretical ratios. This may indicate the presence of omitted temporary factors, since the dynamics in this model are initiated solely by shifts in the stochastic trend.

To explore cyclical interactions we choose hours as a ‘reference variable’ for the cross-correlations in panels A and B of table 2 and in the figures discussed below. In section 2.2, we discussed how consumption – relative to stochastic trend – and hours should be negatively related along the transition path towards a new stationary state. As Barro and King (1984) stress, this is a general implication of intertemporal substitution theories of business cycles.

\(\)Restrictions of this form will hold whenever there is a stochastic steady state [King and Rebelo (1987)]. In the basic neoclassical model, a steady state involves restrictions on preferences and production technologies. However, stationarity of ratios does not involve restricting the aggregate production function to Cobb–Douglas form.

\(\)There are other approaches one might use to render the series of interest stationary. Prescott (1986), for example, imposes a production function specification by calculating technology as \(\log(A, X^n) = \log(Y_t) - \alpha \log(N_t) - (1 - \alpha) \log(K_t)\) for a specific value of \(\alpha\). Then one could condition on technology, thereby eliminating non-stationarity. This procedure involves placing great weight on both the functional form and on accurate measurement of factor inputs.
Table 2
Stochastic growth: Implications for ratios.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Std. dev</th>
<th>Relative std. dev.</th>
<th>Autocorrelations</th>
<th>Cross-correlations with ( \hat{N}_{t-j} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( \log(C/Y) )</td>
<td>1.15</td>
<td>1.44</td>
<td>0.95</td>
<td>0.90</td>
</tr>
<tr>
<td>( \log(I/Y) )</td>
<td>2.86</td>
<td>3.58</td>
<td>0.95</td>
<td>0.90</td>
</tr>
<tr>
<td>( \hat{N} )</td>
<td>0.80</td>
<td>1.00</td>
<td>0.95</td>
<td>0.90</td>
</tr>
</tbody>
</table>

**Panel A:** Population moments: Baseline model \((a_t = 1)\)

**Panel B:** Sample moments: 1948.1–1986.4

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*a* Relative standard deviation is \( \sigma(z)/\sigma(\hat{N}) \).

*b* All variables are taken from the National Income Accounts.
applying to shocks that affect perceived rates of return and wealth. This implication of a negative correlation carries over to the consumption–output ratio and hours. The logarithms of these series are graphed in panel A of fig. 1. This is a marked negative correlation (−0.48 in panel B of table 2) as predicted by intertemporal substitution theory. Further, hours and investment – relative to stochastic trend – should be positively related along the transition path, an implication which again carries over to the ratios plotted in fig. 2. The negative correlation of the investment–output ratio and consumption–output ratio, implied by the model’s requirement that $C + I = Y$ does not show up in the post-war U.S. time series, as the correlation is estimated to be 0.22. This suggests exploring the residual elements omitted from this expression, namely government purchases, consumer durables, inventories and the balance of trade. The temporary character of some government purchases – particularly military expenditure, as stressed by Barro (1981) – make it an important addition to the basic model. In section 4, we explore some alterations in our methods that make it possible to incorporate various aspects of the public sector.
3. Endogenous growth and economic fluctuations

In the basic neoclassical model, long-run growth is exogenously determined by the specified rate of labor augmenting technical change. Recent work by Romer (1986) and Lucas (1988) explores modifications of the basic neoclassical model that permit the long-run growth rate to be an endogenous outcome of a time invariant technology. This section briefly explores the implications of endogenous growth for economic fluctuations.

Our presentation involves the two stages of analysis undertaken in King and Rebelo (1986, 1987). The initial step is to outline the modifications to the basic neoclassical model that are compatible with endogenous steady state growth. Taking these modifications as given, the analysis then considers the implications for the model's response to shocks. Relying heavily on a closed form borrowed from King and Rebelo (1986), we discuss the implications of endogenous growth for economic fluctuations.

3.1. Modifying technology to permit endogenous growth

To incorporate endogenous growth in the basic neoclassical model, it is necessary to eliminate the diminishing returns to capital accumulation in some
manner. Romer (1983, 1986) makes sustained growth feasible through a combination of aggregate increasing returns and diminishing private returns to capital.\(^8\) Alternatively, endogenous growth may arise by assuming that there are no important fixed factors of production. This second approach, pioneered by Uzawa (1965) and Lucas (1988), is adopted in this section since it is consistent with steady state growth. In our extended model there are two capital stocks, physical capital \((K_t)\) and human capital \((H_t)\). As in the basic neoclassical model, consumption and investment are the two uses of ‘goods’ output, so that the commodity resource constraint continues to take the form (2.5), \(C_t + I_t \leq Y_t\), and physical capital accumulation takes the form (2.3), \(K_{t+1} = (1 - \delta_K)K_t + I_t\). Commodity output \((Y)\), however, depends on whether resources are allocated to the production of final goods or to the production of human capital \((H)\). Let \(N_t\) be the fraction of time units allocated to production of final output and \(V_t\) be the fraction of physical capital allocated to this activity. Then, the Cobb–Douglas production function can be written as

\[
Y_t = A_t(V_tK_t)^{1-\alpha}(N_tH_t)^{\alpha}. \tag{3.1}
\]

Symmetrically, let gross investment in new human capital take place according to another constant returns-to-scale (Cobb–Douglas) production function, with inputs being time in efficiency units \((E_tH_t\), where \(E_t\) is the fraction of time devoted to human capital maintenance and formation) and capital \(((1 - V_t)K_t\). Let \(A^*_t\) be a temporary technology shock analogous to \(A_t\) in the production function for final output. Then, the production function for human capital may be written as \(A^*_t((1 - V_t)K_t)^{1-\nu}(E_tH_t)^{\nu}\), where \(0 < \nu < 1\). Letting \(\delta_H\) be the rate of depreciation on human capital, it follows that changes in the stock of human capital are governed by

\[
H_{t+1} - H_t = A^*_t((1 - V_t)K_t)^{1-\nu}(E_tH_t)^{\nu} - \delta_H H_t. \tag{3.2}
\]

3.2. Feasible steady states with endogenous growth

In any feasible steady state, the share parameters \(E\), \(N\) and \(V\) must be constant, since these are bounded between zero and one. Further, as in the basic neoclassical model, since consumption and investment are perfect substitutes \((Y = C + I)\), it follows that the growth rates of consumption, invest-

\(^8\)Romer’s model implies that growth rates should be accelerating over time, rather than constant as in any steady state model. Our perspective is that steady state growth models can capture salient features of macroeconomic time series and that endogenous growth is an important, feasible extension of these models. The restriction to the steady state class is rationalized partly by casual empiricism and partly by pragmatic modeling considerations, in that steady state models can be formulated, simulated and (in principle) estimated with existing technology.
ment, output and physical capital must be equal to the rate of growth of human capital ($\gamma_C = \gamma_Y = \gamma_Y = \gamma_K = \gamma_H$). But, from (3.2), it follows that the growth rate of human capital depends on the fraction of societal resources allocated to this activity and on the ratio of physical to human capital,

$$\gamma_H = A^*(1-V)(K/H)^{1-\nu}(E)^{\nu} - \delta_H.$$  

(3.3)

Unless both production transactions are constant returns to scale it will not be feasible for the economy to sustain constant growth rates.\(^9\)

In specifying preferences compatible with steady state growth, it is necessary to make a choice as to the arguments of utility. If agents consume leisure in pure time units, so that utility is of the form $u(C, L)$ with $N + E + L = 1$, then the restrictions derived in the companion essay are necessary and sufficient for the optimality of a steady state path. If agents consume leisure in efficiency units, so that utility is of the form $u(C, HL)$ with $N + E + L = 1$, then $u(\cdot)$ must be homogeneous, i.e., can be written as $H^{z}u(c, L)$, where $z$ is a parameter and $c = C/H$. Becker (1965), Heckman (1976) and other authors have postulated preferences of this latter form in exploring life-cycle allocations of time to market work, investment in human capital and leisure. In the labor economics context, these preferences permit small responses of effort to wage changes induced by accumulation of skills over the life-cycle, while retaining major inter- and intratemporal responses to variations in wages arising from other sources (e.g., market conditions and government policies). Similarly, preferences of this form should permit enhanced response to temporary shocks in general equilibrium models of the variety studied in this paper.

The endogenous growth economy has a number of general properties that are important for growth and fluctuations. First, although there is no steady state capital stock, there is a steady state ratio of physical to human capital. Second, preferences play a role in determining the efficient growth rate because the optimal social rate of accumulation depends on the utility costs of investments in growth (e.g., time allocated to education) and the rate of intertemporal substitution in consumption. Third, although proportionate increases in both capital stocks do not occasion dynamic adjustments except for a scale change, a change in the initial conditions for one of the capital

\(^9\)For example, suppose that we modify the production function for human capital to be $H_{t+1} - H_t = A^*(1-V)K_t \delta H_t$, without imposing the requirement that $\nu_1 + \nu_2 = 1$. Then, it turns out that $(H_{t+1} - H_t) / H_t = A^*(1-V)(K_t / H_t) \delta H_t$. Unless $\nu_1 + \nu_2 = 1$ it is not feasible to sustain a constant rate of human capital accumulation. Notice that it is the constant returns to scale feature of technology – not the restriction to the Cobb–Douglas form – that is critical. To depart from the constant returns to scale assumption, it is necessary to eliminate the perfect substitutability of final output between consumption and investment. Such extensions are discussed in Rebelo (1987a) and King and Rebelo (1987).
stocks generally sets off a complicated pattern of adjustment to a new growth path. Along this transition path, there are generally changes in effort, as in the neoclassical model, even though $L$, $N$ and $E$ are constant in the steady state. Fourth, temporary disturbances to production possibilities ($A_*$ and $A_*$ above) have permanent effects on the levels of economic activity, because they permit temporary changes in the amount of resources allocated to growth. Thus, endogenous growth models generate integrated time series, even when the underlying shocks are stationary. In our view, all of these properties originate from the underlying constant returns structure that is needed to generate endogenous steady state growth. In macroeconomics as in other areas, constant returns implies that scale may not be uniquely determined in a model without the specification of initial conditions or other side constraints.

3.3. Stochastic growth: An example

To illustrate these general properties, it is useful to consider a variant of the preceding economy for which a closed form solution exists. Like the Long and Plosser (1983) closed form, of which it is a relative, it involves some special characteristics not shared by more general specifications. To generate the closed form, it is necessary to assume that the momentary utility function is logarithmic, $\log(C_t) + \theta_1 \log(L_t) + \theta_2 \log(H_t)$, where choice of the $\theta$ parameters permits us to treat individuals as valuing time in efficiency units ($\theta_1 = \theta_2$) or in pure form ($\theta_2 = 0$). It is also necessary to assume that there is complete depreciation of both forms of capital ($\delta_K = \delta_H = 1$). Then, it is straightforward to show that there is invariance of all of the time allocation decisions $(V, E, N)$ to the state of the economy $(A_t, A^*, K_t, H_t)$ and that the following difference equation system describes the dynamics of capital accumulation:

$$\log(K_{t+1}) = \kappa_1 + (1 - \alpha) \log(K_t) + \alpha \log(H_t) + \hat{A}_t,$$

(3.4a)

$$\log(H_{t+1}) = \kappa_2 + (1 - \nu) \log(K_t) + \nu \log(H_t) + \hat{A}^*_t.$$

(3.4b)

In this expression, $\hat{A}_t$ is the percent deviation of $A_t$ from its normal level, $\hat{A} = \log(A_t/A)$, $\hat{A}^*$ is similarly defined, and the constants $\kappa_1$ and $\kappa_2$ are complicated combinations of the model's underlying parameters ($\alpha, \nu, A, A^*, \theta_1, \theta_2$). Although the example shares some special features of the Long and Plosser (1983) example – notably invariance of labor supply decisions – it serves as a useful device for illustrating the general properties discussed above. For example, multiplying both capital stocks by a scale factor [adding $\log(z)$ to $\log(K_t)$ and $\log(H_t)$] simply shifts up the growth path by the scale factor.

The dynamic system for the two capital stocks also contains a unit root, which leads changes in initial conditions [$\log(H_t), \log(K_t)$] and shocks [$\hat{A}_t, \hat{A}^*_t$] to have permanent effects. From (3.4a)-(3.4b) it follows that univariate
processes for \( \log(K) \) and \( \log(H) \) are

\[
\begin{align*}
[(1 - B)(1 - (\nu - \alpha)B)] \log(K_{t+1}) &= (1 - \nu)k_1 + \alpha k_2 + (1 - \nu B)\hat{A}_t + \alpha B\hat{A}_t^*, \\
[(1 - B)(1 - (\nu - \alpha)B)] \log(H_{t+1}) &= (1 - \nu)k_1 + \alpha k_2 + (1 - \nu)B\hat{A}_t + (1 - (1 - \alpha)B)\hat{A}_t^*.
\end{align*}
\] (3.5a) (3.5b)

In general, these expressions indicate that shocks to either production process impinge on the time profile of both capital stocks.\(^{10}\) Thus, shocks to technologies (\(\hat{A}\) and \(\hat{A}^*\)) will have permanent effects on the levels of variables such as output (\(Y\)) and consumption (\(C\)), since these are just functions of the capital stocks.\(^{11}\)

A particularly simple case of dynamics arises if the production functions have the same share parameters (\(\alpha = \nu\)). Then, growth rates of the capital stocks are functions solely of shocks at date \(t\) and \(t - 1\). For shocks to the own production function, for example \(\hat{A}\) in the physical capital equation, there is a negative effect of lagged shocks on accumulation. But this is less than one for one (so long as \(\alpha = \nu < 1\)), so that the level of capital is permanently higher. Shocks to the other production function (e.g., \(\hat{A}^*\) in the physical capital equation) exert effects on changes only during the period in which they occur, so that capital stocks are always permanently higher as a result of these shocks.

Although the capital stock time series generated by the endogenous growth model are non-stationary, there are transformations that are stationary. With a little algebraic manipulation, it is evident that \(\log(K/H)\) is a stationary stochastic process,

\[
\log(K_{t+1}/H_{t+1}) = (k_1 - k_2) + (\nu - \alpha)\log(K_t/H_t) + (\hat{A}_t - \hat{A}_t^*).
\] (3.6)

Thus, the stationarity of ratios prevails as in the basic neoclassical model, implying that \(\log(H)\) and \(\log(K)\) are co-integrated.

\(^{10}\)An exception is the example presented in early drafts of this paper and the King, Plosser, Stock and Watson (1987) paper, in which \(\nu = 1\), i.e., physical capital played no role in the production of human capital. [See also Lucas (1988).] In this case, there is no impact of \(\hat{A}\) shocks on human capital accumulation. This is one more artifact of the restrictions necessary for a closed form, particularly \(\delta_k = \delta_H = 1\). Shocks to human capital production (\(\hat{A}^*\)) will show up in the physical capital path in Lucas's model (\(\nu = 1\), if \(\delta_k < 1\) and \(\delta_H < 1\).

\(^{11}\)For example, \(\log(Y_t)\) is a constant plus \(\log(\hat{A}_t + \alpha \log(H_t) + (1 - \alpha)\log(K_t))\).
It is also possible to derive solutions for growth rates that express $\Delta \log(K_{t+1})$ and $\Delta \log(H_{t+1})$ as stationary functions of the histories of shocks,

$$
\Delta \log(K_{t+1}) = (1 - (\alpha - \nu) B)^{-1}(1 - \nu B) \hat{A}_t
+ (1 - (\alpha - \nu) B)^{-1}(\alpha B) \hat{A}_t^*,
$$

(3.7a)

$$
\Delta \log(H_{t+1}) = (1 - (\alpha - \nu) B)^{-1}(\nu B) \hat{A}_t
+ (1 - (\alpha - \nu) B)^{-1}(1 - \alpha B) \hat{A}_t^*.
$$

(3.7b)

Versions of these stationary representations hold in more general models, with alternative preference specifications and less than complete depreciation. Thus, steady state model economies with endogenous growth dictate a class of transformations of macroeconomic time series.

3.4. Implications and future developments

Steady state endogenous growth models represent a promising avenue of research, since they enhance the propagation mechanisms of the basic neoclassical model. There are a number of theoretical extensions to the basic economy described above in King and Rebelo (1987), including incorporation of learning by doing. It remains to be seen, however, whether restricted parameterizations of this class of models can generate stochastic processes for endogenous variables that generate moments that resemble those of observed macroeconomic time series.

4. Suboptimal equilibrium

Many economists believe that analysis of economies with suboptimal competitive equilibrium is necessary for understanding various macroeconomic phenomena. Distorting taxation, externalities, market incompleteness, increasing returns to scale and imperfect competition are often invoked as key ingredients in explaining certain features of the data or as a rationale for policy interventions. The objective of this section is to explore methods that allow us to incorporate some of these alternatives.

Early examples of the use of a suitably specified ‘planning problem’ to determine dynamic suboptimal equilibria include Arrow (1962) and Brock (1975) and Hall (1971). Recently, Romer (1983) has provided a detailed analysis of methods for computing suboptimal equilibrium and applied these to the study of economies with externalities and increasing returns to scale [Romer (1986)] and to a class of environments in which there is a monopolistic competition [Romer (1987)].
4.1. Distorting taxation in a static economy

In order to illustrate the essential aspects that underlie the computation of suboptimal equilibrium, we first consider a static economic environment with endogenous labor supply and production. The optimum allocation for this economy can be determined by maximizing $u(C, 1 - N)$ subject to the production function $Y = AF(N)$ and the resource constraint $C = Y$.

Suppose that a tax on output is introduced and its revenue employed to finance a lump sum transfer to the private sector. Denoting the tax rate by $\tau$ and the per capita lump sum transfers by $T$, the government's budget constraint can be written as $T = \tau Y$, where the underline denotes per capita quantities.

In an economy with $m$ private agents the influence of an increase of one unit in a particular individual's production on the economy's per capita output is $1/m$. For $m$ large this influence is negligible so each private agent views per capita output as exogenous. As a result, the transfer each individual receives from the government is also exogenous from his standpoint. The decision problem faced by each representative agent is then given by

$$\max u(C, 1 - N),$$

subject to

$$Y = AF(N), \quad C = (1 - \tau)Y + T.$$

The first-order conditions associated with this problem for an arbitrary value of $T$ are

$$D_2 u(C, 1 - N) = D_1 u(C, 1 - N)(1 - \tau)AD_1 F(N), \quad (4.1)$$
$$C = (1 - \tau)AF(N) + T. \quad (4.2)$$

While the transfer $T$ is viewed as exogenous by each private agent, it is not exogenous for the economy as a whole. Per capita transfers are given by $T = \tau Y$. Since all private agents are alike, per capita quantities will turn out to coincide with the choices of our representative agent, i.e., $N = N, Y = Y, C = C$. Using these two facts we can then write eqs. (4.1) and (4.2) as

$$D_2 u(C, 1 - N) = D_1 u(C, 1 - N)(1 - \tau)AD_1 F(N), \quad (4.1')$$
$$C = AF(N). \quad (4.2')$$

---

12 The computation of suboptimal equilibrium in a static context was pioneered by Bailey (1954).

13 This policy clearly reduces welfare since all it does is create a distortion. However it is a useful thought experiment that allows us to isolate the distortionary effects of taxation from issues related to the use of government revenue.
Eqs. (4.1') and (4.2') determine the competitive equilibrium. The result of combining the representative individuals' resource constraint and the government's budget constraint in (4.2') is identical to the resource constraint of an economy without taxes. This simply reflects the fact that the presence of taxation does not alter the production possibilities of the economy as a whole. The only decision private agents make in this economy is how to allocate their time between work and leisure. This decision is based on the after-tax marginal productivity of labor, consequently, the economy is driven away from the optimal allocation.

Before proceeding to study economies with capital and more sophisticated government policies it is worthwhile to stress two basic points. First, all private agents understand the rule that the government uses to determine their transfer payment. It is the presence of a large number of agents, which makes per capita output exogenous from each agent's standpoint, that leads them to view the path of government transfers as exogenous. Similarly, if aggregate tax rates were set to achieve a fixed revenue objective, this would lead to a dependence of \( \tau \) on \( N \) but not on \( N \). Second, the presence of a representative agent simplifies the computation of the competitive equilibrium but does not play a crucial role. Suppose there are two types of agents in the economy, \( a \) and \( b \), and that the government uses the output tax revenue to finance a lump sum transfer that is identical for both types. This transfer is then given by

\[
T = \tau[\theta_a Y^a + (1 - \theta_a) Y^b],
\]

where \( \theta_a \) is the proportion of the population that is of type \( a \). As before, the presence of many agents of both types makes \( Y^a \) and \( Y^b \) exogenous from the standpoint of each individual. Since all individuals of the same type make the same decisions, \( Y^a = Y^a \) and \( Y^b = Y^b \), we can obtain a system of equations analogous to (4.1') (4.2').

4.2. Distorting taxation and government expenditures

We now consider the basic neoclassical model augmented by government. In particular, there is an output tax whose revenue is used to finance an exogenous path of per capita government expenditures \( \{G_t\}_{t=0}^\infty \) and lump sum transfers \( \{T_t\}_{t=0}^\infty \). For simplicity of exposition, we assume that these expenditures do not affect the economy's production possibilities or the representative agent's marginal utility. The government's budget constraint is \( T_t = \tau Y_t - G_t \). As in the static example, the presence of a large number of agents implies that each individual views the transfer he receives from the government \( T_t \) as exogenous. The representative agent's decision problem is

\[
\max_{t=0}^{\infty} \beta^t u(C_t, 1 - N_t),
\]

For more details on models with heterogeneous agents see section 5 below and Rebele (1987a).
subject to

\[(1 - \tau_t)A_t F(K_t, N_t, X_t) + (1 - \delta_K)K_t + T_t = C_t + K_{t+1},\]

where \(K_0\) is fixed. The efficiency conditions associated with specific transfer and spending paths are\(^{15}\)

\[
\beta'D_1u(C_t, 1 - N_t) - \Lambda_t = 0, \quad (4.3)
\]

\[-\beta'D_2u(C_t, 1 - N_t) + \Lambda_t[\Delta(1 - \tau_t)X_tA_tD_2F(K_t, N_t, X_t) = 0, \quad (4.4)
\]

\[
\Lambda_{t+1}[(1 - \tau_{t+1})D_1A_{t+1}F(K_{t+1}, N_{t+1}X_{t+1}) + (1 - \delta_K)] - \Lambda_t = 0, \quad (4.5)
\]

\[
(1 - \tau_t)A_t F(K_t, N_t, X_t) + (1 - \delta_K)K_t + I_t - C_t - K_{t+1} = 0, \quad (4.6)
\]

\[
\lim_{t \to \infty} \Lambda_tK_{t+1} = 0. \quad (4.7)
\]

for all \(t = 1, 2, \ldots, \infty\).

Though the individual views the paths for \(G_t\) and \(\tau_t\) as being specified exogenously by the government, the tax rate and the level of government expenditure can be functions of a vector of endogenous and exogenous variables. Suppose, for instance, that the tax rates and the level of per capita government expenditures are given by

\[
\tau_t = 1 - \Omega_t(A_t, K_t, G_t) = 1 - \Omega_t, \quad (4.8)
\]

\[
G_t = G_t(A_t, K_t). \quad (4.9)
\]

As before, the representative agent's decisions have a negligible influence on output per capita and so he views the paths of \(\tau_t, G_t\) and \(T_t\) as exogenous, but he views the specific values of the rates, etc. as induced by per capita quantity decisions. When we substitute (4.8)–(4.9) in (4.4)–(4.6) and impose the equilibrium conditions \(\ddot{Y}_t = Y_t, K_t = K_t\) and \(C_t = C_t\), we obtain

\[-\beta'D_2u(C_t, 1 - N_t) + \Lambda_t[\Omega_tX_tA_tD_2F(K_t, N_t, X_t) = 0, \quad (4.4')
\]

\[
\Lambda_{t+1}[(1 - \tau_{t+1})D_1A_{t+1}F(K_{t+1}, N_{t+1}X_{t+1}) + (1 - \delta_K)] - \Lambda_t = 0, \quad (4.5')
\]

\[
F(K_t, N_t, X_t) + (1 - \delta_K)K_t - K_{t+1} - C_t - G_t(A_t, K_t) = 0, \quad (4.6')
\]

\(^{15}\)An existence proof for the models discussed in this section can be constructed along the lines of Romer and Sasaki (1986).
which, together with (4.3) and (4.7), determine the competitive equilibrium under perfect foresight for a given $K_0$.

In order for steady state growth to be a competitive outcome, it is necessary to impose some constraints on the pattern of taxes, transfers and government purchases. If tax rates are to be constant in the steady state, it follows that they must depend on the ratio $K_t/X_t$ rather than on $K$ and $X$ separately. Similarly, the ratios of government purchases to output ($G_t/Y_t$) and transfers to output ($T_t/Y_t$) must also be invariant to the scale of the economy (i.e., to $X$). With these conditions imposed, we can proceed to transform the economy as in section 2 of the introductory essay. Further, in order for the stationary capital stock of the transformed economy to be stable, it is necessary to impose some additional restrictions on the tax functions. With these ‘scale invariance’ and ‘stability’ conditions imposed, we can proceed to approximate the economy around the stationary point using the same sort of argument outlined in section 3 of the first essay. (See the technical appendix for a detailed discussion.) The key observation is that approximate suboptimal equilibria involve a difference equation system of the same variety encountered in the study of optimal equilibria.

4.3. Examples of quantitative analyses

Some recent research makes clear that there is a major quantitative impact of incorporating government and, more generally, factors that lead to suboptimal equilibria.

4.3.1. U.S. economic activity during WW II

Wynne (1987a, b) examines the influence of wartime movements in government spending and tax rates on U.S. economic activity, focusing on the World War II episode. During World War II, as Barro (1981, 1987) has emphasized, there was an important temporary change in military expenditure: total government spending exceeded its steady state value by an average of 240 percent for the four-year period 1942–1945.\footnote{In addition, there was a 3 increase in tax rates: the average marginal tax rate calculated by Barro and Sahasakul (1986) rose from about 4 percent in 1935 to about 26 percent in 1945. Wynne (1987b) studies implications of these movements in tax rules.}

Using the basic neoclassical model parameterized to correspond to an annual time interval, Wynne (1987a) shows that government spending and tax rate movements account for a surprisingly major portion of the U.S. experience under the assumption that the magnitude and duration of World War II was largely anticipated. The model captures the substantial increase in real national product and labor input, as well as the decline in private investment.
At the same time, actual U.S. consumption responds less sharply (negatively) to temporary government spending than suggested by Wynne's parameterization of the basic neoclassical model.

4.3.2. A balanced budget rule

Working within the basic (one-sector) neoclassical model extended to include tax policies in the manner discussed above, Baxter and King (1987a) investigate the implications of adjusting tax rates so that the government finances a constant stream of expenditures \( G \) by raising the required tax revenue \( (\tau Y_r) \) each period.\(^{17}\) It turns out that this stringent policy has major implications for the response of the basic neoclassical economy to a technology shock. Holding factors of production fixed, the balanced budget rule implies that an increase in \( A \), must be accompanied by a decline in \( \tau \), in order to maintain budget balance. From the standpoint of an individual agent, this implies that tax changes reinforce the effect of productivity shocks on the after-tax schedule for the marginal product of labor. In turn, individuals respond more elastically to technology shocks. Further, there are implications of the balanced budget rule for the behavior of the after-tax marginal product of capital, since taxes are lower with higher capital stocks, positive technology shifts, or additional labor input. Taken together, these features serve to enhance the response of the system to shocks, inducing more volatile responses.

Table 3 presents the coefficients of the linear system \( (\mu, \pi) \) under some alternative assumptions about government: (i) no government, (ii) constant and equal tax rates and government shares, and (iii) a point in time balanced budget.

The first line of the table shows the baseline model as presented in the companion essay, in which there is no government. The second line shows the implications of incorporating steady state budget balance with government purchases equal to 0.3 of national product. (That is, we set the steady state tax rate \( \tau \) equal to 0.3, which implies a steady state value of the wedge \( \omega = 1 - \tau \) of 0.7.) The effects of steady state taxation basically work through the following two mechanisms. First, since taxation lowers the after tax marginal product of capital at any given capital stock, there is a lower steady state capital stock and, consequently, a smaller fraction of output must be devoted to replacement investment (i.e., \( s_i \) declines with the steady state taxation imposed in table 3 from about 0.3 of national product to about 0.2 of national product). Further, there must be a one-for-one crowding out of private

\(^{17}\)This rule can imply that there are multiple steady state values of tax rates and output, i.e., multiple points on the same steady state 'Laffer curve'. Baxter and King (1987a) finesse this issue by specifying the steady state.
### Table 3
Implications of public interventions for basic neoclassical model.\(^a\)

<table>
<thead>
<tr>
<th>Policy parameters(^b)</th>
<th>Linear systems coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_g) (\Omega) (\omega_A) (\omega_N) (\omega_K) (\mu_1) (\pi_{kA}) (\pi_{ck}) (\pi_{CA}) (\pi_{Nk}) (\pi_{NA}) (\pi_{yk}) (\pi_{yA}) (\pi_{I_k}) (\pi_{I_A}) (\pi_{I_b}) (\pi_{I_w}) (\pi_{k}) (\pi_{rA})</td>
<td></td>
</tr>
<tr>
<td>Baseline model</td>
<td>0 1.0 0 0 0 0.953 0.166 0.617 0.108 -0.294 1.332 0.249 1.773 -0.629 5.747 0.544 0.441 -0.029 -0.005</td>
</tr>
<tr>
<td>Baseline model with steady state balanced budget</td>
<td>0.3 0.7 0 0 0 0.954 0.230 0.667 0.161 -0.369 1.252 0.206 1.726 -0.595 7.968 0.575 0.474 -0.031 -0.007</td>
</tr>
<tr>
<td>Baseline model with point in time balanced budget</td>
<td>0.3 0.7 0.429 0.249 0.180 0.968 0.350 0.579 0.246 -0.187 2.807 0.312 2.628 -0.112 12.129 0.632 0.947 -0.022 -0.008</td>
</tr>
</tbody>
</table>

\(^a\)The parameter values of the baseline model are: \(\alpha = 0.58\), \(\delta_k = 0.25\), \(\sigma = 1\), \(\beta = 0.988\) and \(\gamma_X = 1.016\). For a full discussion of the source of these values see King, Plosser and Rebelo (1988).

\(^b\)\(s_g\) = steady state share of government purchases in output, \(\Omega = (1 - \tau)\) = steady state tax wedge, \(\omega_x\) = elasticity of wedge with respect to variable \(x\).
consumption in the steady state given the shares of government and any tax-induced changes in investment's share. The net effect of a rise in government's share of national income, \( s_g \), from 0 to 0.3 and the decline in replacement investment is that consumption's share of national income, \( s_c \), falls from about 0.7 to about 0.5 as one moves from the baseline economy to the version with a steady state balanced budget in table 3. Second, when shocks occur, there is a constant level of government spending in the transformed economy. Hence, a given percentage movement in output requires a greater proportionate movement in private spending and, generally, in both of its components.\(^{18}\)

The third line of the table involves a point in time balanced budget with constant government spending, i.e., adjustment of tax rates according to \( \tau = g/y = g/A_t F(K_t, N_t) \). This implies that the elasticities of the wedge function (\( \Omega_t \)) with respect to economic conditions are \( \omega_s = (1 - \Omega_t)/\Omega_t \), \( \omega_k = (1 - \Omega_t)/\Omega_t(1 - \alpha) \), and \( \omega_N = (1 - \Omega_t)/\Omega_t \alpha \), where \( \Omega_t \) is the steady state value of the wedge. Adjustment of tax rates in this manner substantially increases the responsiveness of the neoclassical model to shocks. For example, the impact effect of a technology shock on labor input (\( \pi_{N,t} \)) is about twice as large in the third line of table 3 as in earlier lines. This reflects the quantitative importance of the substitution responses described above, in a fully specified general equilibrium model.

### 4.3.3. Productive externalities and cyclical volatility

In environments that possess a steady state, suboptimal competitive equilibria with productive externalities can be studied with an extension of the methods described in the preceding sections. For example, suppose that one posits that the individual agent works on a production function that has the form \( y_t = A_t F(K_t, N_t, K_t, N_t) \), where \( K_t, N_t \) are quantities chosen by the individual agent and \( K_t, N_t \) are per capita quantities that the agent treats as parametric in deciding on \( K_t, N_t \). Then, as Romer (1986, 1987) has stressed in a growth context, competitive equilibrium will be inefficient but can be computed by finding per capita sequences ((\( K_t \)), (\( N_t \))) that satisfy individual efficiency conditions and rational expectations.

Investigating the relation between cyclical volatility and productive externalities, Baxter and King (1987b) study an environment in which there is (a) constant returns to scale with respect to individual capital and labor, (b) aggregate increasing returns to scale, and (c) aggregate diminishing returns to

\(^{18}\)The resource constraint requires \( s_g \delta t + s_c \xi_t + s_i \xi_t = \xi_t \). Thus, with a one percent change in output, private spending must adjust by \( 1/(1 - s_g) \) percent. This feature of the environment tends to make consumption and investment more highly correlated with output when government is present in the environment, even if it does not adjust tax rates or spending to economic conditions.
capital. This last condition is instrumental in guaranteeing that there is a stable steady state in the presence of increasing returns, in contrast to the unbounded growth of Romer (1986). Taking technology shocks as an impulse, Baxter and King (1987b) conclude that economies with suboptimal equilibria and aggregate increasing returns display more volatility than those with optimal equilibria and constant returns. Increased volatility arises because there are external effects of individual production and accumulation decisions that act to reinforce those of other agents. They also point out, however, that competitive allocations in economies with suboptimal equilibria and increasing returns are likely to be less volatile than the corresponding optimal allocations. Further, they provide examples in which this is dramatically the case. Sample economies display competitive allocations with and without increasing returns that are broadly similar in terms of volatility. However, there is a major increase in volatility when optimal allocations are studied in the increasing returns version.

5. Heterogeneous agents

Like most other modern macroeconomic models, the models discussed in the previous sections assume the presence of a single type of agent in the economy. This has often been viewed as a serious limitation of the class of environments we have discussed. Heckman (1984), in particular, has argued that representative agent constructs are inappropriate for studying labor market issues. Rebelo (1987b) shows that agent heterogeneity can be readily accommodated within the class of models that we consider in this pair of essays, i.e., steady state growth models developed within the certainty equivalence perspective. As with the basic neoclassical model, the insight on how to incorporate heterogeneity comes from looking at optimal competitive equilibria, but there are direct extensions to suboptimal equilibria along the lines discussed in the previous section.

5.1. Optimal steady state growth with heterogeneity

The key result that allows us to compute competitive equilibrium in economies with heterogeneous agents originates in Lange (1942) and Negishii (1960). These authors demonstrate that, under standard assumptions on preferences and technology, any Pareto optimal allocation can be computed by maximizing a weighted sum of agent's utility functions.

In this section we explore briefly the consequences of heterogeneity on the properties of the basic model studied in the companion essay. We consider an economy populated by J types of agents which differ with respect to their labor skills, preferences and wealth. There is a large number of individuals of each type.
Any competitive equilibrium for this economy can be obtained as the solution to the following planning problem:

$$\max \left\{ \sum_{j=0}^{J} \theta_j^* \sum_{t=0}^{\infty} \beta^t u_j(C_{jt}, L_{jt}) \right\},$$

subject to

$$\sum_{j=1}^{J} \theta_j C_{jt} = C_i, \quad (5.1)$$

$$N_{jt} + I_{jt} = 1, \quad (5.2)$$

$$\sum_{j=1}^{J} \theta_j \varphi_j N_{jt} = N_t, \quad (5.3)$$

$$A_i \varphi(K_i, N_i, X_i) = K_{t+1} - K_t(1 - \delta_K) + C_i, \quad (5.4)$$

where restrictions (5.1)–(5.4) and non-negativity constraints on $C_{jt}$, $L_{jt}$, $N_{jt}$, and $K_{jt}$ must hold for $t = 0, 1, 2, \ldots, \infty$. The initial capital stock, $K_0$, is given.

In this problem $\theta_j^*$ denotes the welfare weight associated with agents of type $j$. The sum of these welfare weights can be arbitrarily normalized to one, $\sum_{j=1}^{J} \theta_j^* = 1$. The percentage of individuals of type $j$ in the population is denoted by $\theta_j$ so that $\sum_{j=1}^{J} \theta_j = 1$. In the production structure implicit in (5.3)–(5.4), hours of work supplied by different types of agents are perfect substitutes in production. However, different types have different productivities which we capture by the index $\varphi_j$. For convenience we normalize this index so that $\sum_{j=1}^{J} \varphi_j \theta_j = 1$. The value $\varphi_j N_j$ can be viewed as the labor supplied by individual of type $j$ in efficiency units.

Given $K_0$ and the weights $\theta_1^*, \theta_2^*, \ldots, \theta_J^*$ chosen by the fictitious planner, we can compute the associated Pareto optimal allocation and then the capital stock that would have to be given to each agent of each type at time zero in order to decentralize this optimal solution as a competitive equilibrium under perfect foresight. Our objective, however, is not to solve the planning problem for arbitrary welfare weights. Instead, we want to take as given the vector of initial capital stocks of the various types of agents, $[K_0^1, K_0^2, \ldots, K_0^J]$, and compute the weight $\theta_j^*$ that yields the optimal allocation (for $K_0 = \sum_{j=1}^{J} \theta_j K_{j0}$) which coincides with the competitive equilibrium associated with this vector of initial conditions. This can be a very cumbersome problem. Luckily, for steady state models we can study near steady state dynamics without computing the welfare weights. All that is required is that we be able to compute the steady state consumption and labor supply associated with each type of agent. Since that depends on their initial capital stocks, the problem becomes greatly simplified if we choose the initial capital endowments of both types of agents to be consistent with the economy’s steady state. In the context of this model,
this implies that one cannot study economies in which $K_0^1, K_0^2, \ldots, K_0^d$ are arbitrary, but only those in which $\sum_{j=0}^{d} \theta_j K_{0j} = k X_0$, where $k$ is the steady state value of $K/X$. To put it in a different way, we can explore the effect of alternative distributions of the steady state capital stock $k$ between the different types of agents.

This economy can be transformed to a stationary one by dividing through by the common rate of labor augmenting technical change. Using lower case letters to denote consumption in the transformed economy and using the relevant current valued shadow price, the following conditions must hold for each consumer $j$ in an (optimal) competitive equilibrium:

\[ \theta_j^* D_1 u(c_{jt}, L_{jt}) = \lambda_j, \quad (5.5) \]
\[ \theta_j^* D_2 u(c_{jt}, L_{jt}) = \lambda_j \sigma_j A_d F(K_t, N_t), \quad (5.6) \]

which are individual versions of the efficiency conditions that we discussed earlier.

The utility function must be restricted to be consistent with steady state growth. As in the companion essay each agent's momentary utility function must be isoelastic in consumption. Further, Rebelo (1987b) shows that, in the presence of technological progress ($\gamma_x > 1$), the elasticity of marginal utility of consumption must be common to all agents. Consequently, the class of preferences consistent with steady state growth is

\[ u_j(c_j, L_j) = \frac{1}{1 - \sigma} c_j^{1-\sigma} v_j(L_j), \quad 0 < \sigma < 1, \quad \sigma > 1, \quad (5.7) \]
\[ u_j(c_j, L_j) = \log(c_j) + v_j(L_j), \quad \sigma = 1, \quad (5.8) \]

which involves heterogeneity only with respect to labor supply decisions.

5.2. Linear approximation

Approximating (5.5)–(5.6) near the steady state values and assuming that preferences are (5.8), Rebelo (1987b) shows that aggregate consumption and labor input obey

\[ -\hat{c} = \hat{\lambda}, \quad (5.9) \]
\[ \hat{N} = [\hat{\lambda} + \hat{\lambda}, + (1 - \alpha) \hat{k}, + (\alpha - 1) \hat{N}] \psi, \quad (5.10) \]

where

\[ \psi = \sum_{j=1}^{J} \left( \frac{\theta_j \sigma_j N_j}{N} \right) \left( \frac{N_j}{1 - N_j} \right)^{-1} \quad \text{and} \quad \nu_j = -\frac{D^2 v_j(L_j) L_j}{D v_j(L_j)} > 0. \]
These conditions indicate that the near steady state behavior of aggregate consumption—given shadow prices—is unchanged from the single agent model, which is a consequence of the fact that all agents have the same elasticity of intertemporal substitution. On the other hand, there is an altered aggregate elasticity of labor supply. If all agents are alike, then the supply elasticity is simply equal to $\psi = \left[ \nu N/(1 - N) \right]^{-1}$, which is the elasticity of marginal utility of leisure multiplied by the ratio of hours worked to hours of leisure. In the model with heterogeneity, the aggregate labor supply elasticity ($\psi$) is a share weighted average elasticity across the participants in the economy, where the individual labor supply elasticities are $[\nu_j N_j/(1 - N_j)]^{-1}$ and the shares in aggregate effort supply are $[\theta_j N_j, \theta_j N_j]$.\(^{19}\)

Expressions (5.9) and (5.10) make clear that, in the absence of heterogeneity in preferences ($\nu_j = \nu$ for all $j$), redistributions of capital or mean preserving spreads in skill distribution do not influence the near steady state dynamics of the system. In other words, the basic model studied in the companion essay can be viewed as an economy composed of an arbitrary number of agent types which differ with respect to their wealth and labor skills.

5.3. Quantitative implications

Kydland (1984) studied the effects of labor heterogeneity in the labor market in the context of the Kydland and Prescott (1982) model. Using the preceding results, we can easily discuss the implications of Kydland’s heterogeneity pattern in the basic neoclassical model. Naturally our results cannot be directly compared to those obtained by Kydland since his model has a more complex preference and production structure.

Kydland (1984) divides the population into two groups of agents of equal size—skilled and unskilled workers. Skilled workers are twice as productive as unskilled workers and work 20 percent more time. The two groups share the same preferences. In terms of our notation this corresponds to: $\theta_1 = \theta_2 = 1/2$, $\mathcal{S}_1 = 2/3$, $\mathcal{S}_2 = 4/3$, $N_2 = 1.2 N_1$, where 2 denotes skilled and 1 unskilled workers.

The introduction of this heterogeneity pattern decreases the elasticity of aggregate hours weighted in terms of efficiency units.\(^{20}\) In consequence the variance of labor productivity decreases. Since the elasticity of (unweighted) hours is the same in both economies\(^{21}\) the ratio of the variance of hours to

\(^{19}\)Notice that there are two elasticities of labor supply—the one that pertains to hours in efficiency units and the one that refers to unweighted hours.

\(^{20}\)The value of $\psi$ for the economy without heterogeneity is $[1 - (1.13 N_1)]/\nu 1.1 N_1$ while the comparable value in the absence of heterogeneity (i.e., for $\mathcal{S}_1 = \mathcal{S}_2 = 1$, $N = 1.1 N_1$) is $(1 - 1.1 N_1)/\nu 1.1 N_1$.

\(^{21}\)If we assume the number of hours worked to be identical in the two economies the elasticity of hours worked is in both cases $[1 - (1.1 N_1)]/\nu 1.1 N_1$. 

variance in productivity is higher in the presence of this type of heterogeneity, which accords with Kydland’s findings.

The ability to study economies with heterogeneous agents enlarges substantially the class of environments that we can study in a tractable manner. Extensions of the economies that we consider to multi-country settings or introduction of financial intermediation are two examples of application of the techniques discussed in this section and in Rebelo (1987b).

6. Conclusions

This paper outlines some new directions for research into real business cycles. By focusing on these new directions, we do not mean to downplay the prospects for valuable extensions to multiple consumption and capital goods, along the lines initiated by Kydland and Prescott (1982) and Long and Plosser (1983). Rather it is clear from those earlier efforts, from this paper and from other papers in this volume, that it is feasible to explore a wide range of economic hypotheses within the neoclassical perspective.

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