Abstract: The bond yield dynamics implied by a welfare-maximizing monetary policy, and its credibility, are explored in general equilibrium. Credibility is captured by a regime change from discretion to commitment. Optimal monetary policy determines the inflation risk premium: If the elasticity of intertemporal substitution is lower than the elasticity of substitution across goods, the premium is negative. Credibility reduces the inflation risk and bond risk premia. A calibration implies lower yield spreads, less volatile yields, and reduced deviations from the expectations hypothesis under commitment. The model suggests an explanation for changes in yield dynamics in the U.S. for different policy regimes.
Dear David,

Following your request, I am submitting my paper “Bond Risk Premia and Optimal Monetary Policy” for possible publication in the Review of Economic Dynamics. This paper links previous theoretical work on optimal monetary policy to long-term interest rates. It analyzes the implications of welfare-maximizing policies under discretion and commitment on the compensations for risk in long-term government bonds.

The paper (or closely related research) has not been published or accepted for publication. It is based on the main chapter of my doctoral dissertation at Carnegie Mellon University. I received the “Alexander Henderson Award for Excellence in Economic Theory” at Carnegie Mellon and the “Best Doctoral Tutorial Paper Prize” at the European Finance Association Meeting 2006, for earlier versions of this work.

Sincerely,

Francisco Palomino
Bond Risk Premia and Optimal Monetary Policy

Francisco Palomino *

July 15, 2009

*This paper is based on the first chapter of my dissertation at Carnegie Mellon University. It previously circulated as “Interest Rates, Bond Premia and Monetary Policy.” I am especially grateful to Burton Hollifield and Chris Telmer for their advice and guidance. I also thank Michael Gallmeyer, Vincent Glode, Marvin Goodfriend, Bennett McCallum, Monika Piazzesi, Valery Polkovnichenko, Bryan Routledge, Martin Schneider, Adrien Verdelhan, Marno Verbeek, Stanley Zin, and participants at the Friday Lab Meetings, the International Trade and Finance seminar at Universitat Pompeu Fabra, the EFA Doctoral Tutorial 2006, the Finance seminar series at Carnegie Mellon University, the Federal Reserve Bank of New York, the Board of Directors of the Federal Reserve System, Columbia University, University of Chicago, University of Michigan, University of Texas at Austin, the Western Finance Association Meeting 2007 and the UCLA Conference on the Interaction Between Bond Markets and the Macro-Economy for their helpful comments and suggestions. All errors are my sole responsibility. Address: The University of Michigan, Ross School of Business, Ann Arbor, MI 48109; Tel: (734) 615-4178; E-mail: fpal@bus.umich.edu; http://webuser.bus.umich.edu/fpal/.
Abstract

The bond yield dynamics implied by a welfare-maximizing monetary policy, and its credibility, are explored in general equilibrium. Credibility is captured by a regime change from discretion to commitment. Optimal monetary policy determines the inflation risk premium: If the elasticity of intertemporal substitution is lower than the elasticity of substitution across goods, the premium is negative. Credibility reduces the inflation risk and bond risk premia. A calibration implies lower yield spreads, less volatile yields, and reduced deviations from the expectations hypothesis under commitment. The model suggests an explanation for changes in yield dynamics in the U.S. for different policy regimes.

JEL Classification: D51, E43, E52, G12.

Keywords: Affine term structure, general equilibrium, time-varying term premia, monetary policy, discretion vs. commitment.
1 Introduction

Recent contributions to the term structure literature such as Dai and Singleton (2002) and Duffee (2002) provide models that successfully reproduce salient properties of the term structure of interest rates. Upward-sloping yield curves, volatile long-term bond yields, and time-varying bond risk premia can be captured by no-arbitrage affine models with flexible prices of risk. This achievement, however, has been accomplished by specifying bond yields as functions of latent variables with no evident links to economic fundamentals. As a result, the underlying sources of variation in bond yields and bond risk premia still need to be explained. What are the economic factors driving the behavior of long-term interest rates? An answer to this question requires establishing a link between interest rates, consumption and production decisions, and government policy. In particular, monetary policy, as an important determinant of inflation and, arguably, economic activity, might play an important role in explaining the rich dynamics of long-term interest rates.

Studies such as Piazzesi (2005) have made significant progress in analyzing the link between monetary policy and bond yield dynamics. In the spirit of Taylor (1993), these studies have introduced monetary policy to the analysis of the term structure by incorporating policy rules to yield curve models. This approach captures an implementation aspect of the policy: The monetary authority sets a short-term interest rate using a rule that responds to macroeconomic and/or financial information. However, the approach is silent about the underlying policy objectives. In this sense, it is inadequate to gain insights into how monetary policy objectives affect the behavior of interest rates.

This paper analyzes the effects of optimal monetary policy on the term structure of interest rates. More specifically, it asks how a welfare-maximizing policy and changes in its credibility affect bond risk premia. The paper explores the economic foundations linking
bond yields, a welfare objective for monetary policy, and the policy’s ability to influence economic agents’ expectations. A welfare objective represents a reasonable goal for the monetary authority and, therefore, becomes an appropriate tool to investigate the impact of monetary policy on financial assets. The analysis also can be useful to understand fundamental issues in finance and macroeconomics. For instance, the portfolio choice implications of inflation, the hedging properties of real and nominal bonds, or the bond-market channel of transmission of monetary policy can be explored in this framework.

I develop a general equilibrium model that links monetary policy to bond yields. The model builds on the standard framework for monetary policy analysis presented in Clarida, Galí and Gertler (1999) or Woodford (2003) and extends it to price bonds. Sections 2 and 3 present a simplified model and an extended model, respectively. The simplified model allows us to gain intuition about the main results. The extended model is built to capture salient bond-pricing facts, i.e., upward sloping yield curves and time-varying bond risk premia. This is mainly achieved by incorporating stochastic external habit formation in preferences.¹

A welfare-maximizing goal for monetary policy, in the model, is equivalent to aiming for the joint stabilization of output and inflation. Monetary policy influences real activity due to nominal rigidities in an environment of monopolistic competition in the production sector. These rigidities, in combination with shocks to production markups, generate a tradeoff between output and inflation that makes it difficult to achieve the perfect stabilization of the economy. As a result, markup shocks have effects on real activity and inflation, whose size depends on the credibility of the policy regime. Following Kydland and Prescott (1977), credibility is captured allowing monetary policy to be conducted under commitment or discretion. Under commitment, the policy is perfectly credible and affects private sec-

tor's expectations on future economic conditions. Under discretion, the monetary authority takes private sector's expectations as given. Consequently, the comparison of the equilibrium characteristics of long-term bond yields across the two regimes captures the effects of policy credibility on the term structure of interest rates.

The model delivers the affine term structure presented in section 4, similar to Duffie and Kan (1996). It has the advantage that equilibrium bond yields can be expressed as linear functions of macroeconomic factors, with factor loadings depending on the policy regime and deep economic parameters, e.g., preference and production parameters. It then provides the necessary link to understand the economic sources of risk driving the behavior of long-term bonds.

The macroeconomic and bond-pricing implications of optimal monetary policy are presented in section 5. Welfare maximization prescribes an optimal tradeoff between output and inflation. The tradeoff implies that for each one percent of inflation, the monetary authority should react by contracting output (or output growth if the policy is under commitment) by a number of percentage points equal to the elasticity of substitution across goods (ESG).\(^2\) In equilibrium, inflation and output (consumption) are affected by markup shocks. For instance, a positive markup shock generates positive inflation and, given the policy, a negative effect on output. Since the marginal utility of consumption, in real and nominal terms, depends on consumption and inflation, the real and nominal components of the discount factor and thus the valuation of financial assets with real and nominal payoffs are affected by markup shocks. In short, investors require a compensation for holding assets affected by markup shocks since these shocks are a source of systematic risk, where the sign and size of the compensation are determined by the optimal policy and its credibility.

The optimal nature of the policy determines the sign of the compensation for markup

\(^2\)In the extended model, it is the growth-adjusted elasticity of substitution across goods.
shocks in assets with nominal payoffs (e.g., nominal bonds). The compensation is positive or negative depending on whether the effects of markup shocks on inflation are greater or lower than those on consumption, in marginal utility terms. This can be understood by decomposing the stochastic discount factor in its real and nominal components. A shock that generates inflation decreases consumption and therefore increases the marginal utility of consumption. The effect on the real component is offset by the inflation adjustment generated by the nominal component in the discount factor. Given the optimal inflation-consumption tradeoff, the real component effect outweighs the nominal one if the elasticity of intertemporal substitution of consumption (EIS) is lower than the ESG. In this case, asset payoffs positively correlated with inflation involve a negative risk premium. Inflation has large negative effects on consumption growth, it induces a high demand for assets that hedge inflation risk, and reduces the expected excess return on the assets. The effect is the opposite if the EIS is greater than the ESG. The inflation effect on marginal utility outweighs the consumption effect, and market participants demand high expected excess returns to hold financial assets exposed to inflation risk.

Policy credibility determines the size of the compensation for markup shocks. The compensation is larger under discretion than under commitment. Under commitment, anchoring private sector’s expectations works as an additional mechanism to stabilize output and inflation. Therefore, the impact of markup shocks on economic performance declines and the price of this risk decreases.

Bond risk premia are affected by the optimal policy and its credibility through the implied compensation for markup shocks. Bond returns depend on the evolution of the marginal utility of consumption over time. It implies that bond returns are sensitive to markup shocks, and the optimal policy shapes the covariance of bond returns with the nominal discount factor (i.e., bond risk premia) induced by markup shocks. The effect of credibility
on bond risk premia is reflected in the difference of this covariance across discretion and commitment. The magnitude and direction of this difference are obtained from a model calibration.

The model is calibrated to the U.S. economy for the fiat money period (1971-2007) that followed the collapse of the Bretton Woods agreement. Some macroeconomic and bond yield properties in the data are matched to their model counterparts under discretion. Subsequently, a policy experiment is conducted to observe the changes implied by a regime switch from discretion to commitment. The persistence and standard deviation of inflation significantly decline under commitment. This reduction implies less volatile bond yields and lower average spreads between long-term and short-term bonds. The latter reflects reduced bond risk premia. Under commitment, investors demand lower expected returns for holding long-term bonds since bond returns are less sensitive to markup shocks. In addition, commitment induces changes in the covariance between excess bond returns and bond spreads, resulting in reduced deviations from the expectations hypothesis. These changes can be understood by the different response of inflation to markup shocks under commitment.

Section 6 analyzes yield properties for different monetary policy regimes in the United States. The analysis is performed to provide potential credibility-based explanations to changes in bond yield dynamics over time. High policy credibility might explain the significant inverted yield curve that characterized the Gold Standard regime (1880-1913), or the low volatile yields and low bond spreads during the Bretton Woods agreement. On the other hand, the increased bond risk premia observed during the Greenspan era sheds some doubts about an improved policy commitment during this period. However, an explanation based on policy credibility can be given to the Greenspan conundrum, since higher commitment in monetary policy reduces the response of long-term rates to economic shocks. Section 7 concludes, and the appendix contains relevant proofs.
Related Literature

This paper joins a growing body of work that relates the term structure of interest rates to monetary policy. Diebold, Piazzesi and Rudebusch (2005) summarize recent attempts in empirical and theoretical grounds to understand the joint dynamics of the term structure of interest rates, macroeconomic variables, and monetary policy. For instance, Ang and Piazzesi (2003) and Piazzesi (2005) show how economic information and monetary policy help us improve the empirical fitting of the yield curve relative to successful latent factor models. Simultaneously, Ang, Dong and Piazzesi (2007) and Bikbov and Chernov (2006) recognize the absence of arbitrage in bond prices as an additional restriction to identify and estimate monetary policy rules.

Theoretical contributions such as Bekaert, Cho and Moreno (2006), Hör Dahl, Tristani and Vestin (2006), Wu (2006) and Rudebusch and Wu (2007) link monetary policy and the term structure through structural macro models. This approach provides macro factors and additional restrictions for no-arbitrage term structure models. However, these structural models are not able to capture time-variation in risk premia from first principles and, therefore, their ability to explain deviations from the expectations hypothesis is limited.

McCallum (1994) proposes that observed deviations from the expectations hypothesis are compatible with time-varying risk premia and a monetary policy rule responding to bond yields. Following this direction, Gallmeyer, Hollifield and Zin (2005) explore models implying time-varying risk premia which deliver affine term structures. Ravenna and Seppälä (2007) propose a New-Keynesian model that generates time-varying term premia to show that the systematic component of monetary policy can explain the rejection of the expectations hypothesis.

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3The frameworks in Bekaert, Cho and Moreno (2006) and Wu (2006) imply that the expectations hypothesis holds. Hör Dahl, Tristani and Vestin (2006) impose an exogenous time-varying market price of risk, which is not consistent with the endogenous discount factor. The theoretical foundations in Rudebusch and Wu (2007), as they mention, are tenuous, as an exogenous time-varying market price of risk is imposed.
hypothesis. While they attribute deviations from the expectations hypothesis to technology and preference uncertainty, Buraschi and Jiltsov (2005) find that a time-varying inflation risk premium and monetary policy shocks are important to explain the rejection of the expectations hypothesis. Rudebusch and Wu (2008) examine the recent shift in the dynamics of the term structure of interest rates and suggest a link between this shift and changes in the perception of the dynamics of the inflation target. None of these papers incorporate a monetary policy with the explicit objective of maximizing welfare and, as a consequence, do not allow us to identify the welfare implications of the policy and the effects on the equilibrium characteristics of bond yields of changes in policy credibility. Empirical evidence relating changes in the predictive power of the yield curve to changes in the credibility of the monetary regime is presented in Bordo and Haubrich (2004).

2 A Simplified Model

This section presents a macroeconomic model to characterize the link between compensations for risk in long-term bonds and optimal monetary policy. The model provides the necessary intuition to understand the most general results and can be seen as a simplified version of the model presented in the next section.\(^4\) The micro foundations for the reduced-form equations presented here are derived in the next section in the context of the more general model.

Following Clarida, Galí and Gertler (1999), consider the problem faced by a monetary policy maker. The policy maker maximizes welfare and determines the optimal levels of log-output, \(y_t\), inflation, \(\pi_t\) and the short-term nominal interest rate, \(i_t\). The welfare maximization problem takes into account the restrictions imposed by the optimal behavior of

\(^4\)The difference between this model and the more general model is the specification of preferences and exogenous processes. Extending the model in these two dimensions allows us to capture a significant size and variation in expected excess returns on bonds.
households and firms and can be written as

$$
\max_{\{x_t, \pi_t, i_t\}} \quad - \frac{1}{2} \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t \left( \kappa x_t^2 + \theta \pi_t^2 \right) \right] 
$$

subject to

$$
e^{-i_t} = \mathbb{E}_t [M_{t,t+1}] 
$$

and

$$
\pi_t = \kappa x_t + \beta \mathbb{E}_t [\pi_{t+1}] + u_t, 
$$

where

$$
x_t \equiv y_t - y_f^t 
$$

is the output gap that captures deviations of output from the potential output $y_f^t$. The precise definition of $y_f^t$ is presented in the next section.

Equation (1) is the welfare function. As will be shown for the general model, it can be obtained from an approximation of household’s utility. It can be seen as a loss function that is minimized when the output gap and inflation simultaneously reach their zero targets. For simplicity and consistency, the relative weights of the output gap and inflation are denoted by $\kappa$ and $\theta$, respectively.

Equation (2) represents the relevant optimality condition from the maximization of household’s utility. It provides us with the price of a one-period nominal bond and relates the implied one-period nominal interest rate, $i_t$, to the intertemporal marginal rate of substitution of consumption in nominal terms, $M_{t,t+1}$. The marginal rate of substitution is the discount factor that allows us to price all financial assets and must reflect the compensations for risk in the economy. By relating the discount factor to the marginal rate of substitution,
we obtain compensations for risk that depend on monetary policy.

We assume power utility from consumption in this section and, as a result, the discount factor becomes

$$M_{t,t+1} = \beta \left( \frac{Y_{t+1}}{Y_t} \right)^{-\gamma} \left( \frac{P_{t+1}}{P_t} \right)^{-1}.$$  \hfill (5)

This equation takes into consideration that, in equilibrium, consumption is equal to output, $Y_t$, and the nominal price of consumption is $P_t$. Notice that $y_t \equiv \log Y_t$ and $\pi_t \equiv \log P_t - \log P_{t-1}$.

Equation (3) describes the optimality conditions for firms. This equation is obtained from the maximization of profits in an environment of monopolistic competition with price rigidities. It links inflation to the output gap, expected future inflation and “markup” shocks, $u_t$. These shocks are the only source of risk in the simplified model and follow the autoregressive process

$$u_t = \phi_u u_{t-1} + \sigma_u \varepsilon_{u,t},$$

where $\varepsilon_{u,t} \sim \text{IID} \mathcal{N}(0, 1)$.

The policy maker can solve the maximization problem (1) under discretion or under commitment. Under discretion, the policy maker is unable to affect expectations of households and firms about future economic conditions. The policy problem, therefore, reduces to

$$\max_{\{x_t, \pi_t\}} \quad -\frac{1}{2} \left( x_t^2 + \pi_t^2 \right) \quad \text{subject to} \quad \pi_t = \kappa x_t + F,$$

where $F = \beta \mathbb{E}_t[\pi_{t+1}] + u_t$ is taken as given. Solving the problem, optimality implies

$$\pi_t = -\frac{1}{\theta} x_t,$$
and we obtain solutions for the output gap and inflation given by

\[ x_t^d = -\frac{\theta}{1 + \kappa \theta - \beta \phi_u} u_t \quad \text{and} \quad \pi_t^d = \frac{1}{1 + \kappa \theta - \beta \phi_u} u_t. \]

Under commitment, expectations about future economic conditions are affected by the policy and the policy maker maximizes (1) subject to (3) for all \( t \). It implies

\[ \pi_t^c = -\frac{1}{\theta} (x_t^c - x_{t-1}^c), \]

for all \( t > 0 \). The solutions for the output gap and inflation are

\[ x_t^c = x_m x_{t-1}^c + x_u u_t \quad \text{and} \quad \pi_t^c = x_m \pi_{t-1}^c - \frac{1}{\theta} x_u \Delta u_t, \]

respectively, where

\[ x_m = \frac{1 + \kappa \theta + \beta - \sqrt{(1 + \kappa \theta + \beta)^2 - 4\beta}}{2\beta} \quad \text{and} \quad x_u = -\frac{\theta}{1 + \kappa \theta - \beta \phi_u + \beta(1 - x_m)}. \]

Notice the difference in the processes for the output gap and inflation under the two regimes. Under discretion, the output gap and inflation are proportional to the shock \( u_t \). Under commitment, the processes can be written as an autoregressive process and inflation not only depends on the current shock but also on the lagged value of the shock. In addition, the response of the two processes to the current shock is always lower under commitment than under discretion. This lower response turns out to be critical to understand the implications of the two regimes on asset prices.

\[ ^5 \text{The optimality condition that applies to } t = 0 \text{ is } \pi_0 = -\frac{1}{\theta} x_0. \text{ This optimality condition generates time inconsistency. We abstract from this problem assuming an } x_{-1} \text{ such that } \pi_0 = -\frac{1}{\theta} (x_0 - x_{-1}) \text{ that avoids the time inconsistency. See Woodford (2003) for details.} \]
Market Price of Risk

The market price of risk for this economy is the compensation per unit of a markup shock. The implications for the market price of risk under discretion and commitment can be obtained from the analysis of the marginal rate of substitution under the two regimes. From equation (5), we can represent the discount factor as

\[ -\log M_{t,t+1} = -\log \beta + \gamma(\Delta y_{t+1}^{f} + \Delta x_{t+1}) + \pi_{t+1}, \]

where \( \Delta \) is the difference operator. For simplicity, assume that the potential output \( y_{f}^{t} \) is constant and, therefore, \( \Delta y_{f}^{t} = 0 \) for all \( t \). It allows us to focus on the effects of the policy on the output gap and inflation.

Replacing the processes for the output gap and inflation for the two regimes in the equation above, the stochastic component of the marginal rate of substitution implies the market prices of risk

\[ \lambda_{d} = -\frac{\gamma - 1}{1 + \kappa \theta - \beta \phi_{u}}, \quad \text{and} \quad \lambda_{c} = -\frac{(\gamma - 1)}{1 + \kappa \theta - \beta \phi_{u} + \beta (1 - x_{m})}, \]

---

To see this, consider a financial asset with log-normal return \( r_{t} \). In equilibrium, it satisfies

\[ 1 = \mathbb{E}_{t}[\exp(\log M_{t,t+1} + r_{t+1})]. \]

Since \( M_{t,t+1} \) is log-normal and, from Equation (2), \( i_{t} = -\mathbb{E}_{t}[\log M_{t,t+1}] - \frac{1}{2}\var_{t}(\log M_{t,t+1}) \), the solution to the expectation above implies

\[ \mathbb{E}_{t}[r_{t+1} - i_{t}] + \frac{1}{2}\var_{t}(r_{t+1}) = -\text{cov}_{t}(\log M_{t,t+1}, r_{t+1}) \leq \sqrt{\var_{t}(\log M_{t,t+1}) \var_{t}(r_{t+1})}. \]

Ignoring the Jensen’s inequality term \( \frac{1}{2}\var_{t}(r_{t+1}) \), it follows that the maximum compensation for unit of risk (market price of risk) is given by \( \sqrt{\var_{t}(\log M_{t,t+1})} \).
under discretion and commitment, respectively. It follows that
\[
\frac{\lambda^c}{\lambda^d} = \frac{1 + \kappa \theta - \beta \phi_u}{1 + \kappa \theta - \beta \phi_u + \beta (1 - x_m)} < 1,
\]
since \(0 < x_m < 1\). That is, the market price of risk is always lower (in absolute value) under commitment than under discretion because inflation and the output gap are less sensitive to the shock. This is a direct consequence of the benefits of commitment over discretion.

In addition, the sign of the market price of risk under both, discretion and commitment, depends on whether \(\gamma \theta - 1\) is positive or negative. It is positive if the marginal utility effect of the shock on the output gap, \(\gamma \theta\), is lower than the effect of the shock on inflation. That is, the sign of the market price of risk depends on whether the real effects of the shock are higher or lower than its nominal effects. This sign becomes important to understand the hedging properties of financial assets in terms of deep economic parameters.

**Bond Risk Premia**

We now analyze the effect of the policy on the compensations for risk in two-period bonds. We extend the analysis to bonds of any maturity in the extended model. Consider the interest rate associated to a two-period bond, \(i_t^{(2)}\). From the optimality conditions of the household’s problem we find that the price of this bond is
\[
e^{-2i_t^{(2)}} = \mathbb{E} [M_{t,t+2}] = \mathbb{E} [M_{t,t+1}e^{-i_{t+1}}],
\]
where the second equality results from the equilibrium condition \(M_{t,t+2} = M_{t,t+1}M_{t+1,t+2}\) and an application of the law of iterated expectations.

By solving the expectation above, we can write the average spread between the two-period
and the one-period interest rates (plus a correction term) as

\[ E[i_t^{(2)} - i_t] + \frac{1}{4} E[\text{var}_t(i_{t+1})] = \frac{1}{2} E[\text{cov}_t(\log M_{t,t+1}, i_{t+1})]. \]

The covariance term is the compensation for risk embedded in two-period bonds. From the equilibrium discount factor and interest rates, it can be shown that the compensations for risk in the two regimes satisfy

\[
\frac{E[\text{cov}_t(\log M_{t,t+1}^c, i_{t+1}^c)]}{E[\text{cov}_t(\log M_{t,t+1}^d, i_{t+1}^d)]} = \left( \frac{1 + \kappa \theta - \beta \phi_u}{1 + \kappa \theta - \beta \phi_u + \beta (1 - x_m)} \right)^2 \left( \frac{(\gamma \theta - 1)(1 - \phi_u - x_m)}{(1 - \phi_u)\gamma \theta + \phi_u} \right) < 1.
\]

Therefore, if \( \gamma \theta > 1 \), the compensation is positive and lower under commitment than under discretion. Figure 1 presents the difference in the compensation for risk between commitment and discretion. The parameter values are standard in the monetary policy literature. The figure shows that the difference in compensations for risk increases with the autocorrelation of the shocks \( \phi_u \) and the coefficient of relative risk aversion \( \gamma \), and decreases with the sensitivity of inflation to output gap \( \kappa \).\(^7\) In summary, this section shows us that the stabilization and welfare benefits from commitment on output and inflation translate into lower market prices of risk and lower compensations for risk in two-period bonds.

The model presented here is a very stylized version of the economy that abstracts from consumption growth and the existence of additional shocks. In the next section, a model with different preference specifications, consumption growth, and two additional shocks is derived. It will allow us to gain additional insights on the effects of optimal monetary policy on the time variation of bond expected returns.

\(^7\)However, it does not take into account the dependence of \( \kappa \) on \( \gamma \) as implied by the extended model.
Figure 1: Difference in compensations for risk between commitment and discretion. It corresponds to $E[\text{cov}_t(\log M_{c,t+1}, i_{d,t+1}^c)] - E[\text{cov}_t(\log M_{c,t+1}, i_{d,t+1}^d)]$. The baseline parameter values are $\beta = 0.99$, $\gamma = 0.5$, $\kappa = 0.024$, $\theta = 10$, $\phi_u = 0.88$ and $\sigma_u = 0.001$.

3 The Extended Model

I model a discrete-time closed economy populated by households that derive utility from the consumption of an aggregate of differentiated goods and disutility from labor. Households provide labor to firms that maximize profits in a monopolistic competitive setting characterized by price rigidities and a labor-only technology. In this economy, monetary policy is conducted to maximize welfare under two possible regimes: discretion or commitment. The two regimes are modeled independently and the equilibrium characteristics of the two respective economies are compared.\(^8\) There are three sources of uncertainty: preference, productivity, and markup shocks, which for simplicity are assumed to be uncorrelated. Throughout the paper the subindex $t$ denotes time and $\Delta$ is the difference operator.

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\(^8\)The potential transition between regimes is not modeled. This transition is an interesting extension since agents who can anticipate regime changes may modify their behavior, which in turn may affect economic performance. See Davig and Leeper (2008) for a potential approach to endogenize changes in policy regimes.
3.1 Households

Households exhibit preferences over an infinite horizon on consumption, $C$, and labor, $h(j)$, which is provided to a continuum of firms owned by the households and indexed by $j \in [0, 1]$. The consumption good is the Dixit-Stiglitz aggregate of a continuum of differentiated goods, $C(j)$, given by

$$C_t = \left[ \int_0^1 C_t(j) \frac{\theta_t-1}{\theta_t} \, dj \right]^{\theta_t-1},$$

where $\theta_t > 1$ is the potentially time-varying elasticity of substitution between differentiated goods. The expected utility of households is represented by

$$E \left[ \sum_{t=0}^{\infty} \beta^t \left( \frac{1}{1-\gamma} C_t^{1-\gamma} - \frac{1}{1+\omega} \int_0^1 h_t(j)^{1+\omega} \, dj \right) \right],$$

where $\gamma^{-1}$ and $\omega^{-1}$ are the elasticities of intertemporal substitution of consumption and labor, respectively. The utility derived from consumption is affected by the process $Q$. This process depends on aggregate consumption, $\tilde{C}$ and preference shocks $\varepsilon_q \sim \text{IID} \mathcal{N}(0, 1)$. Specifically, denoting the aggregate consumption growth by $\Delta\tilde{c}_t \equiv \log \tilde{C}_t - \log \tilde{C}_{t-1}$, the process for $q \equiv \log Q$ is

$$q_{t+1} = q_t + \eta \Delta\tilde{c}_t + (1 + K_q \Delta\tilde{c}_t)^{1/2} \sigma_q \varepsilon_{q,t+1}. \tag{8}$$

Given its dependence on aggregate consumption, $Q$ can be seen as an external habit where $\eta$ is the average sensitivity of changes in the habit to lagged consumption growth and $K_q$ captures the dependence of the volatility of the habit on lagged consumption growth. The second term in equation (7) is the aggregate disutility from the labor supplied to the production of differentiated goods.
Households choose contingent consumption and labor streams to maximize their expected utility subject to the contingent budget constraints

$$\int_{0}^{1} P_t(j)C_t(j) dj + W^+_t \leq W^-_t + \int_{0}^{1} w_t(j) h_t(j) dj + \int_{0}^{1} \Psi_t(j) dj - \tau_t,$$

for all $t$, where $P(j)$ denotes the prices of the differentiated goods. According to this equation, consumption and investment in financial instruments, $W^+_t$, are constrained by previous period financial wealth plus return, $W^-_t$, labor income from the production sector, $w_t(j)h_t(j)$, profits from the firms supplying the differentiated goods, $\Psi_t(j)$ and taxes, $\tau_t$. Financial markets are complete and nominal and real zero-net-supply default-free bonds are traded for all maturities.

Denoting the price of the consumption good by $P$, the value of the consumption good is $PC = \int_{0}^{1} P(j)C(j) dj$,\(^9\) and we can replace the set of budget constraints with the intertemporal budget constraint

$$\mathbb{E} \left[ \sum_{t=0}^{\infty} M_{0,t} P_t C_t \right] \leq W^-_0 + \mathbb{E} \left[ \sum_{t=0}^{\infty} M_{0,t} \left( \int_{0}^{1} w_t(j) h_t(j) dj + \int_{0}^{1} \Psi_t(j) dj + \tau_t \right) \right],$$

where $M_{t,t+n} > 0$ is the nominal pricing kernel that discounts nominal cashflows at time $t+n$ to time $t$.

The solution to the household’s problem implies a pricing kernel given by the marginal rate of substitution of consumption in nominal terms. It is

$$M_{t,t+n} = \beta^n \left( \frac{C_{t+n}}{C_t} \right)^{-\gamma} \left( \frac{Q_{t+n}}{Q_t} \right)^{-1} \left( \frac{P_{t+n}}{P_t} \right)^{-1}. \quad (9)$$

\(^9\)Equation (6) can be seen as the production function of $C_t$ with inputs $C_t(j)$ for a competitive producer with optimal profits of zero. It implies $P_t = \left[ \int_{0}^{1} P_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$, which is the minimum cost of the basket of differentiated goods.
In addition, the price at time $t$ of a nominal bond with maturity at $t + n$ can be obtained from the $n$-period marginal rate of substitution as

$$b_t^{(n)} = \mathbb{E}_t [M_{t,t+n}].$$  

(10)

In particular, households can invest in a one-period nominal bond which pays the nominal one-period rate $i_t$. It satisfies

$$e^{-i_t} = \mathbb{E}_t [M_{t,t+1}] = \mathbb{E}_t [\exp(\log \beta - \gamma \Delta c_{t+1} - \Delta q_{t+1} - \pi_{t+1})],$$

(11)

where $\pi_{t+1} \equiv \log P_{t+1} - \log P_t$ is the rate of inflation. This equation shows that habit persistence makes the short-term interest rate depend not only on expectations about future consumption growth and future inflation but also on the current level of consumption growth and preference shocks.

To complete the analysis of the household’s problem, the intratemporal marginal rate of substitution

$$\frac{w_t(j)}{P_t} = h_t(j)\omega C_t Q_t$$

(12)

provides the tradeoff between consumption and labor that must be satisfied in equilibrium.

### 3.2 Production Sector

The production of differentiated goods is characterized by monopolistic competition and price rigidities and is affected by productivity and markup shocks. There is a continuum of suppliers of differentiated goods $j \in [0, 1]$ who have market power to set their product prices
and face demand curves

\[ Y_t(j) = Y_t \left( \frac{P_t(j)}{P_t} \right)^{-\theta_t}, \quad (13) \]

where \( Y_t = \left[ \int_0^1 Y_t(j) \frac{\theta_t - 1}{\theta_t} dj \right]^{\frac{\theta_t}{\theta_t - 1}} \) is the aggregate output.\(^{10}\)

The production function of a differentiated good is

\[ Y_t(j) = A_t h_t(j), \quad (14) \]

where growth in labor productivity, \( \Delta a_t \equiv \log A_t - \log A_{t-1} \) is modeled as the autoregressive process

\[ \Delta a_{t+1} = (1 - \phi_a) g_a + \phi_a \Delta a_t + (1 + K_a \Delta a_t)^{1/2} \sigma_a \varepsilon_{a,t}, \quad (15) \]

with \( \varepsilon_a \sim \text{IIDN}(0, 1) \). The volatility of the productivity shocks is potentially dependent on productivity growth.

Using the production function (14) and the marginal rate of substitution (12) we can write the real marginal production cost of a differentiated good \( s_t(j) \equiv \frac{1}{P_t} \frac{\partial (w_t(j) h_t(j))}{\partial Y_t(j)} \) as

\[ s_t(j) = \frac{1}{Y_t(j)} \left( \frac{Y_t(j)}{A_t} \right)^{1+\omega} Y_t^\gamma Q_t, \quad (16) \]

where \( \omega \) can be seen as the elasticity of the real marginal cost with respect to the supply of a differentiated good.

Now, with the purpose of facilitating the exposition of the production problem with

\(^{10}\)Demand curves can be obtained as the result of a profit maximization problem for a price-taking producer of good \( Y \) with aggregate production costs \( \int_0^1 P_t(j) Y_t(j) dj \).

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price rigidities, it is convenient to derive the aggregate output of a hypothetical economy with full price flexibility. This output is known as the natural rate of output or potential output and is a reference point to conduct monetary policy. Denoting this output by $Y_t^f$, the profit-maximization problem is

$$\max_{P_t(j)} (1 + \tau) P_t(j) Y_t^f(j) - w_t(j) h_t(j)$$

subject to (13) and (14). The parameter $\tau$ can be seen as a subsidy to production provided to eliminate the inefficiency of the potential output due to monopolistic competition. The solution to this problem implies $P_t(j) / P_t = 1 + \tau \theta_t j$, with the time-varying markup $\mu_t \equiv \theta_t / (\theta - 1)$ arising from market power. In order to eliminate (on average) the inefficiency of the natural rate of output, taxes are set such that

$$1 + \tau = \frac{\theta}{\theta - 1} = \mu,$$

where $\theta$ is the average elasticity of substitution across goods and $\mu$ is the implied long-term markup. Denoting the deviation of the markup from the long-term markup by $\epsilon_t \equiv \log(\mu_t) - \log(\mu)$, the optimality condition can be written as

$$P_t(j) / P_t = e^{\epsilon_t j}.$$ 

We refer to this deviation as a markup shock and define the exogenous process

$$\epsilon_{t+1} = \phi_t \epsilon_t + (1 + K_t \epsilon_t)^{1/2} \sigma_t \epsilon_{t,t+1},$$

(17)

with $\epsilon \sim \text{iid} \mathcal{N}(0, 1)$ and the volatility of the shocks depending on the level of $\epsilon_t$. 

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Under price flexibility, \( P_t(j) = P_t \), \( Y_t(j) = Y_t \) and the real marginal cost for all suppliers becomes \( s^f_t = e^{-\epsilon_t} \). Using this condition and equation (16), we obtain the output process

\[
y^f_t = \frac{1}{\omega + \gamma} \left[ -q^f_t + (1 + \omega)a_t - \epsilon_t \right],
\]

where \( y^f_t \equiv \log Y^f_t \). Besides the positive dependence on the level of technology and the negative dependence on the markup shock, the process for the natural rate of output depends on the natural level of habit, \( q^f_t \equiv \log Q^f_t \), which is the external habit that characterizes an economy with price flexibility.

At this point we can incorporate price rigidities. We follow the Calvo (1983) framework for staggered price setting, where only a fraction \( 1 - \alpha \) of suppliers is able to change prices optimally in a given period, while a fraction \( \alpha \) keeps last period prices adjusted by the long-term average inflation \( \Pi^* \). Within this framework, the optimal price set at time \( t \) solves the profit-maximization problem

\[
\max_{P_t(j)} \mathbb{E}_t \left[ \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} \left[ (1 + \tau) P_t(j)(\Pi^*)^{T-t} Y_{T\|t}(j) - w_{T\|t}(j) h_{T\|t}(j) \right] \right]
\]

subject to (13) and (14), for all \( T \). That is, suppliers that are able to reset prices choose a price that maximizes expected discounted profits taking into account the probability of not resetting that price in the future. It implies that output is

\[
Y_{T\|t}(j) = Y_T \left( \frac{P_t(j)(\Pi^*)^{T-t}}{P_T} \right)^{-\theta_T},
\]

with a similar reasoning for \( w_{T\|t}(j) \) and \( h_{T\|t}(j) \).

From the equations above, appendix I.1 shows that optimality in the production sector
implies\textsuperscript{11}

\[
\pi_t - \pi^* = \kappa(y_t - y^*_t) + \frac{\kappa}{\omega + \gamma}(q_t - q^*_t) + \beta_g \mathbb{E}_t[\pi_{t+1} - \pi^*],
\]  

(19)

where \( \beta_g \) is the discount factor adjusted by growth, \( \kappa \equiv \frac{(1-\alpha \beta_g)(1-\alpha)}{\alpha} \) and \( \zeta \equiv \frac{\omega + \gamma}{1 + \beta_g \omega} \).\textsuperscript{12}

This forward-looking equation captures the short-term tradeoff between output and inflation induced by price rigidities. It shows us that inflation is driven by expectations on future inflation, current and lagged deviations of output from the flexible-price output, as captured by \( y_t - y^*_t \) and \( q_t - q^*_t \), respectively.

To facilitate the exposition of the monetary policy problem, we define the output gap as \( x_t \equiv y_t - y^*_t - (\omega + \gamma)^{-1} \epsilon_t \), which is the deviation of actual output from the natural rate of output adjusted by the markup shock, and the habit gap, \( l_t \equiv q_t - q^*_t \), which is the deviation of the actual habit from the natural habit. As a result equation (19) can be written as

\[
\pi_t - \pi^* = \kappa x_t + \frac{\kappa}{\omega + \gamma} l_t + \beta_g \mathbb{E}_t[\pi_{t+1} - \pi^*] + \frac{\kappa}{\omega + \gamma} \epsilon_t,
\]  

(20)

where \( \pi^* \equiv \log \Pi^* \). The optimality condition for firms is similar to the one presented in section 2, adjusted to incorporate the habit gap.

\textsuperscript{11}This condition is known as the standard New-Keynesian Phillips Curve, in this case expanded to take into account the effect on inflation of the external habit.

\textsuperscript{12}The discount factor adjusted by growth is \( \beta_g = \beta \ast \exp \left( \frac{(1+\omega)(1-\gamma-\eta)}{\omega+\gamma+\eta} \beta \right) \). The term \( \zeta \) describes the degree of strategic complementarity between the price setting decisions of the suppliers of differentiated goods. It determines the size of the distortions caused by price rigidities and, therefore, how useful monetary policy can be. An economy is said to be characterized by strategic complementarity when \( \zeta < 1 \), since it implies \( \frac{dP(i)}{dP} > 0 \). In this case, optimal prices tend to follow the aggregate price level, distortions in relative prices caused by price rigidities are significant and, therefore, production decisions tend to differ considerably from the natural rate of output. Some authors refer to \( \zeta \) as the degree of real rigidity in the economy.
3.3 Monetary Policy

Monetary policy is conducted to maximize households’ welfare, that is, the expected utility of households. We assume that the monetary authority is a social planner that sets the levels of output, inflation, and the nominal one-period interest rate, and decides between following a policy under commitment or under discretion. While under discretion the monetary authority is unable to generate credibility, under commitment the policy is perfectly credible and, therefore, affects expectations of households and firms about future economic activity.

The monetary authority knows the structure of the economy, including the functional form (7) of the expected utility, the aggregate demand (11), and supply (20) conditions. As a result, the monetary policy problem can be written as

\[
\max_{\{\pi_t, y_t, i_t\}} \mathbb{E}\left[ \sum_{t=0}^{\infty} \beta^t \left( \frac{1}{1-\gamma} Y_t^{1-\gamma} - \frac{1}{1+\omega} \int_0^1 \left( \frac{Y_t(j)}{A_t} \right)^{1+\omega} dj \right) \right]
\]

subject to (11) and (20), for all \( t \). Appendix I.2 shows that maximizing welfare is equivalent to targeting inflation and output to minimize the loss function\(^{13}\)

\[
\frac{1}{2} \mathbb{E}\left\{ \sum_{t=0}^{\infty} \beta^t \left[ \left( x_t + \frac{1}{\omega + \gamma} l_t \right)^2 + \frac{\theta_g}{\kappa} (\pi_t - \pi^*)^2 \right]\right\},
\]

where \( \theta_g \equiv \theta \left( \frac{1-\alpha\beta}{1-\alpha\beta} \right) \) can be seen as the steady-state elasticity of substitution across goods adjusted by growth. From the loss function, welfare is maximized when inflation reaches the target \( \pi^* \) and output and habit gaps reach their zero targets. The monetary authority must assign a weight of \( \frac{\theta_g}{\kappa + \theta_g} \) to inflation stabilization and \( \frac{\kappa}{\kappa + \theta_g} \) to stabilize output. Habit

\(^{13}\)There is nothing in particular about solving the monetary authority problem using the loss function instead of the (approximated) welfare function. It is simply a convenient change of variable from output in the welfare function to output gap in the loss function to make evident the objective of the monetary authority.
formation in preferences captures the idea that deviations from the target output in the past affect household’s welfare today.

The solution to the monetary policy problem and, therefore, the equilibrium output, inflation, and interest rates depend on whether the monetary authority is committed to the long-term objective of welfare maximization. A policy under commitment has the ability that a policy under discretion lacks to affect expectations of households and firms.

The following two propositions present the equilibrium outcomes for output and inflation of policies followed under discretion and commitment, respectively. These propositions allow us to analyze the effects on interest rates of the two policies in the next two sections.

**Proposition 1.** *Optimal monetary policy under discretion implies that output growth and inflation follow the processes*

\[
\Delta y_t^d = \frac{1}{\omega + \gamma} \left[ -\Delta q_t^d + (1 + \omega)\Delta a_t \right] - \theta_g \Delta \pi_t^d
\]

and

\[
\pi_t^d = \pi^* + \pi_c^d \epsilon_t,
\]

respectively, where \( \pi_c^d = \frac{\kappa}{(\omega + \gamma)(1 + \kappa \theta_g - \beta_g \phi_e)} \).

**Proof.** See appendix I.3.

If the monetary policy is conducted under discretion, deviations of inflation from the target are proportional to markup shocks. It implies that the persistence of inflation is equal to the persistence of the shocks. Output growth depends on lagged output growth and is negatively affected by changes in inflation, with the size of the impact determined by the elasticity of substitution of differentiated goods.
Under commitment, output growth is affected directly by the level of inflation and inflation is determined by lagged inflation and changes in markup shocks, as the following proposition shows.

**Proposition 2.** Optimal monetary policy under commitment implies that output growth and inflation follow the processes

\[
\Delta y_c^e = \frac{1}{\omega + \gamma} \left[ -\Delta q_c^e + (1 + \omega) \Delta a_t \right] - \theta_g (\pi_c^e - \pi^*)
\]

(24)

and

\[
\pi_c^e = (1 - \phi_c^e \pi^*) + \phi_c^e \pi_{t-1}^e + \pi_c^e \Delta \epsilon_t,
\]

(25)

respectively, where

\[
\phi_c^e = \frac{1}{2 \beta_g} \left[ 1 + \kappa \theta_g + \beta_g - \sqrt{(1 + \kappa \theta_g + \beta_g)^2 - 4 \beta_g} \right] \quad \text{and} \quad \pi_c^e = \frac{\kappa}{(\omega + \gamma)(1 + \kappa \theta_g + \beta_g(1 - \phi_c^e \pi^* - \phi_c^e))}.
\]

Proof. See appendix I.4.

We can notice that under commitment the persistence of inflation is no longer equal to the persistence of the markup shocks.

4 The Term Structure of Interest Rates

Equilibrium interest rates for all maturities and their associated term premia are obtained from the equilibrium processes for output growth and inflation described in propositions 1 and 2. Specifically, using the pricing kernels \(M_{t,t+n}\) in equation (9) and the bond price equation (10), interest rates and term premia can be written as linear functions of a small set of macroeconomic variables. As a consequence, we obtain an affine term structure with
analytical solutions for coefficients that depend on deep parameters of the economy. This section starts by describing the affine framework linked to the model of the previous section and concludes by presenting the equilibrium interest rates and term premia in propositions 3 and 4.

Macroeconomic variables follow autoregressive processes with time-varying volatility. Therefore, we can group them in a vector of state variables, $s_t$, and write their evolution in matrix form as

$$s_{t+1} = \psi + \Phi s_t + \Psi(s_t) \Sigma^{1/2} \varepsilon_{t+1},$$

(26)

where

$$\Sigma = \text{diag}\{\sigma^2_a, \sigma^2_\epsilon, \sigma^2_q\}$$

and $\varepsilon = (\varepsilon_a, \varepsilon_\epsilon, \varepsilon_q)^\top$.

The specific content and dimensions of vectors $s$ and $\psi$ and matrices $\Phi$, $\Psi_c$ and $\Psi(s_t)$ depend on the equilibrium characteristics of the particular monetary policies under study. The matrix $\Phi$ has the autoregressive coefficients of the state variables, and the matrices $\Psi_c$ and $\Psi(s_t)$ contain the constant and time-varying components of the volatility of the state variables, respectively. The vector $\varepsilon$ captures the technology, supply, and preference innovations.

Using the specification in (26), the one-period nominal pricing kernel in (9) is

$$-\log M_{t,t+1} = \Gamma_0 + \Gamma_1^\top s_t + \lambda^\top \Psi(s_t) \Sigma^{1/2} \varepsilon_{t+1},$$

(27)

with $\Gamma_0 = -\log \beta + \ell_0^\top \psi$, $\Gamma_1 = \Phi^\top \ell_0 + \ell_1$, and $\lambda = (\Psi_c^\top \ell_0 + \ell_2)$, and vectors $\ell_0$, $\ell_1$ and $\ell_2$ depending on the monetary policy.

Of particular interest is the last term of equation (27). It describes the stochastic char-
acter of the pricing kernel and, as a result, provides a clear idea of the compensations for risk in the economy and their dependence on monetary policy. Given that this term is a function of \( \Psi(s_t) \), risk premia depend on the state of the economy. The analysis of this term and, specifically, of its constant component \( \lambda \) is fundamental to understand the effects of credibility of the policy on the term structure. This analysis is undertaken in the next section.

Denoting the one-period interest rate on the nominal bond maturing at \( t + n \) by \( i_t^{(n)} = \frac{1}{n} \log b_t^{(n)} \), and noticing that the bond price equation (10) can be written recursively as

\[
b_t^{(n)} = \mathbb{E}_t \left[ M_{t,t+1} b_t^{(n-1)} \right],
\]

we conjecture the solution for bond yields of the form

\[
i_t^{(n)} = \frac{1}{n} \left[ A_n + B_n^\top s_t \right].
\]

That is, interest rates are linear functions of macroeconomic variables. Replacing this form in equation (28) we obtain recursive formulas for coefficients \( A_n \) and \( B_n \) given by

\[
A_n = \Gamma_0 + A_{n-1} + B_n^\top \psi - \frac{1}{2} \lambda_n^\top \Sigma \lambda_n,
\]

\[
B_n^\top = \Gamma_n^\top + B_{n-1}^\top \Phi - \frac{1}{2} \lambda_n^\top \Sigma \text{diag}\{\lambda_n\} A_n,
\]

with initial conditions \( A_0 = 0 \) and \( B_0 = 0^\top \), where \( \Psi(s_t)^2 = C + \Sigma \text{diag}\{s_t A\} \) and

\[
\lambda_n^\top \equiv \lambda^\top + B_{n-1}^\top \Psi_c.
\]

The term \( \lambda_n \) plays an important role in the characterization of the risk premia contained in
the term structure of interest rates. It captures maturity-specific adjustments to the constant component of the pricing kernel, \( \lambda \), given by the factor \( B_{n-1}^\top \Psi_c \). This factor captures the sensitivity of one-period returns to the different sources of risk for the \( n \)-period bond. If bond yields do not depend on the state of the economy \( (B_n = 0) \), the expected returns offered by long-term bonds must be the same as the return of a one-period bond, \( i_t \).

This notion is formalized with the definition of bond risk premia. The term premium for an \( n \)-period bond captures the deviation of the \( n \)-period interest rate from a weighted average of the one-period interest rate and expectations of future interest rates. Explicitly, it is defined by

\[
\xi_t^{(n)} \equiv i_t^{(n)} - \frac{1}{n} \left[ i_t + (n - 1)E_t i_{t+1}^{(n-1)} \right].
\]  

(30)

Appendix I.5 shows that we can express the term premium as

\[
\xi_t^{(n)} = \xi_{A,n} + \xi_{B,n}^\top s_t,
\]

(31)

with coefficients given by

\[
\xi_{A,n} = -\frac{1}{2n} (\lambda + \lambda_n)^\top \Sigma (\lambda_n - \lambda) \quad \text{and} \quad \xi_{B,n}^\top = -\frac{1}{2n} (\lambda + \lambda_n)^\top K \Sigma \text{diag}(\lambda_n - \lambda) A.
\]

We can notice from this set of equations that a term premium is zero if \( \lambda_n - \lambda = B_{n-1}^\top \Psi_c = 0 \). In addition, term premia are sensitive to the state of the economy \( s_t \) only if \( K \neq 0 \), that is, only if the state variables are affected by time-varying volatility.

We now can use the affine framework presented above to characterize the equilibrium interest rates and term premia when monetary policy is conducted under discretion or com-
mitment, as the following two propositions show.\textsuperscript{14}

**Proposition 3.** The equilibrium interest rate characteristics for the optimal monetary policy under discretion are described by equations (26), (27), (29), and (31) when

\[
\mathbf{s}_t \equiv (\Delta a_t, \pi^d_t, \Delta y^d_t)^\top,
\]

\[
\psi = \left( (1 - \phi_a) g_a, (1 - \phi_{\epsilon}) \pi^*, \left( \frac{1 + \omega}{\omega + \gamma} \right) (1 - \phi_a) g_a - (1 - \phi_{\epsilon}) \theta_g \pi^* \right)^\top,
\]

\[
\Phi = \begin{bmatrix}
\phi_a & 0 & 0 \\
0 & \phi_{\epsilon} & 0 \\
\frac{1 + \omega}{\omega + \gamma} \phi_a & \theta_g (1 - \phi_{\epsilon}) & -\frac{\eta}{\omega + \gamma}
\end{bmatrix},
\]

\[
\Psi_c = \begin{bmatrix}
1 & 0 & 0 \\
0 & \pi^d & 0 \\
\frac{1 + \omega}{\omega + \gamma} \theta_g \pi^d & -\theta_g \frac{\pi^d}{\omega + \gamma}
\end{bmatrix},
\]

\[
\Psi(\mathbf{s}_t) = \text{diag}\left\{ (1 + K_a \Delta a_t)^{1/2}, (c_{\pi} + K_{\pi} \pi^d_t)^{1/2}, (1 + K_q \Delta y^d_t)^{1/2} \right\},
\]

\[
\ell_0 = (0, 1, \gamma)^\top, \quad \ell_1 = (0, 0, \eta)^\top, \quad \ell_2 = (0, 0, 1)^\top,
\]

\[
\mathbb{C} = \text{diag}\{1, c_{\pi}, 1\}, \quad \mathbb{K} = \text{diag}\{K_a, K_{\pi}, K_q\}, \quad \text{and} \quad \mathbb{A} = \mathbb{I}_{(3 \times 3)},
\]

where

\[
c_{\pi} = 1 - K_{\epsilon} \frac{\pi^*}{\pi^d} \quad \text{and} \quad K_{\pi} = \frac{K_{\epsilon}}{\pi^d},
\]

**Proof.** The processes for $\Delta a_t$ and $\Delta y_t$ are obtained from equations (15) and (22), respectively. The process for $\pi_t$ is obtained replacing (17) into (23). The process for the discount factor is obtained replacing equation (26) into (27). \hfill \Box

\textsuperscript{14}We can notice that using equation (29) the one-period nominal interest rate, $i_t$, can be written as

\[
i_t = \Gamma_0 - \frac{1}{2} \lambda^\top \Sigma \lambda + \left( \Gamma_1 - \frac{1}{2} \lambda^\top \mathbb{K} \Sigma \text{diag}\{\lambda\} \right)^\top \mathbf{s}_t,
\]

resembling a monetary policy rule where the monetary authority sets the short-term rate reacting to the level of macroeconomic variables. The reaction coefficients are functions of preference and production parameters.
This proposition tells us that when monetary policy is conducted under discretion, the dynamics in the term structure of interest rates can be explained by the dynamics of three macroeconomic factors: changes in labor productivity, inflation, and output growth. Proposition 4 shows that under commitment we require an additional factor, markup shocks, to explain interest rates.

**Proposition 4.** The equilibrium interest rate characteristics for the optimal monetary policy under commitment are described by equations (26), (27), (29), and (31) when

\[
s_t \equiv (\Delta a_t, \epsilon_t, \pi_t^c, \Delta y_t^c)^\top,
\]

\[
\psi = \left((1 - \phi_a)g_a, 0, (1 - \phi_{\pi}^c)\pi^*, \frac{1 + \omega}{\omega + \gamma}(1 - \phi_a)g_a + \theta_g\phi_{\pi}^c\pi^*\right)^\top,
\]

\[
\Phi = \begin{bmatrix}
\phi_a & 0 & 0 & 0 \\
0 & \phi_{\epsilon} & 0 & 0 \\
0 & -\pi_{\epsilon}^c(1 - \phi_{\epsilon}) & \phi_{\pi}^c & 0 \\
\frac{1 + \omega}{\omega + \gamma}\phi_a & \pi_{\epsilon}^c\theta_g(1 - \phi_{\epsilon}) & -\theta_g\phi_{\pi}^c & -\frac{\eta}{\omega + \gamma}
\end{bmatrix},
\Psi_c = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & \pi_{\epsilon}^c & 0 \\
\frac{1 + \omega}{\omega + \gamma} & -\pi_{\epsilon}^c\theta_g & -\frac{1}{\omega + \gamma}
\end{bmatrix},
\Psi(s_t) = \text{diag}\left\{(1 + K_a\Delta a_t)^{1/2}, (1 + K_{\epsilon}\epsilon_t)^{1/2}, (1 + K_{\pi}\Delta y_t^c)^{1/2}\right\},
\ell_0 = (0, 0, 1, \gamma)^\top, \quad \ell_1 = (0, 0, 0, \eta)^\top, \quad \ell_2 = (0, 0, 1)^\top,
\]

\[
C = I_{(3 \times 3)}, \quad K = \text{diag}\{K_a, K_{\epsilon}, K_{\pi}\}, \quad \text{and} \quad A = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}.
\]

**Proof.** The processes for the state variables are obtained from equations (15), (17), (24), and (25). The process for the discount factor is obtained replacing equation (26) into (27).
5 Analysis

This section is devoted to understanding the mechanism linking bond yields and monetary policies under discretion and commitment in the model. The analysis shows that there is a specific channel driving the asset-pricing implications of credibility improvements. This channel is the reduction generated by policy credibility of the effects of markup shocks on the economy. To understand this channel, this section first describes the macroeconomic and general asset-pricing implications and then focuses on the effects on the term structure of interest rates. The analysis is complemented by a quantitative exercise where the model is calibrated to match some bond yield and macroeconomic characteristics of the U.S. economy. Although this exercise evidences the difficulty to simultaneously capture (quantitatively) interest rate and macroeconomic properties, it shows that enhancements in policy credibility can be helpful to understand some of the observed changes in the dynamics of interest rates, term premia, and macro variables across periods.

5.1 Macroeconomic Implications

The macroeconomic implications can be understood comparing the output growth and inflation processes in (22) to (25). Inflation and output growth under commitment are less affected by markup shocks than under discretion. The result is that credibility improvements decrease the vulnerability of the economy to markup shocks and therefore affect the nature of macroeconomic risk. A shift in monetary policy from discretion to commitment reduces the variability of inflation and the variability of output growth. It implies that the trade-off between inflation volatility and output volatility is improved under commitment and it translates into higher social welfare.

Propositions 5 and 6 summarize the volatility and persistence properties of inflation,
respectively, for the two policies. The variability of inflation is proportional to the variability of the markup shocks, but commitment in monetary policy reduces the sensitivity of inflation to these shocks. Under discretion, inflation inherits the autocorrelation properties of the markup shocks. Under commitment the persistence of inflation is reduced by the fact that inflation reacts to changes in the level of markup shocks instead of reacting to the level as in the discretion case. This decline in sensitivity is determinant for the asset-pricing implications of the model.

**Proposition 5.** The variance of inflation is \((\pi^d)^2 \var(\epsilon_t)\) and \(\frac{2}{1+\phi_c} \frac{1-\phi_c}{1-\phi_c \phi_c} (\pi^d)^2 \var(\epsilon_t)\) under discretion and commitment, respectively. Therefore, a policy conducted under commitment reduces the volatility of inflation if \(\phi_c > \frac{1}{2+\phi_c^2}\).

**Proof.** The variance of inflation is obtained solving for the unconditional moments of the inflation processes in propositions 3 and 4. The result follows from subtracting the variance under commitment from the variance under discretion.

**Proposition 6.** The first-order autocorrelation of inflation is \(\phi_c\) under discretion and \(\phi_c - \frac{1}{2} (1 - \phi_c^2)(1 + \phi_c)\) under commitment. Therefore, a policy conducted under commitment always reduces the persistence of inflation.

**Proof.** The covariance of inflation is obtained solving for the unconditional moments of the inflation processes in 3 and 4. Algebraic manipulations of the implied correlations provide the results.

The average output growth for the economy, \(\frac{1+\phi_c}{\omega+\gamma} g_a\), does not depend on monetary policy. The effects of credibility improvements on the variance and autocorrelation of output growth, and the correlation of output growth and inflation are difficult to explore analytically. Different calibrations, suggest that the variability of output growth under commitment
is always lower than under discretion, and the autocorrelation of output growth tends to increase with improvements in credibility. Calibrations involving highly persistent markup shocks imply that credibility improvements imply less negative correlation between output growth and inflation. However, the effect is reversed as the persistence of the shocks is reduced.

Summarizing, enhancements in policy credibility increase macroeconomic stability. The change is driven by a reduced reaction of inflation and consumption to markup shocks. Not surprisingly, this change also drives the change in the dynamics of interest rates.

5.2 Risk Premia

Improvements in policy credibility affect risk premia through a very specific channel: Credibility enhancements reduce the effects of markup shocks on inflation and real activity and thus compensations for this risk in financial assets decline.

The stochastic term of the pricing kernel in (27) provides us the time-varying risk premia \( \lambda^\top \Psi(s_t) \). This term has three components since there are three shocks affecting the economy: technology, markup, and preference shocks. The inflation processes in equations (23) and (25) imply that the only source of uncertainty for inflation is markup shocks; therefore, we can refer to markup shocks as inflation risk and label the risk premium for markup shocks as the inflation risk premium.

Average Risk Premia

In order to understand the impact of credibility on the average risk premia we need to compare the constant component \( \lambda \) for the two policies.\(^{15}\) This comparison is presented in

\(^{15}\)To see this, consider \( \mathbb{E} \left[ \text{var}_t(\log M_{t,t+1}) \right] = \lambda^\top \Sigma \mathbb{E} \left[ \Psi(s_t)^2 \right] \lambda \). The matrix \( \Psi(s_t)^2 \) is a diagonal matrix with exogenous components \( \Delta a_t \) and \( \epsilon_t \) in the first two rows and an endogenous component \( \Delta y_t \) in the third row. Since the unconditional expectation of output growth is independent of monetary policy, changes in the policy only affect the average properties of the risk premia through \( \lambda \).
proposition 7.

**Proposition 7.** The risk premium factor $\lambda$ for the optimal monetary policy under discretion is

$$\left(\gamma \frac{1+\omega}{\omega+\gamma}, \frac{\omega}{\omega+\gamma} \right)^\top \pi^d_e \left(1-\gamma \theta_g\right),$$

and for the optimal monetary policy under commitment is

$$\left(\gamma \frac{1+\omega}{\omega+\gamma}, \frac{\omega}{\omega+\gamma} \right)^\top \pi^c_e \left(1-\gamma \theta_g\right).$$

Therefore, the average risk premia for technology and preference shocks are not affected by the credibility of the policy, and the magnitude of the premium for inflation risk is always lower under commitment than under discretion.

**Proof.** Equations (32) and (33) are found from the definition of $\lambda$ and the associated parameters in propositions 3 and 4. Since the first and third components of $\lambda$ are the same for both policies, it follows that technology and preference risk premia are unaffected by the policy. The fact that $\pi^d_e > \pi^c_e$ makes the magnitude of the inflation risk premia under commitment to be always lower than under discretion.

The effects of credibility on the average risk premia are concentrated on the the second component of $\lambda$, the risk premium for markup shocks (inflation risk premium).\textsuperscript{16} Improvements on policy credibility decrease the magnitude of the inflation risk premium because the sensitivity of output growth and inflation to markup shocks is lower under commitment.

---

\textsuperscript{16}The first and third components containing the risk premia for productivity and preference shocks, respectively, depend on intertemporal elasticities of consumption and labor. Although their economic analysis is interesting, I focus only on the component that depends on monetary policy to save space.
than under discretion. That is, the economy is less vulnerable to markup shocks under commitment than under discretion and, therefore, the compensation for facing this risk is also lower (in absolute value).

We also can observe a close connection between the size of the reduction in the magnitude of the inflation risk premium and the reduction in the persistence of inflation. Looking at the definition of $\pi^d_\epsilon$ and $\pi^c_\epsilon$ in propositions 1 and 2, we can see that the reduction in the magnitude of the inflation risk premium is proportional to $1 - \phi^c_\pi$. Since, by proposition 6, the reduction in the persistence is also proportional to $1 - \phi^c_\pi$, the decrease in the magnitude of the inflation risk premium is proportional to the decrease in persistence caused by gains in credibility.

**Time Variation in Risk Premia**

The time-varying component $\Psi(s_t)$ of the volatilities of productivity growth, markup shocks, and the habit generates time variation in risk premia. While the time variation in the premium for productivity shocks is not sensitive to monetary policy, the time variation of the premia for markup and preference shocks depend on the type of policy that is conducted.

Time variation in the premium for productivity shocks is driven entirely by the level of productivity growth of the economy, $\Delta a_t$. Since it is exogenous, it is not affected by monetary policy. Variability in the premium for markup shocks is linked to the level of the shocks, $\epsilon_t$, as the second element in the diagonal of $\Psi(s_t)$ shows. The volatility of the premium is reduced by improvements in credibility of monetary policy through the reduction of the vulnerability of the economy to markup shocks as in the case of the average risk premium. Since markup shocks are the only source of inflation risk and variability $K_\epsilon$ implies that higher inflation in the economy increases the inflation risk premia. Since commitment in the policy reduces the volatility of inflation, improvements in credibility imply less volatile inflation risk premia. The time variation of the preference premium is given by $\sqrt{1 + K_q \Delta y_t}$. Since output growth
is affected by credibility in monetary policy, the volatility of the premium is also affected. For a countercyclical risk premium \((K_q < 0)\), when a positive markup shock affects the economy, output growth under discretion declines more than under commitment and, as a result, the preference premium under discretion increases more than under commitment.

\textit{Asset-Pricing Implications of Monetary Policy}

There is an important asset-pricing implication for real and nominal financial assets linked to the inflation risk premium in the model. It arises from the correlation between output growth and inflation and, thus, is useful to observe the effects of nominal rigidities on asset prices.

Financial assets with real cashflows positively affected by inflation involve a negative premium for inflation risk. To see this, consider the one-period real pricing kernel for the equilibrium of the model,

\[
\log M_{t,t+1}^{\text{real}} = \log \beta - \gamma \Delta y_{t+1} - \Delta q_{t+1}, \tag{34}
\]

and the output growth equations (22) and (24). We can see that the real pricing kernel depends positively on future inflation, \(\pi_{t+1}\). It implies that assets with real payoffs affected positively by inflation have a positive covariance with the pricing kernel and therefore expected returns that are lower than the one-period (risk-free) interest rate. Households are willing to accept low expected returns for these assets because they hedge consumption risk: These assets pay high when output growth is low.

Financial assets with nominal cashflows that are positively sensitive to inflation involve a premium for inflation risk. Whether this premium is positive or negative depends on the relation between the intertemporal elasticity of substitution of consumption and the (growth-adjusted) elasticity of substitution of differentiated goods. Noting that negative
markup shocks reduce inflation, the sign of the embedded premium for inflation risk is given by the sign of $1 - \gamma \theta_g$. That is, if the elasticity of intertemporal substitution of consumption, $\gamma^{-1}$, is lower than the growth-adjusted elasticity of substitution of differentiated goods, $\theta_g$, the embedded risk premium is negative. The reason is that the benefits of a higher cashflow with higher inflation are more than offset by the reduction in output growth caused by the higher inflation. If households’ needs of smooth intertemporal consumption are greater than their needs of smooth intratemporal consumption across goods, the expected returns of holding these assets are lower than the one-period interest rate. On the other hand, if smoothing consumption across goods is more important than intertemporal smoothing, investors require an incentive (positive expected excess returns) to hold assets with inflation-protected cashflows.

5.3 Bond Yields and Bond Risk Premia

The affine term structure framework presented in section 4 shows that bond yields can be expressed as functions of macro variables. Therefore, the effects of credibility on macro variables propagate on the dynamics of bond yields. The average level of bond yields decreases and the volatility and average shape of the yield curve changes. That is, changes in policy credibility generate changes in bond risk premia. More specifically, bond risk premia are affected because the correlation between the nominal pricing kernel and bond returns induced by markup shocks is affected by credibility. This shift in the dynamics of the term premia\(^{17}\) diminishes deviations from the expectations hypothesis.

The One-Period Interest Rate

Policy credibility affects the dynamics of the short-term nominal rate, $i_t$. Proposition 8

\(^{17}\)I use the terms bond risk premia and term premia interchangeably.
shows the difference in the average short-term rates under the two regimes.

**Proposition 8.** *The difference in the average one-period interest rates implied by monetary policies under discretion and under commitment is*

\[
E [i_t^d - i_t^c] = -\frac{1}{2} (1 - \gamma \theta_y)^2 [(\pi^d_{\epsilon})^2 - (\pi^c_{\epsilon})^2] \sigma^2_{\epsilon}.
\]

*Proof.* See appendix I.6. \(\square\)

The difference in the two short-term rates is a precautionary savings effect resulting from markup shocks. The average interest rate under discretion is always lower than the one under commitment and the difference between the two rates increases as the difference between the intertemporal and the intratemporal growth adjusted elasticities of substitution increases.

The volatility of the interest rate implied by the model is difficult to interpret analytically. The calibration exercise below shows that the volatility of long maturity yields decline with gains in credibility, while the volatility for the short-term rate increases.

**Bond Risk Premia and Deviations from the Expectations Hypothesis**

Credibility improvements in monetary policy affect the sensitivity of bond returns to inflation risk. This change, caused by the fact that credibility makes output growth and inflation less vulnerable to markup shocks, affects the joint dynamics of term premia and spreads. In particular, since the responses of spreads and term premia to the state of the economy vary with credibility, the size of the deviations from the expectations hypothesis is affected.

Consider first the average premia in the term structure of interest rates. Equation (45) tells us that the average effects of different monetary policies on the term premia are captured by the effects of the policies on \(\lambda, \lambda_n\), and their interaction. Since the effects of credibility on
the risk premia component $\lambda$ are analyzed above, completing the analysis of the average term premia amounts to describe the effects of discretion and commitment on $\lambda_n - \lambda = B_{n-1}^T \Psi_c$.

That is, we need to analyze the sensitivity to uncertainty of the one-period return of an $n$-maturity bond. In general, the analytical solutions for these terms are complicated. However, the analysis of the two-period component and the numerical properties of the components for bonds with longer maturities provide the necessary intuition to understand the transmission channel of credibility on the premia.

**Proposition 9.** The difference in the $B_1^T \Psi_c$ component between discretion and commitment is

$$\begin{pmatrix}
0, \\
\gamma \theta_g \pi^d + (1 - \phi_{\pi}) \pi_c^c + \left[(1 - \gamma \theta_g) \phi_c - \theta_g \left(\frac{\omega}{\omega + \gamma}\right) \left(\eta - \frac{1}{2} \frac{\omega}{\omega + \gamma} K_{q} \sigma^2_q\right)\right] (\pi^d_c - \pi^c_c) \\
-\frac{1}{2} (1 - \gamma \theta_g)^2 K_{c} \sigma^2_c ((\pi^d_c)^2 - (\pi^c_c)^2), \\
0
\end{pmatrix}^T.$$

Therefore, the average 2-period term premium for productivity and preference shocks is not affected by the credibility of the policy.

**Proof.** Noting that $B_1^T = \Gamma_1^T - \frac{1}{2} \lambda^T K \Sigma \text{diag} \{\lambda\} A$, the equation follows from subtracting the $B_1^T \Psi_c$ implied by proposition 4 from that one implied by Proposition 3.

The second term in the equation of the proposition above is the difference in the compensation for markup shocks between discretion and commitment in 2-period bonds. This term is positive for reasonable parameter values, implying that credibility reduces the sensitivity of the 2-period bond returns to markup shocks. Notice also that the reduction in term premia is more significant for economies where households’ preferences are more affected by the habit (more negative $\eta$).
The analysis can be complemented with numerical computations of $B_{n-1}^T \Psi_c$ for both policies and different maturities, $n > 2$, to see that the first and third elements of this sensitivity vector are not affected by the type of policy, while the second component changes. It makes clear that term premia for all maturities are affected by credibility through changes in the sensitivity of the discount factor and bond returns to markup shocks.

Rejections of the expectations hypothesis of interest rates can be explained by the existence of time variation in term premia that is correlated to time variation in spreads. To see this, we can consider the Campbell and Shiller (1991) regressions

$$i_t^{(n-1)} - i_t^{(n)} = \alpha^{(n)} + \frac{\beta^{(n)}}{n-1} (i_t^{(n)} - i_t) + \varepsilon_{CS,t}^{(n)}. \tag{35}$$

These regressions are a standard exercise to test the expectations hypothesis of interest rates. Under the expectations hypothesis, the Campbell-Shiller coefficients, $\beta^{(n)}$, are equal to one, meaning that spreads, $i_t^{(n)} - i_t$, only reflect expected changes in bond yields. We can use the definition of term premium, as shown in appendix I.5, to show that the coefficients implied by the model can be written as

$$\beta^{(n)} = 1 - n \frac{\text{cov} (i_t^{(n)} - i_t, \xi_t^{(n)})}{\text{var} (i_t^{(n)} - i_t)}. \tag{36}$$

We can infer from this representation that deviations from the expectations hypothesis are explained by the correlation between a time-varying spread and time-varying term premia. Moreover, in order to obtain $\beta^{(n)} < 1$, this correlation has to be positive. That is, according to the model, the expectations hypothesis is rejected because spreads contain information about term premia that depend on the state of the economy. As a consequence, arguing that the size of deviations hypothesis decline with gains in monetary policy credibility is
equivalent to say that credibility affects the positive correlation between term premia and spreads and this is motivated by changes in the joint reaction of term premia and spreads to markup shocks. In addition, if credibility makes the $\beta^{(n)}$ coefficients closer to 1, spreads will reflect expected changes in short-term interest rates and, then, expected changes in monetary policy. Consequently, credibility may improve the ability to extract expectations on future monetary policy from the term structure.

5.4 Quantitative Analysis

In order to gain additional insights into the effects of monetary policy on bond yields, this section presents a calibration of the model and a policy experiment based on United States data. The calibration exercise evidences some difficulties for the model to simultaneously match selected U.S. statistics. However, using it in combination with the policy experiment allows us to understand the main differences in bond yields and bond risk premia between discretion and commitment in the policy. The analysis is also helpful for the discussion in section 6.

5.4.1 Data

The data consist of United States time series for bond yields, consumption, and consumer prices from 1952:2 to 2007:4. The term structure series was obtained from quarterly data on bond yields for yearly maturities from 1 to 5 years from the Fama-Bliss discount bonds database found in CRSP, and the short-term nominal interest rate is the 3-month T-bill from the Fama risk-free rates database. The consumption growth series was constructed using quarterly data on real per-capita consumption of nondurables and services from the Bureau of Economic Analysis. The inflation series was constructed following the methodology used
Table 1: Average Levels (%), Standard Deviations (%), and Campbell-Shiller coefficients for U.S. Government Bond Yields

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[i_t] \times 4$</td>
<td>5.13</td>
<td>3.42</td>
<td>6.02</td>
<td>4.49</td>
</tr>
<tr>
<td>$E[i_t^{(4)}] \times 4$</td>
<td>5.52</td>
<td>3.76</td>
<td>6.45</td>
<td>4.90</td>
</tr>
<tr>
<td>$E[i_t^{(8)}] \times 4$</td>
<td>5.72</td>
<td>3.83</td>
<td>6.71</td>
<td>5.21</td>
</tr>
<tr>
<td>$E[i_t^{(12)}] \times 4$</td>
<td>5.88</td>
<td>3.97</td>
<td>6.88</td>
<td>5.47</td>
</tr>
<tr>
<td>$E[i_t^{(16)}] \times 4$</td>
<td>6.00</td>
<td>4.04</td>
<td>7.04</td>
<td>5.67</td>
</tr>
<tr>
<td>$E[i_t^{(20)}] \times 4$</td>
<td>6.08</td>
<td>4.09</td>
<td>7.13</td>
<td>5.80</td>
</tr>
<tr>
<td>$\sigma(i_t) \times 4$</td>
<td>2.87</td>
<td>1.62</td>
<td>2.98</td>
<td>2.05</td>
</tr>
<tr>
<td>$\sigma(i_t^{(4)}) \times 4$</td>
<td>2.88</td>
<td>1.72</td>
<td>2.93</td>
<td>2.10</td>
</tr>
<tr>
<td>$\sigma(i_t^{(8)}) \times 4$</td>
<td>2.84</td>
<td>1.61</td>
<td>2.86</td>
<td>1.99</td>
</tr>
<tr>
<td>$\sigma(i_t^{(12)}) \times 4$</td>
<td>2.78</td>
<td>1.56</td>
<td>2.75</td>
<td>1.88</td>
</tr>
<tr>
<td>$\sigma(i_t^{(16)}) \times 4$</td>
<td>2.75</td>
<td>1.53</td>
<td>2.69</td>
<td>1.79</td>
</tr>
<tr>
<td>$\sigma(i_t^{(20)}) \times 4$</td>
<td>2.71</td>
<td>1.48</td>
<td>2.62</td>
<td>1.72</td>
</tr>
<tr>
<td>$(E[i_t^{(20)}] - E[i_t]) \times 4$</td>
<td>0.95</td>
<td>0.67</td>
<td>1.11</td>
<td>1.31</td>
</tr>
<tr>
<td>$\beta^{(4)}$</td>
<td>-0.73</td>
<td>-0.08</td>
<td>-0.78</td>
<td>1.23</td>
</tr>
<tr>
<td>$\beta^{(8)}$</td>
<td>-0.88</td>
<td>0.31</td>
<td>-0.91</td>
<td>0.54</td>
</tr>
<tr>
<td>$\beta^{(12)}$</td>
<td>-1.18</td>
<td>1.42</td>
<td>-1.28</td>
<td>-0.28</td>
</tr>
<tr>
<td>$\beta^{(16)}$</td>
<td>-1.31</td>
<td>1.67</td>
<td>-1.45</td>
<td>-0.47</td>
</tr>
<tr>
<td>$\beta^{(20)}$</td>
<td>-1.60</td>
<td>1.74</td>
<td>-1.74</td>
<td>-0.48</td>
</tr>
</tbody>
</table>

in Piazzesi and Schneider (2007) to capture inflation related to non-durables and services consumption only.\textsuperscript{18} Tables 1 and 2 show summary statistics for bond yields and macroeconomic variables, respectively. The statistics are computed for the whole sample period and different sub-samples. The sub-samples are labeled as “Bretton Woods,” “Fiat Money,” and “Greenspan” to capture the idea of potentially different monetary policy regimes. The comparison of these statistics will be helpful for the discussion in section 6.

\textsuperscript{18}The details of the construction of the series can be found in \url{http://www.stanford.edu/piazzesi/nberannualprograms.zip}. One important difference between the series constructed in Piazzesi and Schneider (2007) and the one presented here is the additional adjustment made here to drop the effect of population growth and obtain consumption in per-capita terms. The price index and consumption series, and the population series used for the adjustment were obtained from the Bureau of Economic Analysis.
Table 1 presents average levels and standard deviations for bond yields and their associated Campbell-Shiller coefficients. The average yield curve is upward sloping, and yield volatility decreases with maturity. These two characteristics are robust across sub-samples. In particular, we can notice that the Bretton Woods period is characterized by lower levels, spreads, and volatilities than during the Fiat Money period. The reduction in the spread between long-term bonds and the short-term interest rates indicate lower bond risk premia on average. The Campbell-Shiller coefficients from regression (35) are negative for the whole sample, suggesting a strong positive correlation between bond spreads and bond excess returns. However, this relation is not stable across sub-samples. While they are negative for the Fiat Money period, they are close to zero or positive during the Bretton Woods and Greenspan regimes. This instability suggests that properties of bond risk premia might be affected by monetary policy regimes.

Table 2 shows statistics for consumption growth and inflation. The volatility of consumption growth was higher during the Bretton Woods period than for the whole sample, and the volatility of inflation was lower. This reduction in inflation volatility was accompanied by a reduced persistence of inflation. Consumption growth and inflation are negatively autocorrelated and the autocorrelation was more negative during the Fiat Money period. As for the Bretton Woods period, the Greenspan period is characterized by low volatility and persistence in inflation. A difference between the two periods is the reduction in consumption growth volatility during the Greenspan era.

5.4.2 Calibration and Policy Experiment

The purpose of this exercise is to gain further insights about the effects of credibility on long-term bond yields. The model is calibrated using the U.S. quarterly data described in section 5.4.1 for bond yields, inflation, and consumption growth from 1971:3 to 2007:4.
Table 2: U.S. Consumption Growth ($\Delta c$) and Inflation ($\pi$) Statistics.

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td></td>
<td>(Post-Accord)</td>
<td>(Bretton Woods)</td>
<td>(Fiat Money)</td>
<td>(Greenspan)</td>
</tr>
<tr>
<td>$E[\Delta c_t] \times 4$ (%)</td>
<td>2.05</td>
<td>2.30</td>
<td>1.92</td>
<td>1.78</td>
</tr>
<tr>
<td>$E[\pi_t] \times 4$ (%)</td>
<td>3.70</td>
<td>2.36</td>
<td>4.40</td>
<td>2.98</td>
</tr>
<tr>
<td>$\sigma(\Delta c_t) \times 4$ (%)</td>
<td>1.88</td>
<td>2.16</td>
<td>1.71</td>
<td>1.34</td>
</tr>
<tr>
<td>$\sigma(\pi_t) \times 4$ (%)</td>
<td>2.49</td>
<td>1.49</td>
<td>2.62</td>
<td>1.28</td>
</tr>
<tr>
<td>corr($\Delta c_t, \Delta c_{t-1}$)</td>
<td>0.35</td>
<td>0.27</td>
<td>0.41</td>
<td>0.24</td>
</tr>
<tr>
<td>corr($\pi_t, \pi_{t-1}$)</td>
<td>0.83</td>
<td>0.68</td>
<td>0.82</td>
<td>0.54</td>
</tr>
<tr>
<td>corr($\Delta c_t, \pi_t$)</td>
<td>-0.28</td>
<td>-0.17</td>
<td>-0.32</td>
<td>-0.16</td>
</tr>
</tbody>
</table>

The parameter values for the calibration are shown in table 3, and details of the calibration procedure are presented in appendix II. Since the purpose is to observe the effects of monetary policy on bond risk premia, the calibration focuses on capturing the slope of the yield curve, while matching important properties of the inflation process observed in the data. In particular, the persistence of inflation is important to characterize the differences between discretion and commitment. The model captures the slope of the curve with a

$(the period labeled as “Fiat Money”). The main assumption for this calibration is that the Federal Reserve conducted a policy under discretion during this period and, therefore, the equilibrium properties of the model under discretion characterize the economy. The calibration is complemented with a policy experiment that evaluates the properties of an economy characterized by a policy under commitment using the parameter values of the calibration exercise. This experiment allows us to compare the main differences between the two types of policies and capture the effects of credibility.

On February 3, 2003, Ben Bernanke declared before the Money Marketeers of New York University that “...constrained discretion is an approach that allows monetary policymakers considerable leeway in responding to economic shocks, financial disturbances, and other unforeseen developments. Importantly, however, this discretion of policymakers is constrained by a strong commitment to keeping inflation low and stable.” He added “In my view, constrained discretion characterizes the current monetary policy framework of the United States.”
Table 3: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Subjective discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Curvature parameter</td>
<td>3.51</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Habit parameter</td>
<td>-0.19</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Inverse of elasticity of substitution of labor</td>
<td>10.59</td>
</tr>
<tr>
<td>$\sigma_q$</td>
<td>Habit volatility parameter</td>
<td>0.30</td>
</tr>
<tr>
<td>$K_q$</td>
<td>Habit stochastic volatility parameter</td>
<td>-35.40</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Elasticity of substitution of goods</td>
<td>3.00</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Price stickiness probability</td>
<td>0.67</td>
</tr>
<tr>
<td>$\pi^* \times 10^2$</td>
<td>Average inflation</td>
<td>1.10</td>
</tr>
<tr>
<td>$g_a \times 10^3$</td>
<td>Average productivity growth</td>
<td>5.75</td>
</tr>
<tr>
<td>$\phi_a$</td>
<td>Autocorrelation of productivity growth</td>
<td>0.18</td>
</tr>
<tr>
<td>$\sigma_a \times 10^3$</td>
<td>Productivity growth volatility parameter</td>
<td>6.71</td>
</tr>
<tr>
<td>$K_a$</td>
<td>Productivity growth stochastic volatility parameter</td>
<td>0.00</td>
</tr>
<tr>
<td>$\phi_\epsilon$</td>
<td>Autocorrelation of markup shock</td>
<td>0.82</td>
</tr>
<tr>
<td>$\sigma_\epsilon \times 10$</td>
<td>Markup shock volatility parameter</td>
<td>3.01</td>
</tr>
<tr>
<td>$K_\epsilon$</td>
<td>Markup shock stochastic volatility parameter</td>
<td>3.32</td>
</tr>
</tbody>
</table>

relatively low value for $\gamma$, negative values for $\eta$ and $K_q$, and a positive $K_\epsilon$. This is consistent with habit formation in preferences, countercyclical risk aversion, and a volatility of inflation that increases as the level of inflation increases.

Tables 4 and 5 show some statistical properties of the calibration under discretion and the implied properties under commitment. Quantitatively, the results of the calibration evidence the difficulty to simultaneously capture bond yield and consumption growth properties of the U.S. economy. This is consistent with Rudebusch and Swanson (2008) who find that habits in a production economy do not capture significant bond risk premia unless allowing for significant distortions in labor. These distortions are captured in the calibration by the very inelastic labor supply (high $\omega$) and the abnormally high standard deviation of consumption growth in table 4. The tension between bond risk premia and volatility of consumption growth also is reflected in the high volatility of short-term bond yields in table 5.
The policy experiment provides important implications for the differences between discretion and commitment. Consumption growth and inflation are less volatile under commitment, and there is less persistence in inflation. Simultaneously, the spread between long-term bond yields and the short-term rate is lower under commitment, the volatility of interest rates decreases significantly for long-term maturities, and the Campbell-Shiller coefficients are closer to one. This implies a reduction in the compensation for markup shocks in bonds and a partial explanation for deviations from the expectations hypothesis based on lack of policy credibility.

Figure 2 shows the unconditional expected term premia and the associated one-period expected excess returns on bonds. The positive term premia under discretion decrease under commitment. It implies that lack of credibility makes economic agents demand a high compensation for holding nominal bonds. This is consistent with an upward sloping yield curve. Under commitment, bonds are less risky for investors and, therefore, they are willing to hold them for a lower expected excess return. In summary, the dynamics of bond risk premia in the term structure of interest rates can be dramatically affected by improvements in credibility. In particular, the reduction in inflation risk resulting from gains in policy credibility reduces the compensation for risk in nominal bonds.
Table 5: Model-Implied Bond Yield Properties

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model Discretion</th>
<th>Model Commitment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1971:3-2007:4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[i_t] \times 4$</td>
<td>6.02</td>
<td>6.02</td>
<td>6.21</td>
</tr>
<tr>
<td>$E[i_t^{(4)}] \times 4$</td>
<td>6.45</td>
<td>6.84</td>
<td>6.80</td>
</tr>
<tr>
<td>$E[i_t^{(8)}] \times 4$</td>
<td>6.71</td>
<td>7.03</td>
<td>6.91</td>
</tr>
<tr>
<td>$E[i_t^{(12)}] \times 4$</td>
<td>6.88</td>
<td>7.08</td>
<td>6.99</td>
</tr>
<tr>
<td>$E[i_t^{(16)}] \times 4$</td>
<td>7.04</td>
<td>7.10</td>
<td>7.04</td>
</tr>
<tr>
<td>$E[i_t^{(20)}] \times 4$</td>
<td>7.13</td>
<td>7.11</td>
<td>7.08</td>
</tr>
<tr>
<td>$\sigma(i_t) \times 4$</td>
<td>2.98</td>
<td>9.35</td>
<td>9.67</td>
</tr>
<tr>
<td>$\sigma(i_t^{(4)}) \times 4$</td>
<td>2.93</td>
<td>5.89</td>
<td>4.19</td>
</tr>
<tr>
<td>$\sigma(i_t^{(8)}) \times 4$</td>
<td>2.86</td>
<td>4.40</td>
<td>3.15</td>
</tr>
<tr>
<td>$\sigma(i_t^{(12)}) \times 4$</td>
<td>2.75</td>
<td>3.38</td>
<td>2.62</td>
</tr>
<tr>
<td>$\sigma(i_t^{(16)}) \times 4$</td>
<td>2.69</td>
<td>2.68</td>
<td>2.19</td>
</tr>
<tr>
<td>$\sigma(i_t^{(20)}) \times 4$</td>
<td>2.62</td>
<td>2.19</td>
<td>1.84</td>
</tr>
<tr>
<td>$(E[i_t^{(20)}] - E[i_t]) \times 4$</td>
<td>1.11</td>
<td>1.09</td>
<td>0.87</td>
</tr>
<tr>
<td>$\beta^{(4)}$</td>
<td>-0.78</td>
<td>0.83</td>
<td>0.90</td>
</tr>
<tr>
<td>$\beta^{(8)}$</td>
<td>-0.91</td>
<td>0.87</td>
<td>0.91</td>
</tr>
<tr>
<td>$\beta^{(12)}$</td>
<td>-1.28</td>
<td>0.90</td>
<td>0.91</td>
</tr>
<tr>
<td>$\beta^{(16)}$</td>
<td>-1.45</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td>$\beta^{(20)}$</td>
<td>-1.74</td>
<td>0.92</td>
<td>0.91</td>
</tr>
</tbody>
</table>

In order to gain some insights about the changes in the deviations of the expectations hypothesis it is useful to analyze the impulse responses to markup shocks of spreads and bond risk premia. Figure 3 shows the response of the five-year spread and term premium to positive inflation (positive markup shock). The magnitude and sign of the responses make evident that the joint dynamics of spreads and term premia change under the two regimes. While under commitment the response of the spread to markup shocks is larger than under discretion, the opposite is true for the term premium. This difference can be explained by the different reaction of inflation under the two regimes. Under commitment, the initial increase in inflation is followed by a period of disinflation, such that in the long run,
Bond risk premia are $4 \times \mathbb{E}[\xi_t^{(n)}]$ and one-quarter expected excess returns are $4 \times n \times \mathbb{E}[\xi_t^{(n)}]$.

inflation is zero. That makes the term premium remain stable. Investors do not need large changes in the compensation for holding long-term bonds because inflation is not persistent. Simultaneously, under commitment the equilibrium short-term interest rate drops down dramatically and the spread widens. In summary, deviations of the expectations hypothesis diminish because the positive covariance between spreads and term premia is reduced as policy credibility increases and the volatility of spreads increases.

6 Discussion

This section presents an analysis of bond yield and macroeconomic statistics for historical periods in the United States with different monetary policy regimes. Based on the analysis in section 5, the purpose is to determine whether changes in bond yield and macroeconomic properties across regimes are consistent with differences in the credibility associated with these regimes. This analysis is far from simple for at least two reasons. First, it is not straightforward to determine differences in credibility across regimes, and, second, developments in bond yield dynamics might not be related to monetary policy but can be the result
Figure 3: Impulse responses to a positive markup shock.

of changes in economic fundamentals such as preferences or production conditions. For these reasons, the analysis here limits to present arguments hinting higher or lower credibility for different regimes, and suggest changes in credibility as a potential explanation for the observed changes in the dynamics of bond yields. Alternatively, this analysis can be seen as providing additional information to identify the degree of policy credibility of a particular regime.

The analysis focuses on the Gold Standard regime, the Bretton Woods system, the fiat money regime that followed the collapse of Bretton Woods, and the Greenspan era. In addition, it provides a potential explanation of the Greenspan conundrum, based on gains in credibility in recent years.

The Gold Standard Regime Before and After 1914

According to Bordo and Kydland (1996), the gold standard can be seen as a commitment technology preventing the monetary authorities from changing planned future policy. Under a gold standard regime, the price of gold in dollars is fixed and, therefore, gold is a nominal anchor for the price level. In the United States, such a regime was in effect since 1880 when convertibility from dollars to gold was reestablished, until 1933 when the Federal Reserve
Table 6: U.S. Inflation and Bond Yield Statistics During the Gold Standard. Average and standard deviations are in percentage terms. Persistence is measured as the one-year autocorrelation. The Campbell-Shiller coefficients were computed assuming a 7-year duration for the long-term bond yield.

<table>
<thead>
<tr>
<th></th>
<th>1880 - 1913</th>
<th></th>
<th>1914 - 1932</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>Standard Deviation</td>
<td>Persistence</td>
<td>Average</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.09</td>
<td>2.11</td>
<td>0.27</td>
<td>2.06</td>
</tr>
<tr>
<td>Short-term rate</td>
<td>4.86</td>
<td>1.14</td>
<td>0.87</td>
<td>4.58</td>
</tr>
<tr>
<td>Long-term bond yield</td>
<td>3.66</td>
<td>0.32</td>
<td>0.99</td>
<td>4.39</td>
</tr>
<tr>
<td>Spread</td>
<td>-1.20</td>
<td>1.06</td>
<td>0.86</td>
<td>-0.20</td>
</tr>
<tr>
<td>Campbell-Shiller coeff.</td>
<td>-0.09</td>
<td></td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

decided to devalue the dollar. There are two different periods during this regime with different degrees of commitment to the gold standard. First, the period 1880-1913, which can be considered as a period of classical gold standard with perfect convertibility and high commitment. Second, the 1914-1932 period, when the establishment of the Federal Reserve System and World War I generated deviations from perfect convertibility. For instance, during this period the government financed some amount of expenditures with debt and fiat money and there was an embargo on gold exports. Therefore, the second period can be seen as one with lower credibility in the policy than the first one, and we can analyze if differences in bond yield dynamics in the two periods are consistent with those predicted by the model.

Table 6 shows statistics for the gold standard period before and after 1914.\textsuperscript{20} While the classical gold standard period was characterized by negative inflation and a significantly inverted yield curve, the 1914-1932 period had a positive and persistent inflation, and a mildly inverted curve. These developments in inflation and bond yield dynamics are consistent with

\textsuperscript{20}The inflation series for 1880-1912 is CPI inflation from Historical Statistics (1975), series E135. The inflation series for 1913-1932 is CPI inflation from the Bureau of Labor Statistics. The series for the short-term rate is the U.S. Commercial Paper Rates, New York City supplied by NBER. The series for the long-term bond yield is the Macaulay’s U.S. American Railroad Bond Yields, High Grade supplied by NBER.
the implications of the model for a reduction in policy credibility. As a result, the increase in the slope of the curve during 1914-1932 can be seen as an additional compensation for risk required by investors for deviations from the gold standard.

**Bretton Woods, the Fiat Money Regime, and the Greenspan Era**

Under the Bretton Woods agreement, many countries fixed their exchange rates relative to the U.S. dollar and the United States promised to fix the price of gold at $35 per ounce. According to McKinnon (1996), this system incorporated many of the features of the classical gold standard, suggesting that the golden nominal anchor served to restrain U.S. policy makers. The system collapsed when the government ended the convertibility from dollar to gold in 1971, and was followed by a “Fiat Money Regime” since then.

We can argue that the Bretton Woods system had a higher credibility than the Fiat Money Regime, since the former had the implicit commitment of convertibility. According to the model’s implications, tables 5 and 4 provide support to this hypothesis. Inflation was less volatile and less persistent during the Bretton Woods system than during the Fiat Money Regime. Simultaneously, the spread between the 5-year bond yield and the 3-month T-bill was 67 bps for the Bretton Woods period, significantly lower than the 111 bps during the Fiat Money Regime. With respect to Campbell-Shiller coefficients, they shifted from positive to negative across the two periods, implying a significant change in the positive correlation between spreads and expected excess returns. This evidence provides additional support to the idea of a higher credibility in the policy during the Bretton Woods system and suggests an explanation for the change in the dynamics of bond yields during the Fiat Money regime.

The Fiat Money regime covers the “Great Inflation” period and Paul Volcker’s and Alan Greenspan’s appointments as chairmen of the Federal Reserve. The relatively low and stable inflation observed during the Greenspan era points in the direction of another regime change.
For instance, Clarida, Galí and Gertler (2000) provide empirical evidence that suggests changes in the policy from a passive stance in the pre-Volcker years toward a stronger anti-inflationary stance during the Volcker-Greenspan era. This increase in the response of the Federal Reserve to inflation can be seen as a greater commitment to low inflation. The macroeconomic statistics for the Greenspan era in table 4 provide support for this idea. Inflation and consumption growth are less volatile and there is a significant reduction in the autocorrelation of inflation. However, the bond yield statistics in table 5 show an increase in the average slope of the curve which cannot be explained by an increase in the policy credibility. This increase in the spread between short-term and long-term yields can be interpreted as a fact shedding doubts about the increase in policy credibility during the Greenspan era or, alternatively, as an increase in the riskiness of long-term bonds due to technological changes or market developments not related to monetary policy.

The Greenspan Conundrum

In February 2005 Alan Greenspan declared that, “Long-term interest rates have trended lower in recent months even as the Federal Reserve has raised the level of the target federal funds rate by 150 basis points. Historically, even distant forward rates have tended to rise in association with monetary policy tightening. ... For the moment, the broadly unanticipated behavior of world bond markets remains a conundrum.” Therefore, one way to see the Greenspan Conundrum is as a difficult-to-explain reduction in long-term rates accompanying an increase in the short-term interest rate. We can use the model’s predictions and explain the conundrum as a consequence of credibility improvements in the policy. To see this, consider the impulse responses in figure 3. Under discretion, an inflationary shock is followed by an increase in the short-term rate and wider bond spreads. Spreads are higher as a result of increased risk premia in bonds, given the reduced credibility in the policy. Under commitment, spreads tend to decline after an inflationary shock. Since the policy is credible,
investors perceive an increase in the short-term rate as a measure to decrease inflation in the future. It induces a reduction in the compensation for inflationary risk that reduces the slope of the curve.

7 Conclusion

This paper provides a structural affine term-structure model that links the dynamics of interest rates to optimal monetary policy and macroeconomic risk. The model is useful to understand the effects on long-term interest rates of a welfare-maximizing policy and its credibility. The main finding is that optimal monetary policy affects bond risk premia through a very specific channel, and may determine the hedging properties of nominal bonds. The channel is the optimal tradeoff between the objectives of inflation and output stabilization. A source of inflation risk does not allow the monetary authority to simultaneously stabilize output and inflation, and, as a result, bond investors demand a compensation for this risk. The sign of this compensation depends on intertemporal and intratemporal preferences, and its size depends on the policy credibility. The compensation is positive if smoothing consumption intertemporally is more important than smoothing consumption across goods. In this case, the effect of inflation risk on the marginal utility of consumption outweighs the effect on inflation, making bonds more risky. The size of the compensation is unambiguously reduced by credibility improvements. Credibility helps stabilizing output and inflation, then the economy is less vulnerable to inflation risk and investors require lower compensations for this risk.

The structural framework in the paper also can be useful to understand general asset-pricing implications of inflation. Inflation affects consumption growth, thus the marginal utility of wealth and the price of risk of financial assets with real and nominal cashflows. The
analysis of the effects of inflation on the equity premium, or the study of the intertemporal nature of inflation distortions on welfare are two potential applications.

References


Appendix

I Proofs

I.1 Profit Maximization under Price Rigidities

This appendix contains the derivation of Equation (20).

Writing $M_{t,T} = \beta^{T-t}\Lambda_{t,T}$ and noticing that

\[
\frac{\partial \Psi_{T|t}(j)}{\partial P_t(j)} = \frac{Y_{T|t}(j)}{P_t(j)} e^{-\epsilon_T} \theta_T \left[ P_t(j)(\Pi^*)^{T-t} - e^{\epsilon_T} S_{T|t}(j) \right],
\]

we can write the first order condition for the firm’s problem as

\[
\mathbb{E}_t \left[ \sum_{T=t}^{\infty} (\alpha \beta_g) T-t \frac{\Lambda_T}{\Lambda^{T-t} e^{\Delta y}} \frac{Y_{T|t}}{Y_T} e^{-\epsilon_T} \theta_T \frac{P^*_t}{P_t} \right] = \mathbb{E}_t \left[ \sum_{T=t}^{\infty} (\alpha \beta_g) T-t \frac{\Lambda_T}{\Lambda^{T-t} e^{\Delta y}} \frac{Y_{T|t}}{Y_T} e^{-\epsilon_T} \theta_T \frac{S_{T|t}(j)}{P_t(\Pi^*)^{T-t}} \right],
\]

(37)

where $\beta_g = \beta \delta e^{\Delta y} \Pi^*$ and $\Delta y$ is the long run one-period output growth. It was used the fact that all producers who change prices optimally at $t$ face the same problem, and therefore $Y_{T|t}(j) = Y_{T|t}$, $P^*_t(j) = P^*_t$ and $S_{T|t}(j) = S_{T|t}$. Here $\Lambda^{T-t}$ can be seen as the steady state of $\Lambda_T$. Applying the Taylor expansion $a_t b_t = \bar{a} \bar{b} + b(a_t - \bar{a}) + \bar{a}(b_t - \bar{b})$ to both sides of the equation around a steady-state with $P^*_t = P_t$, $\Lambda_T = \Lambda^{T-t}$, $S_{T|t} = P_t(\Pi^*)^{T-t}$ and $\epsilon_T = 0$, we obtain for the left hand side of the equation

\[
\mathbb{E}_t \left[ \sum_{T=t}^{\infty} (\alpha \beta_g) T-t \frac{\Lambda_T}{\Lambda^{T-t} e^{\Delta y}} \frac{Y_{T|t}}{Y_T} e^{-\epsilon_T} \theta_T \left( \frac{P^*_t}{P_t} - 1 \right) \right]
\]

and for the right hand side

\[
\mathbb{E}_t \left[ \sum_{T=t}^{\infty} (\alpha \beta_g) T-t \frac{\Lambda_T}{\Lambda^{T-t} e^{\Delta y}} \frac{Y_{T|t}}{Y_T} e^{-\epsilon_T} \theta_T \left( \frac{S_{T|t}}{P_t(\Pi^*)^{T-t}} - 1 \right) \right].
\]
Noting that the first and second terms in both sides of the equation are the same, equation (37) becomes

\[
\frac{1}{(1-\alpha \beta_g)} \frac{P_t^*}{P_t} = E_t \left[ \sum_{T=t}^{\infty} (\alpha \beta_g)^{T-t} \frac{e^{\epsilon T} S_{T|t}}{P_t(P_t^*)^{T-t}} \right].
\]

Since \(S_{T|t} = s_{T|t}P_T\), replacing equation (16) in the equation above and re-arranging terms, we obtain

\[
\frac{1}{(1-\alpha \beta_g)} (P_t^*)^{1+\theta \omega} = E_t \left[ \sum_{T=t}^{\infty} (\alpha \beta_g)^{T-t} e^{\epsilon T} \left( \frac{P_T}{P_t^*} \right)^{1+\theta \omega} Y_T^{\omega+\gamma} Q_T A_T^{-(1+\omega)} \right],
\]

where the approximation \(\theta_T \approx \theta\) was used. The equation can be written in terms of the output gap and the habit gap as

\[
\frac{1}{(1-\alpha \beta_g)} (P_t^*)^{1+\theta \omega} e^{\epsilon t + (\omega + \gamma) x_t + l_t} \left( \frac{P_t}{P_t^*} \right)^{1+\theta \omega} = E_t \left[ \left( \frac{P_{t+1}}{P_t} \right)^{1+\theta \omega} \right].
\]

Letting \(p_t^* = \log P_t^*\), \(p_t = \log P_t\) and using the approximation \(e^x \approx 1 + x\), we obtain

\[
\frac{1}{(1-\alpha \beta_g)} (1 + (1 + \theta \omega)(p_t^* - p_t)) = 1 + \epsilon_t + (\omega + \gamma) x_t + l_t + \frac{\alpha \beta_g}{1-\alpha \beta_g} E_t \left[ 1 + (1 + \theta \omega)(p_{t+1} - p_t - p^*_t + p^*_{t+1} - p_{t+1}) \right].
\]

A first order Taylor approximation of \(P_t = \left[ (1-\alpha)(P_t^*)^{1-\theta_t} + \alpha(P_{t-1}^*)^{1-\theta_t} \right]^{1-\theta_t}\) results in

\(p_t = (1-\alpha)p_t^* + \alpha(p_{t-1} + \pi^*).\)

Replacing this equation in equation (38) and noticing that \(\pi_t = p_t - p_{t-1}\), the aggregate supply condition in equation (20) follows.

### I.2 Loss Function

Let \(\bar{Y}_t = Y_0 e^{\epsilon \Delta y}\) be the steady-state output at time \(t\), where \(Y_0\) is defined as the output at time 0 and \(\Delta y\) is the long-run output growth. The steady-state output satisfies

\[
\frac{\bar{Y}_t^{\omega+\gamma}}{Q_t^{-1} A_t^{1+\omega}} = 1,
\]

59
where $\hat{Q}_t$ and $\bar{A}_t$ are the steady-state habit and productivity at time $t$. By defining $\hat{y}_t \equiv \log Y_t - \log \hat{Y}_t$ and $\check{q}_t \equiv \log Q_t - \log \bar{Q}_t$, the utility from consumption in equation (21) can be written as

$$\frac{1}{1 - \gamma} \frac{Y_t^{1-\gamma}}{Q_t} = \frac{1}{1 - \gamma} \frac{\bar{Y}_t^{1-\gamma}}{\bar{Q}_t} e^{(1-\gamma)\hat{y}_t - \check{q}_t}.$$

A second-order log-approximation of this equation results in

$$\frac{1}{1 - \gamma} \frac{Y_t^{1-\gamma}}{Q_t} = \frac{1}{1 - \gamma} \frac{\bar{Y}_t^{1-\gamma}}{\bar{Q}_t} \left(1 + (1 - \gamma)\hat{y}_t - \check{q}_t + \frac{1}{2}(1 - \gamma)^2 \hat{y}_t^2 + \frac{1}{2}\check{q}_t^2 - (1 - \gamma)\hat{y}_t \check{q}_t \right) + \mathcal{O}((||\hat{y}_t, \check{q}_t||)^3).

Replacing equation (13), the disutility from labor in equation (21) is

$$\int_0^1 h_t(j) \frac{1}{1 + \omega} dj = \frac{1}{1 + \omega} \left(\frac{Y_t}{\bar{A}_t}\right)^{(1+\omega)} \int_0^1 \left(\frac{P_t(j)}{\bar{P}_t}\right)^{-\theta_t(1+\omega)} dj.$$

Since $\left(\frac{Y_t}{\bar{A}_t}\right)^{(1+\omega)} = \left(\frac{\Sigma^*}{\bar{A}_t}\right)^{(1+\omega)} e^{(1+\omega)(\hat{y}_t - \check{a})}$ and $\left(\frac{Y_t}{\bar{A}_t}\right)^{(1+\omega)} = \frac{\Sigma_t^{1-\gamma}}{\bar{Q}_t}$, it follows that

$$\int_0^1 h_t(j) \frac{1}{1 + \omega} dj = \frac{1}{1 + \omega} \frac{\bar{Y}_t^{1-\gamma}}{\bar{Q}_t} \int_0^1 e^{(1+\omega)[\hat{y}_t - \check{a}_t - \theta_t \bar{p}_t(j)]} dj,$$

where $\bar{p}_t(j) = \log \frac{P_t(j)}{\bar{P}_t}$. The term under the integral can be log-approximated by

$$\int_0^1 e^{(1+\omega)[\hat{y}_t - \check{a}_t - \theta_t \bar{p}_t(j)]} dj = 1 + (1 + \omega) (\hat{y}_t - \check{a}_t) + \frac{1}{2} (1 + \omega)^2 (\hat{y}_t - \check{a}_t)^2 - \theta_t (1 + \omega) \text{E}_j \bar{p}_t(j) + \frac{1}{2} \theta_t^2 (1 + \omega)^2 \text{var}_j(\bar{p}_t(j)) + \frac{1}{2} \theta_t (1 + \omega) \text{var}_j(p_t(j)) + \mathcal{O}(\bar{p}_t^3) \quad (41)$$

where the second equality comes from the log-approximation $p_t = \int_0^1 p_t(j) dj + \frac{1}{2} (1 - \theta) \text{var}_j(p_t(j)) + \mathcal{O}(p_t^3)$ and $\bar{p}_t(j) = p_t(j) - p_t$. Under Calvo staggered price setting, Woodford (2003), pages 399 and 400, shows that

$$\text{var}_j(p_t(j)) = \omega \text{var}_i(p_{t-1}(j)) + \frac{\alpha}{1 - \alpha} \left(\pi_t - \pi^*\right)^2 + \mathcal{O}(p_t^3)$$

$$= \frac{\alpha}{1 - \alpha} \sum_{s=0}^t \alpha^{t-s} \left(\pi_s - \pi^*\right)^2 + \alpha^{t+1} \text{var}_j(p_{t-1}(j)). \quad (42)$$

In order to obtain a representation in terms of the output gap, define $\tilde{Y}_t^{f} \equiv Y_t^{f} \exp((\omega + \gamma)^{-1} \epsilon_t)$. It is the natural rate of output under flexible prices adjusted by markup shocks. Notice that
\( x_t = y_t - \tilde{y}_t \) where \( \tilde{y}_t \equiv \log \tilde{Y}_t \) and optimality implies

\[
\left( \frac{\tilde{Y}_t}{Q_t} \right)^{\omega+\gamma} = 1.
\] (43)

Combining equations (39) and (43) we can write \( \hat{a}_t \) in terms of \( \hat{y}_t \equiv \log \tilde{Y}_t - \log \bar{Y}_t \) and \( \hat{q}_t \equiv \log Q_t - \log \bar{Q}_t \), such that

\[
x_t = \hat{y}_t - \hat{y}_t \quad \text{and} \quad l_t = \hat{q}_t - \hat{q}_t.
\]

Notice that \( \frac{\tilde{Y}_t^{1-\gamma}}{Q_t} \) does not depend on monetary policy and \( \beta_t = \beta_t^{1-\gamma} \bar{Q}_t^{-1} \). Given the log-approximations for the first and second term in equations (40), (41) and (42), and using the assumption that the habit gap \( l_t \) is external for the monetary authority, the loss function is obtained.

### I.3 Optimal Policy under Discretion

Let \( z_t = x_t - (\omega + \gamma)^{-1} l_t \). Under discretion the monetary policy problem at time \( t \) reduces to

\[
\max -\frac{1}{2} \left[ z_t^2 + \frac{\theta_g}{\kappa}(\pi_t - \pi^*)^2 \right]
\]

subject to

\[
\pi_t - \pi^* = \kappa z_t + F
\]

where

\[
F = \beta_g \mathbb{E}_t[\pi_{t+1} - \pi^*] + \frac{\kappa}{\omega + \gamma} \epsilon_t
\]

is taken as given. From the first order conditions, deviations of optimal inflation from the target can be written in terms of deviations from the target output and habit gaps as

\[
\pi_t - \pi^* = -\frac{1}{\theta_g} z_t.
\] (44)

Replacing equation (44) in equation (20), we obtain the linear rational expectations equation

\[
\pi_t - \pi^* = -\kappa \theta_g (\pi_t - \pi^*) + \beta_g \mathbb{E}_t[\pi_{t+1} - \pi^*] + \frac{\kappa}{\omega + \gamma} \epsilon_t.
\]

Guess a solution for optimal inflation of the form \( \pi_t = \pi^* + \pi^d \). Replacing this form in the equation above and matching coefficients, it can be seen that the coefficient \( \pi^d \) must satisfy the equation

\[
(1 + \kappa \theta_g) \pi^d = \beta_g \phi_t \pi^d + \frac{\kappa}{\omega + \gamma},
\]

with solution

\[
\pi^d = \frac{\kappa}{(\omega + \gamma)(1 + \kappa \theta_g - \beta_g \phi_t)}.
\]
In order to obtain the output growth process, notice that
\[ \Delta z_t = -\theta_g \Delta \pi_t. \]
From the definitions of \( x_t, l_t \) and \( \Delta y^f_t \), the process (22) follows.

Solving the problem for \( x_t \) instead of \( z_t \) and noticing that \( l_t \) can be written in terms of \( x_{t-1} \) provides two solutions. The first solution is consistent with the one presented above. The second solution depends on the lagged output gap \( x_{t-1} \). This dependence survives even when the lagged term does not appear in the rational expectations equation (when there is no habit, \( \eta = 0 \)), and thus violates the minimum state variable (MSV) criterion. Following McCallum (1999), the second solution is ruled out.

### I.4 Optimal Policy under Commitment

Let \( z_t = x_t - (\omega + \gamma)^{-1} l_t \). From equation (20), we obtain
\[ z_t = \frac{1}{\kappa} \left( \pi_t - \pi^* - \beta_g \mathbb{E}_t [\pi_{t+1} - \pi^*] - \frac{\kappa}{\omega + \gamma} \epsilon_t \right). \]
Using this equation, we can replace \( z_t \) in the welfare function to obtain a function that depends only on current and expected future inflation. As a result, the first-order condition with respect to \( \pi_t \) is
\[ \beta_g \left[ \frac{1}{\kappa} z_t + \frac{\theta_g}{\kappa} (\pi_t - \pi^*) \right] - \beta_g^{-1} \frac{\beta_g}{\kappa} z_{t-1} = 0. \]
It implies
\[ \Delta z_{t-1} = -\theta_g (\pi_t - \pi^*). \]
Replacing this condition in equation (20), we obtain
\[ -\frac{1}{\theta_g} (z_t - z_{t-1}) = \kappa z_t - \frac{\beta_g}{\theta_g} \mathbb{E}_t [z_{t+1} - z_t] + \frac{\kappa}{\omega + \gamma} \epsilon_t. \]
Guessing a solution of the form \( z_t = \varphi_1 z_{t-1} + \varphi_2 \epsilon_t \) and matching coefficients, the rational expectations equation has to satisfy the system:
\[ \varphi_1^2 - \frac{1}{\beta_g} (\kappa \theta_g + 1 + \beta_g) \varphi_1 + \frac{1}{\beta_g} = 0, \]
\[ \varphi_2 = -\frac{\kappa \theta_g}{(\omega + \gamma)(1 + \kappa \theta_g + \beta_g - \beta_g \varphi_1 - \beta \phi_\epsilon)}. \]
The equation for \( \varphi_1 \) has two solutions but only one solution is stable \((0 < \varphi_1 < 1)\). We use the stable solution and define \( \phi_c = \varphi_1 \) and \( \pi_c^\epsilon = -\frac{\varphi_2}{\beta_g} \). Using the definition for \( z_t \) we obtain the expressions for output and inflation in proposition 2.
I.5 Term premium derivation

The state variables $s_t$ in equation (26) are conditionally multivariate normally distributed. As a result, bond yields are conditionally normally distributed. By writing bond prices in terms of bond yields and solving the expectation of a log-normal variable, equation (28) becomes

$$ e^{-nt_t} = \exp \left\{ \mathbb{E}_t \left[ \log M_{t,t+1} - (n-1)\bar{i}_{t+1}^{(n-1)} \right] + \frac{1}{2} \text{var}_t \left( \log M_{t,t+1} - (n-1)\bar{i}_{t+1}^{(n-1)} \right) \right\}, $$

then,

$$ -nt_t = -i_t - (n-1)\mathbb{E}_t \left[ \bar{i}_{t+1}^{(n-1)} \right] + \frac{1}{2} \left[ \text{var}_t \left( \log M_{t,t+1} - (n-1)\bar{i}_{t+1}^{(n-1)} \right) - \text{var}_t \left( \log M_{t,t+1} \right) \right]. $$

Comparing the result above with the definition of term premium in equation (30) and the analytical expressions for the variance terms derived from equation (27), we obtain

$$ \xi_t^{(n)} = \frac{1}{2n} \left[ \lambda^\top \Psi (s_t) \Sigma \Psi (s_t)^\top \lambda - \lambda_n^\top \Psi (s_t) \Sigma \Psi (s_t)^\top \lambda_n \right]. \quad (45) $$

While the first term in this equation contains the precautionary savings effect on the one-period interest rate induced by uncertainty in the one-period pricing kernel, the second term contains the precautionary savings effect induced by the interaction of the uncertainty of the one-period pricing kernel and the one-period return risk of the particular bond. Using the definitions of $\lambda_n$ and $\Psi(s_t)$ we obtain the affine form (31).

To obtain the $\beta^{(n)}$ coefficients in equation (36), notice that the $\beta^{(n)}$ is the scaled projection of $i_{t+1}^{(n-1)} - i_t^{(n)}$ on $i_t^{(n)} - i_t$. That is,

$$ \frac{\beta^{(n)}}{n-1} = \frac{\text{cov} \left( i_{t+1}^{(n-1)} - i_t^{(n)}, i_t^{(n)} - i_t \right)}{\text{var} \left( i_t^{(n)} - i_t \right)}.$$

It is convenient now to describe the spread between the yield of a bond maturing at time $n$ and the one-period interest rate using the affine representation

$$ i_t^{(n)} - i_t = C_n + D_n^\top s_t = \left( \frac{1}{n}A_n - A_1 \right) + \left( \frac{1}{n}B_n - B_1 \right)^\top s_t. \quad (46) $$

The expression for $\beta^{(n)}$ in equation (36) follows from replacing $i_{t+1}^{(n-1)} - i_t^{(n)}$ and $i_t^{(n)} - i_t$ in the equation for $\beta^{(n)}$ above, by using equations (30), (31) and (46). It can be simplified, to obtain

$$ \beta^{(n)} = 1 - n \frac{D_n^\top \text{var}(s_t) \xi_B}{D_n^\top \text{var}(s_t) D_n}. $$
I.6 Short-Term Nominal Rates under Discretion and Commitment

The effects of credibility on the short-term rate can be better understood considering first the interest rate observed in a hypothetical economy with flexible prices and inflation at the target level $\pi^*$. This rate is known as the natural rate of interest and is given by

\[
i_t^f = -\log \beta + \pi^* + \gamma \frac{1 + \omega}{\omega + \gamma} (1 - \phi_a) g_a - \frac{1}{2} \gamma^2 \left( \frac{1 + \omega}{\omega + \gamma} \right)^2 \sigma_a^2 - \frac{1}{2} \left( \frac{\omega}{\omega + \gamma} \right)^2 \sigma_q^2 - \frac{1}{2} \left( \frac{\gamma}{\omega + \gamma} \right)^2 \sigma_r^2 \\
+ \frac{1 + \omega}{\omega + \gamma} \left[ \phi_a - \frac{1}{2} \gamma \frac{1 + \omega}{\omega + \gamma} K_a \sigma_a^2 \right] \Delta a_t + \frac{\omega}{\omega + \gamma} \left[ \eta - \frac{1}{2} \frac{\omega}{\omega + \gamma} K_q \sigma_q^2 \right] \Delta y_t^f \\
+ \frac{\gamma}{\omega + \gamma} \left[ 1 - \phi_e - \frac{1}{2} \left( \frac{\gamma}{\omega + \gamma} K_e \sigma_e^2 \right) \right] \epsilon_t.
\]

Besides the positive dependence of the interest rate on the level of labor productivity, the stochastic habit makes the interest rate depend on current output growth. Since $\eta < 0$, the habit reduces the natural rate of output to dissuade households from excessive savings. In addition, a positive markup shock tends to increase the one-period rate given the reduction of output that results from higher market power. The time-variation components in the volatility of the shocks generate additional precautionary savings terms which reduces the natural rate of interest for high levels of productivity, output, and markup shocks.

Given that prices are not perfectly flexible, the interest rate differs from the natural rate of interest and depends on monetary policy. Specifically, the equilibrium level of the interest rate depends on current changes in the output gap and inflation. If the policy is conducted under discretion, the one-period interest rate is

\[
i_t^d = i_t^f + [(1 - \gamma \theta_g)(1 - \phi_e) - 1] \pi^* + \frac{\omega}{\omega + \gamma} \left[ \eta - \frac{1}{2} \left( \frac{\omega}{\omega + \gamma} \right) K_q \sigma_q^2 \right] (y_t^d - y_t^f) \\
+ [(1 - \gamma \theta_g) \phi_e + \gamma \theta_g] \pi_t^d - \left( \frac{\gamma}{\omega + \gamma} \right) (1 - \phi_e) \epsilon_t \\
- \frac{1}{2} \left[ (1 - \gamma \theta_g)^2 (\pi_t^d)^2 - \left( \frac{\gamma}{\omega + \gamma} \right)^2 \right] (1 + K_e \epsilon_t) \sigma_e^2.
\]

\[21\] This rate is found solving the Euler equation for one-period bonds $e^{-i_t^i} = \mathbb{E}_t \left\{ \exp (\log \beta - \gamma \Delta y_t^f - \Delta q_t^f - \pi^*) \right\}$. 

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Under commitment, the interest rate is

\[ i_t^c = i_t^f - (1 - \gamma \theta_g) \phi^c \pi^* + \frac{\omega}{\omega + \gamma} \left[ \eta - \frac{1}{2} \left( \frac{\omega}{\omega + \gamma} \right) K_q \sigma_q^2 \right] (y_t^c - y_t^f) + (1 - \gamma \theta_g) \phi^c \pi^c_t - (1 - \phi^c) \left[ (1 - \gamma \theta_g) \pi^c_t + \frac{\gamma}{\omega + \gamma} \right] \epsilon_t - \frac{1}{2} \left[ (1 - \gamma \theta_g)^2 (\pi^c)^2 - \left( \frac{\gamma}{\omega + \gamma} \right)^2 \right] (1 + K_c \epsilon_t) \sigma^2_c. \] (49)

We can notice from these two equations that when output growth is lower than the growth in the natural rate of output, the external habit raises the interest rate to decrease the excess consumption demand today. The effect of inflation on the short term for the two regimes is unclear given the endogenous link between inflation and markup shocks.

II Calibration and Policy Experiment.

The calibration consists of matching selected properties of the U.S. economy for the 1971:3 to 2007:4 period to the associated unconditional first and second moments of the equilibrium properties of the model under discretion.

The unconditional expectation of the state variables, \( s_t \), is

\[ \mathbb{E}[s_t] = (\mathbb{I} - \Phi)^{-1} \psi \]

and the unconditional covariance matrix is given by the solution \( \text{var}(s_t) \) to the Lyapunov equation

\[ \text{var}(s_t) - \Phi \text{var}(s_t) \Phi^\top = \Psi_c \Sigma \left( C + K \text{diag} \{ \mathbb{E}[s_t] \} \right) \Psi_c^\top. \]

This solution is an approximation since \( s_t \) is constrained to take values that ensures that \( \Psi(s_t)^2 \) is positive definite. The solution to this equation is given by

\[ \text{var}(s_t) = \sum_{t=0}^{\infty} \Phi^t \Psi_c \Sigma \left( C + K \text{diag} \{ \mathbb{E}[s_t] \} \right) \Psi_c^\top \left( \Phi^\top \right)^t. \]

Using the unconditional moments above and the appropriate formulas from the affine framework in Section 4, it is possible to compute unconditional moments for interest rates, term premia and Campbell-Shiller coefficients. The calibration amounts then to find parameter values that minimize deviations of the theoretical unconditional moments from their empirical counterparts.

The are 16 parameters involved in the calibration: seven preference parameters, \( \beta, \gamma, \eta, \omega, \theta, K_q \) and \( \sigma_q \), and nine production parameters, \( \alpha, \pi^*, g_a, \phi_a, \phi_k, K_a, K_k, \sigma_a \) and \( \sigma_k \). In order to obtain values for \( \phi_a, \sigma_a \) and \( K_a \), the labor productivity series from Gomme and Rupert (2007)\(^{22} \) was

\(^{22}\)The data corresponds to GDP per hour of work. The series is found in http://clevelandfed.org/research/Models/rbc/ Index.cfm and the source is Haver Analytics. This series has the inconvenient that it measures productivity of output growth instead of non durables and
used to fit an AR(1) model for growth in labor productivity as in equation (15). Since there is no statistical evidence of heteroscedasticity in the errors, $K_a$ is set to 0. The value of $g_a$ is chosen such that the unconditional expected consumption growth implied by the model perfectly matches the average consumption growth during the period. The values of $\beta$ and $\theta$ were set at 0.99 and 3, respectively. The value for $\pi^*$ was set equal to the average inflation for the sample. The persistence of markup shocks, $\phi_\epsilon$, is set equal to the first-order autocorrelation of inflation. This particular calibration seems appropriate to understand the effects of credibility improvements on the level of interest rates and the term premia, given that, as shown in Section 5.1, inflation persistence is an important indicator of the level of credibility of the policy. The value of $\sigma_\epsilon$ was chosen such that the model matches the standard deviation of inflation. The values for $\gamma$, $\eta$, $\omega$, $K_q$ and $K_\epsilon$ were chosen to minimize a measure of deviations from the sample standard deviation of output growth, the average three-month T-Bill rate, the average slope of the curve and the volatility of the one-year bond yield. The stochastic volatility parameters, $K_q$ and $K_\epsilon$ were constrained such that the volatility of the habit and the markup shock are well defined, respectively, after one standard deviation shocks. That is, $1 + K_q(E[\Delta y_t] + \sigma(\Delta y_t)) \geq 0$ and $1 - K_\epsilon \sigma_\epsilon \geq 0$.

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services consumption. Therefore, it is implicitly assumed that the productivities of consumption goods and services, and aggregate output are the same.