Business cycle accounting for monetary economies

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Abstract

This paper extends business cycle accounting to a class of monetary models to investigate the quantitative importance of various market frictions for the joint dynamics of real and nominal variables over the business cycle. After establishing mappings between frictions in detailed economies and wedges in a prototype economy, the method is applied to the 1973 and the 1982 US recessions, as well as to two outstanding anomalies in the postwar business cycle: the cross-correlations at various leads and lags between output and a short-term nominal interest rate, and between output and inflation. The findings demonstrate that: (i) fluctuations in inflation during the 1973 recession can be accounted for in a model in which the Fed accommodates a decline in output caused by a fall in total factor productivity (TFP); (ii) the fall in inflation during the 1982 recession can be accounted for by a change in monetary policy, but worsening TFP and labour market frictions prevented a much faster decline in inflation; (iii) the two anomalies can be accounted for by fluctuations in TFP and asset market distortions; and (iv) in both recessions, as well as in the entire postwar business cycle, sticky prices and investment distortions played only a minor role.

Key words: Business cycle accounting, monetary models, inflation, nominal interest rate

JEL classification: E31, E32, E43, E52
Summary

[TO BE ADDED]
1 Introduction

Chari, Kehoe and McGrattan (2007a) develop a data analysis method to investigate the quantitative importance of various classes of market frictions for aggregate fluctuations. This method, which they label ‘business cycle accounting’, is intended to help researchers make decisions about where to introduce frictions in their models so that they generate fluctuations like those in the data. Chari et al (2007a), henceforth CKM, focus on fluctuations in four key real variables: output, hours, investment, and consumption. Often, however, economists are also interested in the behaviour of the nominal side of the economy (for example, in fluctuations in the growth rate of prices), and how it co-moves with real economic activity. This paper therefore extends the method to fluctuations in two key nominal variables: inflation and the nominal interest rate. The purpose of this extension is to make the method applicable to investigate the quantitative importance of various classes of frictions for the joint dynamics of real and nominal variables over the business cycle.

Business cycle accounting rests on the insight that a large class of models with various market frictions can be mapped into a prototype model with a number of time-varying ‘wedges’ that distort the equilibrium decisions of agents operating in otherwise competitive markets. (1) Using the equilibrium conditions of the prototype model and data on the model’s endogenous variables, the wedges can be uncovered from the data and fed back into the model, individually and in various combinations, in order to determine their contributions to the observed movements in the data. By establishing mappings between different types of frictions and the wedges, the method can be used to assess the contribution of various frictions to fluctuations in the data. By construction, all wedges together account for all of the movements in the data. (2)

CKM provide mappings between a number of detailed models and a prototype stochastic growth model with four time-varying wedges, henceforth referred to as the CKM economy. At face value, fluctuations in these wedges look like fluctuations in total factor productivity, taxes on labour income, taxes on investment, and government consumption. CKM label these wedges efficiency, labour, investment, and government consumption wedges, respectively. They demonstrate that input-financing frictions are equivalent to

(1) Other researchers besides CKM, for example Hall (1997), Mulligan (2002a) and Mulligan (2002b), also interpret wedges in equilibrium conditions of a competitive economy as reflecting some underlying market distortions.

(2) Other papers besides CKM that discuss the method include Christiano and Davis (2006) and Chari, Kehoe and McGrattan (2007b). Papers that apply the method to various episodes in different countries include Crucini and Kahn (2003), Ahearne, Kydland and Wynne (2005), Chakraborty (2005), Kobayashi and Inaba (2006), and Kersting (2008).
efficiency wedges, labour market distortions are equivalent to labour wedges, investment-financing frictions are equivalent to investment wedges, and frictions in international borrowing and lending are equivalent to government consumption wedges. Applying the method to the Great Depression and the postwar US business cycle they show that promising models of the business cycle have to include frictions that are equivalent to efficiency and labour wedges, but can safely abstract from frictions that are equivalent to investment and government consumption wedges.

In order to make the method applicable to fluctuations in both real and nominal variables, this paper first constructs a prototype monetary economy – a straightforward extension of the stochastic growth model in which consumers hold nominal assets, in addition to physical capital, and in which, in line with much of the current literature, monetary policy is described by a simple feedback rule, like that proposed by Taylor (1993). Besides the four wedges of the CKM economy, the prototype monetary economy has two additional wedges: an asset market wedge, which resembles a tax on nominal bond holdings and distorts a no-arbitrage condition between capital and bonds, and a monetary policy wedge, which resembles a monetary policy shock.

In order to demonstrate that an important class of monetary business cycle models can be mapped into the prototype, the paper provides four examples of mappings between detailed models with frictions and the prototype. These examples include key frictions considered in the literature. In particular, the paper shows that an economy with sticky prices is equivalent to the prototype economy with equal investment and labour wedges, and that an economy with limited participation in asset markets is equivalent to the prototype economy with asset market wedges. The paper also shows that sticky wages are equivalent to labour wedges, and that fluctuations in energy prices in a model with capital utilisation are equivalent to fluctuations in efficiency wedges. Furthermore, it shows that detailed monetary policy rules, such as those with regime changes, are equivalent to a prototype Taylor rule with monetary policy wedges. (3)

The realised wedges are then uncovered using data on output, hours, investment, consumption, the GDP deflator, and the yield on 3-month Treasury bills for the postwar period in the United States. After that the wedges are fed back into the model, one at a time and in various combinations, in order to determine how much of the observed

(3) The equivalence results established in this paper do not provide an exhaustive list of frictions that can be mapped into the prototype model. As stressed by CKM, more than one friction can be mapped into the same wedge. Nevertheless, providing mappings for the key frictions considered in the literature is useful for demonstrating the generality of the prototype economy, as well as for getting a sense of what types of frictions can be mapped into which wedges.
movements in the data can be attributed to each wedge.

The decomposition is applied to two postwar downturns, the 1973 and the 1982 recessions, which are used as case studies in order to demonstrate how the extended method can be used. The two recessions are interesting because they were the two most severe downturns in the postwar US business cycle. In addition, they have very different inflation dynamics and are usually thought to have been caused by different shocks: the 1973 recession by high oil prices (a ‘supply shock’), while the 1982 recession by a tight monetary policy (a ‘demand shock’). We also use the method to shed light on two outstanding anomalies in the nominal business cycle identified in the literature: the cross-correlations at various leads and lags between output and the nominal interest rate (noted, among others, by King and Watson (1996)), and the cross-correlations at various leads and lags between output and inflation (pointed out, for example, by Galí and Gertler (1999)).

For the 1973 recession, we find that the efficiency wedge played a crucial role in the sharp decline in economic activity following the oil-crisis, while the labour wedge played a crucial role in the subsequent slow recovery. In addition, the behaviour of inflation during the recession can be successfully accounted for in a model in which the Fed accommodates a decline in output caused by a fall in the efficiency wedge. Our equivalence result shows that energy price fluctuations and capital utilisation would be consistent with such movements in the efficiency wedge. The decomposition also shows that any model (within the class of models that can be mapped into the prototype) that is to capture the dynamics of the nominal interest rate during this period has to have frictions that are equivalent to asset market wedges.

In the case of the 1982 recession, both efficiency and labour wedges are key for generating both the decline as well as the recovery in real economic activity. These results are the same as those of CKM. In terms of inflation, the monetary policy wedge is crucial for producing the observed fall in the inflation rate by the end of the period. We show that a fall in the implicit inflation target in the Fed’s feedback rule is consistent with the behaviour of the monetary policy wedge in the data. However, the decomposition also shows that without prolonged declines in efficiency and labour wedges during this period, low inflation would have been achieved much faster.

Applying the method to the entire postwar business cycle, we find that any model that is to account for the observed cross-correlations at various leads and lags between output and the nominal interest rate, and between output and inflation, has to have frictions or propagation mechanisms that are equivalent to efficiency and asset market wedges. We
also find that in both recessions, as well as in the entire postwar business cycle, sticky prices (which we show are equivalent to equal investment and labour wedges), as well as frictions that are equivalent to investment or government consumption wedges, played only a minor role in fluctuations in both real and nominal variables. Business cycle models can therefore safely abstract from such frictions without adversely affecting their ability to account for the movements in either real or nominal variables.

The rest of the paper proceeds as follows. Section 2 describes the prototype monetary economy, Section 3 provides two mappings between detailed economies and the prototype, Section 4 describes the procedure for uncovering the wedges from the data and characterises their business cycle behaviour, Section 5 carries out the decompositions, Section 6 investigates the sensitivity of the results to alternative parameterisations of the monetary policy rule, and Section 7 concludes. In addition, Appendix A contains proofs of the equivalence results established in Section 3, while Appendix B provides two additional examples of mappings between detailed economies and the prototype.

2 The prototype monetary economy

2.1 The economic environment

The prototype economy is inhabited by an infinitely lived representative consumer and a representative producer. Both are price takers in all markets. In addition, there is a government that taxes the consumer and issues money. In each period \( t \) the economy experiences one of finitely many events \( z_t \). Let \( z^t = (z_0, ..., z_t) \) denote the history of events up through and including period \( t \), \( Z^t \) the set of all possible histories \( z^t \), \( Z^t \) the appropriate \( \sigma \)-algebra, and \( \mu_t(z^t) \) the probability measure associated with this \( \sigma \)-algebra. The initial event \( z_0 \) is given. The probability space of this economy is thus defined by the triplet \( (Z^t, Z^t, \mu_t(z^t)) \). Furthermore, let \( \mu_t(z^{t+1}|z^t) \) denote the conditional probability \( \mu_t(z^{t+1}|z^t) \). The economy has six exogenous random variables all of which are functions of the history of events \( z^t \): the efficiency wedge \( A_t(z^t) \), the labour wedge \( \tau_{lt}(z^t) \), the investment wedge \( \tau_{xt}(z^t) \), the government consumption wedge \( g_t(z^t) \), the asset market wedge \( \tau_{bt}(z^t) \), and the monetary policy wedge \( \tilde{R}_t(z^t) \). The first four wedges are the same as those in the CKM economy and will therefore be sometimes referred to as the CKM wedges. They distort the same first-order conditions and resource constraints as in the CKM economy. The asset market wedge works like a tax on adjusting bond holdings, and the monetary policy wedge captures all deviations of the nominal interest rate from a rate dictated by a monetary policy rule.
The consumer maximises expected utility over stochastic paths of per capita consumption \( c_t(z^t) \) and per capita leisure \( h_t(z^t) \) \(^{(4)}\)

\[
\sum_{t=0}^{\infty} \sum_{z^t} \beta^t \mu_t(z^t) u \left( c_t(z^t), h_t(z^t) \right) (1 + \gamma_n)^t
\]

where \( \beta \) is a discount factor and \( \gamma_n \) is a population growth rate, subject to three constraints. First, the consumer has to satisfy the time constraint

\[ h_t(z^t) + l_t(z^t) + s_t(z^t) = 1 \] (2)

where \( l_t(z^t) \) is time spent working and \( s_t(z^t) \) is time spent shopping, which is determined by the function

\[ s_t(z^t) = s \left( \frac{c_t(z^t)}{(1 + \gamma_n) m_t(z^t)/p_t(z^t)} \right) \] (3)

Unless specified otherwise, the function \( s(.) \) is assumed to be smooth, increasing and convex. It is also assumed to satisfy the condition \( s(0) = 0 \), i.e., shopping time is zero when the amount of purchases is zero. \(^{(5)}\) Second, the consumer has to satisfy the budget constraint

\[
\begin{align*}
&c_t(z^t) + \left[ 1 + \tau_{zt}(z^t) \right] x_t(z^t) + (1 + \gamma_n) \frac{m_t(z^t)}{p_t(z^t)} \\
&\quad + \left[ 1 + \tau_{lt}(z^t) \right] \left[ (1 + \gamma_n) \frac{b_t(z^t)}{p_t(z^t)(1 + R_t(z^t))} - \frac{b_t(z^{t-1})}{p_t(z^t)} \right] \\
&= \left[ 1 - \tau_{lt}(z^t) \right] w_t(z^t) l_t(z^t) + r_t(z^t) k_t(z^{t-1}) + \frac{m_t(z^{t-1})}{p_t(z^t)} + \frac{T_t(z^t)}{p_t(z^t)}
\end{align*}
\]

Here, \( x_t(z^t) \) is investment, \( m_t(z^t) \) is money balances, \( p_t(z^t) \) is the price of goods in terms of money, \( b_t(z^t) \) is bonds that pay a net nominal rate of return \( R_t(z^t) \) in all states of the world \( z_{t+1} \) and are in net zero supply, \( w_t(z^t) \) is the real wage rate, \( r_t(z^t) \) is the real rental rate for capital, \( k_t(z^{t-1}) \) is capital held at the start of period \( t \), and \( T_t(z^t) \) is government lump-sum transfers. The third constraint is the law of motion for capital

\[
(1 + \gamma_n) k_{t+1}(z^t) = (1 - \delta) k_t(z^{t-1}) + x_t(z^t)
\] (4)

where \( \delta \) is a depreciation rate.

The producer operates an aggregate constant-returns-to-scale production function

\[ y_t(z^t) = A_t(z^t) F \left( k_t(z^{t-1}), (1 + \gamma_A)^t t_t(z^t) \right) \] (5)

where \( \gamma_A \) is the growth rate of labour-augmenting technological progress and where \( F(.,.) \) has all the standard properties. The producer maximises per period profits

\(^{(4)}\) All quantities in the model are in per capita terms.

\(^{(5)}\) Shopping time is one of three alternative ways of introducing money into Arrow-Debreu economies, the other two being a cash-in-advance constraint and money in the utility function. A cash-in-advance constraint is an extreme case of a shopping time constraint, while shopping time and money in the utility function are to a large extent equivalent since money enters utility once we substitute for \( s_t \) from equation (3). Our main results thus do not depend on which of the three ways of introducing money into models we use.
Interest rate is set according to the monetary policy rule such that the allocations are optimal for the consumer and the producer, the nominal equal to \( r_t(z^t) \) and \( w_t(z^t) \), respectively. The aggregate resource constraint requires that

\[
ct(z^t) + xt(z^t) + g_t(z^t) = yt(z^t)
\]  

Following Taylor (1993) and a large empirical literature that estimates monetary policy feedback rules (surveyed, for example, by Woodford (2003), Chapter 1), most existing monetary business cycle models (e.g., McGrattan (1999), Ireland (2004), and Smets and Wouters (2007)) specify monetary policy as following a feedback rule like that proposed by Taylor. In order to preserve the structure of this class of models, the government in the prototype economy also sets the nominal interest rate according to such a rule

\[
R_t(z^t) = (1 - \rho_R)R^*_t(z^t) + \rho_R R_{t-1}(z^{t-1}) + \tilde{R}_t(z^t)
\]  

where

\[
R^*_t(z^t) = R + \omega_y (\ln y_t(z^t) - \ln y) + \omega_\pi (\pi_t(z^t) - \pi)
\]

Here, \( \rho_R \) is a ‘smoothing’ coefficient, \( \pi_t(z^t) \equiv \ln p_t(z^t) - \ln p_{t-1}(z^{t-1}) \) is the inflation rate and a variable’s symbol without a time subscript denotes the variable’s steady-state value.

2.2 Equilibrium and the distortionary effects of wedges

A competitive equilibrium of the prototype economy is a set of allocations \((c_t(z^t), x_t(z^t), y_t(z^t), l_t(z^t), k_{t+1}(z^t), m_t(z^t), b_t(z^t))\) and a set of prices \((p_t(z^t), R_t(z^t), r_t(z^t), w_t(z^t))\) such that the allocations are optimal for the consumer and the producer, the nominal interest rate is set according to the monetary policy rule (7), \( b_t(z^t) \) is equal to zero, and the resource constraint (6) is satisfied.

In equilibrium, the consumer’s optimal behaviour can be summarised by the following first-order conditions for labour, capital, bonds, and money holdings, respectively

\[
[1 - \tau_t(z^t)] A_t(z^t)(1 + \gamma_A)^t F_{lt}(z^t) = \frac{u_{bt}(z^t)}{u_{ct}(z^t)} \{ 1 + s_{ct}(z^t) [1 - \tau_t(z^t)] A_t(z^t)(1 + \gamma_A)^t F_{lt}(z^t) \}
\]

\[
[1 + \tau_{xt}(z^t)] (1 + \gamma_n) = \sum_{z_{t+1}} Q_t(z_{t+1} | z^t) \{ [1 + \tau_{x,t+1}(z^{t+1})] (1 - \delta) + A_{t+1}(z^{t+1}) F_{k,t+1}(z^{t+1}) \}
\]

\[
\sum_{z_{t+1}} Q_t(z_{t+1} | z^t) \frac{[1 + \tau_{x,t+1}(z^{t+1})] (1 - \delta) + A_{t+1}(z^{t+1}) F_{k,t+1}(z^{t+1})}{1 + \tau_{xt}(z^t)}
\]

\[
= \sum_{z_{t+1}} Q_t(z_{t+1} | z^t) \frac{1 + \tau_{h,t+1}(z^{t+1})}{1 + \tau_{bd}(z^t)} \frac{p_t(z^t)}{p_{t+1}(z^{t+1})} [1 + R_t(z^t)] \frac{p_t(z^t)}{p_{t+1}(z^{t+1})}
\]
and
\[(1 + \gamma_A) - \frac{u_{ht}(z^t)s_{mt}(z^t)}{[u_{ct}(z^t) - u_{ht}(z^t)s_{ct}(z^t)]} = \sum_{z_{t+1}} Q_t(z^{t+1}|z^t) \frac{p_t(z^t)}{p_{t+1}(z^{t+1})} \]
(11)
where
\[Q_t(z^{t+1}|z^t) = \beta \mu_t(z^{t+1}|z^t) \frac{u_{ct+1}(z^{t+1}) - u_{ht+1}(z^{t+1}) s_{ct+1}(z^{t+1})}{u_{ct}(z^t) - u_{ht}(z^t) s_{ct}(z^t)} \]
(12)
is the stochastic discount factor. Here, and throughout the paper, \(u_{ct}, u_{ht}, s_{ct}, s_{mt}, F_{kt},\) and \(F_{lt}\) denote the derivatives of the utility, shopping time and production functions with respect to their arguments. Notice that in the absence of shopping time, equations (8)-(9) become the standard optimality conditions in a stochastic growth model.

As in the CKM prototype economy, the labour wedge in the prototype monetary economy distorts the intratemporal optimality condition for labour (8), while the investment wedge distorts the intertemporal optimality condition for investment (9). In addition, for given investment wedges, the asset market wedge distorts the no-arbitrage condition for capital and bonds (10). The efficiency and government consumption wedges play the same role here as in the CKM economy. The efficiency wedge determines the amount of output produced for a given amount of inputs, while the government consumption wedge determines the amount of output available for consumption and investment. As mentioned above, the monetary policy wedge captures all deviations of the nominal interest rate from the rate dictated by the monetary policy feedback rule.

At a mechanical level, money in this economy has real effects only through an inflation tax, which affects shopping time and thus the consumer’s time available for leisure and work. For calibration of the model to US data, these effects are small. It is therefore instructive at this stage to think of the prototype economy as being block recursive: first, the consumer’s optimality conditions (8) and (9), together with the production function (5), the resource constraint (6), and the law of motion for capital (4) determine the equilibrium \(c_t(z^t), x_t(z^t), y_t(z^t), l_t(z^t), k_{t+1}(z^t);\) then the no-arbitrage condition (10) and the monetary policy rule (7) determine equilibrium \(p_t(z^t)\) and \(R_t(z^t);\) and finally, the optimality condition for money (11) determines equilibrium \(m_t(z^t).\) As a result of this (approximately) recursive structure, the CKM wedges affect all endogenous variables, whereas the asset market wedge and the monetary policy wedge have (significant) effects only on inflation, the nominal interest rate and money.

The usefulness of this setup in which money is almost neutral is its generality: a large class of models with various market frictions and propagation mechanisms, including models with nominal rigidities, can be mapped into this prototype economy. The propagation of shocks due to underlying frictions in specific economic environments, including sticky
prices and wages, will show up in the prototype economy as time-varying wedges and our decomposition procedure will determine their contribution to fluctuations in the data. If these frictions show up as CKM wedges, they will affect all variables. If, however, they show up only as asset market or monetary policy wedges, they will have (significant) effects only on nominal variables. It is possible that some frictions show up as more than one wedge. A strong comovement in some wedges in the data will thus indicate that their fluctuations are likely due to the same set of frictions.

2.3 Inflation, the nominal interest rate and underlying frictions: A discussion

This section describes how the dynamics of inflation and the nominal interest rate in a variety of models can be characterised by pricing functions in the prototype economy that link these two variables to the wedges. It is convenient for the current discussion to log-linearise the equilibrium conditions (10) and (7) in the neighborhood of the model’s steady state. Equation (10) becomes

\[ a_1 E_t \hat{\pi}_{x,t+1} - a_2 \hat{\pi}_{xt} + a_3 E_t \hat{A}_t + a_4 E_t \hat{\hat{l}}_{t+1} - a_5 E_t \hat{\hat{k}}_{t+1} = a_6 E_t \hat{\hat{b}}_{t+1} - a_7 \hat{\hat{b}}_{bt} + a_8 \hat{R}_t - a_9 E_t \hat{\pi}_{t+1} \]  \hspace{1cm} (13)

and equation (7) becomes

\[ \hat{R}_t = (1 - \rho_R) \hat{\omega}_y \hat{y}_t + (1 - \rho_R) \hat{\omega}_\pi \hat{\pi}_t + \rho_R \hat{R}_{t-1} + \hat{R}_t \]  \hspace{1cm} (14)

where variables with a ‘hat’ denote percentage deviations from steady state. In equation (13), the coefficients are defined as follows

\[ a_1 \equiv (1 - \delta)/(1 + \tau_x), \]
\[ a_2 \equiv [(1 - \delta)(1 + \tau_x) + AF_k]/(1 + \tau_x)^2, \]
\[ a_3 \equiv F_k A/(1 + \tau_x), \]
\[ a_4 \equiv AF_k l/(1 + \tau_x), \]
\[ a_5 \equiv -AF_{kk} k/(1 + \tau_x), \]
\[ a_6 \equiv (1 + R)/(1 + \pi)(1 + \tau_b), \]
\[ a_7 \equiv (1 + R)/(1 + \pi)(1 + \tau_b), \]
\[ a_8 \equiv 1, \]
\[ a_9 \equiv (1 + R)/(1 + \pi)^2. \]

Notice that they all are positive.

Assuming, for illustration, that each wedge follows an AR(1) process (in the actual application the wedges will follow a vector autoregressive process), and combining equations (13) and (14), inflation in period \( t \) can be expressed as

\[ \hat{\pi}_t = \frac{1}{(1 - \rho_R)\omega_\pi} \left[ -(a_2 - a_1 \rho_x) \hat{\pi}_{xt} + a_3 \rho_A \hat{A}_t + a_4 E_t \hat{\hat{l}}_{t+1} - a_5 E_t \hat{\hat{k}}_{t+1} \right. \]
\[ + \left. (a_7 - a_6 \rho_b) \hat{\hat{b}}_{bt} - (1 - \rho_R) \omega_y \hat{y}_t - \rho_R \hat{R}_{t-1} - \hat{R}_t + a_9 E_t \hat{\pi}_{t+1} \right] \]  \hspace{1cm} (15)

where \( \rho_x, \rho_A \) and \( \rho_y \) are the autocorrelation coefficients of the AR(1) processes for the investment, efficiency and asset market wedges, respectively. It can be shown that the terms \( (a_2 - a_1 \rho_x) \) and \( (a_7 - a_6 \rho_b) \) are positive. The difference equation (15) can then be solved forward to obtain a particular solution for inflation. Notice, that by appearing in the

\[ \text{(6)} \]  

In the case of the investment, asset market, and monetary policy wedges, the inflation rate, and the nominal interest rate, the variables are expressed as percentage point deviations from steady state.
difference equation, investment, efficiency, asset market, and monetary policy wedges have a direct effect on inflation. In addition, the first two wedges, together with labour and government consumption wedges, have also an indirect effect on inflation by affecting equilibrium output, labour and capital.\(^7\) In a similar way we can also characterise the nominal interest rate as a function of the wedges.

Consider now, as a simple example, a real business cycle model, such as that of Dittmar, Gavin and Kydland (2005), in which the only source of fluctuations are shocks to total factor productivity (our efficiency wedge), and in which the central bank follows a Taylor rule. This model is a special case of the prototype economy in which all wedges, except the efficiency wedge, are constant. A persistent fall in total factor productivity in this model reduces the expected real return to capital and current output. As we see in equation (15), a fall in the expected real return to capital, given by 

\[
(a_3 \rho A_t \hat{A}_t + a_4 E_t \hat{l}_{t+1} - a_5 E_t \hat{k}_{t+1}),
\]

leads to a fall in inflation. But when \(\omega_y > 0\), as is the case in most estimated Taylor rules, a fall in the efficiency wedge increases inflation by reducing output. When \(\omega_y\) is sufficiently large, this effect dominates and output and inflation move in opposite directions, following a shock to total factor productivity.

As another example, consider a sticky-price model, such as that used by Ireland (2004). As the next section shows, an economy with sticky prices is equivalent to the prototype economy with equal investment and labour wedges. In a sticky-price economy a negative ‘demand’ shock, due to a positive shock to the nominal interest rate in the Taylor rule, typically leads to a fall in both output and inflation (see Ireland (2004), Figure 1). The propagation of such a shock through sticky prices will show up in the prototype economy as an equal increase in labour and investment wedges. An increase in the two wedges leads to a fall in output due to a decline in labour supply.\(^8\) As in the previous example, a fall in output in equation (15) leads to an increase in inflation. However, since \(\hat{\tau}_{xt}\) appears directly in equation (15), there is also a direct effect of an increase in \(\hat{\tau}_{xt}\) on inflation, which works in the opposite direction of the fall in output. In particular, higher \(\hat{\tau}_{xt}\) reduces inflation (since \(a_2 - a_1 \rho_x > 0\)). When this latter effect is sufficiently strong (or equivalently when \(\omega_y\) is sufficiently small), inflation in a sticky-price model falls, following a negative demand shock. Sticky prices thus generate the traditional ‘Keynesian’ result of a negative demand shock that both output and inflation fall.

\(^7\) Although equation (15) is not a particular solution, we can still use it to discuss the qualitative effects of the wedges on inflation in an equilibrium that excludes explosive paths for inflation. In a particular solution for such an equilibrium the term \(E_t \hat{\pi}_{t+i}\) drops out as \(i \to \infty\), while (since \(a_9 > 0\)) all the other variables have the same qualitative effects on inflation as in the difference equation (15).

\(^8\) For simplicity, we abstract in this example from the effect of the increase in the two wedges on \(E_t \hat{l}_{t+1}\) and \(E_t \hat{k}_{t+1}\).
Using the pricing function (15) we could discuss the qualitative effects of other frictions on inflation and the nominal interest rate. Furthermore, by recovering from the data the realised values of the wedges, we can determine the quantitative importance of different types of frictions for inflation and nominal interest rate dynamics.

3 Equivalence results

This section provides two examples of mappings between the prototype monetary economy and detailed monetary economies. In particular, it demonstrates equivalence between the prototype economy and an economy with sticky prices, and between the prototype economy and an economy with limited participation in the money market (Appendix B provides two additional examples). This section also shows how detailed monetary policy rules can be mapped into the prototype policy rule (7).

Throughout this section we retain the notation of Section 2. For new variables, notation will be introduced as we go. For brevity, this section abstracts from population and technology growth.

3.1 An economy with sticky prices

3.1.1 The underlying economy

Consider an economy with monopolistic competition in product markets and nominal price rigidities. The underlying probability space of this economy is the same as that of the prototype economy described in the previous section; i.e. it is given by \((Z^t, Z^t, \mu_t(z^t))\). There are two types of producers: identical final good producers and intermediate good producers indexed by \(j \in [0, 1]\). Final good producers take all prices as given and solve

\[
\max_{y_t(z^t), \{y_t(j, z^t)\}, j \in [0, 1]} p_t(z^t) y_t(z^t) - \int p_t(j, z^t) y_t(j, z^t) dj
\]

subject to a production function

\[
y_t(z^t) = \left[ \int y_t(j, z^t) \varepsilon_t(z^t) dj \right]^{1/\varepsilon_t(z^t)}
\]

Here, \(y_t(z^t)\) is aggregate output, \(y_t(j, z^t)\) is input of an intermediate good \(j\), \(p_t(j, z^t)\) is its price, and \(\varepsilon_t(z^t)\) is a shock that determines the degree of monopoly power of intermediate good producers.\(^{9}\) The solution to this problem is characterised by a demand function for

---

\(^{9}\) In the context of sticky-price models, a number of different types of shocks have been considered in the literature, including preference, investment-specific, government consumption, mark-up, and monetary policy shocks. Since the shocks are functions of the underlying events, the main result regarding the distortionary effects of sticky prices established here does not depend on the choice of a particular shock.
an intermediate good \( j \)

\[
y_t(j, z^t) = \left( \frac{p_t(z^t)}{P_t(j, z^t)} \right)^{-\frac{1}{\omega_t(z^t)}} y_t(z^t) \quad j \in [0, 1] \tag{16}
\]

and a price aggregator

\[
p_t(z^t) = \left[ \int p_t(j, z^t)^{\omega_t(z^t)} dj \right]^{\frac{1}{\omega_t(z^t)}}
\]

The problem of an intermediate good producer \( j \) can be split into two sub-problems. First, for a given level of output \( y_t(j, z^t) \) the producer solves

\[
\min_{l_t(j, z^t), k_t(j, z^t)} w_t(z^t)l_t(j, z^t) + r_t(z^t)k_t(j, z^t)
\]

subject to

\[
F(k_t(j, z^t), l_t(j, z^t)) = y_t(j, z^t)
\]

where \( l_t(j, z^t) \) and \( k_t(j, z^t) \) are labour and capital, respectively, employed by producer \( j \).

Denoting the value function for this cost minimisation problem by \( \vartheta(y_t(j, z^t), w_t(z^t), r_t(z^t)) \), in the second step of the optimisation problem the producer chooses the price \( p_t(j, z^t) \) to maximise the present value of profits

\[
\sum_{t=0}^{\infty} \sum_{z^t} Q_t(z^t) \left[ \frac{p_t(j, z^t)y_t(j, z^t)}{p_t(z^t)} - \vartheta(y_t(j, z^t), w_t(z^t), r_t(z^t)) - \frac{\phi}{2} \left( \frac{p_t(j, z^t)}{\pi p_{t-1}(j, z^{t-1})} - 1 \right)^2 \right]
\]

subject to the demand function (16). Here, \( Q_t(z^t) \) is an appropriate discount factor and the last term is a price adjustment cost as in Rotemberg (1982).\(^{10}\) Given the symmetry among the producers, all of them choose the same price, capital and labour.

The consumer maximises the utility function (1), subject to the time constraint (2), the law of motion for capital (4) and the budget constraint

\[
c_t(z^t) + x_t(z^t) + \frac{m_t(z^t)}{p_t(z^t)} + \frac{b_t(z^t)}{p_t(z^t)(1 + R_t(z^t))} = w_t(z^t)l_t(z^t) + r_t(z^t)k_t(z^t-1) + \frac{b_{t-1}(z^{t-1})}{p_t(z^t)} + \frac{m_{t-1}(z^{t-1})}{p_t(z^t)} + T_t(z^t) + \psi_t(z^t)
\]

where \( \psi_t(z^t) \) is profits from intermediate good producers.

The government follows a monetary policy feedback rule

\[
R_t(z^t) = (1 - \rho_R) \left[ R + \omega_y (\ln y_t(z^t) - \ln y) + \omega_\pi (\pi_t(z^t) - \pi) \right] + \rho_R R_{t-1}(z^{t-1}) \tag{17}
\]

and its budget constraint is

\[
T_t(z^t) = m_t(z^t) - m_{t-1}(z^{t-1}) + \phi \left( \frac{p_t(z^t)}{\pi p_{t-1}(z^{t-1})} - 1 \right)^2
\]

\(^{10}\) It can be shown that the equivalence result established here also holds for Calvo and Taylor-style price setting behaviour.
An equilibrium of this sticky-price economy is a set of allocations \((c_t(z^t), x_t(z^t), y_t(z^t), l_t(z^t), k_{t+1}(z^t), m_t(z^t), b_t(z^t))\) and a set of prices \((p_t(z^t), R_t(z^t), r_t(z^t), w_t(z^t))\) that satisfy: (i) a set of the consumer’s first-order conditions for labour, capital, bonds, and money, respectively

\[
\begin{align*}
    u_{ct}(z^t)w_t(z^t) &= u_{ht}(z^t) \left[ 1 + s_{ct}(z^t)w_t(z^t) \right] \quad (18) \\
    \sum_{z_{t+1}} Q_t(z^{t+1}|z^t) \left[ 1 + r_{t+1}(z^{t+1}) - \delta \right] &= 1 \quad (19) \\
    \sum_{z_{t+1}} Q_t(z^{t+1}|z^t) \left[ 1 + R_t(z^t) \right] \frac{p_t(z^t)}{p_{t+1}(z^{t+1})} &= 1 \quad (20) \\
    -\frac{u_{ht}(z^t)s_{mt}(z^t)}{u_{ct}(z^t) - u_{ht}(z^t)s_{ct}(z^t)} + \sum_{z_{t+1}} Q_t(z^{t+1}|z^t) \frac{p_t(z^t)}{p_{t+1}(z^{t+1})} &= 1 \quad (21)
\end{align*}
\]

where

\[
Q_t(z^{t+1}|z^t) = \beta \mu_t(z^{t+1}|z^t) \frac{u_{ct,t+1}(z^{t+1}) - u_{ht,t+1}(z^{t+1})s_{ct,t+1}(z^{t+1})}{u_{ct}(z^t) - u_{ht}(z^t)s_{ct}(z^t)}
\]

(ii) a set of optimality conditions for the cost minimisation problem of intermediate good producers

\[
\frac{F_{kt}(z^t)}{F_{lt}(z^t)} = \frac{r_t(z^t)}{w_t(z^t)} \quad (22)
\]

\[
y_t(z^t) = F \left( k_t(z^{t-1}), l_t(z^t) \right) \quad (23)
\]

(iii) a first-order condition for the profit maximisation problem of intermediate good producers (the so-called ‘New-Keynesian Phillips Curve’)

\[
\Phi \left( p_t(z^t), p_{t-1}(z^{t-1}), \eta_t(z^t), y_t(z^t), \varepsilon_t(z^t) \right) + \sum_{z_{t+1}} Q_t(z^{t+1}|z^t) \Psi \left( p_t(z^t), p_{t+1}(z^{t+1}), y_{t+1}(z^{t+1}), \varepsilon_{t+1}(z^{t+1}) \right) = 0 \quad (24)
\]

where \(\eta_t(z^t) \equiv \partial \partial t_l(z^t) / \partial y_t(z^t)\) is a marginal cost and \(\Phi(\ldots, \ldots, \cdot)\) and \(\Psi(\ldots, \ldots, \cdot)\) are smooth functions; (iv) the resource constraint \(c_t(z^t) + x_t(z^t) = y_t(z^t)\); (v) the capital accumulation law (4); (vi) the monetary policy rule (17); and (vii) the bond market clearing condition \(b_t(z^t) = 0\).

Notice that in equilibrium \(r_t(z^t)\) and \(w_t(z^t)\) are not set equal to the marginal products of capital and labour.

3.1.2 The associated prototype economy

Consider now a version of the prototype economy of Section 2. The prototype economy is the same as that of Section 2, except that it has an investment wedge that resembles a tax.
on capital income rather than a tax on investment. (11) The consumer’s budget constraint therefore is
\[
ct(z^t) + xt(z^t) + \frac{mt(z^t)}{pt(z^t)} + [1 + \tau_b(z^t)] \left[ \frac{bt(z^t)}{pt(z^t)(1 + Rt(z^t))} - \frac{b_{t-1}(z^{t-1})}{pt(z^{t-1})} \right] \\
= \left[ 1 - \tau_h(z^t) \right] wt(z^t)lt(z^t) + \left[ 1 - \tau_{kt}(z^t) \right] rt(z^t)kt(z^{t-1}) + \frac{mt(z^{t-1})}{pt(z^{t-1})} + T_t(z^t)
\]
where \( \tau_{kt}(z^t) \) is the capital income tax. In equilibrium, the consumer’s first-order condition for capital accumulation (9) becomes
\[
\sum_{z_{t+1}} Q_t(z_{t+1}|z^t) \left\{ \left[ 1 - \tau_{k,t+1}(z^{t+1}) \right] A_{t+1}(z^{t+1})F_{k,t+1}(z^{t+1}) + (1 - \delta) \right\} = 1 
\]
(25)
where \( Q_t(z_{t+1}|z^t) \) is given as before by equation (12).

**Proposition 1:** Consider equilibrium allocations of the economy with sticky prices \((c_t^*(z^t), x_t^*(z^t), g_t^*(z^t), l_t^*(z^t), k_{t+1}^*(z^t), m_t^*(z^t))\) and prices \((p_t^*(z^t), R_t^*(z^t), r_t^*(z^t), w_t^*(z^t))\) that support these allocations. Let the wedges in the prototype economy satisfy: \( A_t(z^t) = 1, \tau_b(z^t) = g_t(z^t) = R_t(z^t) = 0, \) and
\[
\tau_{kt}(z^t) = \tau_h(z^t) = 1 - \frac{r_t^*(z^t)}{F_{k,t}^*(z^t)}
\]
(26)
for all \( z^t \), where \( F_{k,t}^*(z^t) \) is evaluated at the equilibrium of the sticky-price economy. Then \((c_t^*(z^t), x_t^*(z^t), g_t^*(z^t), l_t^*(z^t), k_{t+1}^*(z^t), m_t^*(z^t))\) and \((p_t^*(z^t), R_t^*(z^t))\) are also equilibrium allocations and prices of the prototype economy.

For the proof, see Appendix A.

The key point here is that sticky prices have the same distortionary effects on equilibrium allocations and prices as capital and labour income taxes. (12) Fluctuations in the data due to sticky prices thus show up in the prototype economy as equal movements in investment and labour wedges.

### 3.2 An economy with limited participation in the money market

#### 3.2.1 The underlying economy

Consider now an economy in which consumers do not participate in the money market, such as that of Christiano and Eichenbaum (1992). The probability space underlying this

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(11) Both types of taxes distort the optimality condition for capital accumulation, but the proof is more straightforward in the case of a capital income tax.

(12) A similar point has been made by Goodfriend and King (1998).
economy is again the same as that of the prototype economy described in Section 2. The consumer chooses plans for consumption \(c_t(z^t)\), investment \(x_t(z^t)\), capital \(k_{t+1}(z^t)\), leisure \(h_t(z^t)\), labour \(l_t(z^t)\), money balances \(m_t(z^t)\), and deposits with financial intermediaries \(q_t(z^{t-1})\) to maximise the utility function (1) subject to four constraints (notice that \(q_t\) depends on \(z^{t-1}\), not \(z^t\)). First, the consumer has to satisfy a time constraint \(h_t(z^t) + l_t(z^t) = 1\). Second, the consumer has to satisfy the budget constraint

\[
c_t(z^t) + x_t(z^t) + \frac{m_t(z^t)}{p_t(z^t)} = \left[1 + R_t(z^t)\right] \frac{q_t(z^{t-1})}{p_t(z^t)} + w_t(z^t)l_t(z^t) + r_t(z^t)k_t(z^{t-1})
\]

\[
+ \frac{m_{t-1}(z^{t-1}) - q_t(z^{t-1})}{p_t(z^t)} + \frac{\psi_t(z^t)}{p_t(z^t)}
\]

where \(\psi_t(z^t)\) is profits from the financial intermediaries. Third, the consumer has to satisfy a cash-in-advance constraint

\[
c_t(z^t) = \frac{m_{t-1}(z^{t-1}) - q_t(z^{t-1})}{p_t(z^t)}
\]

The fourth constraint is the capital accumulation law (4).

The producer has access to an aggregate production function

\[
y_t(z^t) = F(k_t(z^{t-1}), l_t(z^t))
\]

and finances a fraction \(\phi_t\) of the wage bill \(w_t(z^t)l_t(z^t)\) through loans from the financial intermediaries. The intermediaries operate in a perfectly competitive market so that the interest rate on loans is equal to the interest rate on deposits. The producer maximises profits \(F(k_t(z^{t-1}), l_t(z^t)) - [1 + \phi_t(z^t)R_t(z^t)]w_t(z^t)l_t(z^t) - r_t(z^t)k_t(z^{t-1})\) by setting marginal products of capital and labour equal to their effective prices, which in the case of labour is \([1 + \phi_t(z^t)R_t(z^t)]w_t(z^t)\).

The government sets the nominal interest rate according to the feedback rule

\[
R_t(z^t) = (1 - \rho_R) \left[R + \omega_y \left(\ln y_t(z^t) - \ln y\right) + \omega_\pi \left(\pi_t(z^t) - \pi\right)\right] + \rho_R R_{t-1}(z^{t-1}) + \xi_t(z^t)
\]

where \(\xi_t(z^t)\) is a monetary policy shock. The government implements the nominal interest rate dictated by this policy rule through money transfers \(\eta_t(z^t)\) to the financial intermediaries. Total loanable funds at the disposal of the intermediaries are therefore \(q_t(z^{t-1}) + \eta_t(z^t)\). Clearing the money market requires that the supply of loanable funds is equal to their demand

\[
q_t(z^{t-1}) + \eta_t(z^t) = \phi_t(z^t)p_t(z^t)w_t(z^t)l_t(z^t)
\]

(13) In terms of the prototype economy of Section 2, this shock can be considered as a part of the monetary policy wedge, though, as the next subsection shows, the wedge is a much broader object.
Since there are no deposits held by the consumer against the transfer $\eta_t(z^t)$, the gross interest that the intermediaries earn from lending these monetary transfers to the producer is their profit $\psi_t(z^t) = (1 + R_t(z^t))\eta_t(z^t)$, which is paid to the consumer.

An equilibrium of this economy with limited participation is a set of allocations $(c_t(z^t), x_t(z^t), y_t(z^t), l_t(z^t), k_{t+1}(z^t), m_t(z^t), q_t(z^{t-1}))$ and a set of prices $(p_t(z^t), R_t(z^t), r_t(z^t), w_t(z^t))$ that satisfy: (i) a set of the consumer's first-order conditions for deposits, labour and capital, respectively,

$$
\sum_{z_t} \mu_{t-1}(z^t|z^{t-1}) \frac{u_{c,t}(z^t)}{p_t(z^t)} = \beta \sum_{z_t} \mu_{t-1}(z^t|z^{t-1}) \frac{u_{c,t+1}(z^{t+1})}{p_{t+1}(z^{t+1})} (1 + R_t(z^t))
$$

(31)

$$
u_{ht}(z^t) \frac{1 + \phi_t(z^t) R_t(z^t)}{F_{lt}(z^t)} = \beta \sum_{z_{t+1}} \mu_t(z^{t+1}|z^t) u_{c,t+1}(z^{t+1}) \frac{p_t(z^t)}{p_{t+1}(z^{t+1})}
$$

(32)

$$\nu_{ht}(z^t) \frac{1 + \phi_t(z^t) R_t(z^t)}{F_{lt}(z^t)} = \beta \sum_{z_{t+1}} \mu_t(z^{t+1}|z^t) u_{h,t+1}(z^{t+1})
$$

(33)

$$
\times \frac{1 + \phi_{t+1}(z^{t+1}) R_{t+1}(z^{t+1})}{F_{l,t+1}(z^{t+1})} \left[1 + F_{k,t+1}(z^{t+1}) - \delta \right]
$$

(ii) the producer’s first-order conditions $w_t(z^t) = F_{lt}(z^t)/[1 + \phi_t(z^t) R_t(z^t)]$ and $r_t(z^t) = F_{kt}(z^t)$; (iii) the cash-in-advance constraint (27); (iv) the money market clearing condition (30); (v) the aggregate resource constraint $c_t(z^t) + x_t(z^t) = y_t(z^t)$, where $y_t(z^t)$ is given by the production function (28); (vi) the capital accumulation law (4); and (vii) the monetary policy rule (29).

### 3.2.2 The associated prototype economy

Consider now a version of the prototype economy of Section 2. In particular, suppose that the shopping time function (3) has the following form

$$s_t(z^t) = \begin{cases} 
0 & \text{if } p_t(z^t) c_t(z^t) = m_t(z^t) \\
1 & \text{otherwise}
\end{cases}
$$

(34)

Effectively, the consumer faces zero costs of the first shopping trip, and infinite costs of any subsequent trip. Since in equilibrium the consumer always chooses $s_t(z^t) = 0$, this shopping time function implies that in equilibrium the consumer has to satisfy the cash-in-advance constraint $p_t(z^t) c_t(z^t) = m_t(z^t)$.

Given the specification of the shopping time function, the consumer’s first-order conditions with respect to bonds, labour and capital, respectively, now become

$$
\left[1 + \tau_{ht}(z^t) \right] u_{ht}(z^t) \\
\left[1 - \tau_{lt}(z^t) \right] A_t(z^t) F_{lt}(z^t)
$$

(35)
Consider now a special case of Proposition 2. Suppose that the fraction of the wage bill financed through loans from financial intermediaries $\phi_t(z^t)$ fluctuates so as to offset the effects of changes in the interest rate on the effective wage rate $(1 + \phi_t R_t)w_t$. In this case, monetary shocks lead to fluctuations in $\tau_t(z^t)$ but not in $\tau_{lt}(z^t)$. The main idea here is that the propagation of shocks through limited participation in the money market works like fluctuations in a tax on nominal bonds which distort the no-arbitrage condition between capital and bonds in an otherwise competitive economy. Fuerst (1992) labels this
distortion a ‘liquidity effect’. Fluctuations in the data due to liquidity effects thus show up in the prototype economy as fluctuations in the asset market wedge.

3.3 The monetary policy wedge

The monetary policy wedge captures all aspects of monetary policy beyond its responses to output and inflation as summarised by a conventional Taylor rule. As an example, consider a monetary policy rule with fluctuations in an inflation target used by Gavin, Kydland and Pakko (2007). Their monetary policy rule has the form

\[ R_t(z^t) = R + \omega_y \left( \ln y_t(z^t) - \ln y \right) + \omega_\pi \left( \pi_t(z^t) - \bar{\pi}_t(z^t) \right) \]  

(40)

where \( \bar{\pi}_t(z^t) \) is an implicit inflation target that fluctuates around a steady-state inflation rate \( \pi \). This policy rule is equivalent to the prototype rule (7) where the inflation target is constant and the monetary policy wedge is given by \( \tilde{R}_t(z^t) = -\omega_\pi (\pi_t(z^t) - \pi) \). In a similar fashion, responses of the monetary authority to variables other than inflation and output would show up as fluctuations in the monetary policy wedge.

4 Measuring the realised wedges

Our procedure of measuring the realised wedges from the data follows closely that of CKM. In particular, we assume that the events are governed by a stationary Markov process of the form \( \mu(z^t | z^{t-1}) \) and that there is a one-to-one and onto mapping between the events and the wedges. This latter assumption implies that the wedges uniquely identify the underlying events. We can therefore replace in the prototype economy the probability space for the events with a probability space for the wedges. Since the stochastic process for the events is Markov, the stochastic process for the wedges is also Markov. In particular, it is assumed to be a vector autoregressive AR(1) process

\[ \omega_{t+1} = P_0 + P \omega_t + \epsilon_{t+1} \]  

(41)

where \( \omega_t = (\log A_t, \tau_{yt}, \tau_{xt}, \log g_t, \tau_{bt}, \tilde{R}_t) \) and the shock \( \epsilon_{t+1} \) is iid over time and is distributed normally with mean zero and a covariance matrix \( V = BB' \). There are no restrictions imposed on this stochastic process except stationarity. In particular, the off-diagonal elements of \( P \) and \( V \) are allowed to be non-zero.

Uncovering the realised wedges from the data involves three steps. First, we need to choose functional forms of the utility, production and shopping-time functions and their parameter values, as well as the parameter values of the monetary policy rule. Second, we need to estimate the parameters of the stochastic process for the wedges \( P_0, P \) and \( B \). In the third step we use the equilibrium decision rules and pricing functions of the prototype
economy to back out the wedges from the data.

In the second and the third step we need to compute the equilibrium decision rules and pricing functions of the prototype economy. Since the state space is large (there are nine state variables in the model), the equilibrium is computed using a linear-quadratic approximation method described by Hansen and Prescott (1995). The outcome of this method is a set of linear decision rules and pricing functions that express the log deviations of equilibrium allocations and prices from steady state in terms of log deviations of the state vector \((\omega_t, p_{t-1}, R_{t-1}, k_t)\) from steady state. The same approximation method is also used to compute the equilibria in the experiments in Section 5. The rest of this section describes the three steps in more detail and provides some business cycle statistics for the realised wedges.

4.1 Calibration

The model’s calibration is summarised in Table A. For easy comparison with CKM, we follow CKM wherever possible. One period in the model is set equal to one quarter. The utility function has the functional form \(u(., .) = \lambda \log c_t + (1 - \lambda) \log h_t\) and the production function has the form \(F(., .) = k_0^\alpha ((1 + \gamma_A)l_t)^{1-\alpha}\). Following Dittmar et al (2005), the shopping-time function has the form \(s(., .) = \nu_1 (c_t p_t / m_t)^\nu_2\), where \(\nu_1 \in (0, \infty)\) and \(\nu_2 \in [1, \infty)\). The population growth rate \(\gamma_n\) is set equal to 0.0037, the technology growth rate \(\gamma_A\) is set equal to 0.004, the depreciation rate \(\delta\) is set equal to 0.0118, and the capital share of output \(\alpha\) is set equal to 0.35. As in Dittmar et al (2005), the curvature parameter in the shopping time function \(\nu_2\) is set equal to one, which implies a long-run money demand function with interest elasticity of -0.5, found by a number of studies for the United States (see Lucas (2000) for a survey).

The parameters of the monetary policy rule are set equal to fairly standard values in the literature (see Woodford (2003), Chapter 1, for a survey): the weight on output is set equal to 0.125 (which corresponds to 0.5 when output is measured at an annual rate), the weight on inflation is set equal to 1.5 and the smoothing coefficient \(\rho_R\) is set equal to 0.75. Nevertheless, we also study the sensitivity of our findings to alternative parameterisations of the monetary policy rule. The values of the remaining parameters \(\lambda, \beta\) and \(\nu_1\) are chosen so that, for the estimated steady-state values of the wedges, the model matches three calibration targets: \(l\) equal to 0.26, \(k/y\) equal to 11.2 and \(py/m\) equal to 0.58 – the average quarterly velocity of the MZM aggregate in the postwar period.

\(^{14}\) Before computing the equilibrium, the model is transformed so that nominal money balances and the price level are stationary.
4.2 Estimation of the stochastic process

The parameters \( P_0, P \) and \( B \) of the stochastic process for the wedges are estimated using a maximum likelihood method (e.g. McGrattan (1994)). The resulting estimates are contained in Table B. The likelihood function is based on a state-space representation consisting of the stochastic process for the wedges (41) and log-linear approximations of the equilibrium decision rules for \( y_t, x_t, g_t \), and \( l_t \), and equilibrium pricing functions for \( p_t \) and \( R_t \). Estimation is carried out for the period 1959.Q1-2004.Q4 using data on output (the sum of GDP and imputed services from consumer durables), investment (which includes consumer durables), hours from the Establishment Survey, the sum of government consumption and net exports, the GDP deflator, and the yield on 3-month Treasury bills. Data on output, investment, hours, and the sum of government consumption and net exports are in per capita terms. In addition, a common trend of 1.6% at an annual rate is removed from the data on output, investment, and the sum of government consumption and net exports, and a trend of 3.7% (the average postwar inflation rate) is removed from the price level. Capital is computed recursively using the law of motion (4), data on investment, and an initial capital stock.

4.3 Uncovering the realised wedges

The estimated stochastic process for the wedges is then used to compute the equilibrium of the model and to uncover the realised wedges from the data. We denote the vector of the realised wedges by \( \omega^d_t = (\log A^d_t, \tau^d_l, \tau^d_x, \log g^d_t, \tau^d_b, \tilde{R}^d_t) \). Notice that the realised values of \( g_t \) are observed directly from the data as the sum of government consumption and net exports. The realised values of the remaining wedges are obtained from the log-linear approximations to the equilibrium decision rules and pricing functions

\[ y_t = y(\omega_t, p_{t-1}, R_{t-1}, k_t), x_t = x(\omega_t, p_{t-1}, R_{t-1}, k_t), l_t = l(\omega_t, p_{t-1}, R_{t-1}, k_t), p_t = p(\omega_t, p_{t-1}, R_{t-1}, k_t), \] and \( R_t = R(\omega_t, p_{t-1}, R_{t-1}, k_t) \). The log-linear functions constitute a system of five equations that in each period can be solved for the five unknown values of \( \log A_t, \tau_t, \tau_x, \tau_b \), and \( \tilde{R}_t \) using data on \( y_t, x_t, l_t, g_t, p_t, p_{t-1}, R_t, \) and \( R_{t-1} \), and data on \( k_t \), which are generated recursively from the law of motion (4) and data on \( x_t \). As a result of this procedure, putting all six wedges back into the model at the same time exactly reproduces the data.

4.4 Business cycle properties of the realised wedges

Tables C and D provide some business cycle statistics for the realised wedges detrended with the HP-filter. Table C shows their standard deviations, relative to that of HP-filtered
output, and their correlations with HP-filtered output at various leads and lags. Focusing on the CKM wedges first, we see that the efficiency and investment wedges are, respectively, only 63% and 50% as volatile as output, while the government consumption wedge is 1.5 times as volatile as output, and the labour wedge is about as volatile as output. In addition, both the efficiency and investment wedges are procyclical, whereas the labour and government consumption wedges are countercyclical. Notice also that the efficiency wedge is more strongly correlated with output at leads than at lags, while the labour wedge lags output with a negative sign. The government consumption wedge, which is primarily driven by net exports, leads output with a negative sign and the investment wedge has no phase shift. These findings are broadly in line with those of CKM (see their paper for an interpretation).

Looking at the cyclical behaviour of the two new wedges, we see that the asset market wedge is 2.59 times as volatile as output and strongly procyclical. The high volatility of this wedge reflects the well-known failure of Euler equations with power utility functions to price financial assets: since the real return on Treasury bills is more volatile than the marginal rate of substitution, the asset market wedge has to be volatile enough for the first-order condition for bonds to hold. Although it is possible to interpret the asset market wedge as an error that measures the goodness of fit of the Euler equation for bonds, we have provided a more economically useful interpretation of that wedge as summarising some underlying frictions in asset markets. The strong positive comovement of the asset market wedge with output suggests that these frictions worsen in expansions. In contrast to the asset market wedge, the monetary policy wedge is fairly smooth and only weekly correlated with output at all leads and lags.

Table D displays contemporaneous correlations of the HP-filtered wedges with each other. The key finding here is that the labour wedge is negatively correlated with the investment wedge (notice also from Table C that while the labour wedge is strongly countercyclical, the investment wedge is strongly procyclical). This finding is in a sharp contrast with the predictions of sticky price models. According to Proposition 1, if sticky prices were the key friction propagating shocks over the business cycle, we would have to see these two wedges move together. We therefore conclude that sticky prices played at most a modest role in driving aggregate fluctuations in the postwar US economy. This does not mean that by themselves they cannot be an important propagation mechanism. Our finding, however, suggests that other frictions, which drive investment and labour wedges in opposite

\[15\] Another interpretation of the procyclical behaviour of the asset market wedge is that, because of the log-linear approximation of the decision rules and pricing functions, the asset market wedge is picking up countercyclical risk premia: the preference of investors for a safe asset in bad times might be showing up here as a relatively low tax on bonds in downturns, making them relatively more attractive in bad times.
directions, played a more important role.

5 Assessing the contributions of the wedges to fluctuations in the data

This section decomposes fluctuations in the data into movements due to each wedge individually, and in various combinations. For the reasons mentioned in the Introduction, we apply the decomposition to the 1973 and the 1982 recessions, which we use as case studies. In addition, we use the decomposition to shed light on the two anomalies in the postwar nominal business cycle mentioned in the Introduction.

5.1 The procedure

Since the wedges are mutually correlated, it makes little sense to simply put them back into the model one by one, as one would do when assessing the contribution of ‘structural’ shocks. Following CKM, we proceed as follows. Suppose, for example, that we are interested in the movements in the data due to the efficiency wedge alone. In this case, we compare the data to the predictions of a version of the prototype economy in which only the efficiency wedge is a function of the underlying events while all other wedges are set equal to their steady-state values in all states of the world; i.e., the vector of the wedges in period $t$ is $(A_t(z_t), \tau_l, \tau_x, g, \tau_b, \tilde{R})$. As emphasised by CKM, this experiment isolates the distortionary effects of the efficiency wedge, while keeping the underlying probability space of the prototype economy the same (remember that when establishing the equivalence results in Section 3, the underlying probability space was always assumed to be the same across the different economies).

As in Section 4, in the actual implementation of this experiment, the probability space for the events is replaced by the probability space for the wedges implied by the stochastic process (41). We thus solve a version of the prototype economy in which the consumer is faced with the stochastic process (41), with the parameter values reported in Table B, but in which, in the budget and resource constraints, and in the monetary policy rule, all wedges except the efficiency wedge are kept constant at their steady-state values. Let $y^A(\omega_t, p_{t-1}, R_{t-1}, k_t)$, $x^A(\omega_t, p_{t-1}, R_{t-1}, k_t)$, $c^A(\omega_t, p_{t-1}, R_{t-1}, k_t)$, $l^A(\omega_t, p_{t-1}, R_{t-1}, k_t)$, $p^A(\omega_t, p_{t-1}, R_{t-1}, k_t)$, and $R^A(\omega_t, p_{t-1}, R_{t-1}, k_t)$ be the equilibrium decision rules and pricing functions of this modified economy. Starting from $p_{-1}$, $R_{-1}$ and $k_0$ for some base period 0, these decision rules and pricing functions are used, together with $\omega_t^d$ (the vector of the realised wedges), to compute the efficiency wedge component of the variables of interest. In a similar fashion we also compute components of the data due to the other wedges, and due to their various combinations.
5.2 The 1973 recession

The findings for the 1973 recession are displayed in Figures 1-9. We employ a working definition of the 1973 recession as the period from the start of the oil crisis in 1973.Q4 to full recovery in output in 1978.Q4. Figure 1 shows the actual data and the realised wedges for this period. Panel A plots percentage deviations of output, investment, consumption, and the sum of government consumption and net exports from a linear trend of 1.6%, and percentage deviations of hours from their postwar average. The data are normalised so that in 1973.Q3, one quarter before the oil crisis, the deviations are zero. We see that by the first half of 1975 output is about 7% below trend and does not fully recover until the end of 1978. A similar pattern is also observed for hours, investment and consumption, although the fall in investment is much sharper (27% below trend by the end of 1975) while the fall in consumption is milder (5.4%).

Panel B plots the deviations of quarterly inflation and the nominal interest rate (both expressed at annual rates) from their 1973.Q3 levels. The surge in inflation following the oil crisis clearly stands out in the chart: by the end of 1974 inflation is 4 percentage points higher than before the crisis. However, after this initial increase it falls below its pre-crisis level and starts to pick up only towards the end of the recession. Except for the initial peak, the nominal interest rate follows a similar pattern as inflation, although it is less volatile. Panels C and D display the deviations of the wedges. For all six wedges, their relative volatilities and their comovement with output during the recession are broadly in line with their behaviour throughout the entire postwar period, as summarised by Table C.

Plotting the data and the wedges is useful for getting a general idea about the recession. However, what matters for the assessment of the quantitative contribution of the different wedges to fluctuations in the data are the responses of the model once we put the wedges back into the model. Recall that this process involves re-computing the equilibrium of the model under the assumption that only the wedges under investigation distort the equilibrium.

We start by putting the wedges back into the model one at a time. As mentioned in Section 2, the CKM wedges affect all endogenous variables, whereas the asset market and monetary policy wedges have significant effects only on inflation and the nominal interest rate. Since we are interested in the joint fluctuations in real and nominal variables, we feed back individually only the CKM wedges. The contribution of the asset market and monetary policy wedges to fluctuations in inflation and the nominal interest rate will be studied only in combination with the CKM wedges.
Consider the efficiency wedge first. In Figure 2 we see that this wedge alone accounts for nearly all of the decline in output (86%), but generates a more rapid recovery than in the data. The wedge also accounts for a large fraction of the decline in the labour input (68%) and for essentially all of the decline in investment (93%). However, as in the case of output, for both variables it generates a more rapid recovery than actually occurred. We also see that throughout the entire period the efficiency wedge alone generates fluctuations in inflation that closely mimic those in the data. It, however, does not account for the behaviour of the nominal interest rate: while the model predicts an increase, the interest rate falls in the data.

What types of frictions and propagation mechanisms can be mapped into the efficiency wedge? A number of studies demonstrate that a fall in the efficiency wedge can result from frictions or government policies that lead to a reallocation of resources from more to less productive uses (Chari et al (2007a), Chu (2001), Lagos (2006), and Restuccia and Rogerson (2007)). Appendix B provides an additional mapping, which is particularly relevant for the 1973 recession. It shows that fluctuations in the efficiency wedge in the prototype economy can be due to movements in energy prices in an underlying economy with capital utilisation. In the underlying economy, an increase in energy prices leads to a fall in the efficiency wedge. As discussed in Subsection 2.3, a fall in the efficiency wedge generates an increase in inflation if the central bank puts a positive weight on deviations of output from ‘potential’, i.e., it tries to accommodate a fall in output (other things being equal) by cutting interest rates. Our findings here suggest that such a mechanism alone can potentially account for the behaviour of inflation during the 1973 recession.

Figure 3 shows responses to the labour wedge. We see that the model generates a fall in output that is not as sharp as in the case of the efficiency wedge (75% vs 86%). This is despite the fact that the labour wedge accounts for more than the observed decline in hours. The labour wedge, however, produces the slow recovery observed in the data. This suggests that worsening labour market frictions prevented the economy from a quick recovery predicted by the efficiency wedge. However, unlike the efficiency wedge, the labour wedge does not generate the observed movements in inflation. Figure 4 shows responses to the investment wedge. Here we see that the investment wedge generates a mild expansion and produces fluctuations in inflation and the nominal interest rate that are way too smooth. The responses of the model to the government consumption wedge are similar (not shown).

Now we put the wedges back into the model in various combinations. In these experiments, we always put back all wedges except the one whose contribution we want to
assess. Since putting back all six wedges exactly reproduces the data, leaving a wedge out measures its marginal contribution to the movements in the data. We start by leaving out the efficiency wedge. Figure 5 shows that without this wedge the model predicts a recession than is much milder and that occurs a year later than in the data. Furthermore, without the efficiency wedge the model does not capture the behaviour of inflation. Leaving this wedge out, however, has only little effect on the model’s predictions for hours and the interest rate.

Figure 6 plots the responses when we leave out the labour wedge. We see that in this case the timing of the fall in output predicted by the model coincides with that in the data. However, the decline is only half as big as in the data. This is because the model completely misses the decline in hours worked. Leaving out the labour wedge, however, has only a small effect on inflation and the nominal interest rate: although the levels are not exactly as in the data, the model produces the general dynamic pattern of these two variables.

In contrast to the efficiency and labour wedges, leaving out the investment wedge has almost no effect on the ability of the model to reproduce the data, as Figure 7 shows. The marginal contribution of the government consumption wedge is equally small (not shown).

Figures 8 and 9 show the marginal contributions of the monetary policy and asset market wedges, respectively. We see that while both wedges are important for inflation dynamics, only the asset market wedge matters for the dynamics of the nominal interest rate. The importance of the two wedges for inflation dynamics seems to contradict our earlier result that fluctuations in inflation during the 1973 recession can be accounted for by the efficiency wedge alone. There is, however, no contradiction since the current experiment measures the marginal contribution of the two wedges. Taken together, the current and the earlier results suggest that there was an important interaction between the labour, asset market and monetary policy wedges during the 1973 recession: once the labour wedge is included in the model, the asset market and monetary policy wedges also need to be included if the model’s predictions for the two nominal variables are to be in line with the data.

5.3 The 1982 recession

The results for the 1982 recession are displayed in Figures 10-15. Panels A and B of Figure 10 show the data, while panels C and D show the wedges for the period 1979.Q3–1985.Q4. We see that as in the case of the 1973 recession, the volatilities
(relative to output) of the data and the wedges, and their comovement with output, are broadly in line with the entire postwar period as summarised in Table C.

However, there are also important differences between the two recessions. First, while the 1982 recession is characterised by a prolonged decline in output and a relatively fast recovery, the 1973 recession is characterised by a sharp fall in output and a slow recovery. Second, while inflation sharply increased after the oil-price shock in 1973, the 1982 recession is characterised by a sustained decline in the growth rate of prices. Third, during the 1982 recession the nominal interest rate was above inflation, while the opposite was true during the 1973 recession. And fourth, during the 1973 recession the monetary policy wedge fluctuated below its pre-recession level, while during the 1982 recession it fluctuated above its pre-recession level. Notice from our discussion in Subsection 3.3 that one possible explanation for such a behaviour of the monetary policy wedge is that the Fed reduced its implicit inflation target (or the ‘tolerated’ inflation rate), after Paul Volcker became its chairman in 1979.

Due to space constraints we only report the marginal contributions of the wedges. We start by leaving out the efficiency wedge (Figure 11). We see that without this wedge the model produces a recession than is much milder than in the data. In terms of the two nominal variables, the model generates paths that co-move with the data, but whose levels are below those in the data for much of the period. In particular, we see that in the absence of the decline in the efficiency wedge during this period, the ‘conquest’ of American inflation would have been much faster. We obtain similar results when we leave out the labour wedge, as Figure 12 shows. In contrast, leaving out the investment wedge has basically no effect on the ability of the model to reproduce the data (Figure 13).

Figures 14 and 15 show the responses of the model when we leave out either the monetary policy or the asset market wedge, respectively. Figure 14 shows that leaving out the monetary policy wedge leads to fluctuations in the nominal variables that co-move positively with the data, but that have much higher levels and are more volatile. More importantly, however, without the monetary policy wedge the model predicts inflation at the end of the recession about 6 percentage points above the inflation rate in the data. As for the asset market wedge, Figure 15 shows that in contrast to the 1973 recession, leaving this wedge out has a bigger effect on inflation than on the nominal interest rate. However, it does not affect the level of inflation at the end of the recession.
5.4 The dynamics of inflation and the nominal interest rate in the entire postwar business cycle

The existing literature has identified two important anomalies in the behaviour of nominal variables over the business cycle, as summarised by their correlations with output at various leads and lags. One of the anomalies is the lead-lag pattern of a short-term nominal interest rate, pointed out, among others, by King and Watson (1996). King and Watson note that the nominal interest rate in the United States is, what they call, an ‘inverted leading indicator’ – it is strongly negatively correlated with future output and positively correlated with past output. Henriksen, Kydland and Sustek (2008) document that this property also characterises the dynamics of short-term nominal interest rates in other major industrialised economies.\(^{(16)}\)

The second anomaly is the lead-lag pattern of inflation, pointed out, for example, by Galí and Gertler (1999): in contrast to the predictions of existing models, in the US data inflation is strongly positively correlated with past output. Wang and Wen (2007) document that this lead-lag pattern is also observed in other developed economies. In this section we use the prototype economy and the decomposition procedure to determine which wedges drive the observed comovement between output and the two nominal variables. Doing so we hope to provide insight into what types of frictions and propagation mechanisms models of the business cycle need to have in order to account for the two anomalies.

Table E reports the correlations of output in period \(t\) with the nominal interest rate in period \(t + j\) for the US data, and for the prototype model with various combinations of the wedges. We see that in the data, the nominal interest rate is strongly negatively correlated with future output, and positively correlated with past output. In order to determine the importance of the individual wedges for this lead-lag pattern, we start by leaving out one wedge from the model at a time. Besides the lead-lag pattern predicted by the model, we also report the volatility of output and the interest rate in the model, relative to the data, and the correlation between output in the model and output in the data.

We see from the table that the efficiency and asset market wedges are key for the dynamic relationship between output and the nominal interest rate observed in the data. Without the efficiency wedge, the leading indicator property of the nominal interest rate is exactly opposite to that in the data: the nominal interest rate is positively correlated with future output. And without the asset market wedge, the nominal interest rate is negatively

\(^{(16)}\) Backus, Routledge and Zin (2007) is a recent attempt to account for this anomaly.
correlated with output at all leads and lags, with a very strong negative contemporaneous correlation. In contrast to these two wedges, the effects of leaving out any of the other four wedges on the lead-lag pattern are much smaller. In particular, leaving out the investment wedge has hardly any effect at all.

Given the importance of the efficiency and asset market wedges for the output-interest rate dynamics, we ask now whether these two wedges alone can generate the lead-lag pattern observed in the data. The second row from the bottom of Table E reports the results from this experiment. We see that even though the phase shift is not as pronounced as in the data, the model generates a nominal interest rate that is negatively correlated with future output and positively correlated with past output. The reason for why these two wedges generate the phase shift is that the efficiency and asset market wedges themselves have a phase shift with respect to output as discussed in Subsection 4.4. These two wedges also generate output that is about as volatile as in the data and that comoves with the data with a correlation coefficient of 0.76. The nominal interest rate in the model is, however, only half as volatile as in the data. The bottom row of the Table shows that adding the monetary policy wedge increases the volatility of the nominal interest rate (now the model accounts for 82% of the volatility in the data) and also makes its phase shift more pronounced.

Table F reports the results of similar experiments for the lead-lag pattern of inflation. We see that the lead-lag pattern of inflation in the data is very similar to that of the nominal interest rate. In particular, inflation is negatively correlated with future output and positively correlated with past output (the literature has primarily focused on correlations with past output). We would therefore expect the efficiency and asset market wedges to be crucial also for the inflation dynamics. They are. We see that without the efficiency wedge the model predicts exactly opposite lead-lag pattern to that in the data. And without the asset market wedge inflation is negatively correlated with output at most leads and lags, and strongly so contemporaneously. In addition, as in the case of the nominal interest rate, the combination of the efficiency and asset market wedges generates the correct phase shift. Including the monetary policy wedge makes the phase shift more pronounced and also reduces the volatility of inflation closer to that in the data.

6 Alternative parameterisations of the monetary policy rule

A number of researchers have argued that the coefficients of the Fed’s reaction function have changed following the appointment of Paul Volcker as the Chairman of the Federal Reserve in 1979 (see Woodford (2003), Chapter 1, for a brief review of the literature and Sims and Zha (2006) for an alternative view that the coefficients remained broadly
unchanged). There is, however, less agreement on the exact values of the parameters of the reaction function before and after 1979. We therefore investigate the sensitivity of our main results to alternative weights on output and inflation in the monetary policy rule (7). For space constraints we only report the results of the sensitivity analysis for the importance of the efficiency wedge during the 1973 recession. In general, we find that our conclusions regarding the importance of the individual wedges for fluctuations in the data do not change with alternative values of the parameters of the monetary policy rule within a range of values found in the literature.

We consider three alternative values for the weight on inflation, \( \omega_\pi = \{1.3, 1.5, 1.7\} \), and three alternative values for the weight on output \( \omega_y = \{0.08, 0.125, 0.175\} \), where 1.5 and 0.125 are our baseline values. (17) Splitting the sample into two subsamples, 1959.Q1-1979.Q3 (the pre-V olcker period) and 1979.Q4-2004.Q4 (the post-V olcker period), we estimate the stochastic process for the wedges under the six alternative parameterisations of the Taylor rule for each subsample separately. We then back out the wedges and feed them back into the model as before. Figures 16 and 17 show the results for the marginal contribution of the efficiency wedge (for comparison, we also plot the responses from Figure 5). We see that although the responses of the variables differ across the different parameterizations of the Taylor rule, our main results regarding the importance of the efficiency wedge for fluctuations in the data remain broadly unchanged. In particular, regarding inflation dynamics, without the efficiency wedge, the model does not generate the observed surge in inflation following the oil crises.

7 Conclusions

The purpose of business cycle accounting is to guide researchers in making decisions about what types of frictions to introduce into models so that they exhibit fluctuations like those in the data. This paper has extended the method to include two key nominal variables, inflation and the nominal interest rate, in addition to the real variables studied by CKM. The purpose of this extension is to make the method applicable to investigate the quantitative importance of various classes of frictions for the joint dynamics of real and nominal variables over the business cycle.

In order to demonstrate how the extended method can be used, we have applied it to two postwar US downturns, the 1973 and the 1982 recessions, as well as to two outstanding

(17) Some researchers, for example Lubik and Schorfheide (2004), argue that the pre-V olcker period is characterised by \( \omega_\pi < 1 \) and thus indeterminacy of equilibria. We abstract from this possibility here in order to avoid all the complications associated with multiple equilibria. In fact, in our case, values of \( \omega_\pi \) below 1.28 result in indeterminacy.
anomalies in the postwar US business cycle.

We have found that in the case of the 1973 recession, the efficiency wedge alone successfully accounts for the decline in economic activity following the oil-crisis, as well as for the behaviour of inflation during that period. Using our equivalence result between efficiency wedges and energy price fluctuations in a model with capital utilisation, we conclude that a model in which the Fed accommodates a fall in output caused by high energy prices is a promising model of inflation behaviour during the 1973 recession.

In the case of the 1982 recession, we have found that both efficiency and labour wedges are crucial for capturing the behaviour of real economic activity. These results are the same as those of CKM. In terms of inflation, the monetary policy wedge is key for generating the correct inflation rate at the end of the period. Given our interpretation of that wedge, this finding is not surprising. It is consistent with a fall in the Fed’s ‘tolerated’ inflation rate (or an implicit inflation target) following the appointment of Paul Volcker as its chairman. Somewhat less obvious, however, is our finding that without the prolonged declines in efficiency and labour wedges observed in the data, low inflation would have been achieved much faster.

During each of the two recessions, as well as in the entire postwar business cycle, investment and labour wedges co-move negatively with each other. This finding is in a sharp contrast with our equivalence result for sticky prices and suggests that sticky prices played only a minor role in the postwar business cycle. This does not mean that sticky prices, by themselves, cannot be a strong propagation mechanism. However, based on the behaviour of the two wedges in the data, it seems that during the US postwar business cycle other frictions – frictions driving the two wedges in opposite directions – played a more important role.

Application of the method to the entire postwar business cycle also shows that in order to successfully account for the lead-lag pattern between output and inflation, and between output and the nominal interest rate, business cycle models have to have frictions that are equivalent to efficiency and asset market wedges. Frictions that are equivalent to labour, investment, and government consumption wedges are not of a first-order importance. Future work addressing inflation and the nominal interest rate dynamics should therefore focus on developing models with frictions and propagation mechanisms that are equivalent to efficiency and asset market wedges.
Appendix A: Proofs of Propositions 1 and 2

PROOF OF PROPOSITION 1

The proof proceeds by comparing the equilibrium conditions of the detailed economy with those of the prototype economy. Notice that when in the prototype economy \( A_t(z^t) = 1 \), \( \tau_b(z^t) = 0 \), \( g_t(z^t) = 0 \), and \( \bar{R}_t(z^t) = 0 \), the equilibrium conditions in the two economies are the same except: (i) in the prototype economy the capital rental rate is set equal to the marginal product of capital, whereas in the detailed economy this equilibrium condition is replaced by the optimal price-setting condition \( (24) \); and (ii) in the prototype economy the wage and the rental rate are subject to taxes, whereas they are not taxed in the detailed economy. Since in the detailed economy \( r^*_t(z^t) \neq F^*_k_t(z^t) \), it follows from the equilibrium condition \( (22) \) that also \( w^*_t(z^t) \neq F^*_l_t(z^t) \). The two economies thus only differ in terms of the prices of capital and labour services that the consumers face. We can, however, eliminate these differences by appropriately choosing \( \tau_{kt}(z^t) \) and \( \tau_{lt}(z^t) \) in the prototype economy. In particular, let \( \tau_{kt}(z^t) \) satisfy \( r^*_t(z^t) = (1 - \tau_{kt}(z^t))F^*_k_t(z^t) \) and let \( \tau_{lt}(z^t) \) satisfy \( w^*_t(z^t) = (1 - \tau_{lt}(z^t))F^*_l_t(z^t) \) for every history \( z^t \). Then the first-order conditions for capital and labour in the two economies are the same and the equilibrium allocations \( (c^*_t(z^t), x^*_t(z^t), y^*_t(z^t), l^*_t(z^t), k^*_t(z^t), m^*_t(z^t)) \) and the equilibrium prices \( (p^*_t(z^t), R^*_t(z^t)) \) of the detailed economy are also equilibrium allocations and prices of the prototype economy. In addition, since in the detailed economy \( w^*_t(z^t) = [F^*_l_t(z^t)/F^*_k_t(z^t)]r^*_t(z^t) \), the labour income tax satisfies \( r^*_t(z^t) = (1 - \tau_{lt}(z^t))F^*_k_t(z^t) \) and therefore \( \tau_{lt}(z^t) = \tau_{kt}(z^t) \). Q.E.D

PROOF OF PROPOSITION 2

The proof proceeds again by comparing the equilibrium conditions of the detailed economy with those of the prototype economy. Notice that when \( A_t(z^t) = 1 \), \( \tau_{xt}(z^t) = 0 \), \( g_t(z^t) = 0 \), and \( \bar{R}_t(z^t) = \xi_t(z^t) \) in the prototype economy, the two economies differ only in terms of the first-order conditions for bonds (deposits), labour and capital. We will choose \( \tau_{kt}(z^t) \) and \( \tau_{lt}(z^t) \) so that the equilibrium allocations and prices of the detailed economy satisfy the three first-order conditions of the prototype economy. First, compare the first-order conditions for labour and capital. It follows immediately that when \( \tau_{lt}(z^t) \) in the prototype economy is given by the condition \( (38) \) of the Proposition for every history \( z^t \), equilibrium allocations and prices of the detailed economy also satisfy the first-order conditions \( (36) \) and \( (37) \) of the prototype economy. Second, compare the equilibrium conditions for bonds in the two economies. To make them more easily comparable, we substitute in the prototype economy the left-hand side of the first-order condition for labour \( (36) \) into the first-order condition for bonds \( (35) \). The resulting equation can be expressed as

\[
\frac{u_{ct}(z^t)}{p_t(z^t)} \Omega_t(z^t) \left[ 1 + \tau_b(z^t) \right] - \beta \left( 1 + R_t(z^t) \right) \times \sum_{z^{t+1}} \mu_t(z^{t+1} | z^t) \frac{u_{ct,t+1}(z^{t+1})}{p_t(z^{t+1})} \frac{\Omega_{t+1}(z^{t+1})}{\Omega_t(z^t)} \left[ 1 + \tau_{bt+1}(z^{t+1}) \right] = 0 \tag{A-1}
\]

where

\[
\Omega_t(z^t) \equiv \sum_{z^{t+1}} \mu_t(z^{t+1} | z^t) \frac{u_{ct,t+1}(z^{t+1})}{u_{ct}(z^t)} \frac{p_t(z^t)}{p_{t+1}(z^{t+1})}
\]
Then, using the law of iterated expectations, we rewrite the first-order condition for deposits (31) in the detailed economy as

$$\sum_{z_t} \mu_{t-1}(z_t | z_{t-1}) \Lambda_t(z_t') = 0$$  \hspace{1cm} (A-2)

where

$$\Lambda_t(z_t') = \frac{u_{ct}(z_t)}{p_t(z_t')} - \beta \left( 1 + R_t(z_t') \right) \sum_{z_{t+1}} \mu_t(z_{t+1} | z_t) \frac{u_{ct+1}(z_{t+1})}{p_{t+1}(z_{t+1})}$$  \hspace{1cm} (A-3)

Notice that if $\Omega_t(z_t'), \Omega_{t+1}(z_{t+1}), \tau_{bt}(z_t')$, and $\tau_{b,t+1}(z_{t+1})$ were absent from equation (A-1), and if the left-hand side of equation (A-3) was zero, the equilibrium conditions for bonds in the two economies would be the same. Fuerst (1992) calls the term $\Lambda(z_t')$ a ‘liquidity effect’. We will choose $\tau_{bt}(z_t')$ so that it has the same effect on the equilibrium of the prototype economy as the liquidity effect. To do so, consider equilibrium allocations $(c_t^*(z_t'), x_t^*(z_t'), y_t^*(z_t'), l_t^*(z_t'), k_{t+1}^*(z_t'))$ and prices $(p_t^*(z_t'), R_t^*(z_t'))$ of the detailed economy. Evaluating the left-hand side of equation (A-1) at these equilibrium allocations and prices and choosing sequences for $\tau_{bt}(z_t')$ such that the right-hand side is equal to zero for every history $z_t'$ implicitly defines $\tau_{bt}(z_t')$ that has the same effect on the equilibrium as the liquidity effect.

Q.E.D

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Appendix B: Additional equivalence results

This appendix provides two additional equivalence results. First, it shows that an economy with sticky wages considered by CKM is equivalent to the prototype monetary economy with labour wedges. This result complements the mapping established by CKM between the sticky-wage economy and a non-monetary prototype economy. Second, it shows that an economy with exogenous fluctuations in energy prices and capital utilisation, like that of Finn (1996), is equivalent to the prototype monetary economy with efficiency wedges. Unless specified otherwise, the notation in this Appendix is the same as in Section 2 (we abstract from population and technology growth here).

B.1 An economy with sticky wages

B.1.1 The underlying economy

Consider an economy populated by a continuum of infinitely lived consumers differentiated by a labour type $j \in [0, 1]$. They consumers can be thought of as being organised in a continuum of unions indexed by $j$. A representative producer has access to an aggregate production function

$$ y_t(z^t) = F(k_t(z^{t-1}), l_t(z^t)) $$

where

$$ l_t(z^t) = \left[ \int l_t(j, z^{t-1}) \varepsilon_t(z^t) dj \right]^{1/\varepsilon_t(z^t)} $$

is a labour aggregate and $\varepsilon_t(z^t)$ is a shock to the degree of monopoly power of the unions. The producer’s problem can be described in two steps. First, for a given $l_t(z^t)$, the producer solves

$$ \min_{\{l_t(j, z^{t-1})\}_{j \in [0,1]}} \int W_t(j, z^{t-1}) l_t(j, z^t) dj $$

subject to (B-2), where $W_t(j, z^{t-1})$ is the nominal wage rate for labour of type $j$. The solution to this problem gives the producer’s demand function for each labour type

$$ l_t(j, z^t) = \left[ \frac{W_t(j, z^{t-1})}{W_t(z^t)} \right]^{\varepsilon_t(z^t)-1} l_t(z^t) $$

where

$$ W_t(z^{t-1}) = \left[ \int W_t(j, z^{t-1}) \frac{\varepsilon_t(z^t)}{\varepsilon_t(z^t)} dj \right]^{\varepsilon_t(z^t)-1} $$

is the aggregate nominal wage rate. In the second step, the producer chooses $k_t(z^{t-1})$ and $l_t(z^t)$ to maximise profits

$$ F(k_t(z^{t-1}), l_t(z^t)) - r_t(z^t) k_t(z^{t-1}) - \frac{W_t(z^{t-1})}{p_t(z^t)} l_t(z^t) $$

The first-order conditions for this problem equalise the marginal products of capital and labour with their prices.

Union $j$ is a monopolist in the market for labour of type $j$ and it sets the nominal wage rate $W_t(j, z^{t-1})$ before the realisation of $z_t$. In addition, it agrees to supply in period $t$ whatever
labour is demanded at that wage rate. The preferences of a consumer $j$ are characterised by a utility function

$$
\sum_{t=0}^{\infty} \sum_{z^t} \beta^t \mu_t(z^t) u \left( c_t(j, z^t), 1 - l_t(j, z^t) - s_t(j, z^t) \right) \tag{B-4}
$$

The consumer/union’s problem is to choose plans for $c_t(j, z^t)$, $x_t(j, z^t)$, $k_{t+1}(j, z^t)$, $l_t(j, z^t)$, $s_t(j, z^t)$, $m_t(j, z^t)$, $b_t(j, z^t)$, and $W_{t+1}(j, z^t)$ to maximise (B-4) subject to the labour demand function (B-3), a shopping time technology

$$
s_t(j, z^t) = s \left( \frac{c_t(j, z^t)}{m_t(j, z^t)/p_t(z^t)} \right)
$$

a budget constraint

$$
c_t(j, z^t) + x_t(j, z^t) + \frac{m_t(j, z^t)}{p_t(z^t)} + \frac{b_t(j, z^t)}{(1 + R_t(z^t))p_t(z^t)} = W_t(j, z^{t-1})l_t(j, z^t) + r_t(z^t)k_t(j, z^{t-1}) + \frac{m_{t-1}(j, z^{t-1})}{p_t(z^t)} + \frac{b_{t-1}(j, z^{t-1})}{p_t(z^t)} + T_t(z^t) \left( \frac{m_t(z^t)}{p_t(z^t)} \right)
$$

and a capital accumulation law

$$
k_{t+1}(j, z^t) = (1 - \delta)k_t(j, z^{t-1}) + x_t(j, z^t)
$$

Assuming that $k_0$, $m_{-1}$ and $b_{-1}$ are the same for all types, the solution to this problem is symmetric across all consumers.

The government sets the nominal interest rate according to a policy rule

$$
R_t(z^t) = (1 - \rho_R) \left[ R + \omega_y \left( \ln y_t(z^t) - \ln y \right) + \omega_\pi \left( \pi_t(z^t) - \pi \right) \right] + \rho_R R_{t-1}(z^{t-1}) \tag{B-5}
$$

and its budget constraint is given by $T_t(z^t) = m_t(z^t) - m_{t-1}(z^{t-1})$.

An equilibrium of this economy with sticky nominal wages is a set of allocations $(c_t(z^t), x_t(z^t), y_t(z^t), l_t(z^t), k_{t+1}(z^t), m_t(z^t), b_t(z^t))$ and a set of prices $(p_t(z^t), R_t(z^t), r_t(z^t), W_t(z^t))$ that satisfy: (i) a set of the consumer’s first-order conditions for wages, capital, bonds, and money, respectively

$$
W_{t+1}(z^t) = \frac{\sum_{zt+1} \mu_t(z^{t+1}|z^t) u_{ht+1}(z^{t+1})l_{t+1}(z^{t+1})}{\sum_{zt+1} \mu_t(z^{t+1}|z^t) \varepsilon_{t+1}(z^{t+1}) \left\{ \frac{l_{t+1}(z^{t+1})}{p_{t+1}(z^{t+1})} \left[ u_{ct+1}(z^{t+1}) - u_{ht+1}(z^{t+1})s_{ct+1}(z^{t+1}) \right] \right\}}
$$

$$
\sum_{zt+1} Q_t(z^{t+1}|z^t) \left[ 1 + r_{t+1}(z^{t+1}) - \delta \right] = 1
$$

$$
\sum_{zt+1} Q_t(z^{t+1}|z^t) (1 + R_t(z^t)) \left( \frac{p_t(z^t)}{p_{t+1}(z^{t+1})} \right) = 1
$$

$$
- \frac{s_{mt}(z^t)u_{ht}(z^t)}{u_{ct}(z^t) - u_{ht}(z^t)s_{ct}(z^t)} + \sum_{zt+1} Q_t(z^{t+1}|z^t) \left( \frac{p_t(z^t)}{p_{t+1}(z^{t+1})} \right) = 1
$$

where

$$
Q_t(z^{t+1}|z^t) = \beta \mu_t(z^{t+1}|z^t) \frac{u_{ct+1}(z^{t+1}) - u_{ht+1}(z^{t+1})s_{ct+1}(z^{t+1})}{u_{ct}(z^t) - u_{ht}(z^t)s_{ct}(z^t)}
$$

(ii) a set of the producer’s first-order conditions

$$
r_t(z^t) = F_{kl}(z^t)
$$
Then

\[ \frac{W_t(z^{t-1})}{p_t(z^t)} = F_{lt}(z^t) \]

(iii) the resource constraint \( c^t(z^t) + x^t(z^t) = y_t(z^t) \), where \( y_t(z^t) \) is given by the production function \((B-1)\); (iv) the capital accumulation law \( k_{t+1}(z^t) = (1 - \delta)k_t(z^{t-1}) + x_t(z^t) \); (v) the monetary policy rule \((B-5)\); and (vi) the bond market clearing condition \( b_t(z^t) = 0 \).

**B.1.2 The associated prototype economy**

Consider now a version of the prototype economy of Section 2, in which all wedges except the labour wedge are set equal to their steady-state values in all states of the world. Comparing the equilibrium conditions of the detailed economy with those of the prototype economy we obtain the following proposition.

**Proposition 3:** Consider equilibrium allocations of the sticky-wage economy \((c^*_t(z^t), x^*_t(z^t), y^*_t(z^t), k^*_t(z^t), m^*_t(z^t))\) and prices \((p^*_t(z^t), R^*_t(z^t), r^*_t(z^t), W^*_t(z^t))\) that support these allocations. Let the wedges in the prototype economy satisfy: \( A_t(z^t) = 1 \), \( \tau_{xt}(z^t) = \tau_{bt}(z^t) = g_t(z^t) = R_t(z^t) = 0 \), and

\[ \tau_t(z^t) = 1 - \frac{u^*_{ht}(z^t)}{[u^*_{ct}(z^t) - u^*_{ht}(z^t) s^*_{ct}(z^t)] F^*_t(z^t)} \]

for all \( z^t \), where \( u^*_{ht}, u^*_{ct}, s^*_{ct}, \) and \( F^*_t(z^t) \) are evaluated at the equilibrium of the sticky-wage economy. Then \((c^*_t(z^t), x^*_t(z^t), y^*_t(z^t), k^*_t(z^t), m^*_t(z^t))\) and \((p^*_t(z^t), R^*_t(z^t))\) are also equilibrium allocations and prices of the prototype economy.

The key point here is that sticky wages are equivalent to labour income taxes in the prototype monetary economy. This mapping is the same as that between the sticky-wage economy and the CKM prototype economy. Sticky wages thus affect inflation and the nominal interest rate only to the extent to which they distort labour decisions. In terms of the pricing function \((15)\), sticky wages have only an indirect effect on inflation and the nominal interest rate through their effect on the equilibrium allocations since the labour income tax does not appear in the pricing function.

**B.2 An economy with capital utilisation and energy price shocks**

**B.2.1 The underlying economy**

Consider now an economy that purchases an intermediate good, called energy, at the world market at a price \( p^*_t(z^t) \), which it takes as given. In this economy, an infinitely lived representative consumer operates an aggregate production function

\[ y_t(z^t) = \left( v_t(z^t) k_t(z^{t-1}) \right)^\alpha I_t(z^t)^{1-\alpha} \]

where \( \alpha \in (0, 1) \), \( v_t(z^t) \) is a rate of capital utilisation and \( v_t(z^t) k_t(z^{t-1}) \) is a flow of capital services. Energy \( e_t(z^t) \) is related to capital services according to

\[ e_t(z^t) = a \left( v_t(z^t) \right) k_t(z^{t-1}) \]

where \( a'(.) > 0 \) and \( a''(.) > 0 \).
The consumer chooses plans for \( c_t(z^t), x_t(z^t), h_t(z^t), l_t(z^t), s_t(z^t), y_t(z^t), k_{t+1}(z^t), m_t(z^t), b_t(z^t), v_t(z^t), \) and \( e_t(z^t) \) to maximise the utility function (1) subject to the time constraint (2), the capital accumulation law (4) and the budget constraint

\[
c_t(z^t) + x_t(z^t) + p^e_t(z^t) e_t(z^t) + m_t(z^t) + \frac{b_t(z^t)}{p_t(z^t)} = y_t(z^t) + \frac{m_{t-1}(z^{t-1})}{p_t(z^t)} + \frac{b_{t-1}(z^{t-1})}{p_t(z^t)} + \frac{T_t(z^t)}{p_t(z^t)}
\]

where \( y_t(z^t) \) is given by the production function (B-6) and \( e_t(z^t) \) is given by the expression (B-7). The government sets the nominal interest rate according to

\[
R_t(z^t) = [R + \omega_y (\ln y_t(z^t) - \ln y) + \omega_{\pi} (\pi_t(z^t) - \pi)] + \rho R T_{t-1}(z^{t-1})
\]

and its budget constraint is given by \( T_t(z^t) = m_t(z^t) - m_{t-1}(z^{t-1}) \).

An equilibrium of this economy with capital utilisation and energy price shocks is a set of allocations \((c_t(z^t), x_t(z^t), y_t(z^t), l_t(z^t), m_t(z^t), k_{t+1}(z^t), b_t(z^t), v_t(z^t))\) and a set of prices \((p_t(z^t), R_t(z^t))\) that satisfy: (i) the consumer’s first-order conditions for capital utilisation, labour, capital, bonds, and money, respectively

\[
\alpha v_t(z^{t-1}) k_t(z^{t-1})^{-\alpha} l_t(z^{t-1}) = p^e_t(z^t) a'(v_t(z^t)) k_t(z^{t-1})
\]

\[
u_{ct}(z^t)(1 - \alpha) k_t(z^{t-1}) v_t(z^t)^\alpha l_t(z^{t-1})^{-\alpha} = u_{ht}(z^t) \left[ 1 + s_{ct}(z^t)(1 - \alpha) k_t(z^{t-1}) v_t(z^t)^\alpha l_t(z^{t-1})^{-\alpha} \right]
\]

\[
\sum_{z_{t+1}} Q_t(z^{t+1}|z^t) [\alpha v_{t+1}(z^{t+1})^{\alpha} k_{t+1}(z^{t+1})^{-\alpha} l_{t+1}(z^{t+1})^{1-\alpha} + 1 - \delta - p_{t+1}^e(z^{t+1}) a'(v_{t+1}(z^{t+1}))] = 1
\]

\[
\sum_{z_{t+1}} Q_t(z^{t+1}|z^t)(1 + R_t(z^t)) \frac{p_t(z^t)}{p_{t+1}(z^{t+1})} = 1
\]

\[- \frac{u_{ht}(z^t) s_{mt}(z^t)}{u_{ct}(z^t) - u_{ht}(z^t) s_{ct}(z^t)} + \sum_{z_{t+1}} Q_t(z^{t+1}|z^t) \frac{p_t(z^t)}{p_{t+1}(z^{t+1})} = 1 \]

where

\[
Q_t(z^{t+1}|z^t) = \beta \mu_t(z^{t+1}|z^t) \frac{u_{c,t+1}(z^{t+1}) - u_{h,t+1}(z^{t+1}) s_{c,t+1}(z^{t+1})}{u_{ct}(z^t) - u_{ht}(z^t) s_{ct}(z^t)}
\]

(ii) the resource constraint

\[
c_t(z^t) + x_t(z^t) + p^e_t(z^t) a'(v_t(z^t)) k_t(z^{t-1}) = y_t(z^t)
\]

where \( y_t \) is given by the production function (B-6); (iii) the capital accumulation law (4); (iv) the monetary policy rule (B-8); and (v) the bond market clearing condition \( b_t(z^t) = 0 \).

B.2.2 The associated prototype economy

Consider now a version of the prototype economy of Section 2 in which the production function has the Cobb-Douglas functional form as in the underlying economy

\[
y_t(z^t) = A_t(z^t) k_t(z^{t-1})^{\alpha} l_t(z^t)^{1-\alpha}
\]

and in which the investment wedge resembles a tax on capital income rather than a tax on investment.\(^{(18)}\) The consumer’s budget constraint is now

\(^{(18)}\) As in the case of the equivalence result for sticky prices, this assumption is not crucial but makes the exposition easier.
$c_t(z^t) + x_t(z^t) + \frac{m_t(z^t)}{p_t(z^t)} + \frac{b_t(z^t)}{p_t(z^t)(1 + R_t(z^t))} = [1 - \tau_{kt}(z^t)] r_t(z^t) k_t(z^{t-1}) + w_t(z^t) l_t(z^t) + \frac{m_{t-1}(z^{t-1})}{p_t(z^t)} + \frac{b_{t-1}(z^{t-1})}{p_t(z^t)} + \frac{T_t(z^t)}{p_t(z^t)}$

where $\tau_{kt}$ is a tax on capital income, and the first-order condition for capital is

$$\sum_{z_{t+1}} Q_t(z_{t+1}^t | z^t) \left[ (1 - \tau_{k,t+1}(z_{t+1}^t)) \alpha A_{t+1}(z_{t+1}^t) k_{t+1}(z^t)^{\alpha-1} l_{t+1}(z_{t+1}^t)^{1-\alpha} + (1 - \delta) \right] = 1$$

Comparing the equilibrium conditions of the detailed economy with those of the prototype economy we obtain the following proposition.

**Proposition 4:** Consider equilibrium allocations of the detailed economy with capital utilisation and energy price shocks ($c_t(z^t), x_t(z^t), y_t(z^t), l_t(z^t), k_{t+1}(z^t), m_t(z^t), v_t(z^t)$, $e_t(z^t)$) and prices ($p_t(z^t), R_t(z^t)$) that support these allocations. Let the wedges in the prototype economy satisfy: $\tau_{kt}(z^t) = g_t(z^t) = R_t(z^t) = 0$, and

$$A_t(z^t) = v_t(z^t)^\alpha$$

$$\tau_{kt}(z^t) = \frac{p_t(z^t) a (v_t(z^t))}{\alpha A_t(z^t) (k_t(z^{t-1})^\alpha-1 (l_t(z^t))^{1-\alpha}}$$

$$g_t(z^t) = p_t(z^t) a (v_t(z^t)) k_t(z^{t-1})$$

Then ($c_t(z^t), x_t(z^t), y_t(z^t), l_t(z^t), k_{t+1}(z^t), m_t(z^t)$) and ($p_t(z^t), R_t(z^t)$) are also equilibrium allocations and prices of the prototype economy.

Consider now a special case of this proposition. Suppose that in equilibrium fluctuations in capital utilisation $v_t(z^t)$ are such that they offset fluctuations in $p_t(z^t)$ in a way that leaves the energy price costs per unit of capital $p_t(z^t) a (v_t(z^t))$ unchanged. Then fluctuations in energy prices in the underlying economy show up in the prototype economy as fluctuations in the efficiency wedge, but not as fluctuations in the investment wedge or the government consumption wedge. The main idea here is that fluctuations in energy prices (or prices of commodities used to produce energy, such as oil) are equivalent to fluctuations in the efficiency wedge.

---

(19) It is trivial to show that $\partial v_t / \partial p_t < 0$ and thus $\partial a_t / \partial p_t < 0$. 

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References


Chakraborty, S (2005), ‘Business cycles in a neoclassical growth model: How important are technology shocks as a propagation mechanism?’, manuscript.


Henriksen, E, Kydland, F E and Sustek, R (2008), ‘The high correlations of prices and interest rates across nations’, manuscript, University of California at Santa Barbara.


Table A. Baseline parameter values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>0.266</td>
<td>Consumption share in utility</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.995</td>
<td>Discount factor</td>
</tr>
<tr>
<td>( \gamma_n )</td>
<td>0.0037</td>
<td>Population growth rate</td>
</tr>
<tr>
<td>( \gamma_A )</td>
<td>0.004</td>
<td>Technology growth rate</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.0118</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.35</td>
<td>Capital share in production</td>
</tr>
<tr>
<td>( \nu_1 )</td>
<td>0.0319</td>
<td>Level parameter</td>
</tr>
<tr>
<td>( \nu_2 )</td>
<td>1.0</td>
<td>Curvature parameter</td>
</tr>
<tr>
<td>( \pi )</td>
<td>0.0091</td>
<td>Steady-state inflation rate</td>
</tr>
<tr>
<td>( \omega_y )</td>
<td>0.125</td>
<td>Weight on output</td>
</tr>
<tr>
<td>( \omega_{\pi} )</td>
<td>1.5</td>
<td>Weight on inflation</td>
</tr>
<tr>
<td>( \rho_R )</td>
<td>0.75</td>
<td>Smoothing coefficient</td>
</tr>
</tbody>
</table>

Table B. Stochastic process for the wedges

\[
P_0 = \begin{bmatrix} -0.0798 & 0.0072 & -0.0338 & 0.0474 & -0.0119 & -0.0019 \end{bmatrix}
\]

\[
P = \begin{bmatrix}
0.854 & -0.0963 & 0.173 & -0.0061 & -0.0425 & 0.520 \\
-0.0673 & 1.058 & -0.0014 & 0.0097 & 0.0465 & -0.722 \\
-0.857 & -0.0335 & 1.088 & 0.0026 & -0.0116 & 0.402 \\
0.0821 & 0.0587 & -0.0974 & 1.0053 & 0.0241 & 0.341 \\
0.0973 & -0.298 & 0.085 & -0.0076 & 0.826 & 0.12 \\
-0.0217 & 0.0146 & 0.0005 & 0.0004 & 0.0063 & 0.441
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0.0072 & 0 & 0 & 0 & 0 & 0 \\
0.0037 & 0.0092 & 0 & 0 & 0 & 0 \\
0.0058 & -0.0008 & 0.0029 & 0 & 0 & 0 \\
0.0009 & 0.005 & 0.0117 & 0.0087 & 0 & 0 \\
0.0005 & -0.0175 & -0.0013 & 0.0014 & 0.0219 & 0 \\
0.0003 & 8.3e-6 & 0.0001 & -0.0002 & 0.004 & 0.001
\end{bmatrix}
\]

\(a\) The equilibrium conditions of the prototype economy imply that in a steady state the values of \( \tau_b \) and \( \tilde{R} \) are zero. This restriction is imposed in the estimation of \( P_0 \), \( P \) and \( B \).
### Table C. Business cycle properties of the wedges, 1959.Q1-2004.Q4

<table>
<thead>
<tr>
<th>Wedge</th>
<th>Relative std. dev.</th>
<th>Correlations of output in period $t$ with wedges in $t + j$:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log A_{t+j}$</td>
<td>0.63</td>
<td>$j = -4$ 0.33 0.49 0.67 0.77 0.85 0.62 0.38 0.13 -0.05</td>
</tr>
<tr>
<td>$\tau_{l,t+j}$</td>
<td>0.92</td>
<td>-0.17 -0.33 -0.50 -0.67 -0.74 -0.78 -0.74 -0.63 -0.43</td>
</tr>
<tr>
<td>$\tau_{x,t+j}$</td>
<td>0.50</td>
<td>0.16 0.35 0.54 0.68 0.79 0.62 0.44 0.26 0.13</td>
</tr>
<tr>
<td>$\log g_{t}$</td>
<td>1.51</td>
<td>-0.40 -0.42 -0.45 -0.44 -0.35 -0.24 -0.10 0.04 0.20</td>
</tr>
<tr>
<td>$\tau_{b,t+j}$</td>
<td>2.59</td>
<td>0.06 0.27 0.48 0.70 0.82 0.81 0.72 0.58 0.41</td>
</tr>
<tr>
<td>$\tilde{R}_{t+j}$</td>
<td>0.12</td>
<td>0.11 0.15 0.13 0.15 0.11 0.01 -0.09 -0.16 -0.17</td>
</tr>
</tbody>
</table>

- The statistics are computed after the wedges and output have been detrended with HP-filter.
- The standard deviations are measured relative to that of output.

### Table D. Contemporaneous correlations of the wedges with each other: 1959.Q1-2004.Q4

<table>
<thead>
<tr>
<th>Wedge</th>
<th>$\log A$</th>
<th>$\tau_l$</th>
<th>$\tau_x$</th>
<th>$\log g$</th>
<th>$\tau_b$</th>
<th>$\tilde{R}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log A$</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_l$</td>
<td>-0.31</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_x$</td>
<td>0.90</td>
<td>-0.28</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log g$</td>
<td>-0.34</td>
<td>0.45</td>
<td>0.01</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_b$</td>
<td>0.53</td>
<td>-0.88</td>
<td>0.54</td>
<td>-0.40</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>$\tilde{R}$</td>
<td>0.19</td>
<td>-0.02</td>
<td>0.17</td>
<td>-0.19</td>
<td>0.35</td>
<td>1.00</td>
</tr>
</tbody>
</table>

- The statistics are computed after the wedges have been detrended with HP-filter.
Table E. The dynamics of the nominal interest rate with respect to output in the postwar business cycle\(^a\)

<table>
<thead>
<tr>
<th></th>
<th>Rel. Corr</th>
<th>Rel. Correlations of output at (t) with the nominal interest rate at (t + j):</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>std. (y)</td>
<td>(y^d, y^m) std. (R)</td>
<td>(j = -5)</td>
<td>-4</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Data</td>
<td>1.58</td>
<td>1.51</td>
<td>-0.61</td>
<td>-0.50</td>
<td>-0.34</td>
<td>-0.15</td>
<td>0.10</td>
<td>0.32</td>
<td>0.42</td>
<td>0.47</td>
<td>0.46</td>
<td>0.44</td>
<td>0.41</td>
</tr>
<tr>
<td>Combination of wedges</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No (A)</td>
<td>0.62</td>
<td>0.14</td>
<td>8.89</td>
<td>0.39</td>
<td>0.47</td>
<td>0.56</td>
<td>0.56</td>
<td>0.47</td>
<td>0.38</td>
<td>0.33</td>
<td>0.19</td>
<td>0.09</td>
<td>-0.02</td>
</tr>
<tr>
<td>No (\tau_l)</td>
<td>0.72</td>
<td>0.60</td>
<td>9.11</td>
<td>-0.14</td>
<td>-0.19</td>
<td>-0.24</td>
<td>-0.23</td>
<td>-0.19</td>
<td>-0.17</td>
<td>-0.07</td>
<td>-0.01</td>
<td>0.05</td>
<td>0.09</td>
</tr>
<tr>
<td>No (\tau_x)</td>
<td>1.16</td>
<td>1.00</td>
<td>0.53</td>
<td>-0.57</td>
<td>-0.49</td>
<td>-0.34</td>
<td>-0.15</td>
<td>0.09</td>
<td>0.30</td>
<td>0.41</td>
<td>0.44</td>
<td>0.42</td>
<td>0.38</td>
</tr>
<tr>
<td>No (g)</td>
<td>1.18</td>
<td>0.96</td>
<td>0.70</td>
<td>-0.75</td>
<td>-0.70</td>
<td>-0.58</td>
<td>-0.42</td>
<td>-0.21</td>
<td>0.00</td>
<td>0.15</td>
<td>0.25</td>
<td>0.32</td>
<td>0.38</td>
</tr>
<tr>
<td>No (\tau_b)</td>
<td>1.13</td>
<td>1.00</td>
<td>0.52</td>
<td>-0.09</td>
<td>-0.28</td>
<td>-0.46</td>
<td>-0.61</td>
<td>-0.78</td>
<td>-0.85</td>
<td>-0.70</td>
<td>-0.51</td>
<td>-0.30</td>
<td>-0.10</td>
</tr>
<tr>
<td>No (\tilde{R})</td>
<td>1.09</td>
<td>1.00</td>
<td>0.69</td>
<td>-0.55</td>
<td>-0.55</td>
<td>-0.50</td>
<td>-0.41</td>
<td>-0.25</td>
<td>-0.16</td>
<td>-0.02</td>
<td>0.09</td>
<td>0.13</td>
<td>0.18</td>
</tr>
<tr>
<td>(A, \tau_b)</td>
<td>1.04</td>
<td>0.76</td>
<td>0.51</td>
<td>-0.36</td>
<td>-0.38</td>
<td>-0.36</td>
<td>-0.32</td>
<td>-0.25</td>
<td>-0.22</td>
<td>-0.07</td>
<td>0.06</td>
<td>0.09</td>
<td>0.11</td>
</tr>
<tr>
<td>(A, \tau_b, \tilde{R})</td>
<td>0.96</td>
<td>0.74</td>
<td>0.82</td>
<td>-0.47</td>
<td>-0.48</td>
<td>-0.43</td>
<td>-0.34</td>
<td>-0.20</td>
<td>-0.06</td>
<td>0.08</td>
<td>0.19</td>
<td>0.23</td>
<td>0.23</td>
</tr>
</tbody>
</table>

\(^a\) The statistics are for HP-filtered series. Except for the data, the standard deviations are expressed relative to those of the data.
Table F. The dynamics of inflation with respect to output in the postwar business cycle

<table>
<thead>
<tr>
<th>Combination of wedges</th>
<th>Rel. Corr std. $y$, $y^d$, $y^m$</th>
<th>Rel. Corr std. $R$</th>
<th>Correlations of output at $t$ with inflation at $t + j$:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1.58</td>
<td>1.14</td>
<td>-0.42 -0.39 -0.26 -0.11 0.02 0.12 0.27 0.38 0.46 0.48 0.45</td>
</tr>
<tr>
<td>No $A$</td>
<td>0.62</td>
<td>0.14</td>
<td>1.73 0.36 0.41 0.42 0.22 0.03 -0.16 0.06 -0.07 -0.15 -0.20 -0.20</td>
</tr>
<tr>
<td>No $\tau_l$</td>
<td>0.72</td>
<td>0.60</td>
<td>1.80 -0.26 -0.31 -0.24 -0.21 -0.13 -0.26 0.21 0.32 0.31 0.16 0.19</td>
</tr>
<tr>
<td>No $\tau_x$</td>
<td>1.16</td>
<td>1.00</td>
<td>1.14 -0.34 -0.39 -0.30 -0.20 -0.14 -0.05 0.08 0.20 0.30 0.34 0.30</td>
</tr>
<tr>
<td>No $g$</td>
<td>1.18</td>
<td>0.96</td>
<td>1.26 -0.56 -0.57 -0.51 -0.44 -0.32 -0.16 0.04 0.21 0.36 0.48 0.53</td>
</tr>
<tr>
<td>No $\tau_b$</td>
<td>1.13</td>
<td>1.00</td>
<td>3.04 -0.08 -0.25 -0.33 -0.38 -0.46 -0.44 -0.20 -0.06 0.05 0.11 0.16</td>
</tr>
<tr>
<td>No $\tilde{R}$</td>
<td>1.09</td>
<td>1.00</td>
<td>2.79 -0.28 -0.20 -0.14 -0.13 -0.07 -0.09 0.09 0.09 0.13 0.16</td>
</tr>
<tr>
<td>$A, \tau_b$</td>
<td>1.04</td>
<td>0.76</td>
<td>2.81 -0.37 -0.27 -0.23 -0.25 -0.19 -0.20 0.09 0.13 0.10 0.12 0.18</td>
</tr>
<tr>
<td>$A, \tau_b, \tilde{R}$</td>
<td>0.96</td>
<td>0.74</td>
<td>1.55 -0.39 -0.43 -0.36 -0.31 -0.20 -0.18 0.13 0.29 0.34 0.29 0.28</td>
</tr>
</tbody>
</table>

$^a$ The statistics are for HP-filtered series. Except for the data, the standard deviations are expressed relative to those of the data.
Figure 1. The 1973 recession: Data and wedges

A. Deviations of logged data

B. Deviations of data

C. Deviations of wedges

D. Deviations of wedges
Figure 2. The 1973 recession: Efficiency wedge only

Figure 3. The 1973 recession: Labour wedge only
Figure 4. The 1973 recession: Investment wedge only

Figure 5. The 1973 recession: No efficiency wedge
Figure 6. The 1973 recession: No labour wedge

Figure 7. The 1973 recession: No investment wedge
Figure 8. The 1973 recession: No monetary policy wedge

Figure 9. The 1973 recession: No asset market wedge
Figure 10. The 1982 recession: Data and wedges

A. Deviations of logged data

B. Deviations of data

C. Deviations of wedges

D. Deviations of wedges
Figure 11. The 1982 recession: No efficiency wedge

Figure 12. The 1982 recession: No labour wedge
Figure 13. The 1982 recession: No investment wedge

Figure 14. The 1982 recession: No monetary policy wedge
Figure 15. The 1982 recession: No asset market wedge
Figure 16. The 1973 recession: No efficiency wedge – alternative weights on inflation

Legend: Thin dashed – data; thick solid – from Figure 5; thin solid – $\omega_\pi = 1.3$; thick dashed – $\omega_\pi = 1.5$; thick dash-dotted – $\omega_\pi = 1.7$

Figure 17. The 1973 recession: No efficiency wedge – alternative weights on output

Legend: Thin dashed – data; thick solid – from Figure 5; thin solid – $\omega_y = 0.08$; thick dashed – $\omega_y = 0.125$; thick dash-dotted – $\omega_y = 0.175$