Discussion of: Investor Information, Long-Run Risk, and the Duration of Risky Cash Flows

Lars Peter Hansen
Term structure of cash flows

Let $X = \{X_t\}$ be an underlying Markov process, and let $M_{t+j,t}$ be the stochastic discount factor between period $t+j$ and $t$ constructed from the Markov process.

Consider a cash flow:

$$D_t = \hat{D}_t \psi(X_t)$$

where $\hat{D}$ is a positive martingale.

$$P_{n,t} = \hat{D}_t E \left[ M_{t+n,t} \left( \frac{\hat{D}_{t+n}}{\hat{D}_t} \right) \psi(X_{t+n}) | X_t \right]$$

Holding period returns: $R_{n,t+1} = \frac{P_{n-1,t+1}}{P_{n,t}}$. Equity return is a portfolio of holding period returns.
Limit results and slopes

Hansen-Heaton-Li and Hansen-Scheinkman

Initial return: $D_{t+1}$ divided by price - price one period cash flow risk exposure.

Limit return:

$$\exp(-\rho) \left( \frac{\hat{D}_{t+1}}{\hat{D}_t} \right) \left( \frac{\phi(X_{t+1})}{\phi(X_t)} \right)$$

cash flow component and a value component. The term $\phi$ does not depend on the transient contribution to cash flows. Two sources of risk to price. The value component solves “fixed point problem”

$$E \left[ M_{t+1,t} \left( \frac{\hat{D}_{t+1}}{\hat{D}_t} \right) \phi(X_{t+1}) \mid X_t \right] = \exp(-\rho)\phi(X_t).$$
Log Decompositions

Let $d_t = \log D_t$.

Consider the log normal model with moving average representation:

$$d_t - d_{t-1} = \mu d + \sum_{j=0}^{\infty} \kappa_j \epsilon_{t-j}.$$  

The shocks $\epsilon_t$ are multivariate standard normal. The $\{\kappa_j\}$'s are the moving-average coefficients - impulse response functions.
Permanent-transitory composition

Trick

\[ \kappa(z) = \sum_{j=0}^{\infty} \kappa_j z^j \]

Form:

\[ \kappa(z) = \kappa(1) + [\kappa(z) - \kappa(1)] = \kappa(1) + (1 - z)\tilde{\kappa}(z). \]

Then

\[ d_t = t\mu_d + \kappa(1) \sum_{t=1}^{t} \epsilon_j + \tilde{d}_t - \tilde{d}_0. \]

where \( \tilde{d}_t = \tilde{\kappa}(L)\epsilon_t. \)
Full information example

\[
\begin{align*}
    d_{t+1} - d_t &= \mu_d + \phi_x x_t + \sigma_d \epsilon_{t+1} \\
    x_{c,t} &= \rho x_{t-1} + \sigma_x \epsilon_t.
\end{align*}
\]

\[\kappa(z) = \sigma_d + z \frac{\phi_x \sigma_x}{1 - \rho z}\]

In particular,

- shock exposure of the permanent component:

\[\kappa(1) = \sigma_d + \frac{\phi_x \sigma_x}{1 - \rho}\]

- shock exposure of the transient component:

\[\hat{\kappa}(0) = \kappa(0) - \kappa(1) = \kappa_0 - \sum_{j=0}^{\infty} \kappa_j = -\frac{\sigma_x}{1 - \rho}\]
Consumption

Let
\[ c_t = \log C_t \]

Moving-average representation:
\[ c_t - c_{t-1} = \mu_c + \sum_{j=0}^{\infty} \lambda_j \epsilon_{t-j} \]

Permanent-transitory decomposition:
\[ c_t = t\mu_c + \lambda(1) \sum_{t=1}^{t} \epsilon_j + \tilde{c}_t - \tilde{c}_0. \]
Consumption evolution

\[ c_{t+1} - c_t = \mu_c + x_t + \sigma_c \epsilon_{t+1} \]
\[ x_{c,t} = \rho x_{t-1} + \sigma_x \epsilon_t. \]

\[ \lambda(z) = \sigma_c + z \frac{\sigma_x}{1 - \rho z} \]

In particular,

- shock exposure of the permanent component:
  \[ \lambda(1) = \sigma_c + \frac{\sigma_x}{1 - \rho} \]
- shock exposure of the transient component:
  \[ \hat{\lambda}(0) = -\frac{\sigma_x}{1 - \rho} \]
Valuation contribution to the long run

For simplicity, restrict intertemporal substitution elasticity to be unity as in Figure 9.

One period cash flow risk exposure $\sigma_d$

Limit holding period return - long run cash flow contribution: $\sigma_d$. Need to solve:

$$\tilde{E} [\exp(A_{t+1,t})\phi(X_{t+1})|X_t] = \exp(-\rho)\phi(X_t)$$

where

$$A_{t+1,t} = -\log \delta - c_{t+1} + c_t + \left(\sigma_d + \frac{\sigma_x}{1-\rho}\right) \epsilon_{t+1}.$$ 

and $\mu_d = \mu_c$. The operator $\tilde{E}$ includes an adjustment from recursive utility. It adds $(\gamma - 1)\lambda(\delta)$ to the mean of $\epsilon_{t+1}$. 

Discussion of: Investor Information, Long-Run Risk, and the Duration of Risky Cash Flows – p. 9/12
Valuation contribution

Extract temporary component from log consumption. Risk exposure of $\phi(X_{t+1})$ is

$$-\frac{\sigma_x}{1-\rho}$$

Combined risk exposure from the limit return

$$\sigma_d + \frac{\phi_x \sigma_x}{1-\rho} - \frac{\sigma_x}{1-\rho}$$

Need $\phi_x < 1$ for the long run risk to pull down equity term structure.

One period risk price $\sigma_c + (\gamma - 1)\lambda(\delta)$
Information

- Kreps-Porteus - relax the reduction of compound lotteries. Information structures per se matter in valuation.
- Fair game to explore sensitivity to information. Continuation values of future consumption plans depend on information structures in way that cannot simply be integrated as in other applications of the Law of Iterated Expectations. Good question!
- Reduces information by making $\{x_t\}$ unobservable. Use dividend growth and consumption growth as indicators. Remains Markov but with a different state space. Shifts up the term structure. Stochastic steady state solution.
- Unconditional means are known - not necessary
- Parameter values hard to measure and key parameter values do not jump out with much precision from historical evidence
- Hansen - Sargent develop a notion of fragile beliefs in which investors are not confident in the existence of these components.
Cash flows

- Alternative portfolios are viewed as alternative claims on aggregate dividends.
- Applies Lettau-Wachter model - can you match actual portfolio cash flows? Portfolio formation changes long run risk exposure. Specific model may miss important low frequency components of the cash flow.
- Why not look directly at portfolio cash flows?