“Exotic” (Recursive) Preferences & Cyclical Properties of US Asset Returns

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Overview

Time preference

Risk preference
  ▶ Chew-Dekel
  ▶ Risk premiums

Recursive preferences

Applications of recursive preferences
  ▶ Pricing kernels
  ▶ Risk sharing
  ▶ Asset returns
Time preference

Additive preferences

\[ U_t = (1 - \beta)u_t + \beta U_{t+1} = (1 - \beta) \sum_{j=0}^{\infty} \beta^j u_{t+j} \]

Time aggregator \( V \)

\[ U_t = V(u_t, U_{t+1}) \]

(discounting built into \( V_2 \))

Why don’t we care about this?
Risk preference

Basics: states $s \in \{1, \ldots, S\}$, consumption $c(s)$, probabilities $p(s)$

Certainty equivalent function: $\mu$ satisfying

$$U(\mu, \ldots, \mu) = U[c(1), \ldots, c(S)]$$

Properties:

- Sure things: $\mu(c) = E(c) = c$
- FOSD: $\mu(c + a) \geq \mu(c)$ for constant $a > 0$
- SOSD: $\mu(c + a) \leq \mu(c)$ for mean preserving spread $a$
- $\Rightarrow \mu(c) \leq E(c)$
Chew-Dekel preferences

Certainty equivalent function defined by risk aggregator $M$

$$\mu = \sum_s p(s)M[c(s), \mu]$$

Recursive definition unavoidable (you’ll see why)

Generalization of expected utility (weaker independence axiom)
Chew-Dekel examples

Expected utility

\[ M(c, m) = c^\alpha m^{1-\alpha}/\alpha + m(1 - 1/\alpha) \]

Weighted utility

\[ M(c, m) = (c/m)^\gamma c^\alpha m^{1-\alpha}/\alpha + m[1 - (c/m)\gamma/\alpha]. \]

Disappointment aversion

\[ M(c, m) = c^\alpha m^{1-\alpha}/\alpha + m(1 - 1/\alpha) + \delta I(m - c)(c^\alpha m^{1-\alpha} - m)/\alpha \]

\[ I(x) = 1 \text{ if } x > 0, \ 0 \text{ otherwise} \]
Chew-Dekel as adjusted probabilities

Expected utility

\[ \mu = \left( \sum_s p(s)c(s)^\alpha \right)^{1/\alpha} \]

Weighted utility: ditto with

\[ \hat{p}(s) = \frac{p(s)c(s)^\gamma}{\sum_u p(u)c(u)^\gamma}, \]

Disappointment aversion: ditto with

\[ \hat{p}(s) = \frac{p(s)(1 + \delta I[\mu - c(s)])}{\sum_u p(s)(1 + \delta I[\mu - c(s)])}, \]
Small risks

Two states \((1 + \sigma, 1 - \sigma)\), equal probs, Taylor series around \(\sigma = 0\)

Expected utility

\[
\mu(\text{EU}) \approx 1 - (1 - \alpha)\sigma^2/2
\]

Weighted utility

\[
\mu(\text{WU}) \approx 1 - [1 - (\alpha + 2\gamma)]\sigma^2/2
\]

Disappointment aversion

\[
\mu(\text{DA}) \approx 1 - \left(\frac{\delta}{2 + \delta}\right)\sigma - (1 - \alpha) \left(\frac{4 + 4\delta}{4 + 4\delta + \delta^2}\right)\sigma^2/2
\]
Lognormal risks

Let: \( \log c \sim N(\kappa_1, \kappa_2) \), \( \text{rp} = \log[\mathbb{E}(c)/\mu(c)] \)

Expected utility

\[
\text{rp}(\text{EU}) = (1 - \alpha) \kappa_2/2
\]

Weighted utility

\[
\text{rp}(\text{WU}) = [1 - (\alpha + 2\gamma)] \kappa_2/2
\]

Disappointment aversion

\[
\text{rp}(\text{DA}) = E2C2E
\]
Extremes risks

Let: $\log E \exp(\log c) = \kappa_1 + \frac{\kappa_2}{2!} + \frac{\kappa_3}{3!} + \frac{\kappa_4}{4!}$

Expected utility

$$rp(EU) = (1 - \alpha)\frac{\kappa_2}{2} + (1 - \alpha^2)\frac{\kappa_3}{3!} + (1 - \alpha^3)\frac{\kappa_4}{4!}$$

Weighted utility

$$rp(WU) = [1 - (\alpha + 2\gamma)]\frac{\kappa_2}{2} + [1 - (\alpha + 2\gamma)^2 + \gamma(\alpha + \gamma)]\frac{\kappa_3}{3!} + [1 - (\alpha + 2\gamma)^3 + 2\gamma(\alpha + \gamma)(\alpha + 2\gamma)]\frac{\kappa_4}{4!}$$

Disappointment aversion

$$rp(DA) = \text{Another E2C2E}$$
Recursive preferences

General form

\[ U_t = V[u_t, \mu_t(U_{t+1})] \]

Kreps-Porteus/Epstein-Zin/Weil

\[ V(u_t, \mu_t) = [(1 - \beta)u_t^\rho + \beta\mu_t^\rho]^{1/\rho} \]

\[ \mu_t(U_{t+1}) = (E_t U_{t+1}^\alpha)^{1/\alpha} \]

IES \quad = \quad 1/(1 - \rho)

CRRA \quad = \quad 1 - \alpha

\[ \alpha = \rho \quad \Rightarrow \quad \text{additive preferences} \]
Applications

Pricing kernels

Risk sharing

Cyclical properties of US asset returns
Kreps-Porteus pricing kernel

Marginal rate of substitution

\[ m_{t+1} = \beta \left( \frac{c_{t+1}}{c_t} \right)^{\rho-1} \left[ \frac{U_{t+1}}{\mu_t(U_{t+1})} \right]^{\alpha-\rho} \]

Note role of future utility

- Allows role for predictable future consumption growth
- Ditto volatility
Kreps-Porteus pricing kernel (continued)

Example: let consumption growth follow

\[
\log x_t = \log x + \sum_{j=0}^{\infty} \chi_j w_{t-j}
\]

Pricing kernel

\[
\log m_{t+1} = \text{constant} + \left[ (\rho - 1) \chi_0 + (\alpha - \rho)(\chi_0 + X_1) \right] w_{t+1}
\]
\[
+ (\rho - 1) \sum_{j=0}^{\infty} \chi_{j+1} w_{t-j}
\]

\[
X_1 = \sum_{j=1}^{\infty} \beta^j \chi_j \quad (\text{“Bansal-Yaron” term})
\]
Kreps-Porteus risk sharing

Pareto problem with two (different) recursive agents

Issues

- Time-varying pareto weights
- Representative agent may look different from individuals
- Possible nonstationary consumption distribution
Cyclical properties of US asset returns

Data: cyclical properties of US asset prices and returns

Theory: numerical example [“Bansal-Yaron plus”]
Cyclical properties of US asset prices and returns

Cross correlations for financial indicators and economic growth

- Returns: logs of gross returns
- Excess returns: differences in logs of gross returns

Economic growth

- Monthly: \( \log x_t - \log x_{t-1} \)
- Or year-on-year: \( \log x_{t+6} - \log x_{t-6} \)
- Computed from: industrial production, consumption, employment

US data, monthly, 1960 to present
Equity returns (monthly growth)

Backus, Routledge, & Zin (NYU & CMU)

Exotic + Cyclical Returns
Equity returns (yoy growth)
Equity returns (variations)

Year–on–Year Growth

Real

Monthly

1990 and After

Backus, Routledge, & Zin (NYU & CMU)   Exotic + Cyclical Returns
Term spread (monthly growth)
Term spread (variations)

Data

- Term spread
- Short rate
- Year-on-Year Growth
- 1990 and After

Backus, Routledge, & Zin (NYU & CMU)
Exotic + Cyclical Returns
Think about this for a minute...
Excess returns: equity (yoy)
Excess returns: Fama-French portfolios (yoy)

Backus, Routledge, & Zin (NYU & CMU)
Excess returns: industries (yoy)
Excess returns: bonds (yoy)
Theoretical economy

Take a breath

What do we need?

- Variation in risk and/or price of risk
- ... tied to economic growth

Bansal-Yaron plus

- Representative agent exchange economy
- Recursive preferences (Kreps-Porteus/Epstein-Zin/Weil)
- Loglinear process for consumption growth
- Stochastic volatility
- Interaction between growth and volatility
Theoretical economy

Take a breath

What do we need?

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Bansal-Yaron plus

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Kreps-Porteus preferences

Equations

\[ U_t = \left[ (1 - \beta) c_t^\rho + \beta \mu_t(U_{t+1})^\rho \right]^{1/\rho} \]
\[ \mu_t(U_{t+1}) = \left( E_t U_{t+1}^\alpha \right)^{1/\alpha} \]
\[ \alpha, \rho \leq 1 \]

Interpretation

\[ IES = \frac{1}{1 - \rho} \]
\[ CRRA = 1 - \alpha \]
\[ \alpha = \rho \Rightarrow \text{additive preferences} \]
Kreps-Porteus pricing kernel

Marginal rate of substitution

\[ m_{t+1} = \beta \left( \frac{c_{t+1}}{c_t} \right)^{\rho-1} \left( \frac{U_{t+1}}{\mu_t(U_{t+1})} \right)^{\alpha-\rho} \]

If \( \alpha = \rho \)

- Second term disappears
- No roles for volatility or predictable consumption growth
Consumption growth

Consumption growth follows from

\[ \log g_t = g + e^\top x_t \]

\[ x_{t+1} = Ax_t + a(v_t - \nu) + \nu_t^{1/2} Bw_{t+1} \]

\[ \nu_{t+1} = (1 - \varphi_\nu)\nu + \varphi_\nu \nu_t + bw_{t+1} \]

Note

- \(A\) generates predictable component
- \(\nu_t\) is stochastic
- \(a\) generates interaction
Theoretical excess returns

Transparent loglinear solution

- We love this, but won’t bore you with the details
- Still needs some work

Excess returns depend on

- Volatility ($v_t$)
- Innovations in consumption growth and volatility
- Not expected future consumption growth ($x_t$)!
Excess returns: numerical example
Summary and extensions

Summary

- Data: excess returns correlated with future growth
- Model: ditto via stochastic volatility

Fixups and extensions

- Model dividends explicitly
- Production economies: volatility acts like shock to discount factor, affects consumption and labor supply
Related work (some of it)

Evidence on financial indicators of business cycles

- Ang-Piazzesi-Wei, Estrella-Hardouvelis, King-Watson, Rouwenhorst, Stock-Watson

Kreps-Porteus pricing kernel

- Hansen-Heaton-Li, Weil

Stochastic volatility and returns

- Atkeson-Kehoe, Gallmeyer-Hollifield-Zin, Naik, Primiceri-Schaumburg-Tambalotti
Earnings and dividends (yoy)

Extra slides

Earnings (YÖY)

Dividends (YÖY)
Approximation: two flavors

Problem: find decision rule \( u_t = h(x_t) \) satisfying

\[
E_t F(x_t, u_t, w_{t+1}) = 1, \quad w_t \sim N(0, \kappa_2)
\]

Judd + many others

- Taylor series expansion of \( F \)
- \( n \)th moment shows up in \( n \)th-order term

Us + much of modern finance

- Taylor series expansion of \( f = \log F \) in

\[
E_t \exp[f(x_t, u_t, w_{t+1})] = 1
\]
- All moments show up even in linear approximation
Approximation: example

Linear “perturbation” method

- Linear approximation of $F$

\[ F(x_t, u_t, w_{t+1}) = F + F_x(x_t - x) + F_u(u_t - u) + F_w w_{t+1} \]
\[ E_t F = 1 \Rightarrow u_t - u = (1 - F)/F_u - (F_x/F_u)(x_t - x) \]

- Decision rule doesn’t depend on variance of $w$ (or higher moments)

“Affine” finance method

- Linear approximation of $f = \log F$

\[ f(x_t, u_t, w_{t+1}) = f + f_x(x_t - x) + f_u(u_t - u) + f_w w_{t+1} \]
\[ E_t \exp(f) = 1 \Rightarrow u_t - u = -(f + f_w \kappa_2/2)/f_u - (f_x/f_u)(x_t - x) \]

- Note impact of variance $\nu$ (higher moments would show up, too)