Many business cycle models use costs of adjustment to moderate fluctuations in investment. This typically involves both a resource constraint and a law of motion for capital. The versions without adjustment costs would look something like

\[ y_t = c_t + x_t \]
\[ k_{t+1} = (1 - \delta)k_t + x_t, \]

where \( x \) is (gross) investment. I’ll generally keep (1) and add adjustment costs to (2), but treatments vary.

Examples

There was an explosion of work on this topic in the 1950s and 1960s. Some examples of how this has evolved (using something close to original notation) in an order only I understand:

- Hayashi (Econometrica, 1982). He attributes this law of motion to Uzawa:
  \[ k_{t+1} = (1 - \delta)k_t + \psi(x_t, k_t), \]
  with an “installation function” \( \psi \) governing costs of adjustment. Typically \( \psi \) is \( \text{hd}1 \), so that \( \psi(x, k) = \psi(x/k, 1)k \). The latter’s become the standard in business cycle work.

- Baxter and Crucini (AER, 1993). Their law of motion [their eq (7)] is essentially Hayashi’s:
  \[ k_{t+1} = (1 - \delta)k_t + \phi(x_t/k_t)k_t, \]
  with the installation function \( \phi \) positive, increasing, and concave. They specify (see p 421) “\( \phi \) and \( \phi' \) so that the model with adjustment costs has the same steady state as the model without adjustment costs.” The third parameter sets \( \phi'' \), “which governs the elasticity of the response of \( x/k \) to \( q [1/\phi'] \).”

- Chari, Kehoe, and McGrattan (Econometrica, 2000). Their law of motion [their eq (23)] is
  \[ k_{t+1} = (1 - \delta)k_t + x_t - \phi(x_t/k_t)k_t, \]
  with the “adjustment cost function” \( \phi \) positive, increasing, and convex. This is equivalent to (4) if we group the last two terms together: \( \phi_{BC}(z) = z - \phi_{CKM}(z) \). We can use this to go back and forth between installation and cost of adjustment functions. In practice, they use \( \phi(z) = b(z - \delta)^2/2 \), so adjustment costs are zero at the steady state; see bottom of p 1161.

*Working notes. No guarantee of accuracy or sense.
Fernandez de Cordoba and Kehoe (JIE, 2000). They use an interesting functional form [their eq (52)],
\[ \phi(z) = \left[ \delta^{1-\eta} z^\eta - (1-\eta)\delta \right] / \eta, \]  
with \( \eta \leq 1 \). If \( \eta = 1 \), this reduces to \( \phi(z) = z \), but if \( 0 < \eta < 1 \) you get some curvature (aka adjustment costs). This functional form incorporates two common properties: \( \phi(\delta) = \delta \) (so that \( z = \delta \) is the steady state) and \( \phi'(\delta) = 1 \) (so that adjustment costs don’t affect margins at the steady state), and \( \phi''(\delta) = -(1-\eta) / \delta \).

Jerman (JME, 1998). Same as Baxter-Crucini [eq (2.2) + top of p 273], with
\[ \phi(x_{t}/k_{t}) = \frac{b}{1-a} (x_{t}/k_{t})^{1-a} + c. \]  
a chosen to match elasticity of investment to Tobin’s \( q \); see the discussion on p 264. This is the same functional form as above, expressed a little more mysteriously. If I understand correctly, he finds [p 265] \( a = \xi = 0.23 \).

Uzawa (JPE, 1969). Most of what’s described above comes from earlier work. Uzawa’s is one of the best known, in part for the section on endogenous time preference. He also considers (see section 4) investment. Specifically, the investment \( x \) needed to generate a given increase in the capital stock is:
\[ x_{t}/k_{t} = \varphi(k_{t+1}/k_{t} - 1), \]  
with \( \varphi \) nonnegative, increasing, and convex. You need to invert this to get a law of motion.

Lucas and Prescott (Econometrica, 1971). They use the law of motion
\[ k_{t+1} = k_{t} h(x_{t}/k_{t}), \]  
with \( h \) positive, increasing, and concave. See their eq (2) and the following discussion. If we expressed this in (say) Baxter and Crucini terms, we’d set
\[ h(z) = 1 - \delta + \phi(z). \]  
Putting it this way shows that linear terms in \( \phi \) get soaked up in \( \delta \). It’s also similar to Uzawa. Formally, define \( \varphi^*(z) = \varphi(z - 1) \). Then \( h \) is the inverse of \( \varphi^* \).

Cochrane (JF, 1991). He uses [his eq (9)]
\[ k_{t+1} = g(k_{t}, x_{t}). \]  
In practice, he uses an adjustment cost that is quadratic in the ratio of investment to capital [his eq (14)].

Campanale, Castro, and Clementi (‘‘Chew-Dekel,’’ 2007). They incorporate an adjustment cost into the resource constraint, making it
\[ y_{t} = c_{t} + x_{t} + g(k_{t+1}, k_{t}), \]  
with \( g \) hd1.
Computation

Guess. If you do linear approximation of the decision rules, they’re all the same. We just need the value and two derivatives of $c$ at the steady state.