The last decade has seen a renewed interest in the Phillips curve that might be an odd awakening for a macroeconomic Rip van Winkle from the 1980s or even the 1990s. Wasn’t the Phillips curve tradition discredited by the oil prices shocks of the 1970s or by theoretical critiques of Friedman, Phelps, Lucas, and Sargent? It turns out that the New Keynesian Phillips curve (NKPC) is consistent with both the theoretical demands of modern macroeconomics and some key statistical properties of inflation. In fact, the NKPC can take a sufficient number of guises to accommodate a wide range of perspectives on inflation.

The NKPC originated in descriptions of price setting by firms that possess market power. For example, Rotemberg (1982) describes how a monopolist sets prices if it faces a cost of adjustment that rises with the scale of the price change. He shows that prices then gradually track a target price and also depend on expected, future price targets. Calvo (1983) instead describes firms that are monopolistic competitors. They change their prices periodically. Knowing that some time may pass before they next set prices, firms anticipate future cost and demand conditions, as well as current ones, in setting their price. Also, the staggering or nonsynchronization of price setting by firms
creates an aggregate stickiness: The aggregate price level will react only partially on impact to an economy-wide shock, such as an unexpected change in monetary policy.

These theoretical models link prices to a targeted real variable such as a markup on the costs faced by the price-setting firm. Therefore, they also relate the change in prices over time (i.e., the inflation rate) to real variables. So it is natural to label them as Phillips curves. In fact, there are a range of setups called NKPCs that vary depending on (a) the information and price-setting behavior attributed to firms and (b) the measure of costs or demand that firms are assumed to target. Whether a specific version of the NKPC fits inflation data has implications for our understanding of recent macroeconomic history and for the design of good policy. For example, the parameters of the NKPC influence how monetary policy ideally should respond to external shocks. Schmitt-Grohé and Uribe, in this issue, make this connection clear. (King, also in this issue, describes the uses of an older Phillips curve tradition in policymaking.)

Yet, putting the NKPC to use for policy analysis requires that it has a good econometric track record in describing actual inflation dynamics. In this article we review this record using single-equation statistical methods that study the NKPC on its own. These methods stand in contrast to approaches that place the NKPC in larger economic models, sometimes referred to as systems methods, which are reviewed by Schorfheide in this issue. A disadvantage of single-equation methods is that they do not make use of everything known about the economy (e.g., the monetary policy regime), so they generally do not provide the greatest statistical precision. Their advantage is that they allow us to be agnostic about the rest of the economy, and so their findings remain valid and will not be affected by misspecification of other parts of a larger macroeconomic model.

This article asks the following questions: How can we estimate the NKPC and what do we find when we do so for the United States? Are its parameters stable over time and well-identified? Is there a relation between inflation and real activity? Do we reach similar conclusions about the NKPC regardless of the way in which we measure inflation, forecast future inflation, or model the costs or output gap that inflation tracks?

We focus on marginal cost as the real activity variable in the NKPC. We find that the single-equation statistical evidence for this relationship is mixed. Since 1955 there does seem to be a stable NKPC for the United States with positive parameter values as we would expect from economic theory. But our confidence intervals for these parameter values are somewhat wide, the findings depend on how we model expected future inflation, and further research is needed on the best way to represent the marginal cost variable to which price changes react. Before outlining the methods and findings,
though, we begin by introducing the specific NKPC that we will estimate and the inflation history it aims to explain.

1. FOUNDATIONS AND INTERPRETATIONS

The New Keynesian Phillips curve arises from a description of staggered price setting, which is then linearized for ease of study. The result is an equation in which the inflation rate, $\pi_t$, depends on the expected inflation rate next period, $E_t \pi_{t+1}$, and a measure of marginal costs, denoted $x_t$:

$$\pi_t = \gamma f E_t \pi_{t+1} + \lambda x_t.$$ 

Iterating this NKPC difference equation forward gives inflation as the present value of future marginal costs:

$$\pi_t = \lambda \sum_{i=0}^{\infty} \gamma f E_t x_{t+i}.$$ 

This present-value relation shows that firms consider both their current marginal costs, $x_t$, and their expectations or forecasts of future costs when adjusting prices.

Lacker and Weinberg (2007) describe the history and derivation of this key building block of New Keynesian macroeconomic models. Dennis (2007) outlines a range of environments that can underpin a hybrid NKPC. Calvo’s (1983) specific price-setting model is only one of several possible microfoundations for the NKPC. In a Calvo-based NKPC, a fraction, $\theta$, of firms cannot change prices in a given period. Firms also have a discount factor, $\beta$. The reduced-form parameters of the NKPC, $\gamma_f$ and $\lambda$, are related to these two underlying pricing parameters according to

$$\gamma f = \beta, \quad \lambda = \frac{(1 - \theta)(1 - \beta \theta)}{\theta}.$$ 

Because $\beta$ is a discount factor, both it and $\gamma_f$ must range between zero and one. The same holds for $\theta$ because it represents the fraction of firms unable to move prices at any moment.

Many estimates of the NKPC find that lagged inflation helps to explain current inflation. We report much the same in this article. This has suggested to some economists that a better fit to inflation history can be obtained with this equation:

$$\pi_t = \gamma_b \pi_{t-1} + \gamma f E_t \pi_{t+1} + \lambda x_t,$$

which Gál tí and Gertler (1999) call the hybrid NKPC. They develop their NKPC by modifying Calvo’s (1983) description of price-setting decisions. In this case, a fraction, $\omega$, of firms can change prices, but do not choose this option. Define $\phi = \theta + \omega(1 - \theta(1 - \beta))$. Then the mapping between these
structural parameters and the reduced-form parameters is
\[
\gamma_b = \frac{\omega}{\phi}, \quad \gamma_f = \frac{\beta \theta}{\phi}, \quad \lambda = \frac{(1 - \omega)(1 - \theta)(1 - \beta \theta)}{\phi}.
\]
Gálik and Gertler (1999) note that this mapping between the structural, price-setting parameters $\omega, \theta$, and $\beta$ and the reduced-form hybrid NKPC parameters $\gamma_b, \gamma_f$, and $\lambda$ is unique if the former set of parameters lie between zero and one.

This form of the NKPC also is consistent with the incomplete indexing model of Woodford (2003). He assumes that those firms that cannot optimally alter their prices instead index to a fraction of the lagged inflation rate. This feature makes current inflation depend on lagged inflation, and so provides an alternative interpretation of the hybrid NKPC. Christiano, Eichenbaum, and Evans (2005), among others, study the implications of full indexation, which is equivalent to the restriction $\gamma_b + \gamma_f = 1$ in the Gálik-Gertler hybrid NKPC.

Like the original NKPC, the hybrid version can be rewritten in present-value form:
\[
\pi_t = \delta_1 \pi_{t-1} + \left( \frac{\lambda}{\delta_2 \gamma_f} \right) \sum_{k=0}^{\infty} \left( \frac{1}{\delta_2} \right)^k E_t \pi_{t+k},
\]
where $\delta_1$ and $\delta_2$ are stable and unstable roots, respectively, of the characteristic equation in the lag operator $L$:
\[
-L^{-1} + \frac{1}{\gamma_f \gamma_f} = 0.
\]
The present-value version of the hybrid NKPC shows that inflation persistence can arise from the influence of lagged inflation or the slow evolution of the present value of marginal costs.

The NKPC is often derived by log-linearizing a typical firm’s price-setting rule around a mean zero inflation rate. Ascarì (2004) shows that non-zero mean inflation can affect the response of inflation to current and future marginal cost in the NKPC. However, we follow much of the empirical NKPC literature and demean the data.

Cogley and Sbordone (forthcoming) build on Ascarì’s work, among others, by log-linearizing the NKPC around time-varying trend inflation. This procedure assumes that inflation is nonstationary to obtain a NKPC with time-varying coefficients even though the underlying Calvo-pricing parameters are constant. The resulting NKPC is purely forward-looking, assigning no role to lagged inflation. Cogley and Sbordone estimate the structural coefficients of their NKPC using a vector autoregression. Hornstein (2007) assesses the implications of this approach for the stability of the NKPC. Since these studies use system estimators, we omit them from our review.

In sum, the hybrid NKPC is consistent with various pricing or information schemes. This suggests a focus on the reduced-form coefficients $\lambda$, $\gamma_b$, and
γ_f (rather than the structural price-setting parameters ω, θ, and β), which is our emphasis in this article. We also use single-equation estimators to explore the fit of the hybrid NKPC to U.S. data. After all, obtaining a good fit for the hybrid NKPC is a necessary first step in attributing a monetary transmission mechanism to staggered price setting by firms.

Economists have not yet reached a consensus on two key questions concerning the NKPC parameters. First, what is the mixture of forward (γ_f) and backward (γ_b) weights? If γ_f is large, events in the future (including changes in monetary policy) can influence the current inflation rate. If, instead, γ_b is large, inflation has considerable inertia independent of any slow movements in the cost variable. Such inertia affects the design of monetary policy (again, see Uribe and Schmitt-Grohé). Woodford (2007) reviews several explanations for inflation inertia and also discusses whether it could be stable over time.

Second, can we identify a significant ˆλ, the coefficient on marginal costs? In this case, identification simply refers to measuring a partial correlation coefficient in historical data rather than the possibility of misspecification (i.e., whether this coefficient necessarily measures the theoretical parameter studied in New Keynesian models.) Finding a significant value is a sine qua non for empirical work with the NKPC. If we cannot find a way to represent a price-setting target, we cannot hope to identify the adjustment process of inflation to its target. Consequently, much of the research on estimating the NKPC involves exploring the x-variable or how to measure marginal costs.

Before looking at formal econometric methods, let us look at the data. Figure 1 plots the U.S. inflation rate (the black line) and a measure of marginal cost (the gray line) from 1955:1 to 2007:4. We measure the inflation rate, \( \pi_t \), as the quarter-to-quarter, annualized growth rate in the U.S. implicit GDP deflator (GDPDEF from FRED at the Federal Reserve Bank of St. Louis). Marginal cost, \( x_t \), is the update of the series on real unit labor costs used by Galí and Gertler (1999) and Sbordone (2002). It is given by 1.0765 times the logarithm of nominal unit labor costs in the nonfarm business sector (the ratio of COMPFB to OPHNFB from FRED) divided by the implicit GDP deflator. Both series have fallen since 1980, which in itself provides some statistical support for the idea that inflation tracks marginal cost. There also are some obvious divergences, for example around 2000–2001. However, it is possible that these occurred because inflation was tracking expected future marginal cost (as in the present-value model) or because it was linked to lagged inflation (as in the hybrid NKPC). We next describe the statistical tools economists have used to see if either of these explanations fits the facts.

2. ESTIMATION

The fundamental challenge with estimating the parameters of the hybrid NKPC is that expected inflation cannot be directly observed. The most popular
econometric method for dealing with this issue begins from the properties of forecasts. To see how this works, let us label the information used by price-setters to forecast inflation by \( I_t \), so their forecast is \( E[\pi_{t+1} | I_t] \). Economists do not observe this forecast, but it enters the NKPC and influences the current inflation rate. Denote by \( E[\pi_{t+1} | z_t] \) an econometric forecast that uses some variables, \( z_t \), to predict next period’s inflation rate, \( \pi_{t+1} \). Suppose that \( z_t \) is a subset of the information available to price setters. To construct our econometric forecast, we simply regress actual inflation on our set of variables (sometimes called instruments), \( z_t \), like this:

\[
\pi_{t+1} = b z_t + \epsilon_{t+1},
\]

so that our forecast is simply the fitted value

\[
E[\pi_{t+1} | z_t] = \hat{b} z_t.
\]

By construction it is uncorrelated with the residual term \( \hat{\epsilon}_{t+1} \).

A key principle of forecasts (or rational expectations) is the law of iterated expectations. According to that law, our econometric prediction of price-setters’ forecast of inflation is simply our forecast. Symbolically,

\[
E \left[ E[\pi_{t+1} | I_t] | z_t \right] = E[\pi_{t+1} | z_t].
\]

The idea is that our effort to predict what someone with better information will forecast simply gives us our own best forecast. With the law of iterated expectations in hand, we can also imagine regressing the unknown forecast on \( z_t \):
\[ E[\pi_{t+1} \mid I_t] = E[\pi_{t+1} \mid I_t, z_t] + \eta_t, \]
\[ = E[\pi_{t+1} \mid z_t] + \eta_t, \]
in which the residual, \( \eta_t \), also is uncorrelated with the econometric forecast. The econometric forecast does not use all the information available to price setters when they construct forecasts, so it does not capture all the variation in their forecasts. Put differently, the unobserved, economic forecast has some added variation that appears in \( \eta_t \).

With this statistical reasoning behind us, the hybrid NKPC can be rewritten:

\[ \pi_t = \gamma_b \pi_{t-1} + \gamma_f E[\pi_{t+1} \mid I_t] + \lambda x_t \]
\[ = \gamma_b \pi_{t-1} + \gamma_f (E[\pi_{t+1} \mid z_t] + \eta_t) + \lambda x_t \]
\[ = \gamma_b \pi_{t-1} + \gamma_f E[\pi_{t+1} \mid z_t] + \lambda x_t + \gamma_f \eta_t. \]

This is an econometric equation that can be used to estimate the parameters by least squares, for we now have measurements of the three variables on the right-hand side of the equation. In fact, this two-step procedure—forecast using predictors \( z_t \), then substitute and apply least squares—is just two-stage least squares, familiar from econometrics textbooks. Provided that we include the other hybrid NKPC explanatory variables, \( \pi_{t-1} \) and \( x_t \), in the list of first-stage regressors, the error term will be uncorrelated with them, too, and so least-squares will be valid in the second stage. (In contrast, simply estimating the NKPC by least squares, using \( \pi_{t+1} \) in place of \( E[\pi_{t+1}] \), yields inconsistent estimates of the parameters, because \( \pi_{t+1} \) is correlated with the residual, \( \eta_t \).)

Two-stage least squares, in turn, is a special case of a method known as generalized instrumental variables, or generalized method-of-moments (GMM) estimation. To see how this works, take the hybrid NKPC and write it as follows:

\[ \pi_t = \gamma_b \pi_{t-1} - \gamma_f E[\pi_{t+1} \mid I_t] - \lambda x_t = 0. \]

Then imagine forecasting this entire combination of variables:

\[ E[\pi_t - \gamma_b \pi_{t-1} - \gamma_f E[\pi_{t+1} \mid I_t] - \lambda x_t | z_t] = 0, \]
\[ E[\pi_t - \gamma_b \pi_{t-1} - \gamma_f \pi_{t+1} - \lambda x_t | z_t] = 0, \]

where we again have used the law of iterated expectations to replace the unobserved market forecast with our own econometric one. The last part of this equation is the basis for numerous studies of the NKPC. Simply put, there should be no predictable departures from the inflation dynamics implied by the hybrid NKPC or, equivalently, the residuals should have a mean of zero and be uncorrelated with predictor variables \( z_t \). These properties allow for a diagnostic test of the NKPC.
Finding estimates of the hybrid NKPC parameters proceeds as follows. We collect data on inflation and marginal costs. Then we make a list of widely available information, $z_t$, that could include current and lagged values of these same variables, as well as other macroeconomic indicators such as interest rates. These instrumental variables (also known simply as instruments), $z_t$, must have two key properties. First, they must be at least as numerous as the parameters of the model (which here number three) and provide independent sources of variation (i.e., they cannot be perfectly correlated with one another). Intuitively, to measure three effects on inflation there must be at least three independent pieces of information or exogenous variables. Second, they must be uncorrelated with forecast errors that appear as residuals in the hybrid NKPC. Instruments with these properties are called valid.

Next, we use some econometric software to adjust the economic parameters $\{\gamma_b, \gamma_f, \lambda\}$ so that departures from the hybrid NKPC are uncorrelated with $z_t$, and so are unpredictable. The criterion guiding the adjustment is that the moment conditions that consist of the cross-products of the NKPC residuals with the instruments should be as close to zero as possible. Whenever we have at least three valid instruments in the set $z_t$, we can identify and solve for values of the three hybrid NKPC parameters using this criterion. In practice, the algorithm attempts this task by squaring the deviations of the moment conditions from zero and then minimizing the weighted square of this list of deviations. Cochrane (2001, chapters 10–11) provides a lucid introduction to GMM.

We make brief technical digressions on two details of GMM estimation. First, the distance of moment conditions (i.e., forecasts of departures from the hybrid NKPC) from zero is measured relative to their sampling variability, just as with any statistic. For GMM this involves calculating a heteroskedasticity-and-autocorrelation-consistent (HAC) covariance matrix. This article employs either a Newey and West (1994) or an Andrews (1991) quadratic-spectral HAC estimator with automatic lag-length selection. Second, some authors note that the way the hybrid NKPC is written matters for its estimation. For example, multiplying it by the Calvo parameter, $\phi$, might seem to make it easier to estimate $\phi$ as the weight on $\pi_t$ and $\omega$ as the weight on $\pi_{t-1}$. In this article, we use the continuously updated (CU-)GMM estimator of Hansen, Heaton, and Yaron (1996), from which estimates of the hybrid NKPC are independent of any normalization applied to it.

There are many macroeconomic indicators that could be included in $z_t$. We need to place at least three macroeconomic variables in $z_t$ so that the three parameters of the hybrid NKPC are just-identified, in the jargon of econometrics. The parameters are said to be overidentified when four or more macroeconomic variables are included in $z_t$. It turns out that this possibility provides a test of the validity of the NKPC. According to econometric theory, any instrument set should yield the same coefficients except for some random
sampling error. So by estimating with various sets of instruments and comparing the findings (or seeing if the NKPC departures are close to zero even when we use a long list of instruments) we can test whether the NKPC really holds or not. This diagnostic procedure is called a test of overidentifying restrictions. Informally, we refer to an NKPC that passes this test as fitting the data.

In practice, most researchers have used lagged macroeconomic variables as instruments. To see why, recall that an error term, $\gamma_f \eta_t$, arises in the estimating equation. Recall, however, that by including $\text{E}[\pi_{t+1}|z_t]$ we in fact are trying to represent the regressor $\text{E}[\pi_{t+1}|I_t]$ (a forecast of inflation made in the current period), not $\pi_{t+1}$. So one can think of the econometric equation as containing an error term dated $t$ that reflects the difference between these two measures. Moreover, some economists have argued that there are unobserved cost shocks (components of $x_t$) that also can underlie an error term in the NKPC. Recall that a key property of instruments is that they be uncorrelated with the error term. Many researchers studying the NKPC, therefore, have used only lagged variables as instruments, labelled $z_{t-1}$, to try to ensure that this property holds.

Another way to think of this approach is that using instrumental variables is a classic way of dealing with the problem of a regressor that is subject to measurement error. If the marginal cost series, $x_t$, is measured with error, then including $x_t$ as an instrument will lead to the attenuation bias (bias toward zero) familiar in this errors-in-variables problem. Using lagged instruments can avoid this bias.

Next, we present examples of GMM estimation. Instruments include lagged values of inflation and marginal costs. In addition, we also present results with a longer list of instruments. This list includes the term spread of the five-year Treasury bond over the 90-day Treasury bill, $ts$, which is a natural candidate for forecasting inflation. Table 1 gives the complete list of variables and instruments, with the symbols used to represent them and the sources for these data. Different econometricians might measure the output gap, $y_t$, differently. We used linear detrending of real per capita GDP to produce the output gap as an instrument, but the findings are very similar if we use other possible measures of the output gap as an instrument instead. The list of instruments is inspired by Galí and Gertler (1999). They use four lags of this list of six variables for their quarterly 1960–1997 sample. We adopt the same list, though updated to 2007, so that the reader can compare our findings to theirs.

Table 2 reports CU-GMM estimates of the hybrid NKPC on the 1955:1–2007:4 sample. The first column lists the instrument set, $z_t$. The next columns list estimates of the reduced-form parameters $\hat{\gamma}_b$, $\hat{\gamma}_f$, and $\hat{\lambda}$ over their standard errors, followed by the structural Calvo-pricing estimates $\hat{\omega}$, $\hat{\theta}$, and $\hat{\beta}$ over their standard errors. The final column gives a test statistic, denoted $J$, for the
Table 1 Measuring Variables and Instruments

<table>
<thead>
<tr>
<th>Label</th>
<th>Definition</th>
<th>Code/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>π</td>
<td>Inflation rate, implicit GDP deflator</td>
<td>GDPDEF/FRED</td>
</tr>
<tr>
<td>x</td>
<td>Log labor share of income</td>
<td>1.0765 ln[COMPNFB/OPHNFB] – ln[GDPDEF]/FRED</td>
</tr>
<tr>
<td>y</td>
<td>Linearly detrended log per capita real GDP</td>
<td>ln[GDP96/CNP160V]/FRED</td>
</tr>
<tr>
<td>ts</td>
<td>Five-year Treasury constant-maturity interest rate minus 90-day Treasury bill rate, quarterly average</td>
<td>GS5 – TB3MS/FRED</td>
</tr>
<tr>
<td>wi</td>
<td>Wage inflation</td>
<td>ln[COMPNFB × HOANBS]/FRED</td>
</tr>
<tr>
<td>cp</td>
<td>PPI commodity price inflation</td>
<td>BLS</td>
</tr>
</tbody>
</table>

The hypothesis that the overidentifying restrictions hold. The hypothesis implies the same parameters apply for any instrument set. We include the $J$ statistic, along with its degrees of freedom ($df$), over its $p$-value. The $J$ statistic is asymptotically distributed $\chi^2$ with $df$ equal to the number of overidentifying restrictions (the number of instruments minus the number of parameters). Fixing the number of overidentifying restrictions, a larger $J$ statistic yields a smaller $p$-value, which indicates that the residual is predictable and constitutes a rejection of the hybrid NKPC.

The top row of Table 2 presents CU-GMM estimates based on four instruments. With three hybrid NKPC parameters to estimate, this gives one overidentifying restriction. Besides lagged inflation, the set of instruments in the first row contains only lags of marginal cost, $x_t$. Between the first and last rows of Table 2, we add instruments one by one from the longer Galí and Gertler (1999) list. The penultimate row of Table 2 includes the entire set of 24 instruments used by Galí and Gertler. (The last row presents least-squares estimates, discussed in Section 4.)

Table 2 shows that the hybrid NKPC estimates vary with the set of instruments. We return to that finding in Section 4. Meanwhile, once we include lags of marginal cost and of the output gap, $y$, we find that $\gamma_b$ and $\gamma_f$ are significant, positive fractions, with the weight on expected future inflation much greater than the weight on lagged inflation (see the second and sixth rows). Estimates based on these instrument sets indicate that there is little inflation inertia. These estimates also reveal a significant, positive impact of real unit labor costs, measured by $\hat{\lambda}$, but the scale of the inflation response is small. Sbordone (2002) and Eichenbaum and Fisher (2007) show that if firms have firm-specific capital it can lead to a low response to a cost shock, i.e., a small value for $\lambda$. 
Table 2 U.S. New Keynesian Phillips Curve, 1955:1–2007:4

\[
E \left[ \pi_t - \gamma_b \pi_{t-1} - \gamma_f \pi_{t+1} - \lambda x_t \mid z_t \right] = 0
\]

<table>
<thead>
<tr>
<th>Instruments ((z_t))</th>
<th>(\hat{\gamma}_b) (\text{(se)})</th>
<th>(\hat{\gamma}_f) (\text{(se)})</th>
<th>(100 \times \hat{\lambda}) (\text{(se)})</th>
<th>(\hat{\omega}) (\text{(se)})</th>
<th>(\hat{\theta}) (\text{(se)})</th>
<th>(\hat{\beta}) (\text{(se)})</th>
<th>(J(df)) (\text{(p)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi_{t-1}, x_{t-1}, x_{t-2}, x_{t-3})</td>
<td>-0.11 (0.50)</td>
<td>1.14 (0.58)</td>
<td>1.41 (1.00)</td>
<td>-0.08 (0.36)</td>
<td>0.88 (0.03)</td>
<td>1.03 (0.06)</td>
<td>0.30 (1)</td>
</tr>
<tr>
<td>(\pi_{t-1}, x_{t-1}, x_{t-2}, x_{t-3}, y_{t-1}, y_{t-2})</td>
<td>0.29 (0.10)</td>
<td>0.68 (0.10)</td>
<td>0.67 (0.35)</td>
<td>0.36 (0.17)</td>
<td>0.91 (0.03)</td>
<td>0.94 (0.05)</td>
<td>3.42 (3)</td>
</tr>
<tr>
<td>(\pi_{t-1}, x_{t-1}, x_{t-2}, x_{t-3}, \tau_{t-1}, \tau_{t-2})</td>
<td>-0.20 (0.38)</td>
<td>1.25 (0.44)</td>
<td>0.17 (0.82)</td>
<td>-0.15 (0.23)</td>
<td>0.89 (0.02)</td>
<td>1.04 (0.04)</td>
<td>1.05 (3)</td>
</tr>
<tr>
<td>(\pi_{t-1}, x_{t-1}, x_{t-2}, x_{t-3}, w_{t-1}, w_{t-2})</td>
<td>0.17 (0.18)</td>
<td>0.81 (0.20)</td>
<td>0.89 (0.45)</td>
<td>0.18 (0.24)</td>
<td>0.90 (0.02)</td>
<td>0.97 (0.06)</td>
<td>2.26 (3)</td>
</tr>
<tr>
<td>(\pi_{t-1}, x_{t-1}, x_{t-2}, x_{t-3}, c_{t-1}, c_{t-2})</td>
<td>0.02 (0.17)</td>
<td>1.00 (0.19)</td>
<td>1.07 (0.53)</td>
<td>0.02 (0.16)</td>
<td>0.88 (0.03)</td>
<td>1.02 (0.05)</td>
<td>1.45 (3)</td>
</tr>
<tr>
<td>(\pi, x, y, ts, wi, cp_{t-1}, \ldots) (\pi, x, y, ts, wi, cp_{t-4})</td>
<td>0.25 (0.05)</td>
<td>0.73 (0.05)</td>
<td>0.63 (0.31)</td>
<td>0.29 (0.08)</td>
<td>0.92 (0.02)</td>
<td>0.95 (0.03)</td>
<td>17.08 (21)</td>
</tr>
</tbody>
</table>
| \(\pi_{t-1}, \pi_{t+1}, x_t\) \(\text{(NLLS)}\) | 0.25 (0.01) | 0.75 (0.03) | 2.65 (1.33) | 0.27 (0.01) | 0.81 (0.06) | 0.99 (0.06) | \n
Notes: Data are de-meaned prior to estimation. The estimator is CU-GMM, except the last case is NLLS, and all use a Newey-West HAC correction and automatic plug-in lag length.
The second and sixth rows of Table 2 also contain estimates of the structural Calvo-pricing parameters that are positive fractions, significant, and in keeping with the theory. These estimates yield a discount factor of around 0.95 and indicate that about 90 percent of firms are unable to change prices in a given quarter. Of the 10 percent that can change prices, about a third decide against it.

Additional support for the NKPC is provided by the $J$-test statistics in the final column of Table 2. Across all instrument sets, we obtain a large $p$-value associated with this statistic, so that the overidentifying restrictions cannot be rejected.

3. STABILITY

So far, the updated empirical evidence supports the hybrid NKPC. However, as mentioned in the introduction, it also is natural to ask whether the hybrid NKPC parameters are stable over time. To test their stability, we divide the entire sample at a given quarter, called a break date, into two separate subsamples. We consider all possible break dates between 1963:1 and 1999:4, which trims 15 percent from the beginning and end of the entire sample. For each date in this range, we estimate the NKPC twice: first from 1955:1 to the break date quarter and second from one quarter beyond the break date to 2007:4. All CU-GMM estimates employ the instrument vector $\{\pi_{t-1}, x_{t-1}, x_{t-2}, y_{t-1}, y_{t-2}, w_{t-1}, w_{t-2}\}$, which mimics that of Galí, Gertler, and López-Salido (2005) minus $\{\pi_{t-2}, \pi_{t-3}, \pi_{t-4}\}$.

Figure 2 presents the results. The three panels plot estimates of the reduced-form parameters $\gamma_b$, $\gamma_f$, and $\lambda$, respectively, for break dates from 1963:1 to 1999:4. In each panel, the black line graphs estimates from the sample beginning in 1955:1 and ending at the break date shown, while the gray line graphs estimates from a sample beginning at the break date plus one quarter to 2007:4. For any date, the vertical distance between the two lines gives the difference between estimates from the “before” and “after” samples. The figure shows variation in the estimates that is limited for break dates since 1980, but noticeable for earlier break dates, particularly for $\hat{\gamma}_b$ and for $\hat{\lambda}$. The coefficient $\hat{\gamma}_b$ ranges from 0.29 to 0.40 estimating on the before sample and from 0.27 to 0.35 estimating on the after sample. For $\hat{\gamma}_f$, the corresponding ranges are from 0.64 to 0.73 and from 0.60 to 0.69, respectively. A glance at the vertical axes shows that these estimates confirm the earlier finding that the coefficient on expected future inflation, $\hat{\gamma}_f$, exceeds the coefficient on lagged inflation, $\hat{\gamma}_b$. Estimates of $100 \times \hat{\lambda}$ range from $-1.31$ to 1.03 on the before sample and from 0.08 to 0.87 on the after sample. However, ignoring estimates from the 1960s break dates constrains $100 \times \hat{\lambda}$ to range from about zero to around one.
Next, we test one-by-one whether any of the three parameters \( \{ \gamma_b, \gamma_f, \lambda \} \) changes significantly from the first time period to the second time period. The method developed by Andrews (1993, 2003) allows a statistical test along these lines without pre-supposing knowledge of the exact date at which a break or shift in a parameter value took place. Following this method, we calculate the Wald test for the hypothesis that there is a significant difference between the “before” and “after” estimates for \( \gamma_b, \gamma_f, \) and \( \lambda \) over 1963–1999. We record the maximal value for each of these Wald statistics. Andrews (2003) gives critical values for this test statistic, while Hansen (1997) provides a method for computing \( p \)-values. For \( \gamma_b \), the test statistic is 1.50 with a \( p \)-value of
0.91; for \( \gamma_f \), the test statistic is 1.41 with a \( p \)-value of 0.92; and for \( \lambda \), the test statistic is 3.30 with a \( p \)-value of 0.50. Since these \( p \)-values are far above conventional levels of statistical significance such as 0.05 or 0.10, the tests fail to reject the hypothesis that the parameters are stable.

In summary, our tests suggest that the reduced-form hybrid NKPC parameters are stable. This result is striking because the behavior of inflation has changed over time. For example, inflation was on average lower and less volatile after the mid-1980s. Yet despite this change in the statistical properties of inflation, the link between inflation and marginal cost has remained stable. The stability of this relationship is striking because it suggests a flat hybrid NKPC—with the same relatively low slope \( \hat{\lambda} \)—during the business cycles of the 1970s and the Great Moderation and disinflation that took hold in the mid-1980s. Whatever the sources of this Great Moderation in inflation, the single-equation stability tests suggest that they acted through real activity as measured by marginal costs.

4. WEAK IDENTIFICATION?

Section 2 referred to the need to find valid instruments in order to use single-equation methods. Instruments must satisfy two statistical criteria. First, they must be as numerous as the parameters and must help predict or forecast \( \pi_{t+1} \) so that a projection based on them can be reasonably substituted for the unobserved forecast on the right-hand side of the Phillips curve. Second, they must be uncorrelated with the error term in the econometric equation, just like any regressor.

Unfortunately, these two criteria sometimes can conflict. To see how this can come about, recall that researchers often have used lagged instruments, \( z_{t-1} \). The rationale for this choice is that these past outcomes must be exogenous and, therefore, uncorrelated with unobserved shocks to today’s inflation rate, thus satisfying the second criterion. But now satisfying the first criterion can be challenging. The researcher needs to find at least one variable, \( z_{t-1} \), that helps forecast \( \pi_{t+1} \). Also, the list of instruments must include something other than the other two variables that enter the hybrid NKPC, \( x_t \) and \( \pi_{t-1} \). That is because the constructed forecast \( E\pi_{t+1}|z_{t-1} \) has to exhibit some variation independent of \( x_t \) and \( \pi_{t-1} \). Otherwise, there will be no possibility to measure or identify separately the effects of \( \pi_{t-1}, x_t, \) and \( E\pi_{t+1} \) on current inflation. We want to identify these three effects on current inflation so, logically, we need an inflation forecast that sometimes varies separately from \( \pi_{t-1} \) and \( x_t \).

Seen in this way, the problem of finding instruments is recast as the problem of trying to forecast inflation but with a twist. The statistical challenge is to predict next quarter’s inflation rate, \( \pi_{t+1} \), but without using this quarter’s inflation rate, \( \pi_t \) (because it is the variable we seek to explain on the left-hand side of the hybrid NKPC), or last quarter’s inflation rate, \( \pi_{t-1} \), or this
quarter’s costs or aggregate demand, $x_t$ (because they appear separately on the right-hand side of the hybrid NKPC). Forecasting inflation is difficult, even without one hand tied behind one’s back in this way. The statistical studies by Stock and Watson (1999, 2007) and Ang, Bekaert, and Wei (2007) show that it is challenging to find a stable relationship that can be used to forecast U.S. inflation, especially over the past 15–20 years. Perhaps competent central bankers can take some credit for creating a low, stable inflation rate that has not displayed persistent swings or cycles, but that outcome inherently makes it difficult to isolate an inflation forecast that differs from current or lagged inflation.

The hybrid NKPC provides another perspective on how to forecast $\pi_{t+1}$. We lead the present-value version of the hybrid NKPC forward by one time period and forecast to obtain

$$E_t\pi_{t+1} = \delta_1\pi_t + \left(\frac{\lambda}{\delta_2\gamma_f}\right) \sum_{k=0}^{\infty} \left(\frac{1}{\delta_2}\right)^k E_t x_{t+1+k}.$$ 

Next, suppose that $x_t$ can be forecasted only from its own, lagged value. Suppose that marginal costs follow a first-order autoregression, with coefficient $\rho$, so that its multistep forecast is

$$E_t x_{t+1+k} = \rho^{1+k} x_t.$$ 

Combining the last two equations gives the forecasting equation

$$E_t\pi_{t+1} = \delta_1\pi_t + \frac{\lambda}{\gamma_f(\delta_2 - \rho)} x_t.$$ 

In this case, the three reduced-form parameters cannot be identified by GMM because there is no source of variation in $E_t\pi_{t+1}$ other than $\pi_t$ and $x_t$ (which already are included in the hybrid NKPC). Nason and Smith (2008) show that if $x_t$ is an autonomous $p$th-order autoregression, $p$ must equal 2 to just identify and be greater than 2 to overidentify the three hybrid NKPC parameters using GMM. In other words, higher-order dynamics are needed for identification if $x_t$ is predicted from its own past. That is why the first row of Table 2 includes several lags of marginal cost.

Nason and Smith (2008) also show that setting the NKPC in a broader, New Keynesian model does not suggest sources of identification for single-equation estimation. They show analytically the problem facing an econometrician who tries to estimate the NKPC by GMM in a textbook world where the hybrid NKPC combines with a dynamic IS curve and a Taylor rule. It turns out that there may be no valid instruments available. The logic is that the econometrician must lag instruments to make sure that they are uncorrelated with the residual in the NKPC equation. But lagging them enough to satisfy that criterion for instrumental variables also makes them irrelevant for forecasting $\pi_{t+1}$. 
We dwell on the challenges of forecasting inflation because of another statistical issue. For instrumental variables (GMM) estimation to be informative, it turns out that we need a significant amount of predictability. Imagine reconstructing a forecast equation by regressing $\pi_{t+1}$ on $z_{t-1}$. The $F$-statistic for the joint significance of the variables $z_{t-1}$ in this regression must be above some threshold in order for the full GMM estimation of the hybrid NKPC to yield meaningful results. If this $F$-statistic, or inflation predictability, is too low, then the econometrician is said to be using weak instruments. In that case, the subsequent estimates of the hybrid NKPC parameters will be imprecisely estimated (possess large standard errors). Also, hypothesis tests may have the wrong size (probability of type I error); for example, they may not reject often enough. These problems will persist even in large samples.

Another symptom of the syndrome of weak identification is that estimates may vary a great deal with changes to the instrument set. Two economists with the same hybrid NKPC may obtain disparate parameter estimates when they employ different, but apparently equally admissible, instrument sets, $z_{t-1}$. In Table 2, this sensitivity is apparent in the GMM estimates of the reduced-form and structural hybrid NKPC parameters that are grounded on different combinations of the Galí-Gertler instruments. Since different researchers have tended to apply different instrument sets, weak identification might help to explain the current lack of consensus on parameter estimates of the hybrid NKPC.

Recall from Section 2 that least-squares estimation of the NKPC yields inconsistent estimates; we cannot represent $E_t \pi_{t+1}$ simply by replacing it with the actual value, $\pi_{t+1}$. Another useful result from research on weak instruments is that instrumental-variables estimates converge to least-squares estimates as the econometrician adds more and more weak instruments. The last row of Table 2 shows what can happen in that case by reporting least-squares estimates. With the exception of the larger value for $\hat{\lambda}$, the least-squares estimates are similar to those in some previous rows. That similarity shows that finding these plausible values for the coefficients does not necessarily imply they have a sound statistical basis. And it raises the possibility that the GMM estimates are only weakly identified.

Ma (2002), Mavroeidis (2005), Dufour, Khalaf, and Kichian (2006), and Nason and Smith (2008) draw attention to the pitfalls of weak identification in GMM estimation of the hybrid NKPC. One response to this issue has been to reformulate the hybrid NKPC so that it involves fewer parameters. The idea is simply that by trying to measure a shorter list of effects, the investigator might have greater success in precisely measuring them. For example, one could set $\gamma_b = 0$ and so work with the original NKPC rather than the hybrid version. In that case, $\pi_{t-1}$ also would become available as an instrument.

A number of investigators—including Henry and Pagan (2004) and Rudd and Whelan (2006)—suggest restricting the reduced-form, hybrid NKPC
parameters so that \( \gamma_b + \gamma_f = 1 \). Imposing this restriction helps with identification by reducing the number of coefficients to be estimated by one. It turns out, though, that this restriction is inconsistent with one popular interpretation of the hybrid NKPC parameters, namely that they reflect an underlying Calvo-type model of staggered pricing. To show this, we use the earlier equations that Galí and Gertler (1999) outline to connect the hybrid NKPC parameters to those of the Calvo pricing model, namely \( \omega, \theta, \) and \( \beta \). The proposed restriction gives

\[
\gamma_b + \gamma_f = \frac{\omega}{\phi} + \frac{\beta \theta}{\phi} = 1,
\]

where \( \phi = \theta + \omega(1 - \theta(1 - \beta)) \). Some algebra reveals that this restriction implies that the fraction of firms that can change prices but choose not to is \( \omega = 1 \). However, this extreme result forces the reduced-form parameter on marginal costs,

\[
\lambda = \frac{(1 - \omega)(1 - \theta)(1 - \beta \theta)}{\phi},
\]

to equal zero. Although Galí and Gertler point out that \( \beta = 1 \) also is consistent with \( \gamma_b + \gamma_f = 1 \), often this restriction is imposed without recourse to calibrating the firm’s discount factor to one.

More generally, restricting the hybrid NKPC parameters can be problematic because we want to test hypotheses about all relevant values. We next explore statistical methods that apply even if identification is weak. An econometrician also can test the hybrid NKPC parameters (and compute their confidence intervals) using methods that are robust to weak identification, i.e., that remain valid whether the instruments are weak or not.

Many of these robust methods are based on a 60-year-old statistical insight from Anderson and Rubin (1949). Here is their idea, as applied to the hybrid NKPC. Rewrite the equation by taking the future value of inflation to the left-hand side (without forecasting it) and by adding some list of other variables, \( u_t \), on the right-hand side:

\[
\pi_t - \gamma_f \pi_{t+1} = \gamma_b \pi_{t-1} + \lambda x_t + \delta u_t.
\]

To create this composite variable on the left-hand side of the equation, we need to choose a value for \( \gamma_f \), labelled \( \gamma_{f0} \). We cannot use this regression to estimate that value. But it can be used to test any value for this weight on expected future inflation. To test the hypothesis that \( \gamma_f = \gamma_{f0} \), we simply perform a traditional \( F \)-test of the hypothesis that \( \delta = 0 \) so that the auxiliary variables, \( u_t \), are insignificant. The logic is that if we happen to select the correct value for \( \gamma_f \), then the three explanatory variables in the hybrid NKPC will reproduce the time-series pattern in inflation \( \pi_t \), and no systematic pattern in the residuals will be detected by including other macroeconomic variables, \( u_t \).
To illustrate the Anderson-Rubin (AR) test, we collect auxiliary variables that include the 90-day Treasury bill interest rate, $r_t$ (again, a natural variable to consider in forecasting inflation), as well as extra lags of inflation and unit labor costs. The complete list is: $u_t = \{r_t, r_{t-1}, x_{t-1}, \pi_{t-2}\}$. The sample period is 1955:1–2007:4. We run the regression on a fine grid of values of $\gamma_{f0}$ between 0 and 2. For each such value we record the $F$-statistic associated with the restriction that none of the variables in $u_t$ enters the equation, and we calculate the corresponding $p$-value by locating the statistic in the $F(4, 204)$ distribution. Figure 3 graphs the candidate values, $\gamma_{f0}$, on the horizontal axis and the $F$-statistics (the solid black line) and their $p$-values (the dashed gray line) on the two vertical axes.

Figure 3 shows that the AR test rejects the restrictions for low values of the weight on expected future inflation and also for high values. In particular, when $\gamma_{f0}$ is less than 0.5 or greater than 1.5, the $F$-statistics are high and the $p$-values are low. This means that $\hat{\delta}$ is far from zero, the auxiliary variables, $u_t$, enter the equation, and so the candidate values of $\gamma_{f0}$ can be rejected. The test does not reject at intermediate values of $\gamma_{f0}$. The $F$-statistic reaches its minimum and the associated $p$-value its maximum for $\gamma_{f0}$ around 1.0.

We already know that Table 2 has CU-GMM estimates of $\gamma_f$ that are a large positive fraction (though the estimate depends on the instrument set) with a small standard error. Moreover, the $J$ test did not reject the overidentifying restrictions. So what is gained from the AR approach? The answer is that tests in Table 2 may have been affected by weak identification, whereas statistics in Figure 3 apply whether identification is weak or not. To illustrate the effect of this robust method on inference, note that the range of values for which the $F$-statistics in Figure 3 fall below the $\alpha$-percent critical value of the $F$-distribution (equivalently the $p$-values lie above $\alpha$) constitutes the
1 − α-percent confidence interval for γ_f. In this case, the 90 percent confidence interval is (0.66, 1.62) and the 95 percent confidence interval is (0.61, 1.78). For comparison, the traditional, asymptotic confidence intervals for γ_f from the GMM estimates in the second-to-last row of Table 2 are (0.65, 0.81) at the 90 percent level and (0.63, 0.83) at the 95 percent level. These intervals understate the uncertainty, compared with the intervals that are robust to weak instruments. The AR test suggests a positive value for γ_f, but considerable uncertainty or imprecision remains, and values greater than 1 are possible.

How to draw inference with weak instruments is an active area of research by statisticians. Excellent surveys of inference under weak identification are provided by Dufour (2003) and Andrews and Stock (2006). The AR test assumes x_t is exogenous, whereas some more recent methods allow it to be endogenous. These methods allow tests of all the NKPC parameters, whereas we have focused only on γ_f. One important finding in this research is that the AR test also may lack power, especially when there is overidentification. In other words, it may fail to reject a false, assumed value γ_f0 and so give too wide a confidence interval. This outcome is particularly likely if there are many auxiliary variables, u_t, and some are irrelevant as instruments.

Using identification-robust methods, Ma (2002) finds large confidence sets for the hybrid NKPC parameters, which suggests that they are weakly identified. Dufour, Khalaf, and Kichian (2006) apply the AR test and some more recent tests to the United States for a 1970–1997 sample. They too find wide confidence sets. Nason and Smith (2008) reject the hybrid NKPC for the United States—by finding empty confidence intervals—when testing either reduced-form parameters or the underlying ones ω, θ, and β. For no value of γ_f0 does the hybrid NKPC produce unpredictable residuals, so the confidence intervals are empty. (They use a slightly different definition of x_t, described in Section 6.) Kleibergen and Mavroeidis (2008) use identification-robust methods and conclude that γ_f > γ_b. However, they find wide confidence intervals, especially for γ_b and for λ, where the confidence interval includes zero. They also apply a stability test devised by Caner (2007) that is robust to weak identification. This test suggests that the NKPC experienced a structural break around 1984 and subsequently became flatter. Overall, methods that are robust to weak identification suggest more skepticism about the NKPC than do traditional econometric tools. They reveal considerable uncertainty about the NKPC parameters or, in some cases, reject all reasonable values.

One way to gain power in tests like these (or to find more precise estimates of the hybrid NKPC parameters) is to utilize more information on the inflation forecasting equation and the evolution of the exogenous variable x_t, a conclusion that directs us to consider systems of econometric equations that set the hybrid NKPC within a broader economic/statistical model. In these systems, researchers supplement the hybrid NKPC either with (a) an explicit, statistical forecasting model that recognizes that x_t is most likely
endogenous, or (b) additional equations like a policy rule and dynamic IS curve so as to form a coherent New Keynesian model. Either of these approaches can potentially provide more precision at the cost of introducing bias if the added assumptions are misspecified. Studies that use forecasting models (vector autoregressions) include those of Fuhrer (1997), Sbordone (2002, 2005), Kurmann (2005, 2007), and Rudd and Whelan (2005a, 2005b, 2006), while Lindé (2005) uses a three-equation New Keynesian model. On the other hand, Galí, Gertler, and López-Salido (2005) review these approaches and conclude that GMM estimation remains informative. Schorfheide’s article in this issue provides a complete review of systems estimation of the hybrid NKPC.

5. FORECAST SURVEY DATA

As we have noted, many of the statistical challenges with estimating and testing the NKPC arise because inflation expectations cannot be directly observed. There is an alternative to constructing these forecasts with instrumental variables, though, and that is simply to ask some people what they expect the inflation rate to be in the next quarter. The Federal Reserve Bank of Philadelphia does just this in its Survey of Professional Forecasts (SPF). There are other measures of actual forecasts, but they tend to belong to forecasters either with (potentially) more information (in the case of the Federal Reserve’s Greenbook forecasts) or different information (in the case of the Michigan household survey) than we might attribute to a typical, price-setting firm. These issues have helped to make the SPF the most widely used data source in this context. Another reason to favor the survey-based measures is that they are in real time. Unlike our typical, instrumental-variables estimates, their construction does not involve estimation with any data reported subsequent to the date of the forecast. Roberts (1995), Orphanides and Williams (2002, 2005), Adam and Padula (2003), Dufour, Khalaf, and Kichian (2006), Zhang, Osborn, and Kim (2008), and Brissimis and Magginas (2008) use forecasts to estimate the NKPC.

Next, we see what happens when we use the median forecast from the SPF in our estimator. The series on expected inflation, labeled $\pi_{t+1}$, is the median of the one-quarter-ahead forecasts of the GDP deflator growth rate quarter-to-quarter at annual rates, $dpgdp3$ from the SPF file MedianGrowth.xls, and is available for 1968:4–2007:4. In fact, the SPF survey referred to the GNP deflator until the end of 1991. This matters for the actual inflation rate, $\pi_t$, used to estimate the hybrid NKPC when the median SPF inflation is equated with expected inflation. In this case, we measure $\pi_t$ with GNPDEF from FRED for the period prior to 1991.

As a benchmark, we present CU-GMM results similar to those in Table 2, but with a 1969:1–2007:3 sample. The first row of Table 3 has these results.
Table 3  Forecast Surveys in the U.S. NKPC, 1969:1–2007:3

<table>
<thead>
<tr>
<th>Forecast</th>
<th>$\hat{\gamma}_b$ (se)</th>
<th>$\hat{\gamma}_f$ (se)</th>
<th>$100\times\hat{\lambda}$ (se)</th>
<th>$\hat{\omega}$ (se)</th>
<th>$\hat{\theta}$ (se)</th>
<th>$\hat{\beta}$ (se)</th>
<th>$J(df)$ (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[\pi_{t+1} \mid z_t]$</td>
<td>0.27 (0.07)</td>
<td>0.72 (0.07)</td>
<td>2.46 (1.66)</td>
<td>0.30 (0.10)</td>
<td>0.81 (0.05)</td>
<td>0.97 (0.06)</td>
<td>3.58 (0.73)</td>
</tr>
<tr>
<td>$E[\pi_{t+1}^s \mid z_t]$</td>
<td>0.38 (0.12)</td>
<td>0.56 (0.17)</td>
<td>1.36 (4.56)</td>
<td>0.50 (0.15)</td>
<td>0.83 (0.24)</td>
<td>0.83 (0.23)</td>
<td>11.63 (0.07)</td>
</tr>
<tr>
<td>$\pi_{t+1}^s$ (NLLS)</td>
<td>0.36 (0.03)</td>
<td>0.68 (0.03)</td>
<td>-0.14 (0.13)</td>
<td>0.56 (0.04)</td>
<td>0.91 (0.11)</td>
<td>1.16 (0.09)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Data are demeaned prior to estimation. The estimator is CU-GMM with a Newey-West HAC correction and automatic, plug-in lag length. The instrument vector is $\pi_{t-1}, x_{t-1}, x_{t-2}, w_{t-1}, w_{t-2}, y_{t-1}, y_{t-2}, cp_{t-1}, cp_{t-2}$ using a linearly detrended output gap.

The weight on future inflation is greater than the weight on past inflation, and both are estimated precisely. The estimated values for the underlying parameters $\omega$, $\theta$, and $\beta$, are similar to some of those found in Table 2. The $J$ test does not reject, but the coefficient on marginal costs, $\hat{\lambda}$, while positive, is statistically insignificant.

The second row of Table 3 lists results when the median survey value is equated with expected inflation. We continue to estimate with CU-GMM to allow for the possibility that this inflation expectations measure is contaminated with measurement error that is correlated with the survey measure. By comparison, with the CU-GMM estimates in the top row, the weights on past and future inflation tilt, with a larger weight on lagged inflation and a smaller one on expected future inflation in the second row of Table 3. There is now no significant role for marginal costs in explaining the inflation series ($\hat{\lambda}$ is smaller than its standard error) and the $J$ test rejects the overidentifying restrictions at conventional levels of significance.

Brissimis and Magginas (2008) perform a similar exercise but find that the $\hat{\gamma}_f$ and $\hat{\gamma}_b$ weights tilt in the opposite direction, with a large weight on expected future inflation and no statistically significant weight on lagged inflation. They use the Bureau of Labor Statistics measure of the labor share of output as $x_t$, whereas we use the adjusted Sbordone (2002) measure that also is adopted by Galí and Gertler (1999). This sensitivity of the findings with forecast survey data to the measure of marginal costs may show either that we need further research on modeling marginal costs or that this is not a fruitful way to model expectations in the hybrid NKPC.
Finally, we replace the unobservable $E[\pi_{t+1}|I_t]$ with $\pi_s^{t+1}$ and estimate the hybrid NKPC by least squares. Taking this step does not mean assuming these two series coincide. Instead, it yields consistent estimates whenever the median survey is based on less information than that reflected in forecasts driving actual inflation, so that

$$E[\pi_{t+1}|I_t] = \pi_s^{t+1} + \eta_t,$$

in which $\eta_t$ is uncorrelated with $\pi_s^{t+1}$. In other words, it assumes that the median forecast is an unbiased predictor of the broader-based inflation forecast that influences the behavior of Calvo price setters.

The third row of Table 3 contains the results of least-squares estimation with the median report from the SPF. The striking finding is that $\hat{\lambda}$ is negative so that real unit labor costs enter the equation with the wrong sign. However, the point estimate is small and statistically insignificant. This finding can be viewed as evidence against the use of the median survey measure. Perhaps there is an errors-in-variables problem associated with this representation of expected inflation. But it is not straightforward to explain a negative coefficient, albeit an insignificant one, which argues that this finding also can be viewed as evidence against the hybrid NKPC.

A resolution to the question of how to represent expected inflation, that is, with instrumental variables forecasts or survey forecasts, can be found by including both. Smith (2007) and Nunes (2008) include a linear combination of the two measures and ask which combination best explains current inflation. The estimating equation becomes

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f (\mu E[\pi_{t+1}|z_t] + (1 - \mu) \pi_s^{t+1}) + \lambda x_t,$$

Nunes offers an economic interpretation of this mixture as reflecting price-setters’ different forecasting methods. Smith instead has a purely statistical interpretation. Either way, the evidence is that both measures matter. Their estimates place a slightly greater weight on the survey measure than on the econometric measure. The estimated hybrid NKPC parameters $\{\hat{\gamma}^b, \hat{\gamma}_f, \hat{\lambda}\}$ in these studies resemble those in Table 2 and are consistent with theory.

6. WHAT DRIVES INFLATION?

Up to this point we have studied the hybrid NKPC in which inflation tracks real unit labor costs. But several authors have argued that measures of the output gap (i.e., the cyclical component of real GDP) are better explanatory variables for inflation. This section sets both types of $x$-variables in the hybrid NKPC to learn which might be most useful for explaining U.S. inflation. We first describe the properties of nine different candidate $x$-variables. We then use these series to estimate the hybrid NKPC.

We consider two measures of real unit labor costs. RULC1 is the Sbordone (2002) measure described earlier in Table 1. RULC2 measures real unit labor
cost as the cointegration relation of the logarithm of nominal unit labor cost with the logarithm of the GDP deflator (allowing for an intercept and time trend). The estimated cointegrating coefficient is 1.03. Nason and Slotsve (2004) show that RULC2 is consistent with Calvo’s staggered pricing mechanism. Nason and Smith (2008) use this variable in their estimated hybrid NKPC.

The first measure of the output gap, labeled CBO, is published by the Congressional Budget Office. The remaining measures are based on per capita output. LT (QT) is the series of residuals from linearly (quadratic) detrending per capita real GDP. Next, measures UC and BN are based on the unobserved-components model and Beveridge-Nelson decomposition, respectively. Both of these measures treat real per capita output as the sum of a permanent component and a transitory component. These time-series models assume the permanent component of output is a random walk with drift while the transitory component follows a second-order autoregression. The difference is that the UC model imposes a zero correlation between innovations to the permanent and transitory components. The BN decomposition estimates this correlation, which is -0.97. Maximum likelihood estimation of the UC and BN models is undertaken with the Kalman filter, and the associated output gap estimates are filtered, not smoothed. The UC and BN output gap measures rely on the work of Morley, Nelson, and Zivot (2003).

Measure BK is based on the Baxter and King (1999) bandpass filter. Since the technical details for this implementation are not straightforward, we refer statistically inclined readers to Harvey and Trimbur (2003). They show how to estimate the BK cycle or output gap with the Kalman filter. Finally, measure HP is the cycle that remains after applying the Hodrick and Prescott (1997) filter to output growth, as implemented by Cogley and Nason (1995).

Figure 4 plots the nine (demeaned) measures. All series are shown since 1955 (omitting the volatile Korean war period), but the vertical scale varies across the three panels. In the top panel the two measures of marginal cost have different trends, but RULC1 tends to be dominated by low-frequency movements. The middle panel shows the CBO output gap and the two deterministically detrended output gaps. These three generate more cycles than the marginal cost measures and are also more volatile than RULC1 and RULC2. The CBO, LT, and QT output gaps behave similarly except during the late 1960s and since 1999. The bottom panel of Figure 4 presents UC, BN, BK, and HP measures of the output gap. The BN and BK output gaps have most of their variation between two and four years per cycle, while relatively lower-frequency fluctuations produce most of the variation in the UC and HP output gaps. Volatility varies from one measure to another, with the UC and HP output gaps exhibiting the most variance.

Nason and Smith (2008) show that being able to predict future $x_t$ can be key for the viability of single-equation approaches to the NKPC. Recall
that (a) according to the hybrid NKPC, inflation is related to lagged inflation and to the present value of current and future $x_t$, and (b) finding instruments involves predicting next quarter’s inflation rate, $\pi_{t+1}$. Combining these two facts means that we must predict future values of $x_t$ in order to identify the NKPC.

One possibility discussed in Section 4 is that $x_t$ can be forecasted using its own lagged values. In that case, higher-order dynamics are needed for identification. The idea that some complicated dynamics in $x_t$ help us learn about the NKPC makes intuitive sense. If there are predictable movements in these fundamentals, they should be matched by swings in inflation. The
extent to which they are matched can shed light on whether the NKPC is a
good guide to inflation. If, instead, there are no predictable movements in \( x_t \)
that inflation is theorized to be tracking, there will be no way to identify the
response of inflation.

We test for lag length in univariate autoregressions for each of the nine
\( x \)-variables using the Akaike information criterion, Hannan-Quinn informa-
tion criterion, Schwarz or Bayesian information criterion, and likelihood ratio
(LR) test. The evidence is that for most of these series there are complicated
dynamics in which three to five lags contain forecasting information. The
two measures of unit labor costs, RULC1 and RULC2, and the BN output
gap appear to be exceptions, because LR tests suggest high-order dynamics
that reach 10 to 12 lags. These results suggest the RULCs and output gaps
have the requisite dynamics to overidentify the three structural price-setting
or reduced-form parameters of the hybrid NKPC.

Of course, our main reason for studying RULC1 and RULC2 or the output
gaps is to use them in the hybrid NKPC. Thus, the main goal of this section is
to estimate this NKPC with the nine \( x \)-measures. Table 4 contains the results.
When we estimate using RULC1 or RULC2 we find a small positive effect
of marginal costs on inflation. The significance of this effect depends on the
instrument set, as we documented in Section 2.

The remaining rows of Table 4 present NKPC estimates using the output
gaps. We find no significant role for any of these \( x \)-measures. The coefficient
on \( x_t, \hat{\lambda} \), is small and negative (albeit statistically insignificant) for all the output
gaps. However, most economists would predict the opposite effect: a positive
output gap leading to a rise in prices. The other hybrid NKPC coefficients
also take surprising values that are difficult to interpret. The coefficient on
lagged inflation is negative, while the coefficient on expected future inflation
is greater than one. These coefficients on past and future inflation most likely
are affected by omitted-variables bias. Without some confidence in one’s
measure of the \( x \)-variable that inflation tracks in the NKPC, there cannot be
much confidence in estimates of inflation inertia or other properties of inflation
dynamics.

Investigators who work with an output gap might sometimes wonder
whether their findings depend on the specific filtering or detrending procedure
they use to measure this variable. We have used measures that are commonly
adopted and that have been used in forecasting or explaining inflation, yet
found no role for any of them. Our evidence suggests little support for the
idea that the output gap drives U.S. inflation.

Some recent studies work with inflation and output gaps and do find statisti-
cal links between them. Harvey (2007) adopts an unobserved-components
model of both inflation and output and finds a link between the cyclical com-
ponents of the two series. Basistha and Nelson (2007) use the NKPC to define
or measure the output gap so that it fits into the NKPC by construction.
Table 4  U.S. New Keynesian Phillips Curve, 1955:1–2007:4

\[
E \left[ \pi_t - \gamma_b \pi_{t-1} - \gamma_f \pi_{t+1} - \lambda x_t \mid z_t \right] = 0
\]

<table>
<thead>
<tr>
<th>(\lambda)-Measure</th>
<th>(\hat{\gamma}_b)</th>
<th>(\hat{\gamma}_f)</th>
<th>100×(\hat{\lambda})</th>
<th>(J(df))</th>
</tr>
</thead>
<tbody>
<tr>
<td>RULC1</td>
<td>0.28</td>
<td>0.68</td>
<td>0.56</td>
<td>1.28(1)</td>
</tr>
<tr>
<td>(0.16)</td>
<td>(0.18)</td>
<td>(0.27)</td>
<td>(0.26)</td>
<td></td>
</tr>
<tr>
<td>RULC2</td>
<td>0.34</td>
<td>0.61</td>
<td>0.70</td>
<td>1.64(1)</td>
</tr>
<tr>
<td>(0.16)</td>
<td>(0.18)</td>
<td>(0.37)</td>
<td>(0.20)</td>
<td></td>
</tr>
<tr>
<td>CBO</td>
<td>−0.57</td>
<td>1.70</td>
<td>−3.90</td>
<td>0.02(1)</td>
</tr>
<tr>
<td>(0.84)</td>
<td>(1.02)</td>
<td>(3.13)</td>
<td>(0.89)</td>
<td></td>
</tr>
<tr>
<td>LT</td>
<td>−0.58</td>
<td>1.76</td>
<td>−2.90</td>
<td>0.02(1)</td>
</tr>
<tr>
<td>(0.70)</td>
<td>(0.88)</td>
<td>(2.20)</td>
<td>(0.90)</td>
<td></td>
</tr>
<tr>
<td>QT</td>
<td>−0.66</td>
<td>1.82</td>
<td>−2.60</td>
<td>0.11(1)</td>
</tr>
<tr>
<td>(0.77)</td>
<td>(0.94)</td>
<td>(1.83)</td>
<td>(0.73)</td>
<td></td>
</tr>
<tr>
<td>UC</td>
<td>−0.51</td>
<td>1.59</td>
<td>−0.97</td>
<td>0.80(1)</td>
</tr>
<tr>
<td>(0.55)</td>
<td>(0.65)</td>
<td>(0.65)</td>
<td>(0.37)</td>
<td></td>
</tr>
<tr>
<td>BN</td>
<td>−0.43</td>
<td>1.56</td>
<td>−1.78</td>
<td>0.39(2)</td>
</tr>
<tr>
<td>(0.83)</td>
<td>(1.02)</td>
<td>(2.89)</td>
<td>(0.94)</td>
<td></td>
</tr>
<tr>
<td>BK</td>
<td>−0.66</td>
<td>1.83</td>
<td>−2.15</td>
<td>0.34(3)</td>
</tr>
<tr>
<td>(1.68)</td>
<td>(2.03)</td>
<td>(2.88)</td>
<td>(0.84)</td>
<td></td>
</tr>
<tr>
<td>HP</td>
<td>−0.54</td>
<td>1.70</td>
<td>−1.84</td>
<td>0.06(1)</td>
</tr>
<tr>
<td>(0.69)</td>
<td>(0.87)</td>
<td>(1.34)</td>
<td>(0.80)</td>
<td></td>
</tr>
</tbody>
</table>

Each equation includes a constant term and each instrument set includes a vector of ones. Instruments are \(x_t, \ldots, x_{t-J+1}\) and \(\pi_{t-1}\), where \(J\) is the lag length estimated from the AIC. Estimation is by CU-GMM with a quadratic-spectral kernel HAC estimator.

These studies do not test for the role of conventionally measured output gaps in the standard NKPC. Conversely, there also is statistical work that, like ours, questions the links between measures of the output gap and inflation. For example, Orphanides and van Norden (2005) find that output gaps do not help forecast inflation when both are measured realistically in real time (rather than in revised data).

Marginal Costs Revisited

We have seen that the NKPC that uses the labor share to represent RULCs is relatively successful empirically. This measure of costs is easy to construct and has intuitive appeal. But some labor market arrangements imply that this measure is misspecified, so some recent research augments this model of marginal costs.

Macroeconomic models contain descriptions of the production technology that firms use. Models that contain different technologies will predict different ways to measure the marginal cost variable toward which firms adjust their prices. In particular, if a firm faces other frictions besides the costs of adjusting
prices, those may affect how it sets prices. For example, imagine a firm that must borrow from a bank to finance its wage bill. An increase in the interest rate it pays then will act like a cost shock and affect how it prices its goods. These additional frictions are sometimes called “real rigidities.” They can include the financing constraint just mentioned, sticky real wages, or costs of hiring new employees.

Table 5 lists several recent studies that augment the labor-share measure of real unit labor costs with additional variables. Moreover, several of these studies estimate the NKPC by GMM with the revised measures of $x_t$ and find statistical support for the added terms or right-hand-side variables. Few economists would argue that our model of firms’ costs should be chosen according to how well it explains inflation in the NKPC, and these studies also examine other empirical evidence. But it is promising that a range of plausible modifications have improved the fit of the NKPC (its success in passing tests of overidentifying restrictions or stability tests) without significantly altering the findings about forward-looking and backward-looking weights.

7. MEASURES OF INFLATION

Conclusions about the NKPC also might depend on how the inflation rate is measured. The statistics so far have been based on the GDP deflator, so it seems natural to wonder whether they change if we measure inflation using another index such as the consumer price index (CPI), the deflator for personal consumption expenditure, or the producer price index. To check on the first of these alternatives, we average the monthly CPI (all items, all urban consumers, seasonally adjusted), CPIAUCSL from FRED to find the quarterly value, then construct the inflation rate as the annualized, quarter-to-quarter growth rate in percentage points.

Figure 5 shows this CPI inflation rate (the dashed gray line) and the inflation rate measured with the GDP deflator (the solid black line) used so far.
in this article. The figure shows a common, low-frequency cycle in the two measures of quarterly inflation. But the CPI inflation rate is more volatile. The only persistent difference between the two series occurred in the late 1970s when CPI inflation exceeded deflator inflation for several consecutive quarters.

When we estimate the NKPC with CPI inflation and RULC1 and RULC2, the results change modestly. The coefficient on lagged inflation, $\hat{\gamma}_b$, is slightly larger, and the coefficient on expected future inflation, $\hat{\gamma}_f$, is slightly smaller. The coefficient on marginal costs is smaller and is estimated less precisely. Finally, when we combine the CPI inflation rate with the seven output gaps, the results are quite negative for that approach, just as in the previous section. Overall, we conclude that the evidence summarized so far does not depend significantly on how the inflation rate is measured.

8. INTERNATIONAL EVIDENCE

Researchers also have used single-equation methods to study the NKPC in other countries. Galí, Gertler, and López-Salido (2001) find the hybrid NKPC fits well in quarterly Euro-area aggregate data for 1970–1998. As in the U.S. data, $\hat{\gamma}_f > \hat{\gamma}_b$, $\hat{\lambda}$ is statistically significant, and the $J$ test does not reject overidentifying restrictions. Neiss and Nelson (2005) compare estimates of the NKPC for the United States, United Kingdom, and Australia. They also propose a new measure of the output gap that statistically explains inflation as well as measures of marginal costs. Leith and Malley (2007) estimate hybrid NKPCs for the G7 countries for 1960–1999, while Rumler (2007) does so for eight Euro-area countries for 1980–2003. Both of these studies discuss the role of the terms of trade, in addition to the labor share, in measures of marginal costs. They also report on differences in parameter estimates (such as those measuring price stickiness or inflation inertia) across countries. The
international evidence on the hybrid NKPC may provide a guide to reform the measurement of marginal costs in open economies, in that the effects of foreign trade may be easier to detect in small, open economies than in the United States.

Batini, Jackson, and Nickell (2005) extend the model of marginal costs to reflect the relative price of imports, varying markups, and costs of adjusting employment. They arrive at a hybrid NKPC that nests the standard version and improves on its fit for the United Kingdom for 1972–1999. Bårsden, Jansen, and Nymoen (2004) also estimate more general statistical models of inflation, for the Euro area. They find that the hybrid NKPC can be improved on in terms of forecasting inflation even though it passes the $J$ test.

Nason and Smith (2008) estimate the NKPC for the United Kingdom and Canada and provide tests that are robust to weak instruments. As in U.S. data, they find that the robust tests and traditional single-equation GMM estimation give different messages. The robust tests provide little evidence of forward-looking dynamics in these NKPCs. This international research thus conveys a similar message to the work on U.S. data.

9. CONCLUSION

This article outlines single-equation econometric methods for studying the NKPC and offers a progress report on the empirical evidence. How successful is the NKPC when estimated and tested on U.S. inflation? Enter the proverbial two-handed economist. On the one hand, the hybrid NKPC estimated by GMM on a quarterly 1955–2007 sample has coefficients that have signs and sizes that accord with economic theory and are statistically significant. The structural coefficients ($\hat{\omega}$, $\hat{\theta}$, and $\hat{\beta}$) are positive fractions, as are the reduced-form coefficients on past inflation and expected future inflation ($\hat{\gamma}_b$ and $\hat{\gamma}_f$), while the slope of the reduced-form Phillips curve ($\hat{\lambda}$) is positive. The hybrid NKPC also passes statistical tests based on the unpredictability of its residuals (the $J$ test) and its stability over time (the sup-Wald test). The findings are not sensitive to alternative measures of inflation. Real unit labor costs are much better at statistically explaining inflation than are a plethora of output gap measures.

On the other hand, the $t$-statistic on real unit labor costs usually is not much above two. This indicates that there is not a close relationship between inflation and this measure of marginal costs. Estimates of the NKPC using surveys of forecasts give very different coefficients from those using instrumental-variables estimation. The confidence interval that is valid even with weak instruments gives a wide range of possible values for the parameter on expected future inflation. Moreover, other tests that are robust to weak identification often yield unreasonable values for the other hybrid NKPC parameters or reject the NKPC entirely. Our macroeconomic “Rip van Winkle,” accustomed to the
evidence against the Phillips curve garnered during the 1970s and 1980s, might find that the world has not changed much after all.

How will we learn more from single-equation methods? One promising and active avenue of research focuses on measurement of the cost variable, $x_t$, toward which prices adjust. Econometric tools for drawing inferences with weak identification also continue to advance. And the simple accumulation of macroeconomic data over time may help with precision, too.

Of course, systems methods of estimation also continue to be fruitful ways to identify and estimate the NKPC. Another complement to traditional, single-equation methods is to look at microeconomic data from individual firms or industries. Economists increasingly ask whether macroeconomic models of price stickiness are consistent with data on how prices are adjusted at the microlevel. It may be possible to measure cost shocks in microeconomic data and estimate pricing equations at that level, too.

The NKPC continues to be a key building block for macroeconomic models that require a monetary transmission mechanism. Our econometric work shows that marginal costs may be superior to many output gaps as a guide to inflation. We also obtain GMM estimates that give an important role to expected future inflation in explaining current inflation, while lagged inflation receives less weight. But measuring the effect of expected, future inflation on current inflation can be problematic because of weak instruments. Future research on this key response would be valuable because such forward-looking effects continue to have implications for the design of good monetary policy.

REFERENCES


