Identifying Taylor Rules in Macro-Finance Models

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BI–SHoF Conference | Oslo | June 5-6, 2015

This version: June 7, 2015

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Identifying the Taylor rule

John Cochrane

- In the context of a new-Keynesian model, ... I show that the parameters of the Fed's policy rule are not identified.

- [The Taylor rule parameter] is not identified from data on $\{i_t, \pi_t\}$ [the short rate and inflation] in the equilibrium of this model.

- The crucial Taylor rule parameter is not identified in the new-Keynesian model.

Identifying the Taylor rule

Scott Joslin, Anh Le, and Ken Singleton

- The parameters of a Taylor rule are not econometrically identified within [affine macro-finance models].
- Several recent studies interpret the short-rate equation as a Taylor-style rule. ... However, without imposing additional economic structure, ... the parameters are not meaningfully interpretable as the reaction coefficients of a central bank.
- Absent additional economic structure, there appears to be no basis for interpreting any one of these equivalent rotations as the Taylor rule of a structural model.

Identifying the Taylor rule

What we do

- Illustrate the problem
- Characterize its solution

Identifying the Taylor rule

What we do

- Illustrate the problem
- Characterize its solution
- Monetary policy and asset pricing can once more safely coexist

Two examples

Cochrane's example

Model

$$
\begin{aligned}
i_t &= r + E_t \pi_{t+1} && \text{(Euler equation)} \\
i_t &= r + \tau \pi_t + s_t && \text{(Taylor rule)}
\end{aligned}
$$

State and shock

$$
\begin{aligned}
x_{t+1} &= A x_t + B w_{t+1}, \quad \{w_t\} \sim \mathrm{NID}(0, I) \\
s_t &= d^\top x_t
\end{aligned}
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****Information structure****

- Agents observe everything
- We (economists) observe (π, i, x) — **but not the shock** s

Cochrane's example: solution

Equate EE and TR

$$E_t \pi_{t+1} = \tau \pi_t + s_t$$

Solution: guess $\pi_t = b^\top x_t$

$$\begin{aligned} b^\top A x_t &= \tau b^\top x_t + d^\top x_t \\ b^\top A &= \tau b^\top + d^\top \end{aligned}$$

$$\Rightarrow \quad b^\top = -d^\top (\tau I - A)^{-1}$$

Unique stationary solution if $\tau > 1$, A stable

Cochrane's example: identification

What do we observe?

- State x: so we can estimate A
- Inflation rate π: so we can estimate b
- **Not the shock** s: so we **cannot** estimate d

Can we infer Taylor rule parameter τ?

$$\underbrace{b^\top A}_{\text{EE}} = \underbrace{\tau b^\top + d^\top}_{\text{TR}}$$

Affine example

Model

$$
\begin{aligned}
i_t &= -\log E_t \exp(m^{\$}_{t+1}) = \delta^\top x_t && \text{(Euler equation)} \\
m^{\$}_{t+1} &= -\lambda^\top \lambda - \delta x_t + \lambda^\top w_{t+1} && \text{(pricing kernel)} \\
x_{t+1} &= A x_t + B w_{t+1} && \text{(state transition)}
\end{aligned}
$$

We observe x and i, so can estimate δ

Suppose first two elements of x are inflation and GDP growth

Can we interpret short rate equation as a Taylor rule?

Thinking out loud

Would an extra shock help? (Gertler)

$$\begin{aligned} i_t &= E_t \pi_{t+1} + s_{1t} && \text{(Fisher equation)} \\ i_t &= \tau \pi_t + s_{2t} && \text{(Taylor rule)} \\ s_{it} &= d_i^\top x_t && \text{(shocks)} \end{aligned}$$

Let's say we observe s_1 but not s_2

If shocks are orthogonal, use s_1 as an instrument

Why does this work?

Representative agent model

Representative agent model

- Model

$$\begin{aligned}
i_t &= -\log E_t \exp(m_{t+1} - \pi_{t+1}) && \text{(Euler equation)} \\
i_t &= r + \tau \pi_t + s_{2t} && \text{(Taylor rule)} \\
m_{t+1} &= -\rho - \alpha \log g_{t+1} && \text{(real pricing kernel)} \\
\log g_t &= g + s_{1t} && \text{(consumption growth)} \\
x_{t+1} &= A x_t + B w_{t+1} && \text{(state transition)} \\
s_{it} &= d_i^\top x_t && \text{(shocks)}
\end{aligned}$$

- Solution: guess $\pi_t = b^\top x_t$

$$\underbrace{b^\top A + \alpha d_1^\top}_{\text{EE}} = \underbrace{\tau b^\top + d_2^\top}_{\text{TR}} \Rightarrow b^\top = (\alpha d_1 - d_2)^\top (\tau I - A)^{-1}$$

Representative agent model: identification

What do we observe?

- State x: so we can estimate A
- Inflation rate π: so we can estimate b
- Consumption growth g: so can estimate d_1
- **Not the Taylor rule shock** s_2: so we **cannot** estimate d_2

Can we infer Taylor rule parameter τ?

$$b^\top A + \alpha d_1^\top \quad = \quad \tau b^\top + {d_2}^\top$$

Representative agent model: identification

Reminder

$$b^\top A + \alpha d_1^\top \quad = \quad \tau b^\top + d_2{}^\top$$

What if we set one element of d_2 equal to zero?

Other linear restrictions: $d_2^\top e = 0$

What about Gertler's suggestion that s_1 and s_2 are orthogonal?

Representative agent model: identification

Reminder

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$$d_2^\top \underbrace{\text{Var}(x) d_1}_{=e} \quad = \quad 0$$

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If shock isn't observed, we need one restriction to infer TR (identification is never free)

Stan: "If a rule depends on everything in an arbitrary way, it's not a rule."

More generally, we need one restriction for every TR parameter (ditto other equations)

How do we observe the state?

- Use Kalman filter, replace x with $\widehat{x}$
- Harder than it sounds

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It's now safe again to link finance and macro, charge ahead