Notes on optimal unemployment insurance with hidden savings

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December 21, 2002

1 The Problem

Hoppenhein and Nicolini (1998) show that if the agent cannot save, it is optimal for an agent’s consumption to fall as his unemployment duration increases. The possibility of unobserved saving for an unemployed agent substantial changes the way a planner can provide incentives to this agent. It is then interesting to investigate how the features of the optimal unemployment insurance contract change in this case. This is the question posed by Werning (2002).

1.1 Brief summary of findings with no savings

Hoppenhein and Nicolini’s result is an application of a general feature of optimal dynamic contracts in repeated moral hazard environments (Green (1987), Thomas and Worral (1990), Atkeson and Lucas (1992,1995)).

The planner (principal) uses both "static" and "dynamic" ways to provide incentives to the agent. The static way to provide incentives is to make current consumption depend on an agent’s current shock realization. The dynamic way corresponds to making continuation utility dependent on the current shock realization. In particular, it is optimal to make current consumption and continuation utility relatively low for an agent that experiences a shock that reduces his current income.

As an example, consider the case of an agent that, in each period, experiences an iid cost of effort shock, $\theta_1 < \theta_2$, with probability $\pi_1$ and $\pi_2$, and exerts effort $a \in [0, \bar{a}]$. The planning problem for this example is described in
the appendix. Let $\eta$ be the multiplier of the implementability constraint and $C(u)$ the cost for the planner of providing a utility from current consumption equal to $u$ for the agent. Then the first order conditions for the optimal contract, with separable utility between consumption and effort, imply:

$$C'(w, \theta_1) - C'(w, \theta_2) = \eta \left( \frac{1}{\pi_1} + \frac{1}{\pi_2} \right),$$  \hspace{1cm} (insurance wedge)

$$C'(w, \theta_i) = \sum_i \pi_i C'(w', \theta_i),$$  \hspace{1cm} (intertemporal wedge)

if the planner and the agent discount at the same rate. Here, $w$ is promised utility and $w'_i$ is the continuation utility for an agent reporting a value of the shock $\theta_i$.

The first condition states that current consumption will be lower for an agent reporting higher current cost of working. The second condition can be rewritten using duality - $C'(u) = 1/u'(c)$ - to yield:

$$\frac{1}{u'(c(w, \theta_i))} = \sum_i \pi_i \frac{1}{u'(c(w'_i, \theta_i))},$$

which by Jensen’s inequality implies:

$$u'(c(w, \theta_i)) \leq \sum_i \pi_i u'(c(w'_i, \theta_i)).$$  \hspace{1cm} (Rogerson condition)

The Rogerson condition (due to Rogerson (1985)) implies consumption decreases over the agents’ lifetime (immiseration) and can be understood as follows. Assume that the planner wants to reduce the amount of consumption awarded in the current period and increase the amount awarded in the following period. To ensure incentive compatibility, more consumption needs to be awarded in the $\theta_1$ state (low cost of working) than in the $\theta_2$ state in the following period. This is because the agents receives more current consumption in the $\theta_1$ state, from (insurance wedge) and values marginal increases in consumption less. Given that utility is concave, the cost of providing a given level of expected utility is convex. Then, the presence of an insurance wedge increases the resource cost of increasing expected utility in the following period and encourages the planner to award consumption and utility in the current versus the future periods.

This reasoning easily extends to the case of more than two shocks and to the case of a continuum of shocks.
Applying the two principles above to optimal unemployment insurance implies\(^1\):

\[
\begin{align*}
&c^E_t > c^U_t, \\
u'(c^U_t) \leq p(e_{t+1}) u'(c^E_{t+1}) + (1 - p(e_{t+1})) u'(c^U_{t+1}),
\end{align*}
\]

(RC)

where the second condition implies \(c^U_{t+1} < c^U_t\).

1.1.1 Extension to observed savings

If the agent can save it becomes harder for the planner to provide incentives to exert effort, since the cost of unemployment is smaller and it is harder to spread utility across the unemployment and the employment states.

If savings are observed by the planner, then the optimal for the planner to make an unemployed agent savings constrained. This corresponds to a non-interior solution to the agent’s savings problem, resulting in an intertemporal Euler inequality as in (RC). To achieve this outcome in an optimal unemployment insurance contract, the planner could for example, impose a tax on savings. The optimal tax rate would depend on the duration of the agent’s unemployment spell.

2 Optimal Unemployment Insurance with Unobserved Savings

To gain insight on optimal unemployment insurance with unobserved savings, one could conjecture that the principles governing the solution with observed savings apply.

The features of the solution with observed savings are based on the fact that having outstanding wealth makes providing incentives more costly. Consider the case of an agent that enters an unemployment spell with a positive level of unobserved savings. Assume the planner wants to implement a particular level of effort \(e^*\).

The solution would feature limited unemployment insurance i.e. \(c^U_t < c^E_t\) due to the incentive compatibility constraint. Then if the agent’s optimal savings strategy follows a permanent income logic, one could conjecture that

\(^1\)The problem of providing unemployment insurance can be interpreted as a repeated moral hazard problem in which the shock is an "unemployment spell of length s".
the agent will run down his savings to finance consumption during the unemployment state, if this state is non-absorbing. But then, an unemployed agent will have lower savings at the end of a longer unemployment spell. This will provide in part the dynamic incentive to exert effort and should reduce the planner’s reliance to falling unemployment benefits to implement the desired level of effort $e^*$.

This intuition matches the findings in Werning (2002). Kocherlakota (2002) shows that this intuition is not valid. He shows that in general with unobserved savings it is optimal to leave unemployed agents borrowing-constrained, so that:

$$u'(c_t^U) \geq p(e_{t+1}) u'(e_t^E) + (1 - p(e_{t+1})) u'(c_{t+1}^U).$$

(Kocherlakota condition)

Werning uses the first order approach to derive his results. Kocherlakota shows that it is not correct to use the first order approach in this case. His critique extends to all models of optimal unemployment insurance with utility separable in effort and consumption.

2.1 The failure of the first order approach (FOA)

To see why the FOA fails, let’s follow Kocherlakota and analyze a two period example.

Agent preferences:

$$\ln(c_1) + \ln(c_2) - \alpha p(e_2).$$

Technology:

$$y = E \text{ with probability } p(e_2)$$
$$= U \text{ with probability } 1 - p(e_2)$$

$$e_2 \in [0, \infty), \quad p' > 0, \quad p(0) = 0$$

The planner can condition $c_2$ on the realization of $y$ only.

The planner wishes to implement $e_2^*$ at minimum expected cost:

$$\min_{c_1, c_E, c_U} \quad c_1 + p_2^* c_E + (1 - p_2^*) c_U$$

subject to
\((S, e_2^*) \in \max_{e_2, S \geq 0} \ln (c_1 - S) + p(e_2) \ln (c_E + S) + (1 - p(e)) \ln (c_U + S) - \alpha p(e_2) \)

\[(IC)\]

\[\ln (c_1) + p_2^* \ln (c_E) + (1 - p_2^*) \ln (c_U) - \alpha p_2^* \geq u^*. \]  

\[(PK)\]

**Remark 1** If \((c_1, c_E, c_U)\) is a solution, then \((c_1 - S, c_E + S, c_U + S)\) is also a solution for which the agent finds it optimal not to save.

Without loss of generality, we can then look at this problem with a smaller constraint set:

\[
\min_{c_1, c_E, c_U} c_1 + p_2^* c_E + (1 - p_2^*) c_U \tag{P1}
\]

subject to

\[
(0, e_2^*) \in \max_{e_2, S \geq 0} \ln (c_1 - S) + p(e_2) \ln (c_E + S) + (1 - p(e)) \ln (c_U + S) - \alpha p(e_2) \]

\[(IC-P1)\]

\[\ln (c_1) + p_2^* \ln (c_E) + (1 - p_2^*) \ln (c_U) - \alpha p_2^* \geq u^*. \]  

\[(PK)\]

Since \(S\) is unobserved, this problem has both moral hazard and adverse selection. There is not standard way to attack this using Lagrangian methods.

### 2.1.1 The FOA

Take derivatives of IC with respect to \(e_2\) and \(S\). This yields:

\[1/c_1 \geq p_2^*/c_E + (1 - p_2^*) /c_U. \]  

\[(IC-P2-2)\]

\[\ln (c_E) - \ln (c_U)) p' (e_2) = \alpha p' (e_2), \]  

\[(IC-P2-1)\]

Solve the following problem:

\[
\min_{c_1, c_E, c_U} c_1 + p_2^* c_E + (1 - p_2^*) c_U \tag{P2}
\]

subject to (IC-P2-1), (IC-P2-2) and (PK).

Problem P2 is amenable to Lagrangian methods and can be solved recursively in multiperiod versions, as done by Werning. The only difference
relative to a problem with no savings is the presence of constraint (IC-P2-2), which is the agent’s intertemporal Euler equation. Werning specifies the problem so that the planner chooses a current consumption and next periods’ continuation utility to deliver given continuation utility and subject to a pre-specified upper bound on the marginal utility of consumption. This constraint guarantees that the planner satisfies the agent’s intertemporal Euler equation in (IC-P2-2).

The FOA with no savings Rogerson (1985) shows that it is valid under two sufficient conditions for general principal-agent problems:

1. Higher effort increases the posterior probability of observing higher output (MLRC)
2. The probability of an output level less then or equal to some y decreases as the agent works harder (CDFC).

The monotone likelihood ration condition (MLRC) implies that increases in effort cause output to increase in the sense of stochastic dominance.

It is easy to check that for the environment we are analyzing here, these two conditions amount to $p'(e) \geq 0$ and $p''(e) \leq 0$.

The FOA with hidden savings

Claim 2 Solving $P2$ is not the same as solving $P1$.

Solving P2. At an optimum, the two inequalities in the constraint set for P2 must hold with equality. Then, the solution to P2 is the unique triple $(c_1^*, c_E^*, c_U^*)$ that satisfies all constraints with equality.

This triple is not in the constraint set for P1. If it was, then for given $(c_1^*, c_E^*, c_U^*)$ the agent’s optimization problem should be solved at $S = 0$ and $c_2 = c_2^*$ with the agent’s first order conditions satisfied with equality by construction. But the agent’s Hessian is:

$$-1/(c_1^*)^2 + p_2^*/(c_E^*)^2 + (1 - p_2^*)/(c_U^*)^2 - (1/c_E^* - 1/c_U^*) p'(e_2^*)$$

A necessary condition for optimality is that this Hessian is negative-semidefinite, but this is not true, since the determinant is negative. The
agent can experience a second order gain by \textit{jointly} increasing $S$ and decreasing $e_2$.

To see this, note that a necessary condition for the Hessian to be negative semi-definite is that the product of the diagonal terms is positive. Even if the bottom right term were different from 0, this is not guaranteed under the two sufficient conditions derived by Rogerson. If, under the contract, $c_E > c_1 > c_U$ or $c_1 > c_E > c_U$, the top left term of the Hessian is positive. Then, the bottom right term should be negative for the Hessian to be negative semi-definite. The bottom left term is the derivative of (IC-P2-1) with respect to $e_2$, given by: $(\ln (c_E) - \ln (c_U) - \alpha) p'' (e_2)$. This is identically equal to 0 for any interior effort choice, irrespective of the sign of $p''$. If $p'' < 0$, then a negative semidefinite Hessian requires $\ln (c_E) - \ln (c_U) - \alpha < 0$, which will be the case only if the agent finds it optimal to exert 0 effort ($e_2 \geq 0$ is implicitly imposed here). The intuition for this is as follows. If $c_E > c_1 > c_U$ or $c_1 > c_E > c_U$ the agent has an incentive to save. So there are two $(S, e_2)$ that satisfy first order conditions: $(0, e_2^*)$ and $(S^*, 0)$, with $S^* > 0$. If, on the other hand, under the contract $c_E > c_U \geq c_1$, the agent would like to borrow and the top left term of the Hessian is negative. So there is no deviation that involves setting $e_2$ to 0 that improves utility for the agent.

2.2 Solving the Insurance Problem

2.2.1 A General UIP with Unobservable Savings

The principal (or planner):

$$- \sum_{t=1}^{\infty} \beta^{t-1} c_t.$$ 

The agent:

$$\sum_{t=1}^{\infty} \beta^{t-1} \left[ u (c_t) - \alpha p (e_t) \right], \quad 0 < \alpha < 1,$$

$p (0) = 0, p' > 0$, range of $p$ is $[0, 1], u' > 0, u'' < 0, u$ bounded below and above.

The agent can store at rate $1/\beta - 1$ and his storage is unobserved.

The contract:

$$\{c^E_t, c^U_t\}_{t=1}^{\infty}$$

The employment state is absorbing.
For an arbitrary sequence: $e^* = \{e^*_t\}_{t=1}^{\infty}$, with $e^*_t > 0$ for all $t$, and incentive compatible contract solves:

$$\{S_t^*, e^*_t\}_{t=1}^{\infty} \in \arg \max_{\{S_t, e_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^{t-1} \Pi^{t-1}_{s=1} (1 - p(e^*_s)) \{p(e_t) \left( \frac{\zeta^E_t}{(1 - \beta - \alpha)} \right) \}
+(1 - p(e_t)) u \left( \zeta^U_t \right) \}$$

subject to

$$\zeta^E_t = c^E_t + S_{t-1} (1 - \beta) / \beta \text{ for all } t$$
$$\zeta^U_t = c^U_t + S_{t-1} / \beta - S_t \text{ for all } t$$
$$S_t, e_t, S^E_t, \zeta^U_t \geq 0 \text{ for all } t$$

**Remark 3** For any IC contract, there exists an equivalent contract that induces $0-$savings.

The cost minimization problem for the principle is:

$$\min_{c^E_t, c^U_t} \sum_{t=1}^{\infty} \beta^{t-1} \Pi^{t-1}_{s=1} (1 - p(e^*_s)) \{p(e^*_t) \left( \frac{c^E_t}{(1 - \beta - \alpha)} \right) \}$$

subject to

$$\{0, e^*\} \in \arg \max_{\{S_t, e_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^{t-1} \Pi^{t-1}_{s=1} (1 - p(e^*_s)) \{p(e_t) \left( \frac{c^E_t + S_{t-1} (1 - \beta) / \beta}{(1 - \beta - \alpha)} + (1 - p(e_t)) u \left( \zeta^U_t \right) \}$$

$$\sum_{t=1}^{\infty} \beta^{t-1} \Pi^{t-1}_{s=1} (1 - p(e^*_s)) \{p(e_t) \left( \frac{c^E_t}{(1 - \beta - \alpha)} \right) + (1 - p(e_t)) u \left( \zeta^U_t \right) \} \geq u^*, \quad c^E_t, c^U_t \geq 0 \text{ for all } t$$

### 2.2.2 Solving the UIP

K proceeds by constructing a relaxed problem and showing that the solution to this problem solves the general problem.

**Claim 4** Any IC contract must satisfy:

$$(1 - \beta) \sum_{s=0}^{\infty} \beta^s u \left( \zeta^U_{t+s} \right) = u \left( c^E_t \right) - \alpha (1 - \beta) \text{ for all } t. \quad (R1)$$
By the linearity of the agent’s problem in $p(e_{t+s})$ for all $s \geq 0$, if an agent chooses $p(e_t) > 0$ in every period, he must be indifferent between all possible $p(e)$ sequences, including $p(e_t) = 1$ and $p(e_{t+s}) = 0$ for $s > 0$.

**Claim 5** Any IC contract must satisfy

$$c_t^U \leq c_{t+1}^U$$  \hspace{1cm} (R2)

for all $t$.

Suppose to the contrary that $c_{t+1}^U < c_t^U$. Then an agent unemployed in $t$, would prefer to set $S_t > 0$ and $e_{t+1} = 0$ to $S_t = 0$ and $e_{t+1} = 0$. But (R1) implies that an agent is indifferent between $(S_t = 0, e_{t+1} = 0)$ and $(S_t = 0, e_{t+1} = e^*_t)$. Then, if $c_{t+1}^U < c_t^U$ it is not optimal for an agent to set $S_t = 0$ and $e_t = e^*_t$.

The first order condition for savings is:

$$u'(c_t^U) \geq p(e^*_t)u'(c_{t+1}^E) + (1 - p(e^*_t+1))u'(c_{t+1}^U).$$  \hspace{1cm} (R3)

Condition (R2) implies that (R1) and (R2) do not guarantee incentive compatibility of the strategy $(S_t = 0, e_t = e^*_t)$ because they do not take into account the second order consequences of simultaneous changes in savings and effort.

- The relaxed problem

Impose (R1) and (R2) and (PK). Then, immediately $c_t^U < c_{t+1}^E$ for all $t$.

**Claim 6** In any solution to the relaxed problem: $c_t^U = c_{t+1}^U$.

Suppose instead $c_t^U < c_{t+1}^U$. Then, it is possible to construct a new contract:

- raise $u(c_t^U)$ by $\varepsilon$
- lower $u(c_{t+1}^U)$ by $\varepsilon \beta^{-1}$
- lower $u(c_{t+1}^E)$ by $\varepsilon \beta^{-1} (1 - \beta)$

that satisfies (R1) and (R2) for small $\varepsilon$. 

9
The difference between the cost of the old contract and the cost of the new contract for the planner is:

\[ \frac{\varepsilon}{u'} (c^U_t) - \varepsilon p (c^*_t) / u' (c^U_{t+1}) - \varepsilon (1 - p (c^*_t)) / u' (c^E_{t+1}) \]

\[ \leq \frac{\varepsilon}{u'} (c^U_t) - \varepsilon p (c^*_t) / u' (c^U_{t+1}) - \varepsilon (1 - p (c^*_t)) / u' (c^U_{t+1}) \]

\[ \leq 0 \]

since \( u'' < 0 \), \( c^E_t > c^U_t \) all \( t \).

Then, the contract that solves the relaxed problem has \( c^U_t = \bar{c}^U \) for all \( t \).

Then, from (R1) we can solve for \( c^E_t = \bar{c}^E \).

Is this contract IC?

By construction:

\[ u' (\bar{c}^U) \geq pu' (\bar{c}^E) + (1 - p) u' (\bar{c}^U) , \]

for any \( p \). The agent never wants to save under this contract and is indifferent between any effort sequence. So the contract is IC and minimizes the planner’s cost of providing it.

Why is this optimal?

Increasing \( \bar{c}^U \) by \( \varepsilon \) and reducing \( \bar{c}^E \) by \( \varepsilon \beta^{-1} \) raises the agent’s ex ante utility without affecting the planner’s ex ante costs. But the previous reasoning shows that this implies that the agent would save and set \( e_{t+1} = 0 \) in response. So such a deviation would not be IC.
3 Appendix: Limited Insurance and the Rogerson Condition

Consider an optimal contracting problem for a planner facing a population of agents with preferences:

$$E_\theta \sum_{t=0}^{\infty} \beta^t \left( u(c_t) - \theta_t v(a_t) \right),$$

where \(a_t\) is effort, \(\theta_t\) is an iid cost of working shock, and \(u\) and \(v\) satisfy standard properties.

The planner discounts at rate \(\beta\).

The planner’s cost minimization problem when the shock can take on two values is:

$$J^* (w) = \inf E \left[ (1 - \beta) \left\{ C(u(\theta)) - A(v(\theta)) + \beta J^* (w(\theta)) \right\} \right]$$

s.t.

$$w = E \left[ (1 - \hat{\beta}) \left\{ u(\theta) - \theta v(\theta) \right\} + \hat{\beta} w(\theta) \right],$$

$$\left( 1 - \hat{\beta} \right) \left\{ u(\theta) - \theta v(\theta) \right\} + \hat{\beta} w(\theta) \geq \left( 1 - \hat{\beta} \right) \left\{ u(\hat{\theta}) - \theta \hat{v}(\hat{\theta}) \right\} + \hat{\beta} w(\hat{\theta}),$$

where \(w\) is promised utility.

In the 2 shock case, the temporary IC constraint will only bind for \(\theta = \theta_1\). Only this constraint need be included. The choice variables for the problem are: \(u_i, v_i\) and \(w'_i\) where \(w'_i\) is the continuation utility attribute to an agent who has received a shock \(\theta_i\). The Lagragian for this problem is:

$$\Lambda = \sup \left\{ - \sum_i \pi_i \left[ (1 - \beta) \left( C(u_i) - A(v_i) \right) + \beta J^* (w'_i) \right] \right.$$

$$- \chi \left[ w - \sum_i \pi_i \left[ \left( 1 - \hat{\beta} \right) (u_i - \theta_i v_i) + \hat{\beta} w'_i \right] \right]$$

$$+ \eta \left\{ (1 - \hat{\beta}) (u_1 - \theta_1 v_1) + \hat{\beta} w'_1 - \left( 1 - \hat{\beta} \right) (u_2 - \theta_1 v_2) - \hat{\beta} w'_2 \right\}. $$

The first order necessary conditions for this problem are:

$$\pi_1 (1 - \beta) C'(u_1) = (\chi \pi_1 + \eta) \left( 1 - \hat{\beta} \right),$$

(1)
\[ \pi_2 (1 - \beta) C''(u_2) = (\chi \pi_2 - \eta) \left(1 - \hat{\beta}\right), \]  
\[ \pi_1 \frac{A'(v_1)}{\theta_1} (1 - \beta) = (\chi \pi_1 + \eta) \left(1 - \hat{\beta}\right), \]  
\[ \pi_2 \frac{A'(v_2)}{\theta_2} (1 - \beta) = (\chi \pi_2 - \eta) \left(1 - \hat{\beta}\right), \]  
\[ \beta J^* (w_0') \pi_1 = (\chi \pi_1 + \eta) \hat{\beta}; \]  
\[ \beta J^* (w_0') \pi_2 = (\chi \pi_2 - \eta) \hat{\beta}; \]  
\[ J^* (w) = \chi. \]  

- **Effort wedge**

Combine (1) and (3) and (2) and (4), respectively, to yield:

\[ C'(u_1) - \frac{A'(v_1)}{\theta_1} = 0, \]  
\[ C'(u_2) - \frac{A'(v_2)}{\theta_2} = -\frac{\eta}{\pi_2} \frac{1 - \hat{\beta}}{1 - \beta} \left(1 - \frac{\theta_1}{\theta_2}\right). \]

Note that the wedge is set at the first best for the agent with the low \( \theta \) shock (corresponds to 0 marginal tax on income irrespective of wealth).

- **Insurance wedge**

Combine (1) and (2) to get:

\[ C'(u_1) - C'(u_2) = \eta \left(1 + \frac{1}{\pi_1} + \frac{1}{\pi_2}\right) \frac{1 - \hat{\beta}}{1 - \beta}. \]

The need to provide incentives (the presence of an incentive compatibility constraint) determines incomplete insurance. This spread in marginal utilities is the cost of providing incentives.

- **Intertemporal wedge**
First note that combining (1) and (5) and (2) and (6), respectively, yields:

\[ C' (u_i) = J'' (w'_i) \frac{\beta (1 - \hat{\beta})}{\hat{\beta} (1 - \beta)}. \]  

(11)

By (10) this implies:

\[ J'' (w'_1) - J'' (w'_2) = \eta \left( \frac{1}{\pi_1} + \frac{1}{\pi_2} \right) \frac{\beta}{\hat{\beta}}. \]  

(12)

This expression underlines that the need to provide incentives determines inequality in promised utilities.

To obtain the intertemporal wedge, sum (5) and (6), to get:

\[ \beta \sum_i \pi_i J'' (w'_i) = \hat{\beta} J'' (w). \]  

(13)

This is the intertemporal condition for the planner. Using (11), this can be rewritten as:

\[ \frac{1 - \beta}{1 - \hat{\beta}} \sum_i \pi_i C' (u_i) = J'' (w), \]

or, more precisely, as:

\[ \frac{1 - \beta}{1 - \hat{\beta}} \sum_i \pi_i C' (u (w, \theta_i)) = J'' (w). \]  

(14)

Define:

\[ C' (u (w)) \equiv \sum_i \pi_i C' (u (w, \theta_i)). \]

Then, \( C' (u) \frac{1 - \beta}{1 - \hat{\beta}} = J'' (w) \), and \( C' (u) \) is the the marginal cost in terms of expected consumption equivalents of providing a promised utility level of \( w \) for the planner. Using this definition in (13):

\[ \beta \sum_i \pi_i \sum_i \pi_i C' (u (w'_i, \theta_i)) = \hat{\beta} C' (u). \]  

(15)

Using (14) in (11):

\[ C' (u_i) = \sum_i \pi_i C' (u (w'_i, \theta_i)) \frac{\beta}{\hat{\beta}}. \]  

(16)
Expression (15) in an unconditional version of (16). Expression (16) is the standard Rogerson intertemporal wedge. Using $C'(u) = 1/u'(c)$ by duality:

$$
\frac{1}{u'(c(w, \theta_i))} = \frac{\beta}{\beta} \sum_i \pi_i \frac{1}{u'(c(w'_i, \theta_i))}.
$$

(17)

By concavity of utility, applying Jensen’s inequality to the RHS implies:

$$
u'(c(w, \theta_i)) < \frac{\beta}{\beta} \sum_i \pi_i u'(c(w'_i, \theta_i)),
$$
as long as $u'(c(w, \theta_1)) \neq u'(c(w, \theta_2))$ for any $\omega$.

This means that if there is a spread in the marginal utility of consumption over agents with different $\theta$’s, then a feature of the efficient mechanism is that the intertemporal rate of transformation is greater than the expected intertemporal rate of substitution. This would correspond to a positive tax on income deriving from assets held in this economy.

Notice that (16) can be rewritten as follows:

$$
\frac{1}{C'(u_i)} = \frac{\hat{\beta}}{\beta} \sum_i \pi_i C'(u(w'_i, \theta_i))
$$

The RHS can be manipulated to yield:

$$
\frac{1}{\sum_i \pi_i C'(u(w'_i, \theta_i))} = \frac{\pi_1}{C'(u'(w'_i, \theta_1))} + \frac{\pi_2}{C'(u'(w'_i, \theta_2))} = \frac{\pi_1}{C'(u(w'_i, \theta_1))} - \frac{\pi_2}{C'(u(w'_i, \theta_2))} - \pi_1 \pi_2 \frac{[C'(u(w'_i, \theta_1)) - C'(u(w'_i, \theta_2))]^2}{C'(u'(w'_i, \theta_1)) C'(u'(w'_i, \theta_2)) C'(u_i)}.
$$

Then:

$$
\frac{1}{C'(u_i)} = \frac{\hat{\beta}}{\beta} \left\{ \frac{\pi_1}{C'(u(w'_i, \theta_1))} + \frac{\pi_2}{C'(u(w'_i, \theta_2))} - \pi_1 \pi_2 \frac{[C'(u(w'_i, \theta_1)) - C'(u(w'_i, \theta_2))]^2}{C'(u'(w'_i, \theta_1)) C'(u'(w'_i, \theta_2)) C'(u_i)} \right\},
$$

and the intertemporal wedge is given by:

$$
\frac{\hat{\beta}}{\beta} \pi_1 \pi_2 \frac{[C'(u(w'_i, \theta_1)) - C'(u(w'_i, \theta_2))]^2}{C'(u'(w'_i, \theta_1)) C'(u'(w'_i, \theta_2)) C'(u_i)}.
$$
This expression clarifies that the presence of an intertemporal wedge depends on the need to generate a spread in marginal utilities across $\theta$'s, stemming from the incentive compatibility constraint. Features of $C''()$ will determine how big the spread is.

An important feature of this problem is that implementable effort ranges depend on outstanding wealth.