

Final: Short Answers
(December 17, 1998)

1. Definitions.

- (a) A forward rate agreement specifies the exchange of a fixed “contract” rate for a floating rate at a future date.
- (b) A collar is a long position in a cap and a short position in a floor. It can be used to put upper and lower bounds (a “collar”) on a floating rate payment.
- (c) Reg 144A is a standard exemption from SEC registration for issues sold to institutional investors (Qualified Institutional Buyers).
- (d) Spread duration measures the sensitivity of (say) a corporate bond to changes in the spread of its yield over riskfree debt:

$$D_s = \frac{dp/ds}{p},$$

where p is the price and s is the spread.

- (e) The blended yield is the yield to maturity on a Brady bond. The term refers to the use of a single yield to discount both guaranteed and sovereign cash flows.

2. Puttable bond.

- (a) The spot rates are $y_1 = 4.500\%$ and $y_2 = 4.597\%$. The first is listed at the start of the tree. You can compute the second either with Duffie’s formula or by computing the value of a 2-period zero and finding its yield.
- (b) The usual: use the 50-50 rule to compute the state prices and find the value by summing the pieces. For the first up node, the short rate is $r = 3.600\%$ so state prices are

$$q_u = q_d = 0.5/(1 + .03600/2) = 0.4912.$$

The value of the bond at this node is

$$\begin{aligned} \text{Price} &= 2.5 + q_u \times 102.50 + q_d \times 104.66 \\ &= 104.25. \end{aligned}$$

- (c) We have the option to sell the bond for 102.50 in periods 1 and 2. The value generated by exercising the option at each node is

$$\begin{array}{r} \text{(na)} \quad \left\langle \begin{array}{l} 1.40 \\ 0.00 \end{array} \right\rangle \begin{array}{l} \left\langle \begin{array}{l} 2.08 \\ 0.00 \end{array} \right\rangle \\ \left\langle \begin{array}{l} 0.00 \\ 0.00 \end{array} \right\rangle \end{array}$$

- (d) The next step is to figure out when to exercise the option. If we get to period 2, we would exercise in the top node, not in the others (this is what generates the cash flows above). In period 1, the value in the down node is computed from

$$\begin{aligned} q_u &= q_d = 0.5/(1 + .03600/2) = 0.4912 \\ \text{Price} &= q_u \times 0.00 + q_d \times 0.00 = 0.00. \end{aligned}$$

The value in the up node if we exercise is $1.40 = 102.50 - 101.10$ (see above). If we continue to hold the bond, the value is

$$\begin{aligned} q_u &= q_d = 0.5/(1 + .05800/2) = 0.4859 \\ \text{Price} &= q_u \times 2.08 + q_d \times 0.00 = 1.01. \end{aligned}$$

We take the large number and exercise. In the initial node, the value is

$$\begin{aligned} q_u &= q_d = 0.5/(1 + .04500/2) = 0.4890 \\ \text{Price} &= q_u \times 1.40 + q_d \times 0.00 = 0.68. \end{aligned}$$

The price path for the option is therefore

$$0.68 \begin{cases} \swarrow & 1.40 \\ \searrow & 0.00 \end{cases} \begin{cases} \swarrow & 2.08 \\ \searrow & 0.00 \\ \searrow & 0.00 \end{cases}$$

The put option is worth 0.68 and the putable bond $100.42 + 0.68 = 101.20$. Remember: the investor is long the option in this case.

3. LIBOR² swap.

- (a,b) The value of the fixed rate leg is

$$\text{Value} = (d_1 + d_2 + d_3) \times C/2 + d_3 \times 100.$$

With $C = 24$, the value is 127.68. To get the same value as the floating rate leg (126.53), you need $C = 23.20$.

- (c) The usual approach, with one twist: the “current cash flow” is the discounted value of next period’s payment of LIBOR²:

$$\text{Cash Flow} = \frac{3.6^2/2}{1 + .036/2} = 6.37.$$

The value of the floating rate leg is computed as

$$\begin{aligned} q_u &= q_d = 0.5/(1 + .03600/2) = 0.4912 \\ \text{Price} &= 6.37 + q_u \times 109.33 + q_d \times 102.26 = 110.29. \end{aligned}$$

The swap is therefore worth $110.29 - 119.76 = -9.47$.

- (d,e) The idea is that the floating rate leg of this swap has perverse interest sensitivity: the value goes up when rates rise. You can see that by moving up a column in the tree. Another way to see it is by replication. A vanilla swap is a short position in the fixed rate bond and a long position in an FRN (which is the same as a one-period zero, since it trades at par in all nodes of the tree). What about this swap? Replication works this way. We find quantities that generate the same values as the swap in the two succeeding nodes:

$$\begin{aligned} 12.75 &= x_a \times 108.16 + x_b \times 100 \\ 0.00 &= x_a \times 109.32 + x_b \times 100 \end{aligned}$$

The answer is $x_a = -10.98$ and $x_b = 12.00$. That is: there's 12 units worth of a short position in the long bond, given the swap very strong negative interest sensitivity.

4. Bond futures.

- (a) The basis is the difference between the price of the bond in the cash and futures markets. The price in the futures market is the futures price times the conversion factor, so the basis is

$$\text{Basis} = \text{Cash Price} - \text{Conversion Factor} \times F$$

For the three bonds, the numbers are

Bond	Basis
5-1/4's	14.21
6's	8.89
9-7/8's	0.81

- (b) The last bond is the cheapest to deliver: it has the smallest basis and therefore the lowest net cost to delivery.
- (c) The DV01 of the futures is the DV01 of the bond divided by its conversion factor. For the cheapest to deliver, the DV01 is $0.151/1.170 = 0.129$. For a 50 BP increase, the futures price falls by 6.45 dollars.