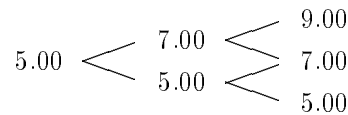


## Assignment 4: Answers

(November 19, 1998)

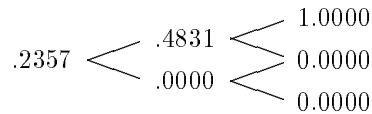
1. An application of the Ho and Lee model similar to those studied in class.

(a) The short rate tree is



Ie, at each node, we add 1% ( $\mu$ ) then either add or subtract 1% ( $\sigma$ ).

(b) The price path for the state-contingent claim is

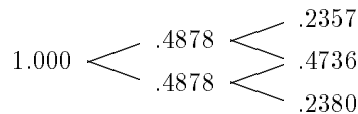


The last column is the pure state-contingent claim [1 in state (2,2), zero in the others]. The earlier columns give us the value of this claim at prior dates. Consider the calculation of 0.4831: (i) The fifty-fifty rule gives us  $q_u = q_d = 0.5/(1 + .07/2) = 0.4831$ . (ii) The pricing relation gives us

$$\text{Price} = 0.4831 \times 1 + 0.4831 \times 0 = 0.4831.$$

Other nodes work the same way: start at the end, and work backwards until we've filled them all in.

(c) Duffie's formula puts the current value of one dollar in state (2,2) in the (2,2) node of the tree, and tells us what current prices of other states are, too. The complete set of state prices is



(d) Duffie's formula is helpful in generating discount factors (sums of state prices down columns) and spot rates (last one is optional):

Maturity (Half Years)	Discount Factor	Spot Rate
1	0.9756	5.000
2	0.9472	5.497
3	0.9152	5.992

## 2. Another application of the Ho and Lee model.

- (a) As in the examples in the notes, the price path for the bond is

$$91.52 \begin{cases} < 92.90 \\ < 94.72 \end{cases} \begin{cases} < 95.69 \\ < 96.62 \\ < 97.56 \end{cases}$$

Details for the last row: discount 100 back one period using the short rate in the same position in the tree. For the top right node:  $95.69 = 100/(1+.09/2)$ . For the top node of the second column: (i) The fifty-fifty rule gives us  $q_u = q_d = 0.5/(1 + .07/2) = 0.4831$ . (ii) The pricing relation gives us

$$\text{Price} = 0.4831 \times 95.69 + 0.4831 \times 96.62 = 92.90.$$

And so on for other nodes.

- (b) The call option generates a cash flow of 1.72 in state (0,1), zero otherwise. Its price path is

$$0.84 \begin{cases} < 0.00 \\ < 1.72 \end{cases} \begin{cases} < (\text{na}) \\ < (\text{na}) \\ < (\text{na}) \end{cases}$$

The callable bond is a long position in the underlying zero [see (a)] and a short position in the option (above), and is therefore worth  $90.69 = 91.52 - 0.84$  (numbers subject to rounding). Its price path is

$$90.69 \begin{cases} < 92.90 \\ < 93.00 \end{cases} \begin{cases} < 95.69 \\ < 96.62 \\ < 97.56 \end{cases}$$

The lower right node is irrelevant: the bond would be called before reaching it.

- (c) We can reproduce the price moves of the callable bond between periods 0 and 1 with positions in the underlying bond (“a”) and a one-period zero (“b”). The positions must generate the same value in the up and down states:

$$\begin{aligned} 92.90 &= x_a \times 92.90 + x_b \times 100 \\ 93.00 &= x_a \times 94.72 + x_b \times 100 \end{aligned}$$

The solution is  $x_b = 0.880$  and  $x_a = 0.053$ . Hence the callable bond has lower duration than the zero itself (you’re holding only 0.05 units of it).