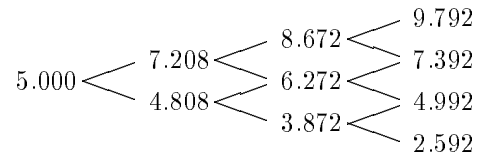


Assignment 5: Answers

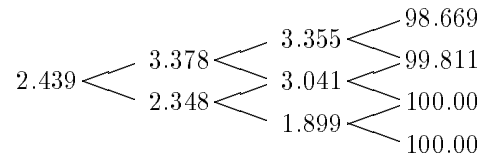
(November 30, 1998)

1. An application of the Ho and Lee model.

- (a) We calibrate the short rate tree by choosing the drift parameters μ_t to reproduce the given spot rates. (This is just like the procedure we followed in class using the spreadsheet.) The resulting drift parameters are $(1.008, 0.264, -0.080)$. The short rate tree is



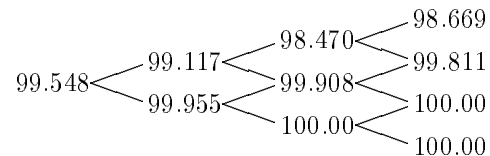
- (b) The cash flows (principal plus interest) have been shifted back one period and discounted at the short rate:



For example:

- $2.439 = 0.5 \times 5.000 / (1 + .05000/2)$
- $3.355 = 0.5 \times 7.000 / (1 + .08672/2)$
- $99.811 = (100 + 0.5 \times 7.000) / (1 + .07392/2)$

- (c) The price path for the note is



These calculations follow the usual rules, which I illustrate for the first “up” node:

- Since the short rate is $r = 7.208\%$, state prices are $q_u = q_d = 0.5 / (1 + .07208/2) = .4826$.

- The price of the asset in the same node is

$$\begin{aligned}
 \text{Price} &= \text{Current Cash Flow} + q_u p_u + q_d p_d \\
 &= 3.378 + 0.4826 \times (98.470 + 99.908) \\
 &= 99.117,
 \end{aligned}$$

as stated.

We follow the same steps at every node, starting at the end.

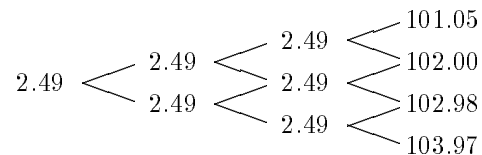
- (d) Without the cap, the note would trade at 100 in every node. Hence numbers that are different from 100 indicate the impact of the cap. Clearly, the impact is greater at the top, where the 7% cap is binding (or could bind in the future).
- (e) Cost is $0.452 = 100 - 99.548$. A standard way to offset this is to put a floor on the rate, as well as a ceiling.
2. Swaption. This example is an option to terminate the swap after one period. It's confusing enough that everyone gets full marks.

- (a) We can use Duffie's formula (or a more onerous method if you prefer) to compute discount factors and spot rates:

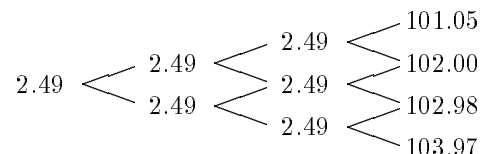
	1	2	3	4
Discount Factor	0.9756	0.9518	0.9287	0.9062
Spot Rate	5.000	4.998	4.994	4.988

Using the par yield formula, the 2-year swap rate is 4.988, virtually indistinguishable from the 2-year spot rate.

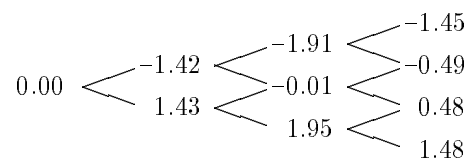
- (b,c) In valuing the swap, it's convenient to think of it as a long position in a fixed rate note (with coupons equal to half the swap rate, or 2.494) and a long position in an FRN (trades at 100 on reset dates). The fixed rate note is the only part that isn't obvious. Its cash flows are



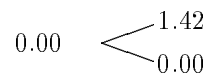
Its price path is



The price path for the swap is the price path for the note (above) minus the FRN (100 in all nodes) minus the coupon (recall that our approach to valuing bonds includes the *current* coupon). The result is



(d) We can settle the swap at 0 in one period, which generates “cash flows” of



Its current value is 0.69:

$$q_u = q_d = 0.5 / (1 + .05/2) = 0.4878$$

$$\text{Price} = q_u \times 1.42 + q_d \times 0 = 0.69.$$