

Midterm: Short Answers
(October 26, 1998)

1. Standard corporate bond calculations (30/360 day-count convention).

(a) We owe 4 days accrued interest (Oct 16 to Oct 20), so

$$\text{Accrued Interest} = \frac{4}{180} \times \frac{5.35}{2} = 0.059.$$

(b) The invoice price is $100.631 + 0.059 = 100.69$.

(c) There are $n = 4$ payments remaining and the current period is fraction $w = 176/180 = 0.978$ of a half-year. The yield then satisfies

$$\text{Invoice Price} = \frac{5.35/2}{(1+y/2)^w} + \frac{5.35/2}{(1+y/2)^{w+1}} + \frac{5.35/2}{(1+y/2)^{w+2}} + \frac{100 + 5.35/2}{(1+y/2)^{w+3}}$$

The answer is $y = 5.012\%$.

(d) Duration is

$$D = (1+y/2)^{-1} [w_1 \times w + w_2 \times (w+1) + w_3 \times (w+2) + w_4 \times (w+3)] / 2,$$

where

$$w_j = \frac{(5.35/2)/(1+.05012/2)^{w+j-1}}{100.69} \quad \text{for } j = 1, 2, 3$$

$$w_4 = \frac{(100 + 5.35/2)/(1+.05012/2)^{w+3}}{100.69}.$$

The answer is $D = 1.866$. Alternatively, you could apply the enormous formula in the class notes.

2. Arbitrage and discount factors.

(a) Find discount factors by the one-at-a-time recursive method described in class: Bond A implies $d_1 = 101.46/104 = 0.9756$. Given this answer, bond B then solves

$$103.85 = d_1 \times 5 + d_2 \times 105 \Rightarrow d_2 = 0.9426$$

Now compute d_3 from bond C:

$$95.83 = d_1 \times 2 + d_2 \times 2 + d_3 \times 102 \Rightarrow d_3 = 0.9019$$

- (b) We need to equate the cash flows of a one-period zero and a “portfolio” with x_a units of A and x_b units of B:

$$\begin{aligned} 0 &= x_a \times 104 + x_b \times 5 \\ 100 &= x_a \times 0 + x_b \times 105 \end{aligned}$$

The solution is $x_a = -0.046$ and $x_b = 0.952$. Intuitively: you buy slightly less than one unit of B (the second-period cash flow of 105 is greater than 100) and short A to offset the first-period coupon for B.

- (c) The logic is the arbitrage-free, “frictionless” theory of asset pricing: two assets with the same cash flows should sell for the same price. If they do not, people would buy the cheaper one and drive up its price. The question is how easy the replication is to do. In practice, bid/ask spreads, transaction costs, and the difficulty of shorting bonds suggest that there may be differences between the prices of a zero and its synthetic replication. Only rarely, however, are differences substantial for examples like the one above.

3. Inverse floater.

- (a) I’ll work this out in general, then set $C = 10$. We can replicate the floater with (i) two long positions in a bond with coupon rate $C/2$ and (ii) a short position in a floating rate note, all of these positions with principals of 100. The value of each of the long bond positions is

$$\text{Value} = (d_1 + d_2 + d_3 + d_4) \times C/4 + d_4 \times 100$$

With $C = 10$, the value of the bond is 99.073. The value of the FRN is, of course, 100. The value of the inverse floater is the sum of the values of the components:

$$\text{Value} = 2 \times 99.073 - 100 = 98.145.$$

- (b) The duration of the floater is the value-weighted average of the durations of the components. The duration of the long bond positions is 1.876 (the yield is 5.496%), computed the usual way for a coupon bond. The duration of the FRN is 0.487. The duration of the inverse floater is therefore

$$D = \left(\frac{2 \times 99.073}{98.145} \right) \times 1.876 + \left(\frac{-100}{98.145} \right) \times 0.487 = 3.291.$$

This is typical of an inverse floater: the implicit long bond positions give it a long duration.

- (c) Having the inverse floater trade at 100 is equivalent to having the long bond trade at 100. Hence we solve [see (a)]

$$100 = (d_1 + d_2 + d_3 + d_4) \times C/4 + d_4 \times 100,$$

giving us $C = 10.991 = 2 \times 5.496$.

4. Two approaches to risk measurement.

- (a) Conventional approach (duration). The fund's value is

$$10 \times 92.73 + 3 \times 74.86 = 1151.88.$$

Its duration is a weighted average of the durations of the two components:

$$D = \left(\frac{10 \times 92.73}{1151.88} \right) \times 1.96 + \left(\frac{3 \times 74.86}{1151.88} \right) \times 6.867 = 2.915.$$

- (b) Statistical approach. We find the standard deviation of the value from the standard deviations and correlation of the spot rates. The link between spot rates and value for individual positions is provided by their DV01's. The one-day standard deviation of the fund's value is

$$\begin{aligned} \sigma^2 &= (10 \times 0.0182 \times 12)^2 + (3 \times 0.0513 \times 7)^2 \\ &\quad + 2 \times 0.7(10 \times 0.0182 \times 12)(3 \times 0.0513 \times 7) \\ &= 3.04^2. \end{aligned}$$

A two-standard deviation move, then, is about \$6 on a total of \$1152 — roughly half a percent.

- (c) Duration is a first-approximation method. For the current situation, the most relevant assumption is that yields of all maturities change the same amount. The statistical measure is less restrictive, since the standard deviations can vary across maturities and the correlation between them can be less than one. A problem here, though, is that the estimated values of standard deviations and correlation may not be representative of the near-term future. Things can change very quickly in financial markets, and estimates based on the recent past may not be indicative of the future (“past performance is no indication...”).