

Assignment 3: Answers

(March 3, 1998)

1. DM swap rates.

- (a) Standard swap rate (par yield formula). Use the spot rates to compute discount factors: $d_n = 1/(1 + y/2)^n$, with n the maturity in half-years. For the 2-year swap, the rate is computed from

$$\begin{aligned} \text{Swap Rate} &= \frac{1 - d_4}{d_1 + d_2 + d_3 + d_4} \times 200 \\ &= 4.62. \end{aligned}$$

For 3 years, a similar calculation gives us 5.01.

- (b) This is harder, and requires you to modify the logic that led to the swap rate. The idea is that a low rate early must be countered by a higher rate later on to keep the overall value the same. The value is par (the same as the floating rate leg of the swap), so we're looking for the rate C that satisfies

$$100 = (d_1 + d_2) \times 2/2 + (d_3 + d_4 + d_5 + d_6) \times C/2 + d_6 \times 100.$$

The solution is $C = 6.63$, which is substantially higher than the straight 3-year swap rate.

2. Inverse floater.

- (a) For variety, replicate the cash flows with (i) a long position in a 15% fixed rate note, (ii) a short position in a floating rate note, and (iii) a long position in a zero, with all three having a maturity of three years. This combination reproduces the interest payments and principal of the inverse floater. We approximate the value of the inverse floater from the values of the positions that replicate its cash flows:

$$\begin{aligned} p &= p_{\text{fix}} - p_{\text{frn}} + p_{\text{zero}} \\ &= 127.67 - 100.00 + 86.13 = 113.80. \end{aligned}$$

- (b) Two equivalent ways to do this: (i) find the formula for C and (ii) compute the value of the note for arbitrary C and vary C until the value is 100. The latter gives us an answer of 10.02, which makes sense: we need

a smaller coupon to get the price down to par. We can compute this directly by setting the value of the inverse floater equal to 100:

$$\begin{aligned} p &= p_{\text{fix}} - 100.00 + 86.13 \\ &\Rightarrow p_{\text{fix}} = 113.87. \end{aligned}$$

Then the coupon satisfies (this is a variant of the par yield calculation):

$$113.87 = (d_1 + d_2 + d_3 + d_4 + d_5 + d_6) \times C/2 + d_6 \times 100.$$

The answer is $C = 10.02$.

3. Yield spread trade.

- (a) For the 5-year: price is 124.30 (use the yield to find this) and PVBP is 0.0494. For the 10-year: price is 121.54 and PVBP is 0.0860. As you might expect, the 10-year bond has greater interest sensitivity.
- (b) You'd probably guess (correctly) that you want to go long the 5-year and short the 10-year. The trick is to find quantities x_5 and x_{10} of the two bonds that leaves you with no sensitivity to equal movements in both yields. The answer is anything that satisfies

$$0 = x_5 \times 0.0494 + x_{10} \times 0.0860.$$

- (c) You can short bonds, but it's easier for big players than small ones. But it's easy for anyone to go short a bond futures contract. Swaps work, too, but typically come in larger sizes than futures.